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مهدی قاسمی\*<sup>۱</sup> و مهدی کریمی راد<sup>۲</sup>

( // // // )

( )

[ ] [ ]  $t$   $n$   $[0, t]$   $(t > 0)$

$\Delta t$   $\Delta t = t/n$   $\Delta t$

[ ] [ ]

[ ] [ ]

[ ]

[ ]

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( ) ( )

( ) ( )  $\alpha \delta$

$t + \Delta t$

${}^{t+\Delta t}\ddot{U}$  ( ) ( )

$t + \Delta t$

$t + \Delta t$

$t - \Delta t$   $t$

${}^0\dot{U}$   ${}^0U$

( )

${}^0\ddot{U}$   $t = 0$

( ) ( )

$\Delta t$   $t$

$M\ddot{U} + C\dot{U} + KU = P$  (1)

$K$   $C$   $M$

$P$

$\ddot{U}$   $\dot{U}$   $U$

( ) ( )

$t$   ${}^{t+\Delta t}\dot{U}$   ${}^{t+\Delta t}U$

$n\Delta t$  .....  $3\Delta t$   $2\Delta t$

( ) ( )

$\delta$   $\alpha$

( ) ( ) ( ) ( )

$t - \Delta t$

$t + \Delta t$

( ) ( )

( ) ( ) ( ) ( )

${}^{t+\Delta t}U = {}^tU + \Delta t {}^t\dot{U} + \frac{\Delta t^2}{2} {}^t\ddot{U} + \frac{\Delta t^3}{6} {}^t\dddot{U} + \dots$  (2)

${}^{t+\Delta t}\dot{U} = {}^t\dot{U} + \Delta t {}^t\ddot{U} + \frac{\Delta t^2}{2} {}^t\dddot{U} + \frac{\Delta t^3}{6} {}^t\ddot{\ddot{U}} + \dots$  (3)

${}^{t+\Delta t}U = {}^tU + \Delta t {}^t\dot{U} + \frac{\Delta t^2}{2} {}^t\ddot{U} + \frac{\Delta t^3}{6} {}^t\dddot{U} + \alpha\Delta t^4 {}^t\ddot{\ddot{U}}$  (4)

${}^{t+\Delta t}\dot{U} = {}^t\dot{U} + \Delta t {}^t\ddot{U} + \frac{\Delta t^2}{2} {}^t\dddot{U} + \delta\Delta t^3 {}^t\ddot{\ddot{U}}$  (5)

${}^t\ddot{\ddot{U}} = \frac{1}{2\Delta t} ({}^{t+\Delta t}\ddot{\ddot{U}} - {}^{t-\Delta t}\ddot{\ddot{U}})$  (6)

${}^t\ddot{\ddot{\ddot{U}}} = \frac{1}{\Delta t^2} ({}^{t-\Delta t}\ddot{\ddot{U}} - 2{}^t\ddot{\ddot{U}} + {}^{t+\Delta t}\ddot{\ddot{U}})$  (7)

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$${}^{t+\Delta t}\dot{U} = {}^t\dot{U} + [(\delta - \frac{1}{4}) {}^{t-\Delta t}\ddot{U} + (1 - 2\delta) {}^t\ddot{U} + (\delta + \frac{1}{4}) {}^{t+\Delta t}\ddot{U}] \Delta t \tag{۸}$$

$${}^{t+\Delta t}U = {}^tU + {}^t\dot{U} \Delta t + [(\alpha - \frac{1}{12}) {}^{t-\Delta t}\ddot{U} + (\frac{1}{2} - 2\alpha) {}^t\ddot{U} + (\alpha + \frac{1}{12}) {}^{t+\Delta t}\ddot{U}] \Delta t^2 \tag{۹}$$

$$T = 2\pi / \omega$$

( ) ( ) [ ]

x

( ) ( )  
( ) ( ) ( )

p k c m  
t + Δt

$${}^{t+\Delta t}\ddot{x} \quad {}^{t+\Delta t}\ddot{x}$$

( ) ( )

$${}^{t+\Delta t}\ddot{x} + 2\xi\omega {}^{t+\Delta t}\dot{x} + \omega^2 {}^{t+\Delta t}x = {}^{t+\Delta t}r \tag{۱۰}$$

: ( )

$$\omega^2 = k/m \quad 2\xi\omega = c/m$$

$$\omega \quad {}^{t+\Delta t}r = {}^{t+\Delta t}p/m$$

$${}^{t+\Delta t}\dot{x} = {}^t\dot{x} + [(\delta - \frac{1}{4}) {}^{t-\Delta t}\ddot{x} + (1 - 2\delta) {}^t\ddot{x} + (\delta + \frac{1}{4}) {}^{t+\Delta t}\ddot{x}] \Delta t \tag{۱۱}$$

$${}^{t+\Delta t}x = {}^tx + {}^t\dot{x} \Delta t + [(\alpha - \frac{1}{12}) {}^{t-\Delta t}\ddot{x} + (\frac{1}{2} - 2\alpha) {}^t\ddot{x} + (\alpha + \frac{1}{12}) {}^{t+\Delta t}\ddot{x}] \Delta t^2 \tag{۱۲}$$

$$\begin{Bmatrix} {}^{t+\Delta t}\ddot{x} \\ {}^t\ddot{x} \\ {}^{t+\Delta t}\dot{x} \\ {}^{t+\Delta t}x \end{Bmatrix} = [A] \begin{Bmatrix} {}^t\ddot{x} \\ {}^{t-\Delta t}\ddot{x} \\ {}^t\dot{x} \\ {}^tx \end{Bmatrix} + \{L\} {}^{t+\Delta t}r \tag{۱۳}$$

$$\lambda^3 + \frac{(-2 + h(\delta - 3\alpha + \frac{2}{3}))}{(1 + h(\alpha + \frac{1}{12}))} \lambda^2 + \frac{(1 + h(3\alpha - 2\delta + \frac{5}{12}))}{(1 + h(\alpha + \frac{1}{12}))} \lambda + \frac{h(\delta - \alpha - \frac{1}{6})}{(1 + h(\alpha + \frac{1}{12}))} = 0 \tag{۱۴}$$

$$\rho(A) \leq 1$$

$$\{L\} [A]$$

[ ]

[ ]

{L}

[A]

$$\rho(A) [A]$$

α δ

:

[A]

$$\rho(A) = \max_i |\lambda_i| \tag{۱۵}$$

[A]

( )

[A]

λ<sub>i</sub> ( )

α δ

( )

λ<sub>i</sub>

$$|\lambda_i| = (\lambda_i \cdot \bar{\lambda}_i)^{1/2}$$

λ<sub>i</sub>

$\delta = 1/3$

( )

$\rho(A) \leq 1$

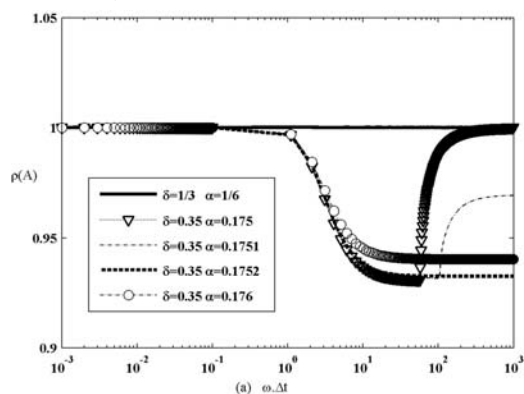
[A]

$\alpha \quad \delta$

$\alpha = 1/6$

$\delta \geq 1/3; \delta/2 \leq \alpha \leq \delta - 1/6$  (16)

[ ]



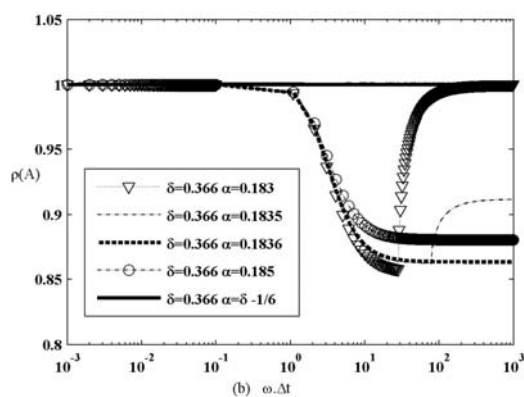
$\alpha = 1/6 \quad \delta = 1/3$

(a)

$\delta = 0.4 \quad \delta = 0.366 \quad \alpha = \delta - 1/6$

[A]

(c) (b)



$(\rho_\infty = \lim_{\omega \cdot \Delta t \rightarrow \infty} \rho(A)) \rho_\infty$

[ ]

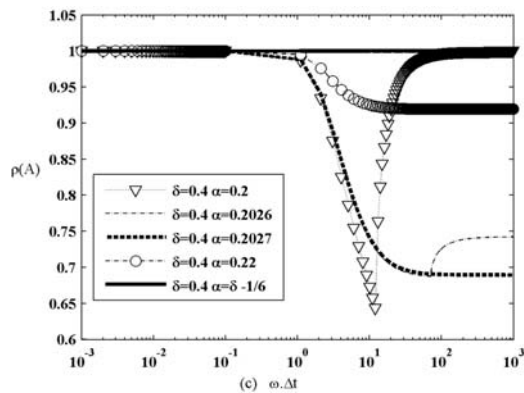
$\alpha = 1/6 \quad \delta = 1/3$

$\alpha$

$\delta > 1/3$

$\rho_\infty$

(a)



$\delta = 0.35$

$\alpha = 0.1752$

( )

$\delta = 0.4 \quad \delta = 0.366$

(c) (b)

$\alpha = 0.1836$

$\rho_\infty$

$\alpha = 0.2027$

شکل ۱: منحنی شعاع طیفی ماتریس [A] بر حسب  $\omega \cdot \Delta t$ .

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[ ]

$$\alpha \quad \delta$$

( )

[A]

$$\alpha = 1/6 \quad \delta = 1/3$$

( )

$\delta$

$\alpha$

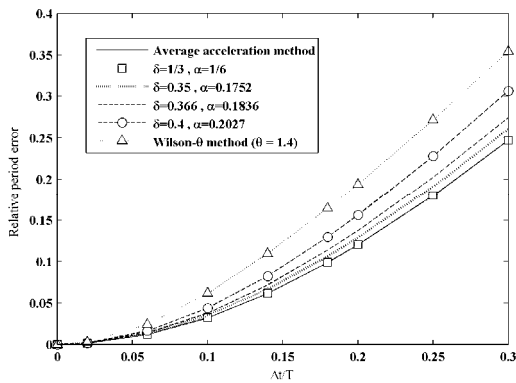
[ ]

$$\delta = 0.366)$$

$$(\alpha = 0.2027 \quad \delta = 0.4) \quad (\alpha = 0.1836$$

$\alpha$

$\delta > 1/3$



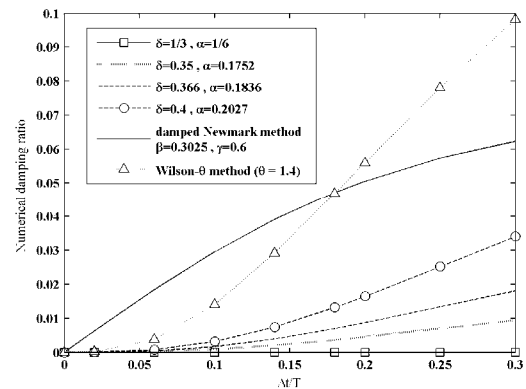
[ ]

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شکل ۲ : مقایسه نسبت میرایی عددی روش پیشنهادی، روش ویلسون و روش نیومارک.

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$$\Delta t/T \quad \alpha = 0.1836 \quad \delta = 0.366$$

$$\alpha = 0.2027 \quad \delta = 0.4 \quad 0.115$$

$\bar{\xi}$

[ ]

$$AD = 2\pi\bar{\xi}$$

$$T = 2\pi \quad x_{exact} = \cos(t) \quad 0.107 \quad \Delta t/T$$

$$(\quad) \quad (\quad) \quad (\quad) \quad (\quad)$$

$$(\quad) \quad \Delta t = T/10 \quad [A]$$

$$\alpha = -0.3$$

$$[ \quad ] \quad \theta = 1.4$$

$$0.04 \quad 0.08 \quad 0.1 \quad \Delta t/T$$

$$[ \quad ]$$

$${}^t e = \frac{|{}^t x - {}^t x_{exact}|}{{}^t x_{exact}} \quad (18) \quad \alpha = 0.1836 \quad \delta = 0.366$$

$${}^t x \quad {}^t x_{exact}$$

$$t$$

$$(\quad)$$

$$\ddot{x} + x = 0 \quad (17)$$

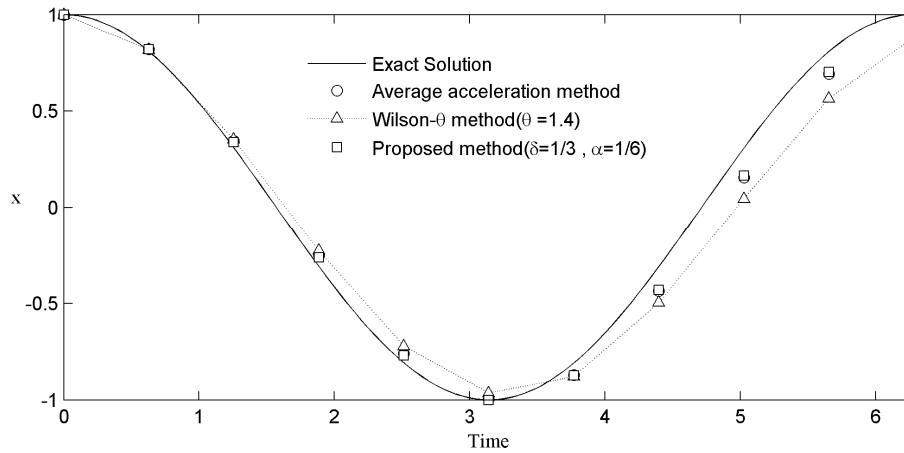
$$(\quad) \quad \dot{x} = 0 \quad x = 1$$

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( ) [ ]

جدول ۱: حل عددی معادله دیفرانسیل (۱۷) با استفاده از روش‌های شتاب متوسط، ویلسون و پیشنهادی.

Time			$(\theta = 1.4)$		$(\delta = 1/3, \alpha = 1/6)$	
	${}^t x$	${}^t e$	${}^t x$	${}^t e$	${}^t x$	${}^t e$
$\Delta t$	0.8203	0.013968	0.8187	0.01199	0.8203	0.013968
$2\Delta t$	0.3459	0.119411	0.3529	0.142063	0.3405	0.101936
$3\Delta t$	-0.2528	-0.18187	-0.2273	-0.26439	-0.2616	-0.15339
$4\Delta t$	-0.7607	-0.0597	-0.7220	-0.10754	-0.7698	-0.04845
$5\Delta t$	-0.9952	-0.0048	-0.9651	-0.0349	-1.0013	-0.0013
$6\Delta t$	-0.8722	-0.07812	-0.8785	-0.08578	-0.8731	-0.07923
$7\Delta t$	-0.4357	-0.41001	-0.4968	-0.60741	-0.4311	-0.39512
$8\Delta t$	0.1573	0.490912	0.0464	0.850115	0.1658	0.463729
$9\Delta t$	0.6938	0.142395	0.5649	0.301724	0.7031	0.1309
$10\Delta t$	0.9810	0.019	0.8843	0.1157	0.9878	0.0122



شکل ۴: منحنی پاسخ معادله دیفرانسیل (۱۷) با استفاده از حل دقیق و روش‌های شتاب متوسط، ویلسون و پیشنهادی.

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