

---

$$* \quad \quad \quad t$$

$$( \quad / / \quad \quad , \quad / / \quad \quad , \quad / / \quad \quad )$$

$$\begin{array}{c} \text{Ritz} \\ \text{Ritz} \end{array} \quad \quad \quad \begin{array}{c} \text{Ritz} \\ \text{CFRP} \end{array}$$

$$[ \quad ]$$

$$[ \quad ] \text{ Betti } .$$

---

$$\begin{array}{c} \text{Doebling} \\ [ \quad ] \\ [ \quad ] \end{array}$$

$$[ \quad ]$$

---

Bonfiglioli		FRP	
[ ]	Ibarra	[ ]	[ ]
			FRP
Bonfiglioli			
[ ]	Ibarra	[ ]	[ ]
			[ ]
			Biswas Pandey
mm	mm	mm	[ ] Aktan Zhang
			[ ]
			CFRP
			Ritz
			Ritz
			[ ]
			Ritz
			Ritz
Ritz			
			Ritz
			FRP

---

---


$$( ) \quad \quad \quad \text{Ritz}$$

FRP

(Ferbenouse norm)

$$[ ] \quad :[ ] \quad M\ddot{x} + C\dot{x} + Kx = 0 \quad ( )$$

$$\begin{array}{ccc} K & C & M \\ \dot{x} & x & \ddot{x} \\ f & & \end{array}$$

$$\begin{array}{ccc} :[ ] & & \\ \overline{M} = \phi^T M \phi & , & \overline{K} = \phi^T K \phi \\ \overline{K} & & \overline{M} \\ \phi & & \end{array} \quad ( )$$

Pandey [ ] ( )  $\Lambda$

[ ] Biswas :[ ] ( )

$$\Lambda = \overline{M}^{-1} \overline{K} \quad ( )$$

( ) ( )  $\Lambda$

$$[\Delta] = [F^d] - [F] \quad [ \Delta ] \quad (\ ) \quad \phi^T K \phi - \phi^T M \phi \Lambda = 0 \quad ( )$$

$$\begin{array}{cc} F^d & F \end{array} \quad (M) \quad (\phi)$$

$\bar{\delta}_j$  :[ ] ( )  $v$

$$\bar{\delta}_j = \max |\delta_{ij}| ; i=1,2,\dots,n \quad j \quad (\ ) \quad v = (\phi^T M \phi)^{1/2} = \overline{M}^{1/2} \quad ( )$$

$\bar{\delta}_j$  :[ ] ( )  $v$

$\bar{\delta}_j$  ( )  $\phi^T K \phi - v^2 \Lambda = 0 , \phi^T K \phi = v^T \Lambda v \quad ( )$

n : ( )

Ritz :[ ]  $K = (\phi^T)^{-1} v^T \Lambda v \phi^{-1} \quad ( )$

Ritz :[ ]

$$F = \phi v^{-1} \Lambda^{-1} v^{-1} \phi^T \quad ( )$$


---

$(f)$	$(\phi v^{-1})$	$r = F \times f$	$F$	$( )$
Ritz	.	.	Ritz	r
Ritz	.	.	Ritz	( )
.	.	Ritz	.	.
CFRP	.	.	( )	.
$r_{ij} = F_{mj} \times f_{ij}$	.	( )	[ ]	Leger
$D_{ij} =  r_{ij} - r_{i0} $	.	( )	Ritz	.
Ritz	$r_{ij}$	( )	Ritz	.
$F_{mj}$	j	i	.	Ritz
$f_{ij}$	j	j	.	.
j	i	.	.	.
$D_{ij}$	.	( )	( )	[ ]
Ritz	.	.	.	.
$D_{ij}$	.	.	.	.
.	.	.	.	.
$\varphi_t^b(x_i)$	$\varphi_m^b(x_i)$	( )	.	.

.....

---

CFRP                           $\rho_{\max}$                           [ ] ACI318-05

10mm

Ibarra

[ ]                          CFRP                          (C-Sheet 240)

[ ]                          CFRP                          [ ] D3039

( )                          2845MPa                          0.117mm

                                237GPa

150mm

Epoxy

5kN    10kN

Epoxy

CFRP

)

(

CFRP                          Epoxy

( )

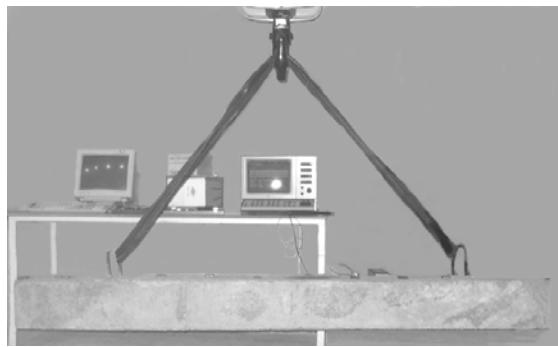
	(mm)	(mm)	(mm)	$f'_c$ (MPa)					CFRP
					$\Phi$	$\Phi$	$\Phi$ @ mm		
					$\Phi$	$\Phi$	$\Phi$ @ mm		
					$\Phi$	$\Phi$	$\Phi$ @ mm		
					$\Phi$	$\Phi$	$\Phi$ @ mm		

		(kN)									
						...	...	...	...	...	...
									...	...	

---



#### **a- Static Test Set-Up**



### **b- Dynamic Test Set-Up**

.[ ]

( )

( )

4096 Hz

0-1600 Hz

(FRF)

GPIB

[ ] MATLAB [ ] STAR

(noise)

.[ ]

[ ]

( ) . [ ]

.[ ]

CFRP

CFRP

[ ]

Ndambi

( )

Ritz

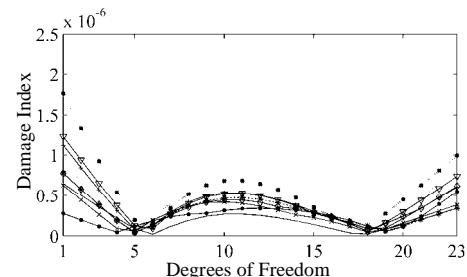
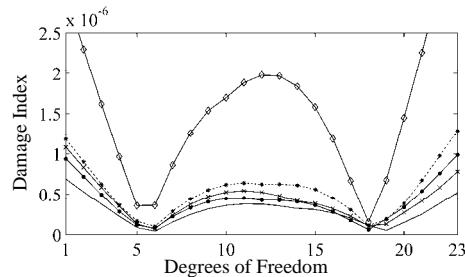
[ ] Biswas Pandey

( ) ( )

( )

[ ] Biswas Pandey

( )

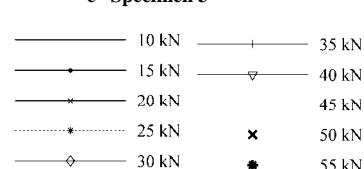
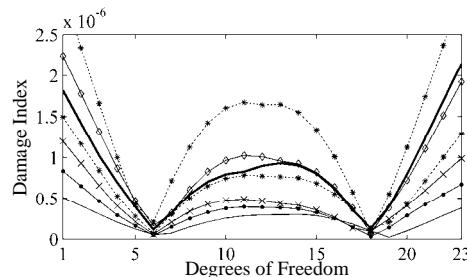
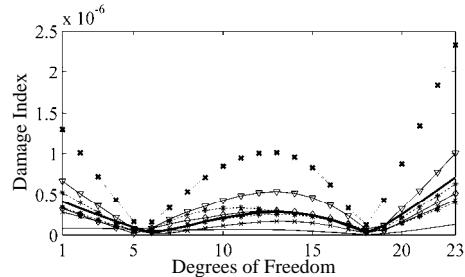


a -Specimen 1

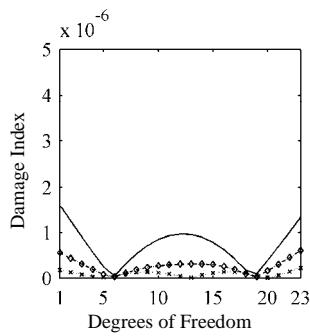
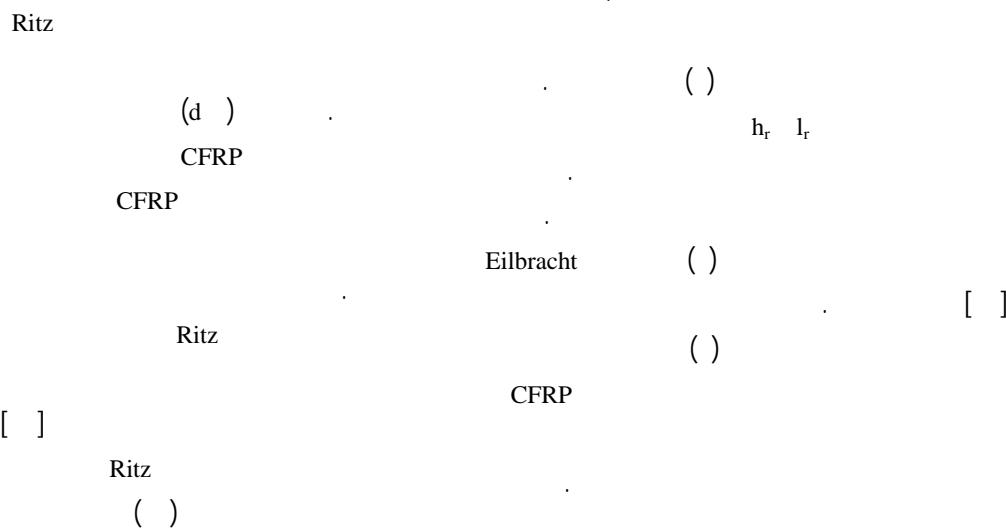
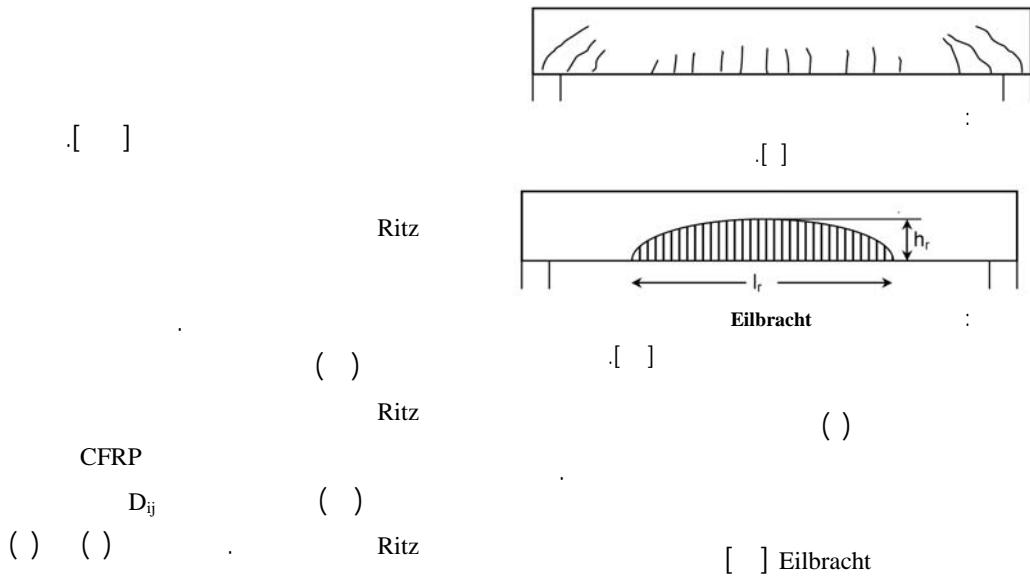
b -Specimen 2

c -Specimen 3

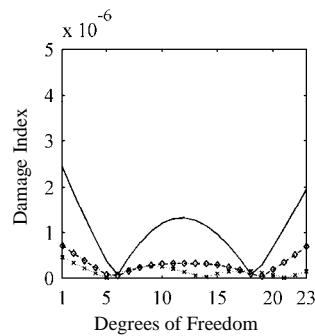
d -Specimen 4



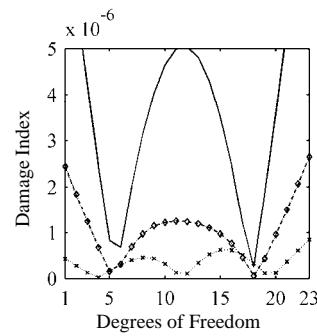
[ ] Biswas Pandey



a- Load Step 1 (10 kN)



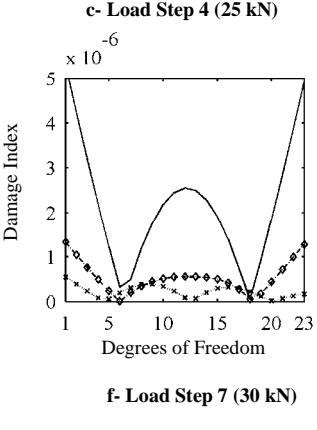
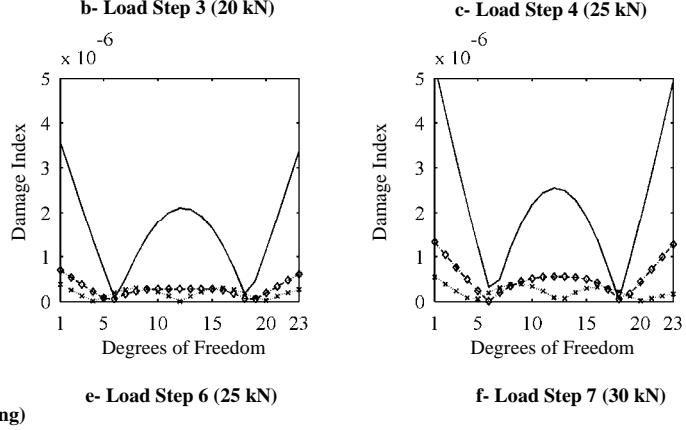
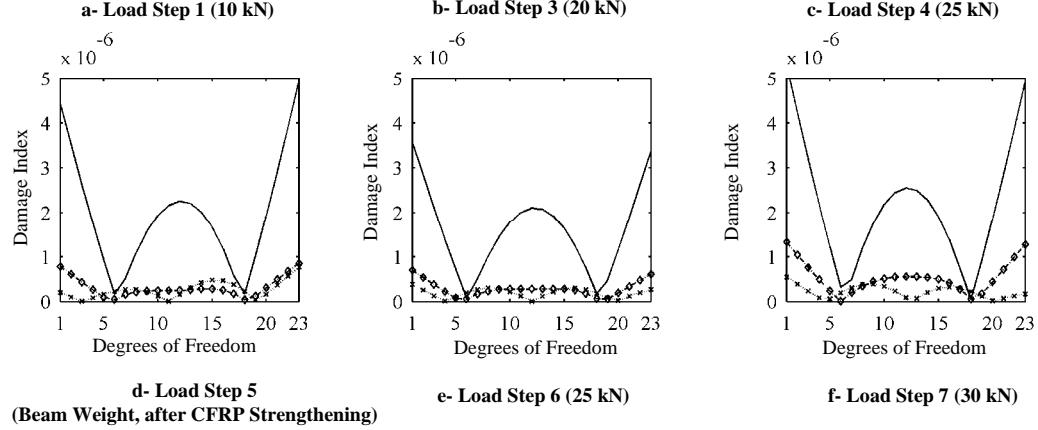
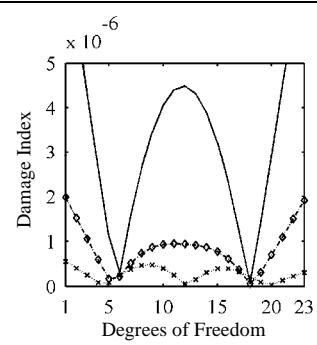
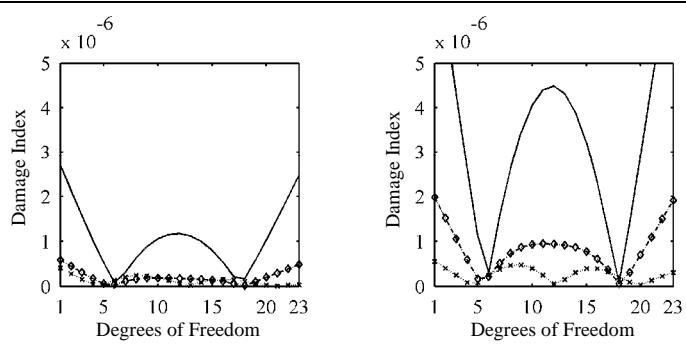
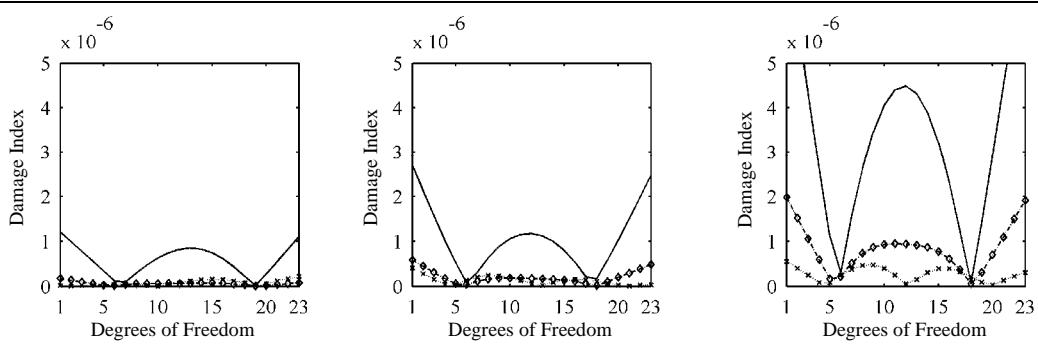
b- Load Step 3 (20 kN)



c- Load Step 4 (25 kN)

— | $r_{1j} - r_{10}|$  ..... x | $r_{2j} - r_{20}|$  ..... ♦ | $r_{3j} - r_{30}|$

Ritz



— | $r_{1j} - r_{10}|$  ..... x | $r_{2j} - r_{20}|$  ..... ♦ | $r_{3j} - r_{30}|$

Ritz

( )

CFRP

[ ] Eilbracht

( )

(a )

(b )

CFRP

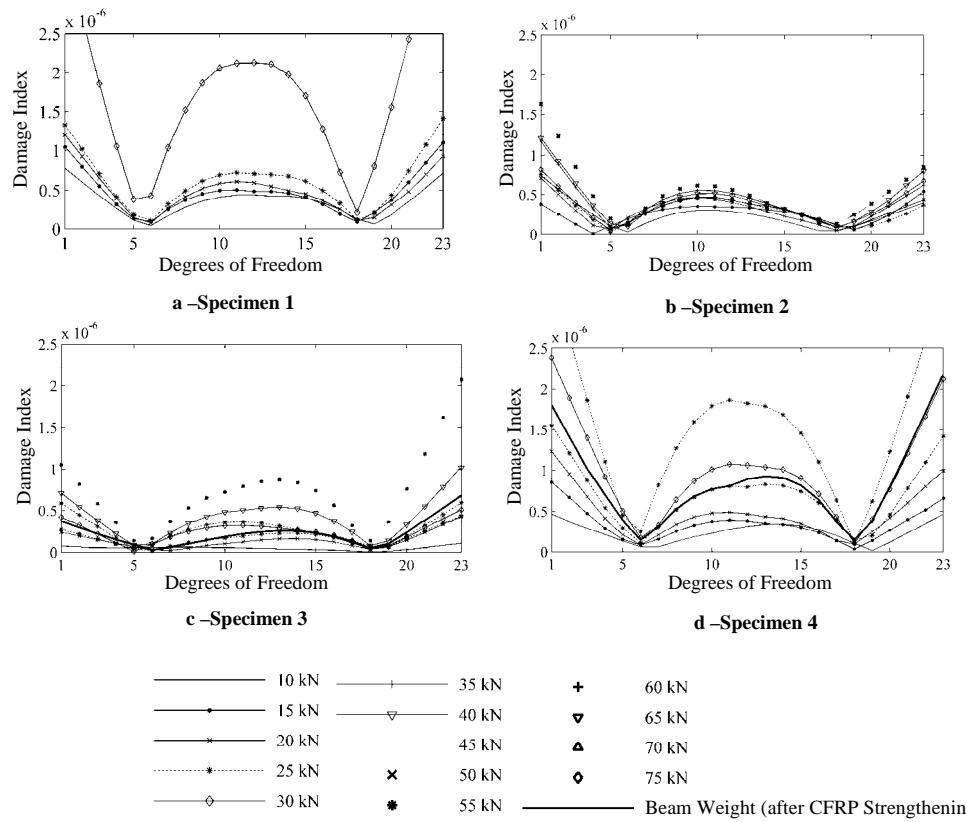
(d ) (c )

CFRP Ritz

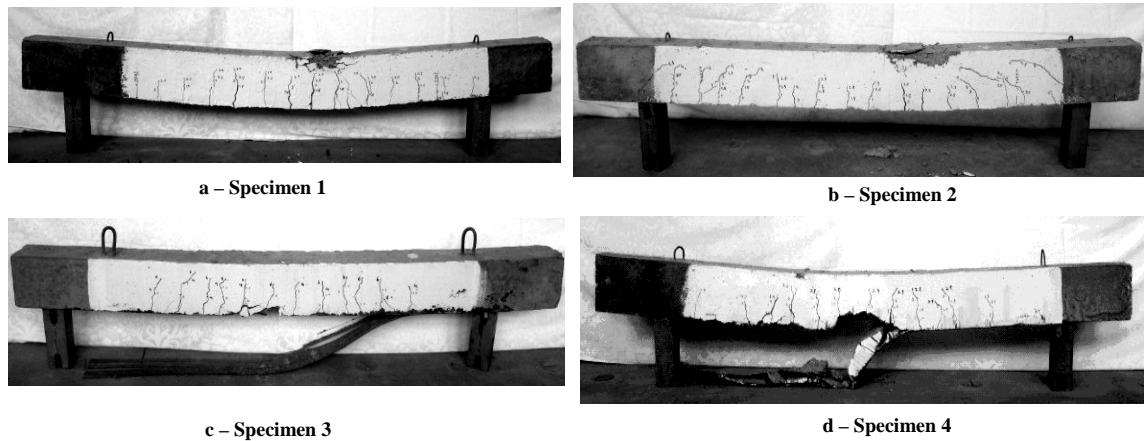
[ ] Biswas Pandey

(d )

( ) ( )



Ritz



Ritz

---

CFRP

Biswas Pandey

Ritz

Ritz

CFRP

Eilbracht

- 1 - Maeck J. (2003). *Damage assessment of civil engineering structures by vibration monitoring*, Ph.D. thesis, Belgium: Department of Civil Engineering, K.U.Leuven.
  - 2 - Betti, R. (2005). *Dynamic Methods for damage detection in structures*, Advanced Course on dynamic methods for damage detection in structures, CISM (Center International des Sciences Mechaniques), , Italy.
  - 3 - Doebling, S. W., Farrar, C.R. and Prime, M. B. (1998). “A summary review of vibration-based damage identification methods.” *The Shock and Vibration Digest*, Vol.30, No. 2, PP.91-105.
  - 4 - Bernal, D. (2002). “Load vectors for damage localization.” ASCE, *Journal of Engineering Mechanics*, Vol. 128, No. 1, PP. 7-14.
  - 5 - Zonta, D., Lanaro, A. and Zanon, P. (2003). “A strain-flexibility-based approach to damage location.” *Key Engineering Materials*, Vol. 245-246, PP. 87-94.
  - 6 - Gao, Y., Spencer Jr., B. F. and Bernal, D. (2007). “Experimental verification of the flexibility-based damage locating vector method.” ASCE, *Journal of Engineering Mechanics*, Vol. 133, No. 10, PP. 1043-1049.
  - 7 - Pandey, A. K. and Biswas, M. (1994). “Damage detection in structures using changes in flexibility.” *Journal of Sound and Vibration*, Vol.169, No. 1, PP.3-17.
  - 8 - Zhang, Z. and Aktan A. E. (1995). “The damage indices for constructed facilities.” *Proceedings of the IMAC*, Vol. 13, PP. 1520-1529.
  - 9 - Sohn, H. and H.Law, K. (2001). “Damage diagnosis using experimental Ritz vectors.” ASCE, *Journal of Engineering Mechanics*, Vol. 127, No. 11, PP. 1184-1193.
  - 10 - Bonfiglioli B., Pascale, G., and Martinez de Mingo S. (2004). “Dynamic testing of reinforced concrete beams damaged and repaired with CFRP sheets.” ASCE, *J. Mater Civil Eng*, Vol. 16, No. 5, PP. 400–406.
  - 11 - Ibarra J., Bonfiglioli B. and Pascale G. (2001). *Assessment of reinforced concrete beams damaged and repaired with externally bonded FRP sheets*, FinalReport\_Jorge1100. Bologna (Italy): Univ. of Bologna.
  - 12 - Alvandi, A. and Ceremona, C. (2006). “Assessment of vibration-based damage identification techniques.” *Journal of Sound and Vibration*, Vol. 292, No. 1, PP. 179-202.
  - 13 - Nour-Omid, B. and Clough, R.W. (1984). “Dynamics analysis of structures using Lanczos coordinates.” *Earthquake Engineering and Structural Dynamics*, Vol. 12, PP. 565-577.
-

- 
- 14 - Leger, P., Wilson, E. and Clough R. W., *The use of load-dependent Ritz vectors for dynamic and earthquake analyses*, Technical Report. UBC/EERC-86/04, Earthquake Engineering Research Center, University of California Berkeley, Berkeley, California.
- 15 - Baghiee, N., Esfahani, M. R. and Moslem, K. (2009). "Studies on damage and FRP strengthening of reinforced concrete beams by vibration monitoring." *Engineering Structures*, Vol. 31, PP. 875–893.
- 16 - Baghiee, N., Esfahani, M.R. and Moslem, K. (2009). "Assessment of behavior of reinforced concrete beams strengthened by CFRP sheets by experimental modal data analysis." *International Journal of Engineering Science*, Iran University of Science and Technology, Vol. 19, No. 2-8, PP. 99-112.
- 17 - ACI 318-05 (2005), *Building Code Requirements for Structural Concrete (318M-05) and Commentary (318RM-05)*, American Concrete Institute (ACI), Farmington Hills, Michigan, USA.
- 18 - ASTM D3039. (1995). *Standard test method for tensile properties of polymer matrix composite materials*, American Society for Testing and Materials, West Conshohocken, Pennsylvania.
- 19 - Ndambi, J-M., Vantomme, J. and Harri, K. (2002). "Damage assessment in reinforced concrete beams using eigenfrequencies and mode shape derivatives." *Engineering Structures*, Vol. 24, PP.501–15.
- 20 - Ndambi JM, Peeters B, Maeck J, De Visscher Y, Wahab M.A. and Vantomme J. (2000). "Comparison of techniques for modal analysis of concrete structures." *Engineering Structures*, Vol. 22, PP. 1159–1166.
- 21 - Maeck J, Abdel Wahab M, Peeters B, De Roeck G, De Visscher J. and De Wilde WP. (2000). "Damage identification in reinforced concrete structures by dynamic stiffness determination." *Engineering Structures*, Vol. 22, No. 10, PP. 339–349.
- 22 - Garaygordobil JCA. (2003). *Dynamic assessment of structural building components*, Ph.D. thesis. Barcelona (Spain): Department of Construction Engineering, Catalunya.
- 23 - STAR. (1990). Structural Testing Analysis and Report, Structural Measurement System, SMS, Version 3.1.
- 24 - MATLAB. (2004).The language of technical computing, Version 7,(Release 14-1).
- 25 - Baghiee, N., Esfahani, M. R., Moslem, K. and Rezaee Pazhand, J. (2009). "Structural damage identification of reinforced concrete beams strengthened by CFRP sheets." *Proceedings of the 8<sup>th</sup> International Congress on Civil Engineering*, May 11-13, Shiraz University, Shiraz, Iran.
- 26 - Eilbracht, G. (1997). *Identifikation von rissbereichen in stahlbetonblken mit hife von schwingungstestdaten*, Fortschritt Berichte VDI; Reihe 18, Ph.D. dissertation, Dusseldorf, Germany.
- 27 - Baghiee, N., Moslem, K., Esfahani, M. R. and Rezaee Pazhand, J. (2010). "Damage Detection in Reinforced Concrete Beams Based on the Ritz Vectors." *Proceedings of the 5<sup>th</sup> National Congress on Civil Engineering*, May 4-6, Ferdowsi University of Mashhad, Mashhad, Iran.

- 
- 1 - Monitoring  
2 - Basic Modal Parameters  
3 - Derived Modal Parameters  
4 - Stiffness  
5 - Normal  
6 - Noise  
7 - Digital filter  
8 - Frequency response function