Controlling Nonlinear Processes, using Laguerre Functions Based Adaptive Model Predictive Control (AMPC) Algorithm

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Abstract
Laguerre function has many advantages such as good approximation capability for different systems, low computational complexity and the facility of on-line parameter identification. Therefore, it is widely adopted for complex industrial process control. In this work, Laguerre function based adaptive model predictive control algorithm (AMPC) was implemented to control continuous stirred tank reactor (CSTR) process temperature runaways. Simulation result reveals that AMPC has a good performance in set-point tracking and load rejection. For comparison, a nonlinear model predictive control based on Laguerre- wiener model was also applied to the process. Simulation result demonstrates that the two controllers have the same performance in set point tracking and load rejection problem.

Keywords: Laguerre function; predictive control; nonlinear process; Laguerre-Wiener model

Introduction
Model Predictive Control (MPC) refers to a class of control algorithm in which a dynamic process model was used to predict and optimize process performance. The first MPC techniques were developed in the 1970s because conventional single-loop controllers were unable to satisfy increasingly stringent performance requirements [1]. Linear models were successfully employed to solve control problems. However, many processes were sufficiently nonlinear. This led to the development of Adaptive Model Predictive Control (AMPC) and Nonlinear Model Predictive Control (NMPC) which were more accurate for process prediction and optimization. Models are a decisive factor in MPC algorithm.

Zervos and Dumont in 1988 proposed a novel linear MPC algorithm based on Laguerre series in which the control horizon equals one [2]. From 2000 to 2004, Zhang presented a lot of successful industrial application of Laguerre functional series based control algorithm on high temperature semiconductor diffusion furnace, double water tank and distillation columns [3-5]. However, for high nonlinear process, it is better to use adaptive Laguerre model to approximate the behavior of the system.

The behavior of many systems could be approximated by a static nonlinearity cascaded with a linear part in particular form. These models are known as Hammerstein and wiener block cascade models. These model structures have been successfully utilized to represent nonlinear system in a number of practical applications in the area of chemical and biological process, signal processing and control [6]. From an identification point of view, pH process has often been considered in the literature as having a wiener structure. Distillation process have been modeled using both Hammerstein and wiener models [7].

Nonlinear model predictive control (NMPC) has been proposed as an alternative to LMPC for plants with highly nonlinear behavior. NMPC offers the same capabilities for interaction compensation and constraint handling as its linear counterpart. The key difference is that NMPC utilizes a nonlinear model to predict and optimize process performance. The use of NMPC for plant-wide control is problematic due to complications associated with dynamic modeling, state estimation and on-line

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optimization. A nonlinear dynamic model of the entire plant is required for controller design. Such large-scale nonlinear models are extremely difficult to obtain using fundamental modeling and available techniques for empirical nonlinear modeling. Another complication is that unmeasured state variables must be estimated from available on-line measurements. This requires the design of a nonlinear observer, which is a difficult task despite recent advances. Even if a suitable nonlinear model is available, a nonlinear programming problem must be solved at each sampling period to generate the control moves. For large-scale systems, the optimization problem may be computationally intractable due to the large number of decision variables and the complexity of the constraints resulting from the nonlinear model equations. While it can be argued that cheaper and faster computers soon will be available to solve plant-wide nonlinear optimization problems in real-time, a simple calculation has shown that a NMPC problem with 20 inputs and 20 outputs will not be able to be solved on-line until well into this century given expected advances in computer technology. As a result, the judicious use of modeling assumptions and simplified controller formulations are required even for problems of moderate size and complexity [8].

In this paper, we have considered a temperature control problem of CSTR with first order exothermic reaction. Two controllers are designed for this purpose. The controllers are constructed through a Laguerre function based adaptive linear model predictive control and a Laguerre-wiener model based nonlinear model predictive control.

Laguerre function

The Laguerre functions are an orthonormal set of functions that are complete in the function space. The Laplace transform of any of these functions is a rational function of the Laplace variable that has all of its poles at the point on the negative real axis. In the time domain, the Laguerre functions are polynomials multiplied by a decaying exponential. As such, the Laguerre functions can be used to approximate stable transfer functions, and/or reasonably behaved functions that decay to zero in the time domain. Laguerre function, φ_i (t),is defined as a functional series [9]:

$$\varphi_i(t)=\sqrt{2p}\,\frac{e^{-pt}}{(i-1)!}\cdot\frac{d^{i-1}}{dt^{i-1}}\left[t^{i-1}.e^{-2pt}\right] \quad i=1,2,\ldots,\infty \tag{1}$$

where p is a constant called time scaling factor and $t\in[0,\infty]$ is a time variable.

The Laplace transformation of Laguerre function is defined as:

$$\varphi_i(s)=L\left[\varphi_i(t)\right]=\sqrt{2p}\,\frac{(s-p)^{i-1}}{(s+p)^{i}} \quad i=1,2,\ldots,\infty \tag{2}$$

Open loop stable system can be approximated by N order Laguerre series as shown in Figure (1):

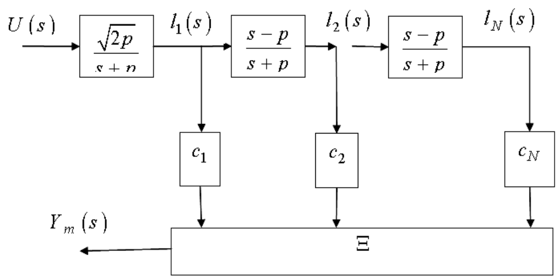


Figure 1: Laguerre series model structure

Open loop stable system can be approximated by N order Laguerre series.

$$Y_m(s)=\sum_{i=1}^{N}c_i\varphi_i(s)U(s)=\sum_{i=1}^{N}c_i l_i(s) \tag{3}$$

The state space expression of incremental mode Laguerre functional model after discretization is:

$$\Delta L(k+1)=A\Delta L(k)+b\Delta u(k) \tag{4}$$
$$\Delta y_m(k)=C^T\Delta L(k)$$
$$\Delta L(k)=L(k)-L(k-1)=\left[\Delta l_1(k)\ \Delta l_2(k)\ldots\Delta l_3(k)\right]^T$$
$$\Delta y_m=y_m(k)-y_m(k-1),\Delta u(k)=u(k)-u(k-1)$$

where $\Delta L(k)$ and Δy_m are the state vector of the incremental mode Laguerre functional model and the input and output of this model in kth sampling period, respectively; and

$C^T = [c_1, ..., c_N]$ is the Laguerre coefficients vector. Matrices, A and b, are calculated as follows:

$$A = \begin{bmatrix} \tau_1 & 0 & \cdots & 0 \\ \frac{-\tau_1\tau_2 - \tau_3}{T} & \tau_1 & \cdots & 0 \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \frac{(-1)^{N-1}\tau_2^{N-2}(\tau_1\tau_2+\tau_3)}{T} & \cdots & \frac{-(\tau_1\tau_2+\tau_3)}{T} & \tau_1 \end{bmatrix} \quad \begin{matrix}(5a)\\(5b)\end{matrix}$$

$$b^T = \left[\tau_4, (-\tau_2/T)\tau_4, ..., (-\tau_2/T)^{N-1}\tau_4\right]$$

where

$$\tau_1 = e^{-pT}, \tau_2 = T + \frac{2}{p}\left(e^{-pT} - 1\right), \tau_3 = -Te^{-pT} - \frac{2}{p}\left(e^{-pT} - 1\right)$$

$$, \ \tau_4 = \sqrt{2p}\,\frac{(1 - \tau_1)}{p}$$

and T is the sampling period.

In the above equation Δu were calculated as a replacement for u in the controller. Owing to the fact that this method could import integral mechanism, which in terms could guarantee zero steady-state error in the closed-loop system [10].

Laguerre-Wiener model

The Laguerre-wiener model of a nonlinear system is constructed by a nonlinear gain cascaded after Laguerre functional model as linear part. The block cascade structure of Laguerre-wiener model is demonstrated in Figure 2:

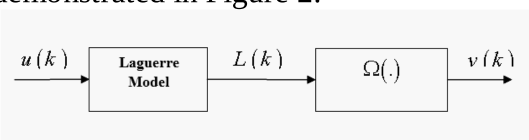


Figure 2: Laguerre-wiener model

The input-output relationship using this model could be presented as follows:

$$L(k+1) = AL(k) + bu(k) \quad (6a)$$

$$y(k) = \Omega(L(k)) \quad (6b)$$

In this model, linear and nonlinear parts in Laguerre function could be represented by various models such as polynomial, NARMA and neural network.

I. Laguerre-Wiener model using second order polynomial

The nonlinear gain of many processes can be approximated by second order polynomial. Khaksar and co-workers used second order polynomial for Hammerstein model in controlling the unstable reactor by MPC algorithm [11].After using this polynomial for Laguerre- wiener model, the relationship between input-output can be shown as following equations:

$$L(k) = c_0 + \sum_{i=1}^{N} c_i l_i(k) \quad (7a)$$

$$y(k) = \Omega(L(k)) = \sum_{i=0}^{2} \gamma_i (L(k))^i \quad (7b)$$

where $C = [c_1, ..., c_N]$ are coefficients of Laguerre functional model and $\gamma = [\gamma_0, \gamma_1, \gamma_2]$ are coefficients of second order polynomial. Offline least square optimization technique was utilized for parameter identification in this model.

Model predictive controller (MPC)

MPC is an optimization-based control strategy which is well suited for constrained, multivariable process. A sequence of control move was computed to minimize an objective function which includes predicted future values of the controlled output. The predictions are obtained from a process model. The various MPC algorithms proposed different cost function for obtaining the control law. A general expression for such an objective function is shown as:

$$\min\ J = \sum_{j=0}^{H-1} \left\| y(k+j|k) - r(k+j|k) \right\|_{Q_j} + \sum_{j=0}^{Nu} \left\| \Delta u(k+j|k) \right\|_{R_j} \quad (8)$$

$$\Delta u(k|k), \Delta u(k+1|k), ..., \Delta u(k+Nu-1|k)$$

$$\Delta u(k+j|k) = u(k+j|k) - u(k+j-1|k)$$

where Nu is the control horizon, H is the prediction horizon, Q is a symmetric positive semi definite penalty matrix on the output, R is symmetric definite penalty matrix on the rate of input and y (k+j|k) is the prediction output. An important characteristic of process control problem is the presence of constrains on input state and output variable. In this work, input constrain was only considered which can be represented as:

$$U^L \le u(k+j|k) \le U^U \quad 0 \le j \le Nu-1 \tag{9}$$

The superscript L and U represents the admissible lower and upper bounds for the input variable, respectively. To compensate for the mismatch between the process and the model and to consider unmeasured disturbance in the process, a term such as one shown below must be added to predicted output of the plant:

$$d(k) = y(k) - y_m(k) \tag{10}$$

where y (k) is the output of the real process and y_m (k) is the model output. The modified predicted output can be represented as:

$$y_{pred}(k+i) = y_m(k+i) + d(k) \quad i = 1,...,H \tag{11}$$

To employ the MPC strategy, it was necessary to obtain vector of future output from the model. For Laguerre functional model, the prediction output could be obtained using the following equations which yield from Equation 4

$$\begin{aligned} &\Delta L(k+2) = A^2 \Delta L(k) + Ab\Delta u(k) + b\Delta u(k+1) \\ &\quad . \\ &\quad . \\ &\quad . \\ &\Delta L(k+H) = A^H \Delta L(k) + \sum_{i=0}^{Nu} A^{H-1-i} b\Delta u(k+i) \end{aligned} \tag{12}$$

And

$$\begin{aligned} &\Delta y_m(k+1) = C^T \Delta L(k) + C^T b\Delta u(k) \\ &\quad . \\ &\quad . \\ &\quad . \\ &\Delta y_m(k+H) = C^T A^H \Delta L(k) + \sum_{i=0}^{Nu-1} C^T A^{H-1-i} b\Delta u(k+i) \end{aligned} \tag{13}$$

Considering that

$$\begin{aligned} &y_m(k+1) = y_m(k) + \Delta y_m(k+2) \\ &y_m(k+2) = y_m(k) + \Delta y_m(k+1) + \Delta y_m(k+2) \\ &\quad . \\ &\quad . \\ &\quad . \\ &y_m(k+H) = y_m(k) + \Delta y_m(k+1) + ... + \Delta y_m(k+H) \end{aligned} \tag{14}$$

Thus

$$Y_m(k+1) = SH_l \Delta L(k) + SH_u \Delta U_{Nu}(k) + Qy_m(k) \tag{15}$$

where

$$Y_m(k+1) = \left[y_m(k+1),...,y_m(k+H) \right]^T \tag{16}$$

$$\Delta U_{Nu}(k) = \left[\Delta u(k),...,\Delta u(k+Nu-1) \right]^T \tag{17}$$

$$S = \begin{bmatrix} 1\ 0 \ldots 0 \\ 1\ 1 \\ . \\ . \\ . \\ 1\ 1 \ldots 1 \end{bmatrix}_{H\times H} \quad Q = \begin{bmatrix} 1 \\ 1 \\ . \\ . \\ . \\ 1 \end{bmatrix}_{H\times 1} \quad H_l = \begin{bmatrix} C^T A \\ C^T A^2 \\ . \\ . \\ . \\ C^T A^H \end{bmatrix}_{H\times Nu} \tag{18}$$

$$H_u = \begin{bmatrix} C^T b & 0 & \ldots & 0 \\ C^T Ab & C^T b & & \\ C^T A^{Nu-1} b & \ldots & \ldots & C^T b \\ . & & & \\ . & & & \\ . & & & \\ C^T A^{H-1} b & \ldots & \ldots & C^T A^{H-Nu} b \end{bmatrix}_{H\times Nu} \tag{19}$$

Coefficients of Laguerre model were identified on-line by RLS (Recursive Least Square) algorithm with forgetting factor [12]. For Laguerre-wiener model, the signal L (k) was defined as follows:

$$L(k) = \left[L(k+1),...,L(k+H) \right]^T \tag{20}$$

Therefore, Laguerre-wiener model based output prediction can be computed as:

$$Y_m(k+1) = \begin{bmatrix} \Omega(L(k+1)) \\ . \\ . \\ . \\ \Omega(L(k+H)) \end{bmatrix} \tag{21}$$

Finally, adaptive Laguerre based model predictive control and Laguerre –wiener models were transferred to sequential quadratic programming problem.

Simulation results

Reactors are the heart of many chemical processes thus dynamic simulation of these significant units is essential for the safe and profitable operation of the entire plant [13]. In reactors with exothermic reactions which are irreversible, the most challenging

problem is the potential for temperature runaways. The case study which was considered in this paper is highly nonlinear CSTR reactors which is the most common type of reactor used in industry.
Consider a reactor in which the following exothermic reaction takes place:

$$A \rightarrow B \quad (22)$$

The reaction rate can be given by:

$$-r_A = kc_A \quad (23)$$

where k is the reaction constant dependant on temperature and is defined as:

$$k = k_o \exp(-E/RT) \quad (24)$$

Using the mass and energy balance equations, the reactor can be modeled as

$$\frac{dc_A}{dt} = \frac{q}{V}(c_{Af} - c_A) - k_o c_A \exp(-E/RT) \quad (25a)$$

$$\frac{dT}{dt} = \frac{q}{V}(T_f - T) + \frac{(-\Delta H)k_o c_A}{\rho c_p}\exp(-E/RT) \quad (25b)$$

$$+\frac{\rho_c c_{pc}}{\rho c_p V} q_c \left[1 - \exp\left(\frac{-hA}{q_c \rho_c c_{pc}}\right)\right](T_{cf} - T)$$

where c_A is concentration of A and T is reactor temperature. T_{cf} is the coolant temperature which was considered as a manipulated variable and V is the reactor volume considered constant. The main objective of this work was to control the reactor temperature. The reactor characteristics adopted in this study are given in Table1 [14].

Figure 3 demonstrates the open-loop response of the process for ±20 % step change in the coolant temperature.

To categorize the process, a uniform random signal was generated in MATLAB as an excitation signal. The switching time between different levels were selected for 16 samples. This signal was applied as the input signal to the process. Input and output data are gathered with sampling time of 0.06 min and with 2000 samples for identification purpose. Figure 4 shows the input and output which were collected for the identification of the process.
Subsequent to identifying the process, initial parameters for Laguerre functional model and parameters for Laguerre-wiener model were obtained. These parameters are shown in Table 2:

Table1: Nominal CSTR operating condition and parameters

$q = 100\,\text{min}^{-1}$	$\rho, \rho_c = 1000\text{gl}^{-1}$
$c_{Af} = 1\,\text{moll}^{-1}$	$c_p, c_{pc} = 1\,\text{cal g}^{-1}\text{K}^{-1}$
$T_f = 350\,\text{K}$	$q_c = 103.41\,\text{l min}^{-1}$
$T_{cf} = 350\,\text{K}$	$T = 440.2\text{K}$
$V = 100\text{l}$	$c_A = 8.16 \times 10^{-2}\,\text{moll}^{-1}$
$hA = 7 \times 10^5\,\text{cal min}^{-1}\,\text{K}^{-1}$	$E/R = 9.95 \times 10^3\,\text{K}$
$k_o = 7.2 \times 10^{10}\,\text{min}^{-1}$	$-\Delta H = 2 \times 10^5\text{calmol}^{-1}$

Table2: Model's parameter

Model	Parameter
Laguerre Functional Model	$c_1 = 1.0444, c_2 = -0.1125, c_3 = -0.2575,$ $c_4 = -0.3192$ $N = 4, p = 1.9608, \tau_1 = 0.889,$ $\tau_2 = -0.0532, \tau_3 = 0.0599, \tau_4 = 0.1121$
Laguerre-Weiner Model	$c_1 = 1.0444, c_2 = -0.1125, c_3 = -0.2575,$ $c_4 = -0.3192$ $N = 4, p = 1.9608, \tau_1 = 0.889, \tau_2 = -0.0532,$ $\tau_3 = 0.0599 \tau_4 = 0.1121, \gamma_2 = -0.4819,$ $\gamma_1 = 1.469, \gamma_0 = -0.06401$

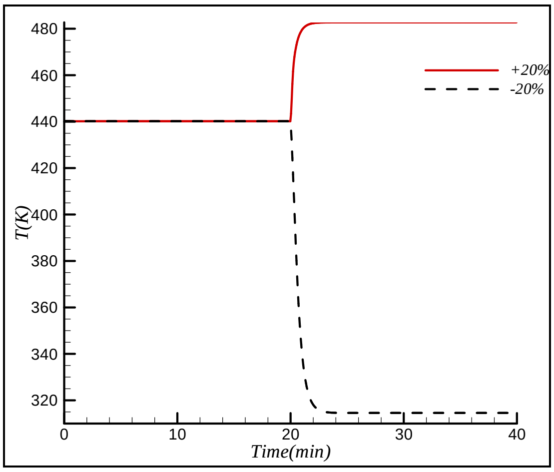
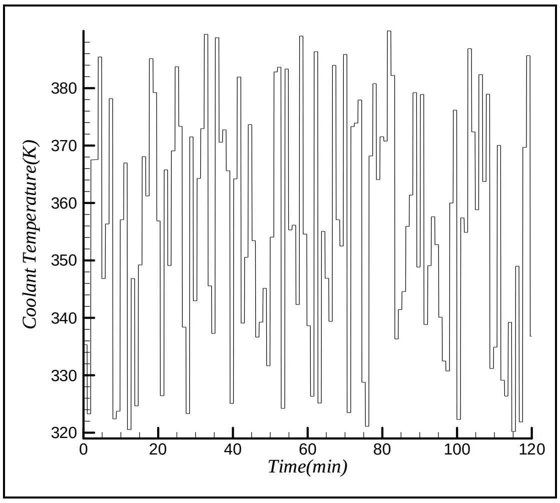
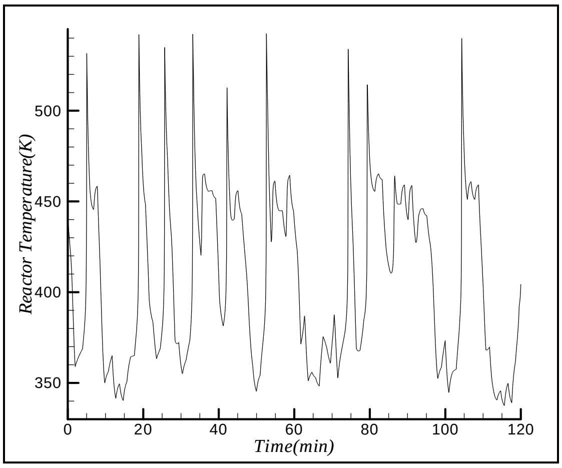


Figure3: Open-loop step-response of the CSTR reactor for change in the coolant temperature



(a)



(b)

Figure4: Signals for identification of CSTR reactor a) input and b)output

By applying the Equation 20, the forgetting factor which has been utilized in RLS algorithm was updated.

$$\lambda = 1 - \frac{e^2(k)}{1+e^2(k)} \tag{26}$$

where e is error between process output and model output.

Figures 5 and 6 illustrate the performance of temperature tracking for the proposed controllers as well as the control action. The transient response of the system for load rejection was also studied in this work. The temperature transient response for the controllers and their corresponding control actions are shown in Figure 7 and 8. The performance of the proposed controller and control action in presence of model mismatch are shown in Figures 9 and 10, respectively. The control horizon and prediction horizons were tuned by trial and error 5 and 10, respectively. The weighting matrices were selected as Q = 100 I and R = 0.3 I. To imposing saturation constraints in manipulated variable, a lower limit of 297 K and an upper limit of 372K were chosen. These figures demonstrate that AMPC based on Laguerre function has a good performance in set-point tracking and load rejection. In this work, the robustness of AMPC model mismatch was also examined and the deviation in the heat of reaction was considered as the model uncertainty.

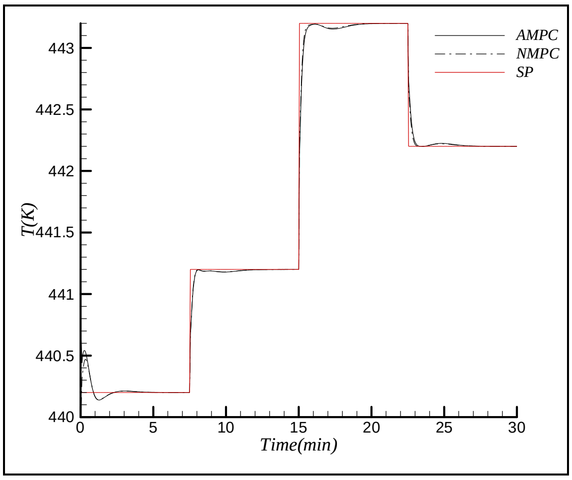


Figure 5: Set-point tracking

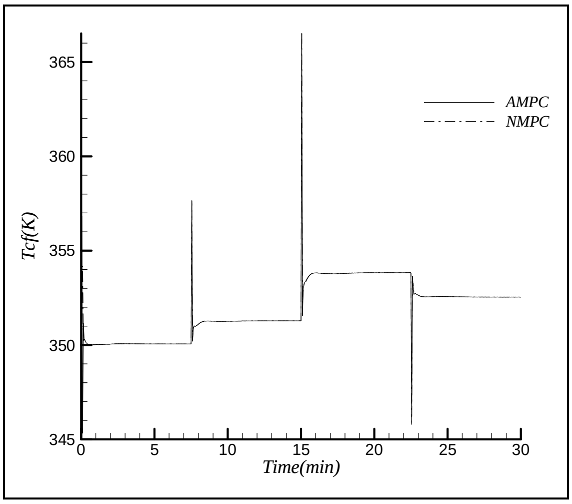


Figure 6: control action for set-point tracking

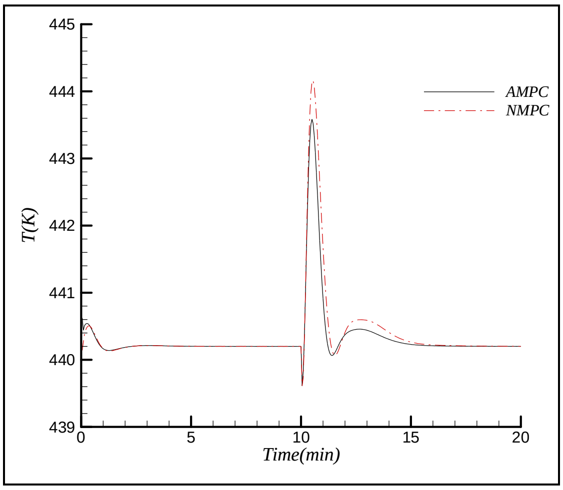


Figure7:Load rejection

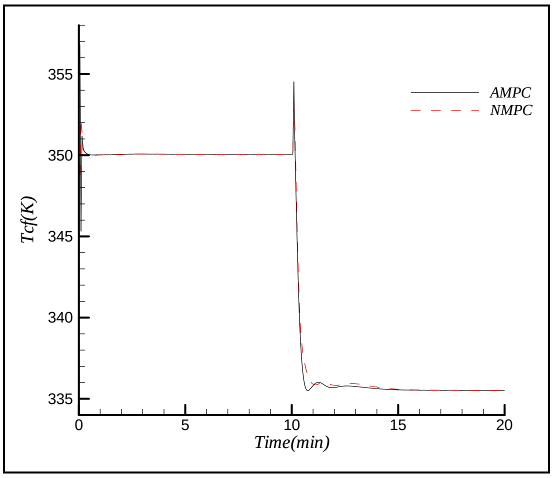


Figure 8: Control action for load rejection

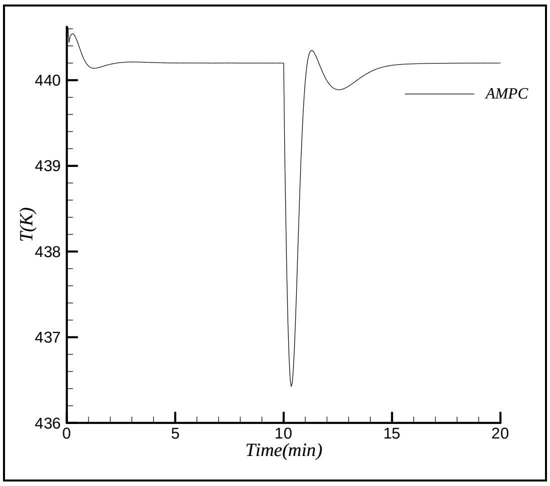


Figure 9: Performance of AMPC for deviation in the heat of reaction

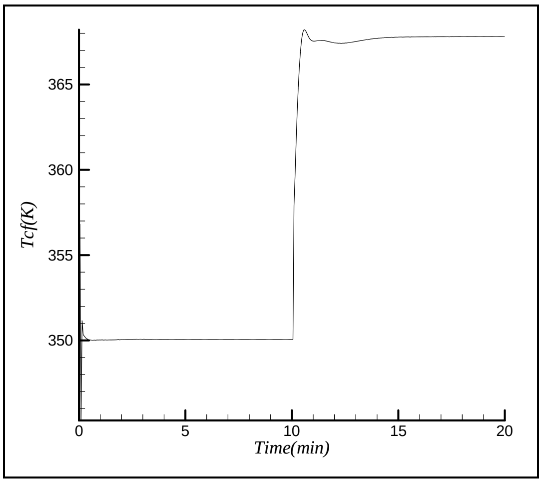


Figure10: Control action in presence of model mismatch

Conclusion

There are many situations in which it is necessary to approximate the transfer function model of a physical system from input/output data. This problem arises in adaptive control, deconvolution, fault detection and many other areas. In this paper, an adaptive model predictive control using Laguerre functional model was presented. This controller was applied and simulated for the control of CSTR reactor process. The simulation result reveals that AMPC has a good performance in set-point tracking and load rejection. For comparison purposes, a nonlinear model predictive control based on Laguerre- wiener model was also applied to the process. Simulation results demonstrate that two purposed controller almost have the same performance.

Notation

c_p Heat capacity of the reactor content, cal kg^{-1} K^{-1}

c_{pc} Heat capacity of cooling water, cal kg^{-1} K^{-1}

C_A Concentration of component A, mol/l^3

C_{Af} Inlet concentration of the reactant, mol/l^3

E Activation energy of reaction, cal l^{-1}

q Inlet volumetric flow rate, l^3/min

$-\Delta H$	Heat of reaction, cal/mol
k_0	Reaction rate constant
R	Universal gas constant, cal/mol K
T	Reactor temperature, K
T_{cf}	Inlet coolant temperature, K
T_f	Reactor inlet temperature, K
h	Overall heart transfer coefficient, cal/min l^2 K
A	Area of heat transfer, l^2
V	Reactor volume, l^3

Greek letter

ρ	Density of the reactor content, g/l^3
ρ_{cp}	Density of jacket fluid, g/l^3

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