Accounting for Secondary Variable for the Classification of Mineral Resources using Cokriging Technique; a Case Study of Sarcheshmeh Porphyry Copper Deposit

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Abstract
Due to substantial effect of classification of resource models on future mine planning, one should come with an accurate method of estimation to guarantee that the minimum error is acquired in the estimation process. The known world class Cu-Mo deposit, Sarcheshmeh Porphyry deposit (central Iran) selected as the study area. The Hypogene zone of the deposit was chosen as the space in which estimation processes should be done. The mean value of Molybdenum and Copper extracted from the top part of this zone, where sampling operations have been done on a dense grid. The correlation coefficient of 0.45 allowed going through the process of interpolation. It was shown that taking account Cu as an auxiliary variable the interpolation process, the estimation had been improved. Simple Cokriging interpolation technique is applied and it was proved that using Cu, with mean value of 0.61 percent, as secondary variable will decrease the estimation variance of Mo interpolation which has the mean value of 0.022 percent. The chief influence of this reduction appeared when the resource should be classified. Only 1% decrease was obtained when Cu used as secondary variable, but in an industrial aspect it can be of great importance as a high number of voxels in “Indicated” class changed into “Measured” one. This led to 133 Mt more Mo-ore that were added to the previous “Measured” class blocks. Also, the transition zones where the changes in class of cells have occurred are identified; these zones are mainly the places where Mo has fewer samples than Cu.

Keywords: Simple Co-Kriging, Secondary Variable, Estimation Variance, Porphyry Copper Deposit, Resource Classification

Introduction
Classifying the resources based on varying confidence categories is important for mine planning. In this way miners and experts have proposed several means by which one can do so, because they have found out the importance as Emery et al. [1] refers to the importance of such procedure as a reliable information is required by financial institutions, investors, and authorities for fixing royalties and taxations, and for strategic decisions and investment planning for mine extraction in the upcomming years.

Different classification methods are proposed, e.g., American USGS Circular 831 and SME Guide, the South-African SAMREC Code, the Canadian CIM Guidelines and National Instrument 43-101, the European Code, and the Australasian JORC Code [2-7]. Among these approaches, the JORC code stands out as it has been used and applied more than other code proposed earlier. Based on the JORC Code mineral deposits can be classified into two main groups of:

a. Mineral resources that bear the potential of being valuable in future when the knowledge or other factors make it reasonable for eventual economic extraction.

b. Mineral reserves or Ore reserves; that are economically valuable and legally and technically feasible to extract.

Sinclair and Balckwell [8] refer to different methods that have been proposed to classify the resources such as geologic continuity, distance from a sample site,
sample density in the vicinity of each voxel, the geometric array of data relative to each voxel, and finally refers to the contributions to classification from Geostatistics. In geostatistical methods several criteria can be used to classify resources such as neighborhood restrictions, kriging variance, conditional variance, relative kriging variance, relative conditional variance [1].

In this study, a geostatistical approach is followed that uses the kriging error [9] and is applied on Sarcheshmeh porphyry Cu-Mo Deposit, SE Iran. In proposed method, based on the kriging variance two important factors are taken into account which are firstly the quantity and configuration of the neighboring data and also the spatial continuity of the grades measured by their variogram. The resources have been classified based on three methods that were recalled by Mwasinga [10]. The first is based on the voxel estimation error at the 90% confidence limits as defined as follows:

\[ \text{Est Error} = \frac{(1.645 \times \text{Krig Std Dev})}{Z_{\text{Est}}} \]  \hspace{1cm} (1)

Clearly, \( Z_{\text{est}} \) is the estimated value and \( t \) is the score of \( t \) distribution for a given confidence level (CL), and degrees of freedom \( (n - 1) \). When estimation errors are less than 10% the voxel is classified as “Measured”, between 10% and 25% the voxel is classified as “Indicated” and when higher than 25% the voxel is classified as “Inferred”. Dividing the kriging variance by variance of each voxel, one can pass the second way of classification. In this procedure, the threshold (of estimation error) of classification is based on the kriging variance defined as follows:

\[ \frac{\delta^*_{\text{CL},n}(x_o)}{Z_{\text{OK}}(x_o)^*n_n} \times 100 \% \]  \hspace{1cm} (3)

where, \( S_o \) is the ordinary kriging standard deviation and \( n \) is the number of neighbor samples used to the estimate \( Z_{\text{OK}}(x_o) \) and \( t \) is the score of \( Z_{\text{OK}}(x_o) \) distribution for a given confidence level (CL), and degrees of freedom \( (n - 1) \).

**Methodology**

On the next the theory of Simple Kriging (SK) and Simple Cokriging (CSK) are described shortly:

**Simple Kriging**

Between ordinary and simple kriging estimation methods, the former one has been a more common estimation practice that may be due to the fact that there is no need to know the mean value of the variable under study [8]. However, there are situations where the mean value (of the field of data in which estimation is undertaken) is well known. For example, the deposits which have been under mining operation for several years in which the mean values of variable of interest can be defined with a convenient certainty. In such cases, the kriging equations reduce to the situation of an unconstrained set of equations (i.e., the weights are not constrained to sum to 1). In this mode of kriging we should use the information that have been acquired by huge amount of

In this case, the thresholds, on which the classification should be based are 0.5 and 0.3 as for values of Ke higher than 0.5 the block is classified as “Measured”; between 0.3 and 0.5 is “Indicated”, and “Inferred” when it is lower than 0.3. Geostatistical classification of a mineral resource is often based on the various ‘errors’ associated with the kriging results. The geostatistical approach for resource classification is well proposed and applied on PGE deposits in Merensky and UG2 Reefs orebodies, South Africa, by Young [11]. Yamamoto computed the estimated relative error with follow from:

\[ \delta^*_{\text{CL},n}(x_o) = \frac{S_o \times \sqrt{t_{0.05,n_n-1}}}{Z_{\text{OK}}(x_o)^*n_n} \times 100 \% \]  \hspace{1cm} (3)
costs and time to improve our estimates which can be done using simple kriging. Our kriged estimate is still a linear sum, but now incorporating the mean, of the process, which must be second-order stationary. Prediction by simple kriging is not an option for processes that are intrinsic only, a variogram with an upper bound is needed [12]. Equation 4 can be used for punctual kriging:

\[
Z_{\text{sk}}^*(X_0) = \sum_{i=1}^{N} \lambda_i Z(X_i) + \left(1 - \sum_{i=1}^{N} \lambda_i\right) \mu
\]  

(4)

where the mean, \( \mu \), of the process is incorporated; \( Z(X_i) \) are the sample values, and the \( \lambda_i \) are the weights, as before, but they are no longer constrained to sum to 1. The unbiasedness is assured by inclusion of the second term on the right-hand side of Equation 4. Also, because the weights no longer sum to 1 we have to work with the covariances, \( C \), instead of the semivariances. Equation 5 shows the simple kriging system:

\[
\sum_{N=1}^{N} \lambda_i C(X_i,X_j) = C(X_0,X_j) \text{ for } j = 1, 2, ..., N.
\]

(5)

where the semivariance is replaced with \( C \) which is the covariances. There is no Lagrange multiplier: there are only \( N \) equations in \( N \) unknowns. The kriging variance is given by Equation 6:

\[
\sigma_{sk}^2(X_0) = C(0) - \sum_{i=1}^{N} \lambda_i C(X_i,X_0).
\]

(6)

where \( C(0) \) is the variance of the process.

**Cokriging**

The possibility of considering secondary variable in the kriging equation has led to the emergence of group of cokriging methods. Suppose there the primary variable of \( Z_1(u) \) that is going to be estimated having the related variables of \( Z_2, \ldots, Z_{j+1} \) accounted as the auxiliary variables. The cokriging estimator is given by Equation 7:

\[
Z_1^*(u) - m_1(u) = \sum_{i=1}^{j+1} \lambda_{\alpha} Z_1(u_{\alpha}) - \sum_{i=1}^{j+1} \lambda_{\beta} Z_i(u_{\beta}) - m_j(u_{\beta})
\]

where \( \lambda_{\alpha} \) and \( \lambda_{\beta} \) are weights used for kriging equations for first and secondly variables, respectively; \( Z_1 \) and \( Z_j \) are primary and secondary variables. If the means \( m_1(u), \ldots, m_{j+1}(u) \) are known and constant that would be the simple cokriging (SCK), the algorithm that is used in this study. It should be noted that the means are locally constant but unknown. The weights \( \lambda_{\alpha} \) and \( \lambda_{\beta} \) must then satisfy:

\[
\sum_{\alpha} \lambda_{\alpha} = 1
\]

\[
\sum_{\beta} \lambda_{\beta} = 0 \forall j \in \{1, \ldots, J\}
\]

The variogram of each variable and all cross-variograms between any two variables for the inference and modeling of multiple variograms are needed for cokriging system. The estimation variance can be calculated using Equation 8:

\[
\sigma_{uk}^2(B) = \sum_{i=1}^{N} \sum_{j=1}^{n} \lambda_{i} \tilde{\gamma}_{ul}(X_j,B) + \psi_u
\]

(8)

where, \( \tilde{\gamma}_{ul}(B) \) is the integral of \( \gamma_{ul}(h) \) over \( B \), i.e. the within-block variance of \( u \).

**Ore-Resource Classification**

In this paper we will adopt the ore-resource classification as proposed by Yamamoto as given in Table 1.

<table>
<thead>
<tr>
<th>Measured</th>
<th>Indicated</th>
<th>Inferred</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error: 0-20%</td>
<td>Error: 20-50%</td>
<td>Error: &gt; 50%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The error in this table is the estimated relative error and is computed as Equation 9 [9]:

\[
\delta_{ct}^*(x_o) = \frac{S_ch(0.05,n_v)}{Z_{ct}(x_o)} \times 100 \% 
\]

(9)

where, \( S_ch(0.05,n_v) \) is the width of 90% confidence interval around \( Z_{ct}(x_o) \).
$t_{(-2.58, -1)}$ is the $t$ value for 90% two-sided critical region and $n_v^{-1}$ degrees of freedom; $n_v$ is the number of sub-blocks used to discretize a block. The estimated relative error after equation 8 is inversely proportional to the square root of the number of sub-blocks. Then, the larger the number of sub-blocks the smaller is the estimated relative error. Constants also decrease as the number of neighbor data increases and consequently estimated relative errors (Equation 9) go down. Estimated relative errors (Equation 9) decreases as number of sub-blocks or number of neighbor data increase and or combination of both numbers go up.

This method is used to classify Sarcheshmeh deposit where the borders between classes with different certainty levels, if drawn, can be of paramount importance and applicability. At first a summary of study area is presented:

**Study Area**

Sarcheshmeh Cu-Moporphyry deposit, which is largest copper mine in Iran, is located 160 km SW of Kerman, SE Iran. In addition to the Sarcheshmeh ore body, there are some other porphyry Cu deposits that occur in the so called Urumia-Dokhtar magmatic belt, as shown in Figure 1 [13,14]. The main ore body contains 1,200 million tons of ore with an average grade of 0.69% for Cu and approximately 0.03% Mo [15].

**Geological Setting**

The geology of the deposit is dominated by Eocene basic to intermediate volcanic rocks including trachybasalt, trachyandesite, and/or andesite [16]. Mineralization at Sarcheshmeh deposit mainly forms stockworks and veins that are equally distributed between Eocene volcanic and Oligo-Miocene quartz diorite, quartz monzonite, and granodiorite units [15,17]. Waterman and Hamilton [13], Shahabpour [17] and Hezarkhani [14] did a comprehensive study on Sarcheshmeh. In terms of the geology of the deposit, they concluded that the orebody is oval shaped with a long dimension of about 2,000 meters, a width of about 900 meters that the center is located on the late Tertiary Sarcheshmeh granodiorite porphyry stock. There is a complex of seris of magmatically related intrusives emplaced in the Tertiary Volcanics a short distance from the edge of an older near-batholith-sized granodiorite mass which Sarcheshmeh porphyry belongs to Waterman and Hamilton [13] and Hezarkhani [14].

**Materials**

The dataset of Cu and Mo values was prepared based on collected samples from drilled boreholes in hypogene zone (Figure 2). Mean values of the elements were selected from the 200 meters upper part of the borehole data in hypogene zone (Figure 2).
Figure 2: Drill hole sample data

Figure 3: Histogram of data used in this study before and after logarithmic transformation

Table 2: Statistical parameters of Cu and Mo

<table>
<thead>
<tr>
<th>Parameter/Statistics</th>
<th>Cu</th>
<th>Mo</th>
<th>Cu-Log</th>
<th>Mo-Log</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>15720</td>
<td>6269</td>
<td>15720</td>
<td>6269</td>
</tr>
<tr>
<td>Mean</td>
<td>0.610</td>
<td>0.022</td>
<td>-0.398</td>
<td>-1.826</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.494</td>
<td>0.024</td>
<td>0.464</td>
<td>0.381</td>
</tr>
<tr>
<td>CV</td>
<td>0.810</td>
<td>1.074</td>
<td>1.166</td>
<td>0.209</td>
</tr>
<tr>
<td>Maximum</td>
<td>9.860</td>
<td>0.326</td>
<td>0.994</td>
<td>-0.487</td>
</tr>
<tr>
<td>Upper Q.</td>
<td>0.898</td>
<td>0.028</td>
<td>-0.056</td>
<td>-1.553</td>
</tr>
<tr>
<td>Median</td>
<td>0.510</td>
<td>0.015</td>
<td>-0.292</td>
<td>-1.824</td>
</tr>
<tr>
<td>Lower Q.</td>
<td>0.220</td>
<td>0.008</td>
<td>-0.658</td>
<td>-2.097</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.020</td>
<td>0.002</td>
<td>-1.699</td>
<td>-2.699</td>
</tr>
</tbody>
</table>
In this part, the sample per volume number, 8.06, is much more than the same parameter for deeper parts, 0.31; hence, the mean value is supposed to represent the overall deposit means. The distribution of both Mo and Cu are lognormal (positively skewed), the datasets are transformed into normal distribution using Logarithmic (Log) function (Figure 3). The statistical parameters are provided in Table 2. When the mean value of variables under study are known, among the ordinary and simple modes of kriging, the second one is preferred due to the lower amount of estimation variance for each point. Therefore, in this study simple kriging is used while Cu is the secondary variable.

**Results and Discussion**

At first, the block model on which the estimation process is going to be performed, should be defined. One may think of topography and hypogene wireframe to generate the domain in which estimation can be done, but as the sole of goal of this study is to compare the results when secondary variable is considered, and as long as the conditions of using these methods are equal, no problem may rise, and a fair comparison can be done. The bench height is one of the most important factors in the generation of block model. In Sarcheshmeh deposit this factor’s value is 12.5 m that means the estimation voxel size is better to be the same. In this way, the results of estimation is of great importance in an industrial point of view. The dimensions of estimation grid should be in the domain that we avoid the extrapolation as much as possible which depends on the density of drill hole data. Finally, the estimated block model was defined as 138, 72 and 40 voxels along each of the X, Y and Z axes with a 12.5 meter voxel size (Figure 5a).

![Variogram features used in this study.](image)

(a) Cu variogram: \( \gamma = 0.1 + 0.13 \times \text{Sph}(190) \),
(b) Mo variogram: \( \gamma = 0.08 + 0.067 \times \text{Sph}(360) \),
(c) Cu-Mo cross variogram: \( \gamma = 0.02 + 0.028 \times \text{Sph}(236) \)
Based on elemental mean values and the search parameters from the variography, the SK estimates and corresponding estimation variances were calculated for each of the voxels in the block model. The variography analysis was carried out on the available data set to get the relevant variogram for estimation steps. Based on the aim of this study which is just to compare the classified resources when a secondary variable is taken into account, just the omnidirectional variograms were drawn and inputted to estimation process. In this way, the anisotropy features of data are not assessed. Variogram models for Cu, Mo and the cross variogram between them are provided in the form of the fitted theoretical variogram. All of the empirical and theoretical fitted variograms (using spherical models) are shown in Figure 4.

The strong spatial cross-structure between Cu and Mo permits going through the study and see how considering Cu to estimate Mo may influence the results. It should be noted that all the estimates are back-transformed into the original scale (of sample data) and then presented in figures. The estimation variance is the key factor on which the comparisons can be based when a secondary variable is considered in kriging equations. The lower values are calculated for the points that are located near the borehole data points. On the contrary, on the corners where the density of sample data points is lower than the other areas, the estimation variance has surged; in other words, the node is estimated with more uncertainties. In the same way, simple cokriging estimation method was applied on the given Mo data, but we used its paragenesis element, Cu, as an auxiliary variable too. Based on simple kriging and simple co-kriging, it is expected to reduce the estimation variance at each grid point.

Figure 5. a- SK estimates, b- SK estimation variance
The vast number of Cu sample data points and their even distribution make it an ideal case to apply the simple co-kriging equations on both the given datasets.

Figure 6a shows the estimation grid on which the value of Mo estimated grade are illustrated. The estimated grid and estimation variance is shown in Figure 6. The estimation variance changes by considering Cu as secondary variable. The estimation variances before and after considering secondary variable are plotted for each grid point (Figure 7). The mean of estimation variance has met a fall in value, from 0.1244 to 0.1233, which is about 1%. This low value of reduction is directly dependent on the correlation coefficient between Cu and Mo. Since 0.45 value of correlation coefficient is very low, 1% of reduction can be justifiable. For each voxel of estimated block model, the estimation variance is decreased that can be seen in Figure 7. The red oval on Figure 7 refers to the area that shows how all the estimation variances are fallen. For higher grades the amount of reduction is surged. In an industrial point of view, it can be of great importance.

Resource Classification at Sarcheshmeh

The resource classification procedure was applied on the estimated values. To pass this step, the inputs include estimated value and estimation variance to be considered for the previously mentioned formula of Yamamoto [9] (Equation 8).

<table>
<thead>
<tr>
<th>Class</th>
<th>SK</th>
<th>CSK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured</td>
<td>300634</td>
<td>326322</td>
</tr>
<tr>
<td>Indicated</td>
<td>72682</td>
<td>53093</td>
</tr>
<tr>
<td>Inferred</td>
<td>24124</td>
<td>18025</td>
</tr>
</tbody>
</table>

Table 3: Resource classification comparison between SK and CSK results
In this study, 90% level of confidence was considered and a constant n = 20, was considered to calculate the error of estimation. For 90% of CL the value equals with 1.645. Using this formula and the boundaries of error in Table 1, each of the estimated cells was labeled with one of the three classes of resource. At first, this method was applied on SK estimates which its results are shown in Figure 8a where the three colors indicate each class introduced earlier. Secondly, the SCK estimates were considered as an input for resource classification procedure. As can be seen in Figure 8b, the classes are illustrated in the same colors. It should be considered that by considering Cu as an auxiliary variable higher number of voxels are labeled with Measured class, and this is due to the lower amount of estimation variance, clearly. Since the amount of error has a vice versa relationship with estimation variance, the higher number of nodes in Measured is justifiable.

Figure 7: Scatter plot of CSK estimation variance vs. SK estimation variance

Figure 8: Resource classification on a- SK and b- CSK (Nodes colored in red: Measured, Green: Indicated and Blue: Inferred)
The mean value for SK and SCK error are 67.9, 40.8, respectively. The summary of classification is provided in Table 3. 25688 cells, which accounts for 9.5% of all voxels, are changed from Indicated and Inferred into Measured class. This amount of reduction can be of many importance at an industrial scale. The tonnage of ore that can be changed into Measured class is 133 Mt. Assuming that all of the changed blocks are within the economic boundaries of grade, if they can be labeled as Proved class, by considering the average grade of Mo in this deposit the mass of Mo would be 0.4 Mt.

Figure 9: Transition between Inferred and Indicated from different perspectives (a & b) and some sections and plans (c) (The drill hole data are labeled for Mo and the gray colored pixels refer to the non-sampled points)
Figure 10: Transition between Indicated and Measured from different perspectives (a & b) and some sections and plans (c) (The drill hole data are labeled for Mo and the gray colored pixels refer to the non-sampled points)
Accounting for such secondary variable bears the potential to change the resource classification results; also, how the blocks which are changed in class may locate in spatial domain are examined, and an evaluation is done on transition zones. In this way, the blocks with changes in class may pass each of the following transitions: (I) Inferred → Indicated (II) Indicated → Measured

These two states are followed, and the relevant figures are illustrated too. Primarily, the first, I, transition is identified and shown in Figure 9 on which the hard conditioning data are overlaid. It can be seen that this transition is occurred where the density of primary variable (Mo) is too low; moreover, higher concentrations of Cu can be found in the same location. The same procedure was followed for transition type II, and the results are shown in Figure 10 in which two perspectives and some plans and sections are provided, also. As can be seen, higher number of blocks which are changed in class are in this type. Clearly, the places in which lower (comparing with Cu samples) number of Mo samples exist, more reduction is occurred, and accordingly the transition has happened.

Conclusions

A secondary variable was considered when Mo grade was to be estimated by simple co-kriging technique. The low correlation coefficient between Mo and Cu had made it worthless to apply the cokriging estimation method on given data, but at an industrial scale, it has had its big effect on the resource classification results. Just 1% reduction in estimation variance had major effects on the resource classification results because 8.5% of previously non-Measured classes changed into Measured using Cu as secondary variable. Extracting the mean value from the upper parts of the area and its further application in the simple kriging was tried too, but it should be noted that this can be done just when ones are sure about the stationarity conditions of the environment in which the estimations are undertaken. The transition zone were identified which are located in the areas in which the difference between the number of primary and secondary variable is significant; such as, at the corner of the estimation grid where Mo is not sampled enough, but on the other points the role of Cu was decreased. One might use some validation data such as blasthole data to test the results, but as far as this study theoretically dealt with classification based cokriging, the focus is just how a given previously-classified cell, using only the primary variable, may change in resource class by considering auxiliary variable and the amount of reduction in kriging error.

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