DESERT

DESERT
Online at http://jdesert.ut.ac.ir

DESERT 14 (2009) 53-62

Statistical model of daily flows in arid and semi-arid regions

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Received: 31 May 2008; Received in revised form: 12 October 2008; Accepted: 15 February 2009

Abstract

Stochastic model is necessary for planning and implementation of water works in ephemeral rivers of arid and semi-arid regions. Accurate estimation of stream flow in arid and semi-arid regions is an important, critical and crucial task in many cases, however daily discharge data is usually limited in such regions. It even occurs for short term rainfall series, whereas monthly and yearly rainfall series usually do not show such manner In this study a model is considered for daily discharge data of Firoozabad basin of Kerman province, Iran which includes identification of some factors including occurrence of flow time, flow increment, flow increment magnitude and calculation of flow reduced decrement. The Markov chain, increments of the rising limb and an exponential recession of the hydrograph were used. The observed daily gauging flow data were used for parameter estimation as well. The model is then applied to daily flow series records which showed that not only the model can show the long-term characteristics of the hydrograph but also it can predict its short-term characteristics.

Keywords: Daily Flows; Stochastic Modeling; Markov Chains ; Hydrograph; Kerman; Firoozabad Basin; Iran

1. Introduction

Water scarcity is one the major threatening factors in arid and semi-arid regions. Water resources planning programs in arid and semi-arid regions have significance influences on water demands of different sectors including agriculture, environment, domestic and industry. Therefore, prediction of available and reliable water resources is of high importance in ephemeral flows which have insufficient data period. Therefore, relevant and appropriate mathematical models needs to be developed to generate synthetic data in water resources planning. Recently, hydrologists have been interested in modeling of flow in ephemeral rivers, which is significantly important in the development of arid zones (Aksoy and Bayazit, 2000).

A stochastic hydrological parameter takes a value greater than or sometimes zeros (Yevjevich, 1972). It is also true for daily rainfall series, whereas monthly and yearly rainfall series usually do not show such manner (Aksoy and Bayazit, 2000). Stream flow records in arid and semi-arid regions are very limited and usually include several zero Then, it influences on the characteristics of data series of arid and semi-arid regions which leads to such a complex behavior, too (Salas, 1993).

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Only a few attempts have been made in the past to model stream flow of arid regions (Aksoy and Bayazit, 2000). Rainfall–runoff modeling is an important aspect in arid and semi-arid areas and there are significant differences from humid climates. The available methods for humid regions are commonly applied almost without distinction to arid regions. However, models developed for ephemeral streams show higher complexity with higher number of parameters considered. Yevjevich (1972) has described the steps used in the hydrological modeling of arid and semi-arid regions as truncated, spell and Boolean approach.

In the study related to intermittent flows, Kisiel et al. (1971) have suggested two statuses including with and without flow for stream flow records. Kisi and Cigizoglu(2007) compared of different artificial neural network (ANN) techniques in short- and long-term continuous and intermittent daily streamflow forecasting and concluded that radial basis function-based neural networks (RBF) to be superior to the other ANN techniques and a time series model in terms of the selected performance criteria. An approach based on the regression model of Thomas and Fiering (1962) has been introduced in order to estimate monthly stream flows. The studies related to the analysis of intermittent river flows are very limited compared in spite of several studies in other climatic conditions with permanent river flows (Rodriguez-Iturbe et al. 1987). In this regard, Yakowitz (1973) specifically has focused on modeling of the daily flows of arid regions. There are other attempts to develop a model in Australia to forecast total flow.

In this study it is aimed to apply a stochastic model for simulating daily stream flows of ephemeral streams in the southeastern Iran due to the scarcity of data available and several observations with zero values in the time series. The complexity of hydrological system in arid and semi-arid regions is so high, and then analysis of daily time series would enable to acquire reliable description and modelling of the intermittent flow in the intermittent streams. The model developed is based on Markov chains.

2. Materials and Methods

2.1. Study area

Application of the model to a daily flow series taken from Firoozabad basin in the Kerman province of Iran is described. Location of the study area shows in Iran map in Figure 1. The climate of study area is mostly semi-arid. Figure 2 shows the location of gauging station in the study area. The gauging station is 2080 m above sea level. The area above the gauging station is 1176 km^2. The long-term mean flow is 1.65m^3/s, 27.4%) of the days have no flows during the recording time.

Fig1. Location of the study area in Kerman province and Iran

2.2. Daily stream flow model

In many hydrological studies, the use of series of short term flow data are necessary not only for planning of water supply projects but also for ecology, ecohydrology and water quality monitoring during low flows which is mostly apparent in arid regions (Kavvas and Delleur, 1984). In such conditions, it is preferred to use daily stream flow data for hydrological model simulation calibration. However, difficulties with daily data, has led to application of the models which use monthly data. When, the time scale of data decreases, it increase the time and processing of data, however, processing of daily data provides much more reliable and in some case applicable models results. When intermittency is added to the periodic and stochastic components of a hydrologic time series, it becomes very complex and further characteristics are needed for understanding and describing the time series. (Kisi & Cigizoglu, 2007). According to Aksoy and Bayazit (2000), the daily stream flow models present three classifications as autoregressive (AR), shot noise (SN) and transition probabilities (TP).

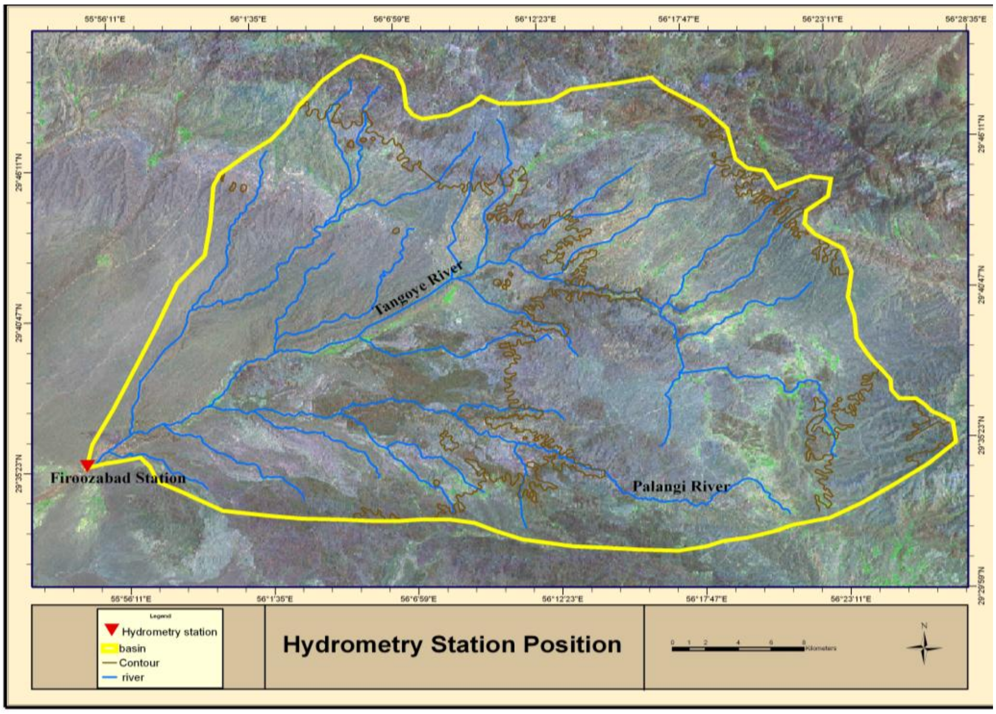

Fig2. Location of the hydrometry station in Firoozabad basin

In the works related to structural analysis of daily stream flow, Quimpo (1968) studied an auto regressive model and suggested that extracted deterministic components from standardized daily flow values could be defined by a second-order auto regressive model (Aksoy and Bayazit, 2000).The basin characteristics usually provide useful information in daily flow simulation. Some researchers have used basin characteristics to reconstruct or simulate daily stream flow. One of the characteristics of autoregressive models (AR) is the fact that they show the long-term characteristics and measures of central tendency and lag-one and higher lag of correlated data coefficients.

The shot noise (SN) or Poisson noise models can be modeled by a Poisson process. It can represent both short-term and long-term characteristics of daily flow. Shot noise models, based on filtered Poisson processes, the concept of shot noise was first introduced in 1918 by Walter Schottky who studied the fluctuations of current in vacuum tubes. Shot noise may be dominant when the finite number of the particles is sufficiently small so that uncertainties due to the Poisson distribution of data such as stream flow data series in this case. Criticizing the autoregressive (AR) and shot noise (SN) models, O'Connel and Jones (1979) proposed a non-linear AR model to overcome deficiency of the models (Aksoy and Bayazit, 2000).

The third type of developed models of daily stream flow is based on the probabilities of transition status of dry / wet days. The probability of a wet or dry was defined with respect to the status of the previous day. The transition probabilities were assumed to be stationary within a given day. The model describes the observed sequence of wet and dry periods. The use of Markov chains in describing daily rainfall occurrences firstly introduced by Gabriel and Neumann (1962). Another study used Markov chains to find the persistency of rainfall data during the rainy season over three tropical countries in Southeast Asia, including Sri Lanka. The research used a simple first order Markov chain model which was suggested for the dry zones while higher order Markov chains models were suggested for the wet zone of the country, especially in case of dry spells (Dahale et al. 1994).It is tried to distinguish between different parts of stream flow series by the daily stream flow in a relatively different approach as a step by step method which is described as follow: (i) days with the stream flow increase; (ii) magnitude of the increment in the stream flow data series; and (iii) the observed stream flow hydrograph recession curves.

It has been introduced an approach for modeling stream flow by determination of dry /wet days based on the system transfer function. The transfer function of a system is the relationship of the system's output to its input, represented in the complex Laplace domain. It is a representation in terms of spatial or temporal frequency, of the relation between the input and output of a linear time-invariant system with zero initial conditions and zero-point equilibrium. Kottegoda et al. (1995) have also tried to model the daily low flow data of a river basin in Italy by using the observed only stream flow data and no rainfall data were used.

2.3. Model Development

The model proposed in this study has the following steps (i) days with the stream flow increase; (ii) magnitude of the increment in the stream flow data series; and (iii) the observed stream flow hydrograph recession curves and (iv) Stream flow recession curve (Figure 3) (Akosy 1999; Aksoy and Bayazit, 2000). An intermittent or seasonal stream is one that only flows for part of the year. Then it may show different states and even zero values for some days of the year. This can be explained by a Markov chain which is a random process that undergoes transitions from one state to another on a state space. It must possess a property that is usually characterized as"memorylessness": the probability distribution of the next state depends only on the current state and not on the sequence of events. The approach has gained applications in hydrometeorological studies (Raudkivi, 1979).

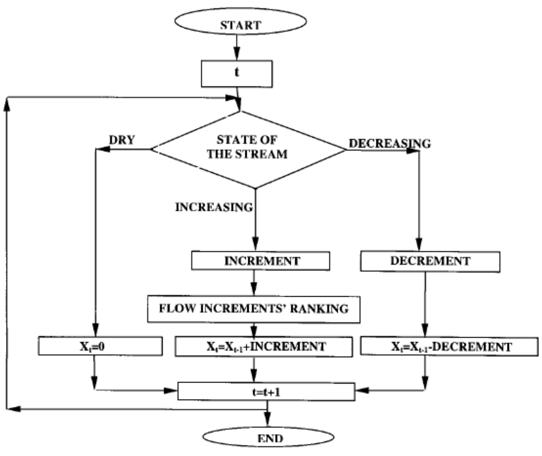

Fig 3. Flowchart of suggested model (Aksoy and Bayazit, 2000).

2.4. State of the stream flow

The transition matrix of three-state Markov chain describing the probabilities of particular transitions, and an initial state (or initial distribution) across the state space can be written as:

$$P = \begin{matrix} A11 & A12 & A13 \\ A21 & A22 & A23 \\ A31 & A32 & A33 \end{matrix} \qquad (1)$$

Where Pij represents probability of transition matrix. A stochastic model describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event provides a useful insight to the hydrological processes. While the time parameter is usually discrete, the state space of a Markov chain does not have any generally agreed-on restrictions: the term may refer to a process on an arbitrary state space. However, many applications of Markov chains employ finite or countably infinite (that is, discrete) state spaces, which have a more straight forward statistical analysis. Besides time-index and state-space parameters, there are many other variations, extensions and generalizations. The probability of transition matrix is calculated as

$$Pxy = \frac{nxy}{\sum nxy} \quad \text{x,y= 1,2,3} \qquad (2)$$

As a two-state Markov chains any position there are two possible transitions, to the next or previous integer. The transition probabilities depend on the current position, not on the state in which the position was reached (Aksoy, 1998).

2.5. The rising limb of the hydrograph

On days with the increased stream flow, the magnitudes of the increments occurring in the flow are determined (Aksoy and Bayazit, 2000). We need to have discharge data as time series to illustrate hydrographs, since there are no enough discharge information and hydrometric stations and also because increments between the flow values of successive days are calculated from the observed data for each month. Along with the rising limb of the hydrograph if the flow values of two consecutive days equals then it is assumed that the rising part of the hydrograph mainly depends on the magnitude of the increment which is equal to zero (Aksoy and Bayazit, 2000). The probability density function (pdf) is a function of a continuous random variable, whose integral across an interval gives the probability that the value of the variable lies within the same interval. The appropriate probability density function is then needed to fit the daily flow series. The gamma distribution is a two-parameter family of continuous probability distributions. The common exponential distribution and chi-squared distribution are special cases of the gamma distribution. Therefore, it is suggested that the gamma distribution provides better results in this case (Aksoy and Bayazit, 2000). The pdf of the gamma distribution in this situation has the form of:

$$f(x) = \frac{1}{BT(a)} * x^{a-1} * e^{-\frac{a}{B}} \quad a \geq 0 \qquad (3)$$

The variance and expected value are then presented as:

$$e(x) = aB \;\; var(x) = aB^2 \qquad (4)$$

The observed series of flow series can be used to estimate the shape (α) and scale (β) parameters of the distribution for each time scale, i.e. day, month, and year.

2.6. Determination the hydrograph falling limb

Recession curve of the hydrograph extends from the peak flow rate onward. The end of flow and the return to groundwater-derived flow (base flow) is often taken as the point of inflection of the recession limb. This part represents the withdrawal of water from the storage stored in the watershed during the previous phases of the hydrograph. The falling limb of the hydrograph contains valuable information the hydrogeological characteristics of the watershed (Tallaksen, 1995).

This process of estimating the recession-curve displacement can be used to find out groundwater recharge from stream flow records. The method is based on the premise that the stream flow-recession curve is displaced upward during periods of groundwater recharge. Ground-water recharge during an event can be estimated based on the Boussinesq's equation for aquifers.

A recession curve starts with a peak and continues with an increment in the flow (Aksoy and Bayazit, 2000). There are high and low discharge values in the records. One useful idea is to use master recession curve in this case which is a smoothed composite of the recessions of several hydrographs to represent the characteristic of decreasing total runoff for a drainage area after passage of a peak flow which includes different parts such as exponential one.

After investigating various approaches to calculate the recession curve it seems that the exponential equation provides better results as recommended by Aksoy and Bayazit, 2000. Accordingly, the recession curve segmentation has two steps. The monthly mean flow describes the average hydrological conditions during the preceding month and therefore is suitable scale in this study. The observed stream flow data can be used to determine the recession coefficients of each month. According to Aksoy and Bayazit (2000), the first part of recession segment can form of:

$$Q = QOe^{-rt} \qquad (5)$$

Where r is the recession coefficient for the first segment of the recession curve, t shows the number of days after a given peak, Q is flow in days after the peak, and Q0 is the previous peak flow of data series. Meanwhile, the second segment of the recession curve is as follow:

$$Q = Q0e^{-r(t-to)} \qquad (6)$$

Where t_0 is the time from the start of the second segment of the recession curve and Q0 is the initial flow of the recession curve (Aksoy and Bayazit, 2000). The recession curve of a stream flow records represents a useful tool in hydrological analysis and design, giving integrated information on long term perspective of low and drought flow. Generally, two parts or segments of the recession curve tend to be gradually lower and lower in a river until it reaches to very small values with very limited variations which mainly depend on the watershed characteristics such as geology, landuse, topography, etc. In such cases, the equation 5 is used to calculate the discharge but in the circumstances that the peak flow of stream flow series is smaller than the observed monthly stream flow data series, the coefficient of stage II is used until it ends to zero (Aksoy, 1998, Aksoy and Bayazit, 2000).

2.7. Application approach

Testing of the proposed model is considered for daily stream flow data of Firoozabad hydrometery station in Kerman province, Iran for the 20-year period as given in Figure 4.

The transition probabilities of the Markov chain for three-state condition along with the parameters of the model are calculated for the data series which are presented as Table 1.

Calculated transition probabilities indicate that P_{21} is greater than P_{32}; P_{11} is greater than P_{23} and P_{31}.

The shape and scale parameters are calculated from the gamma distribution which is the best fitted (Aksoy and Bayazit, 2000) for each month as shown in Table 2.

Using the approach already described in the previous sections, the recession curve parameters of Firoozabad hydrometry station are calculated and presented (Table 3). All daily observed stream flows were checked as suggested by Aksoy and Bayazit (2000) by using the estimated coefficients.

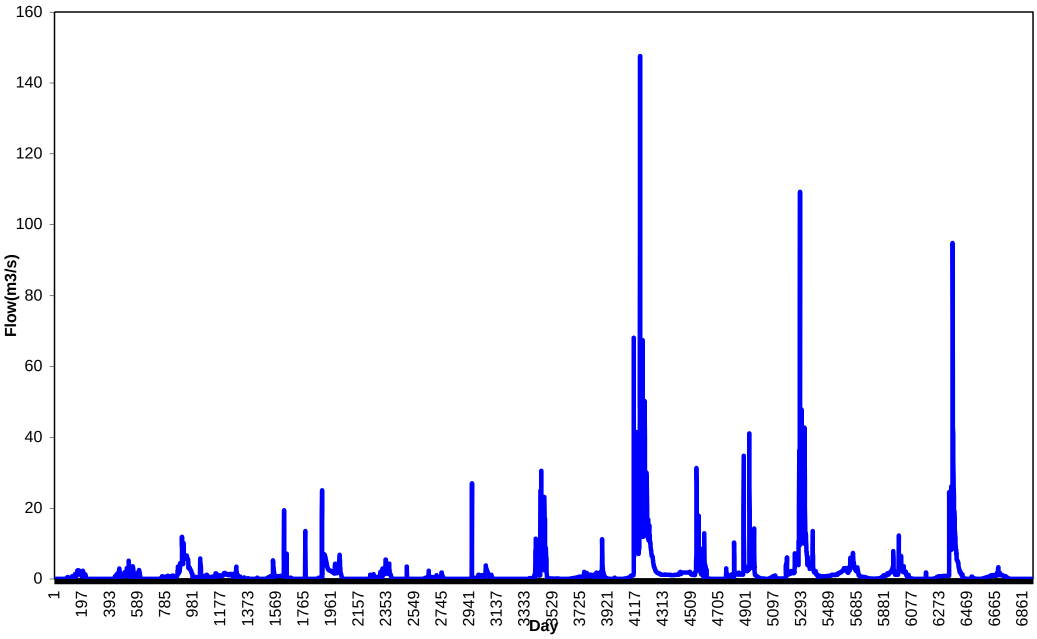

Figure 4. Daily time series of stream flow in Firoozabad basin

Table1. Transition probabilities matrix of in the studied station (Mehr= October)

Month	P_{11}	P_{22}	P_{33}	P_{31}
Mehr	0.523	0.040	0.890	0.124
Aban	0.574	0.179	0.803	0.106
Azar	0.378	0.356	0.575	0.156
Dey	0.272	0.458	0.436	0.164
Bahman	0.314	0.488	0.341	0.216
Esfand	0.417	0.613	0.446	0.204
Farvardin	0.369	0.626	0.355	0.197
Ordibehesht	0.286	0.562	0.645	0.102
Khordad	0.118	0.491	0.776	0.069
Tir	0.194	0.447	0.841	0.072
Mordad	0.538	0.261	0.873	0.045
Shahrivaar	0.366	0.207	0.867	0.035

Table 2. Shape and scale parameters of the gamma distribution in the study region

Month	α	β
Mehr	0.673	0.417
Aban	0.670	0.995
Azar	0.512	0.262
Dey	0.181	0.173
Bahman	0.447	0.134
Esfand	0.289	0.309
Farvardin	0.110	0.616
Ordibehesht	0.434	0.764
Khordad	0.407	0.398
Tir	0.218	0.256
Mordad	0.516	0.203
Shahrivar	0.225	0.250

Table 3. The recession curve parameterers of Firoozabad basin

Month	Recession coefficients		No. of recession		Mean flow of recession curve (cms)		Percent of Relative Error	Monthly average flow (cms)
	First	Second	First	Second	Observed	Estimated		
Mehr	0.045	0.1946	19	18	0.159	0.367	-0.8	0.359
Aban	0.024	0.0289	25	18	0.593	0.530	0.1	0.531
Azar	0.018	0.0037	17	23	0.752	1.722	-0.3	1.719
Dey	0.042	0.0094	16	18	3.220	0.427	-0.8	0.419
Bahman	0.348	0.0144	15	19	1.437	0.797	0.7	0.804
Esfand	0.080	0.0388	14	20	2.703	1.604	0.4	1.608
Farvardin	0.046	0.1035	24	19	6.656	4.665	-0.2	4.663
Ordibehesht	0.046	0.0548	18	17	0.468	2.524	-0.1	2.523
Khordad	0.009	0.0243	18	22	1.709	0.882	-0.9	0.873
Tir	0.044	0.0698	17	19	1.453	0.289	0.7	0.296
Mordad	0.009	0.0726	15	18	0.059	0.698	5.0	0.748
Shahrivar	0.005	0.0262	14	17	0.095	0.274	5.0	0.324

Mean flow values of the estimated and observed recession as shown in Table 3 along with the percentage of relative errors (RE) of each month indicates that the total number of recession curves are 212 and 228 while the recession coefficients for the first and second steps are 0.060 and 0.053, respectively. Accordingly, the observed and estimated mean flow recession curves were 1.239 and 1.232 cms. It means that the estimation procedures represent the actual condition of the basin which shows its characteristics in recession curve behavior. Figure 2 indicates the two step process in which the fitted exponential function plotted for the estimated and observed data series. Table 3 shows the number of recession, estimated and observed values of the mean flow and the corresponding percentage of relative errors(RE) of the first and second steps for recession curve estimation process in the Firoozabad gauging station.

Using the parameters determined, totally 16 simulations were conducted for the observed time series in order to have a better and clear understanding of the assumption of the work and simulations while to sufficiently determine the desired and optimum simulation is the assumption would be relatively enough for hydrological studies.

3. RESULTS

3.1. State of stream flow

Whether the model preserves state of stream, transition probabilities in the observed and estimated matrices will be controlled. The probability of transitioning from one state to another in a single step in the Markov chain is said to be time homogeneous if the transition probabilities are independent of time index. The average of simulations for 20 years length is provided in Table 4. The comparison of Tables shows that the largest differences are in the months of Mordad and Sharivar that observed recession are smallest.

3.2. Ascending hydrograph

Two-parameter gamma distribution random values or two gamma probability density functions are used to find the statistical characteristics in the ascending limb hydrographs of stream flow series obtained as an average of simulations presented in Table 5. In Table 5, peak flow values as average of 16 simulations are presented with the observed ones.

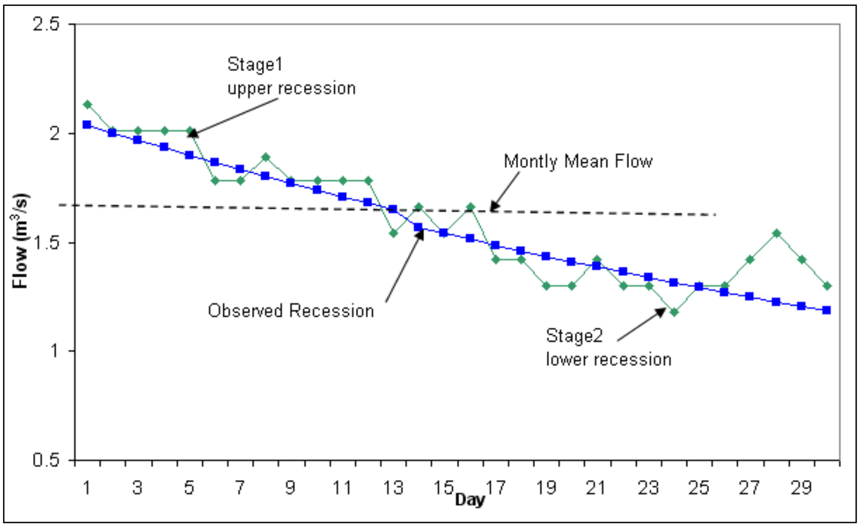

Fig5. Two step recession curve (based on Aksoy and Bayazit, 2000)

Mean and standard deviation of the flow increments can be seen in the series history and the average of simulated series is 2.955 cms which is related to the observed series mean of 2.960 cms. Meanwhile it is clear that the two-parameter probability distribution is used, the skew ness coefficient is generally close to the observed value. However, it is not true in highly skewed series. In other cases, the predicted series simulate the observed values of the series very good.

Table 4: Transition probabilities resulted from the simulations

Month	P_{11}	P_{22}	P_{33}	P_{21}
Mehr	0.528	0.033	0.897	0.115
Aban	0.580	0.178	0.804	0.106
Azar	0.369	0.358	0.573	0.156
Dey	0.286	0.458	0.436	0.164
Bahman	0.316	0.485	0.344	0.216
Esfand	0.411	0.615	0.444	0.204
Farvardin	0.374	0.626	0.355	0.197
Ordibehesht	0.289	0.559	0.648	0.102
Khordad	0.108	0.488	0.779	0.069
Tir	0.203	0.435	0.852	0.072
Mordad	0.484	0.222	0.919	0.044
Shahrivaar	0.369	0.186	0.893	0.029

Table 5: Values of observed (O) and estimated (E) ascending hydrographs

Month	Average Flow (cms)		Standard dev.(cms)		Skew ness coefficient		Peak flow (cms)	
	obs.	est.	obs.	est.	obs.	est.	obs.	est.
Mehr	0.28	0.11	0.43	0.14	3.69	2.85	2.90	1.30
Aban	0.67	0.63	2.11	2.05	12.15	11.97	27.00	26.66
Azar	1.34	0.98	2.62	1.98	6.30	4.48	24.98	21.52
Dey	3.14	3.94	7.55	8.94	5.72	9.70	68.00	60.44
Bahman	6.00	6.07	16.38	16.51	5.99	6.37	147.50	148.22
Esfand	8.93	8.39	16.06	15.11	3.58	0.86	109.25	114.42
Farvardin	6.78	6.48	8.64	8.12	2.53	1.05	47.80	44.99
Ordibehesht	3.31	3.70	3.79	4.47	1.60	3.54	14.20	10.51
Khordad	2.91	3.08	3.32	3.62	2.54	3.40	13.54	15.17
Tir	0.56	0.61	1.11	1.19	3.83	4.07	5.73	6.19
Mordad	1.05	0.91	2.32	2.08	4.51	3.81	13.53	12.20
Shahrivaar	0.56	0.57	0.36	0.37	0.70	0.74	1.73	1.81

3.3. Monthly and annual flow

Monthly average flows computed from the daily flow series for simulated ones presented in Figure 6 with observed flows.

3.4. Daily flow

Daily characteristics of data such as common statistical of the simulated river data were calculated and compared with the corresponding observed flow series.
Daily zero flow percentages of generated and observed flows were compared and the average values of are shown in Table 6. In the case of zero flow percentage, it indicates that Markov chain can acceptably simulate the river flow series in different conditions (low and high flows of hydrograph). Figures 7 present the mean values of the estimated series which shows relatively similar behavior and statistical characteristics compared to the observed values. The pattern similarity to the observed values mostly becomes apparent in sharp rises followed by the slow recession.

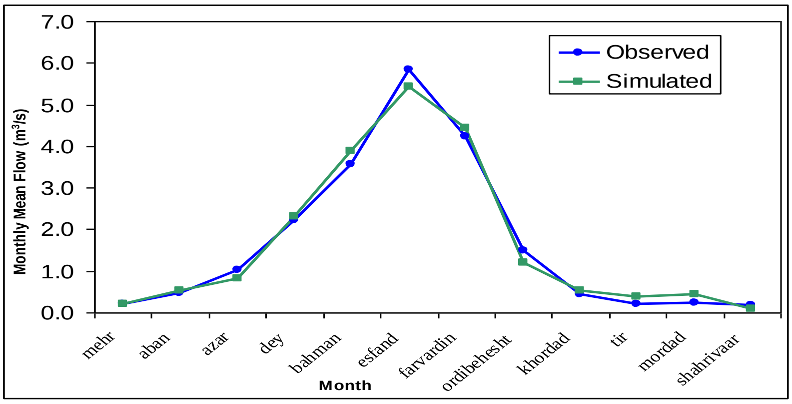

Fig 6. Observed and simulated monthly mean stream flow of the study area

Table 6: Characteristics of daily stream time series in the study region

Data series	Average (cms)	Standard dev. (cms)	Coefficient of skewness	% of Zero flow
Observed	1.656	5.362	10.105	27.440
Estimated 1	1.568	5.487	6.580	25.457
Estimated 2	1.665	6.142	5.619	23.845
Estimated 3	1.432	7.010	7.354	28.351
Estimated 4	1.724	8.194	8.416	27.539
Estimated 5	1.495	4.147	6.454	26.127
Estimated 6	1.565	5.295	7.622	24.519
Estimated 7	1.812	7.537	8.302	29.953
Estimated 8	1.965	8.248	6.648	25.427
Estimated 9	1.326	7.138	7.531	22.575
Estimateds 10-16	1.917	6.462	7.015	28.216
Mean	1.647	6.566	7.154	26.201

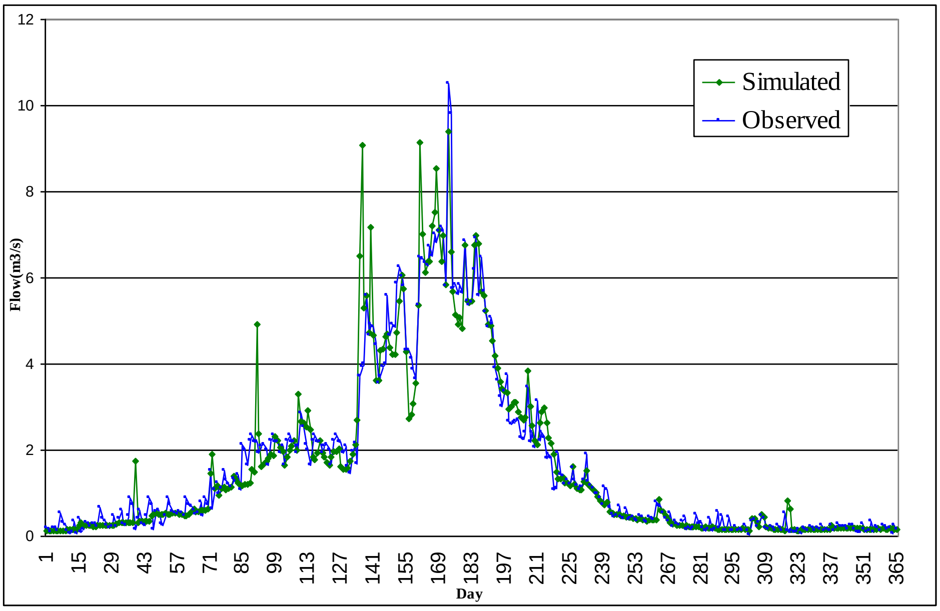

Fig 7. Observed and simulated daily mean flow

4. Conclusions

Although intermittent streams play an important role in providing the scarce water resources in arid regions, however they have attracted limited scientists. Intermittent river in which surface water ceases to flow at some point in space and time. Covering approximately a third of the earth's surface and for many years, determination of the best relationship between hydrograph components and river flow characteristics is the main objective of this research. With these relationships, the hydrograph components can be calculated using physical characteristics of the catchment. Finding of the best relationship between hydrograph components and stream flow characteristics of the basin is the main objective of this research. Evidently in equal condition, the models with more adjusted coefficient of determination, less estimation and approval error and less number of independent variables were selected as the best models which have been followed in the current study.

The application of the model to an intermittent river in southeastern Iran using a Markov chain approach showed that the capability of the method to preserve the statistical characters of the daily river flow for different conditions of the hydrograph in the rising and falling limbs. Maximum flows are well estimated while minimum flows lower than the river observed values were generated.

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