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Contra-?ł-Continuous Functions between Topological Spaces

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Abstract

In this paper, we apply the notion of α-open sets in topological spaces to present and study contra-α-continuity as a new generalization of contra-continuity (Dontchev, 1996).

Key words*: ?Х-open, ?Х-closed, contra-?Х-closed, ?Х-compact, strongly S-closed, contra-?Â-continuity.*

1. Introduction

In 1996, Dontchev (Dontchev, 1996) introduced a new class of functions called contra-continuous functions. Recently, Dontchev and Noiri (Dontchev and Noiri, 1999) introduced and studied, among others, a new weaker form of this class of functions called contra-semicontinuous functions. They also introduced the notion of RC-continuity (Dontchev and Noiri, 1999) which is weaker than contra-continuity and stronger than ?Ý-continuity (Tong, 1998). The present authors (Jafari and Noiri, 1999) introduced and studied a new class of functions called contra-super-continuous functions which lies between classes of RC-continuous functions and contra-continuous functions.

This paper is devoted to introduce and investigate a new class of functions called contra-?e-continuous functions which is weaker than contra-continuous functions and stronger than both contra-semicontinuous functions and contra-precontinuous functions (Jafari and Noiri, 2001).

2. Preliminaries

Throughout this paper, all spaces X and Y (or (X,ι) and (Y, σ)) are topological spaces. A subset A is said to be *regular open* (resp. *regular closed)* if A = Int(CI(A)) (resp. A=CI(Int(A))) where CI(A) and Int(A) denote the closure and interior of A.

Definition 2.1. A subset A of a space is called:

(1) ?¦*open* (Abd El-Monsef et al., 1983) if A⊂ CI(Int(CI(A))),

(2) *preopen* (Mashhour et al., 1982) if A⊂ Int(CI(A)),

(3) *semi-open* (Levine, 1963) if A⊂ CI(Int(A)),

(4) ?*-open* (Njåstad, 1965) if A⊂ Int(CI(Int(A))),

The complement of a preopen (resp. semi-open, ?-open, ?-open) set is said to be *preclosed* (resp. *semi-closed,* ?(*closed,* ?(*closed*) The collection of all closed (resp. preopen, semi-open, ?±open and ?±open) subsets of X will be denoted by C(X) (resp. PO(X), SO(X), ?(X), ?Ã(X)). It is shown in (Njåstad, 1965) that ?Ã(X) (or ι^{α}) is a topology for X and it is stronger than the given topology on X. By αCI(A), we denote the closure of a subset A with respect to ?(X). We set C(X, x) ={ V ∈ C(X) | x ∈V }for x ∈ X. We define similarly PO(X, x) SO(X, x), α(X, x) and ?Ö(X, x). Recall that a subset A of X is said to be *generalized closed* (briefily *g-closed* (Levine, 1970)) (resp. ?á *generalized closed* (briefly *ag-closed)* (Maki et al., 1994) if CI(A) ⊆ U (resp. αCI(A) ⊆ U) whenever A ⊆ U and U is open. Recall that a subset A of X is called *NDB-set* (Dontchev, preprint), if it has nowhere dense boundary. A subset A of X is called ?4*open* if it is the union of regular open sets. The complement of a ?Fopen set is called ?F*closed*. Equivalently, A ⊂ X is called ?Xclosed (Velicko, 1968) if A = *Cl*?(A), where *Cl*?(A) = {x ∈ X | Int(CI(U)) ∩ A ≠ ∅, U is an open set and x ∈ U}. A subset A of X is called ?¦*generalized closed* (Dontchev and Ganster, 1996) if *Cl*?(A) ⊆ U, whenever A ⊆ U and U is open in X.

Definition 2.2. A function f : X → Y is called *perfectly continuous* (Noiri, 1984) (resp. RC-*continuous* (Dontchev & Noiri, 1999) if for each open set V of Y, $f^{-1}(V)$ is clopen (resp. regular closed) in X.

Definition 2.3. A function f: X → Y is called *precontinuous* (Mashhour *et al.*, 1982) (resp. *semi-continuous* (Levine, 1963), ?ý *continuous* (Abd El-Monsef *et al.*, 1983) if for each x ∈ X and each open set V of Y containing f(x), there exists U∈ PO(X, x) (resp. U ∈ SO(X, x) , U∈ ?O(X, x)) such that f(U) ⊂ V.

Definition 2.4. A function f : X → Y is called *contra-super-continuous* (Jafari & Noiri, 1999) if for each x ∈ X and each closed set V of Y containing f(x), there exists a regular open set U in X containing x such that f(U) ⊂ V.

Definition 2.5. A function f : X → Y is called *contra-?-continuous* (resp. *contra-continuous* (Dontchev, 1996), *contra-semicontinuous* (Dontchev & Noiri, 1999), *contra-precontinuous* (Jafari & Noiri, 2001) if $f^{-1}(V)$ is ?-closed (resp. closed, semi-closed, preclosed) in X for each open set V of Y.

Remark 2.1. Every contra-continuous function is contra-α-continuous but not conversely as the following example shows.

Example 2.1. Let X = {a, b, c}, = {X, ∅, {a}} and σ = {X, ∅, {b}, {c}, {b, c}}. Then the identity function f : (X,) → (X,σ) is contra-?-continuous but not contra-continuous.

3. Some properties

Definition 3.1. Let A be a subset of a space (X,). The set ∩ {U ∈ | A ⊂ U } is called the *kernel* of A (Mrsevic, 1986) and is denoted by Ker (A).

Lemma 3.1. The following properties hold for subsets A, B of a space X:

(1) x ∈ Ker(A) if and only if A ∩ F ≠ ∅ for any F ∈ C(X, x).

(2) A ⊂ Ker(A) and A = Ker(A) if A is open in X.

(3) A ⊂ B, then Ker(A) ⊂ Ker(B).

Theorem 3.1. The following are equivalent for function f : X → Y :

(1) f is contra-?-continuous;

(2) for every closed subset F of Y, $f^{-1}(F)$?t?(X);
(3) for each x ∈ X and each F ∈ C(Y, f(X), there exists U ∈ ?ý(X, x) such that f(U) ⊂ F;
(4) *f(?Cl(A))* ⊂ Ker *(f(A)) for* every subset A of X;
(5) ?˜Cl($f^{-1}(B)$) ⊂ f^{-1} (Ker(B)) for every subset B of Y.

Proof. The implications (1) ⇔ (2) and (2) ⇒ (3) are obvious.
(3) ⇒ (2): Let F be any closed set of Y and x ∈ $f^{-1}(F)$. Then f(x) ∈ F and there exists U_x ∈ ?ß(X, x) such that f(U_x) ⊂ F. Therefore, we obtain $f^{-1}(F)$ = ?¼{U_x | x ∈ $f^{-1}(F)$} ∈ ?½(X).
(2) ⇒ (4): Let A be any subset of X. Suppose that y ∉ Ker(f(A)). Then by Lemma 3.1 there exists F ∈ C(X, y) such that f(A) ∩ F = ∅. Thus, we have A ∩ $f^{-1}(F)$ = ∅ and ?ŴCl(A) ∩ $f^{-1}(F)$ = ∅. Therefore, we obtain *f(?àCl(A))* ?à *F* = ∅ and y ∉ *f(?àCl(A))*. This implies that *f(?kCl(A))* ⊂ Ker *(f(A))*.
(4) ⇒ (5): Let B be any subset of Y. By (4) and Lemma 3.1 we have *f(?£Cl* ($f^{-1}(B)$)) ⊂ Ker (B) and ?£Cl(f^{-1}(B)) ⊂ f^{-1}(Ker(*B*)).
(5) ⇒ (1): Let V be any open set of Y. Then, by Lemma 3.1 we have ?Cl ($f^{-1}(V)$ ⊂ $f^{-1}(Ker(V))$ = ($f^{-1}(V)$ and ?Cl(($f^{-1}(V)$) = $f^{-1}(V)$. This shows that $f^{-1}(V)$ is ?+closed in X.

Theorem 3.2. A function f : (X,) → (X, σ) is contra-?«continuous if and only if f : (X, $^{\alpha}$) → (X, σ) is contra-continuous.

Recall that a subset of a topological space (X,) is called a ?Ï-*set* if it is the intersection of open sets.

Theorem 3.3. A function f : (X,) → (X, σ) is contra-?Ï-continuous if and only if inverse images of Λ-sets are closed.

Lemma 3.2. (Mashhour et al., 1983). Let A ∈ PO(X) and B ∈ ?(X),The A ∩ B ∈ ?(A).

Theorem 3.4. If f: X→ Y is contra-?•continuous and U ∈ PO(X), then f│U : U → Y is contra-?¢continuous.

Lemma 3.3. (Mashhour et al., 1983). If A ∈ ?'(Y), and Y ∈ ?4(X),Then A ∈ ?4(X).

Theorem 3.5. Let f: X→ Y be a function and {U_i | i ∈ I }be a cover of X such that U_i ∈ ?ÿ(X) for each i ∈ I. If f | U_i : U_i → Y is contra-?ý continuous for each i ∈ I, than f is contra-?‡continuous.

Proof. Suppose that F is any closed set of Y. We have

$$f^{-1}(F) = \bigcup_{i \in I} f^{-1}(F) \cap U_i = \bigcup_{i \in I} (f|U_i)^{-1}(F)$$

Since $f|U_i$ is contra-?žcontinuous for each i∈I, it follows that $f|(U_i)^{-1}(F)$ ∈ ?(U_i). Then, as a direct consequence of Lemma 3.3 we have $f^{-1}(F)$ ∈ ?(X) which means that f is contra-?Œontinuous.

Now we mention the following well-known result:

Lemma 3.4. The following properties are equivalent for a subset A of a space X:

(1) A is clopen;

(2) A is ?-closed and ?-open;

(3) A is ?Žclosed and preopen.

Theorem 3.6. For a function f: X→Y the following continuous are equivalent:

(1) f is perfectly continuous;

(2) f is contra-?ècontinuous and ?ècontinuous;

(3) f is contra-?úcontinuous and precontinuous.

Proof. The proof follows immediately from Lemma 3.4.

Remark 3.1. In Theorem 3.6, (2) and (3) are decompositions of perfect continuity. The following example shows that contra-?-continuity and precontinuity (or ?-continuity) are independent of each other.

Example 3.1. The identity function on the real line with the usual topology is continuous and hence ?ácontinuous and precontinuous. The inverse image of (0, 1) is not ?éclosed and the function is not contra-?-continuous .

Example 3.2. Let (Z,κ) be the digital line (Khalimsky et al., 1990) and define a function f: (Z,κ) →(Z,κ) by f(n) = n + 1 for each n ∈Z .

Then f is contra-?-continuous. But Int(Cl(f^{-1} ({1})))= ∅ and f^{-1} ({1}) ∉ PO (Z,κ), hence f is neither precontinuous nor ?-continuous .

Theorem 3.7. Let Y be a regular space. For a function f: X→Y, the following properties are equivalent:

(1) f is perfectly continuous;
(2) f is RC-continuous;
(3) f is contra-continuous;
(3) f is contra-?-continuous.

Proof. The following implications are obvious: perfect continuity ⇒ RC-continuity ⇒ contra-continuity ⇒ contra-?-continuity. We show the implication (4) ⇒ (1). Let x be an arbitrary point of X and V an open set of Y containing f(x). Since Y is regular, there exists an open set W in Y containing f(x) such that Cl(W) ⊂ V. Since f is contra-?-continuous, so by Theorem 3.1 there exists U ∈ ?(X, x) such that f(U)⊂ Cl(W). Then f(U) ⊂ Cl(W) ⊂ V. Hence, f is ?-continuous. Since f is contra-?-continuous and ?-continuous, by Theorem 3.6 f is perfectly continuous.

Corollary 3.1. If a function f: X→Y is contra-?-continuous and Y is regular, then f is continuous.

Remark 3.2. The converse of corollary 3.1 is not true. Example 3.1 shows that continuity does not necessarily imply contra-?-continuity even if the range is regular.
Recall that a space X is said to be *rim-compact* if each point of X has a base of neighborhoods with compact frontiers.

Lemma 3.5 (Noiri (1976), Theorem 4]). Every rim-compact Hausdorff space is regular.

Corollary 3.2. If a function f: X→Y is contra-?-continuous and Y is rim-compact Hausdorff, then f is continuous.

Definition 3.2. A function f: X→Y is called *contra-?g-continuous* if the preimage of every open subset of Y is ?g-closed.

Recall that a space X is $T_{½}$ - *space* (Levine, 1961) if every generalized closed set is closed.

Lemma 3.6 (Dontchev, 1997). For a space X the following conditions are equivalent:

(1) X is $T_{½}$- space.

(2) Every ?ǵ-closed subset of X is ?óclosed.

Theorem 3.8. If a function f: X→Y is contra-?ǥ-continuous and X is $T_{½}$ - space, then f is contra-?¼continuous.

Recall that a function f: X→Y is *NDB-continuous* (Dontchev, preprint) if the preimage of every open set is an NDB-set

Lemma 3.7 (Dontchev, preprint) For a subset A of a space X the following conditions are equivalent:

(1) A is ?ƚclosed..

(2) A is a preclosed NDB-set.

Theorem 3.9. For a function f: X→Y, the following conditions are equivalent:

(1) f is contra-?+continuous.

(2) f is contra- precontinuous and NDB-continuous.

Definition 3.3. A function f: X→Y is said to be

(1) *I.c.?Úcontinuous* if for each x∈X and each closed set F of Y containing f(x), there exists an ?ì-open set U in X containing x such that Int[f(U)] ⊂ F.

(2) *(?, s)-open* if f(U) ∈ SO(Y) for every U ∈ ?(X).

Theorem 3.10. If a function f: X→Y is I.c.?•continuous and (?ȿ s)-open, then f is is contra-?¢continuous.

Proof. Let x be an arbitrary point of X and V∈C(Y, f(x)). By hypothesis f is I.c.?4continuous which implies the existence of a set U ∈ ?(X, x) such that int[f(U)] ⊂ V. Since f is (?Ƒs)-open, then f(U) ∈SO(Y). It follows that f(U) ⊂ Cl(Int(f(U))) ⊂ Cl(V) and therefore f is contra-?ɉ-continuous.

Definition 3.4. A filter base Λ is said to be ?-*convergent* (Jafari, 2001) (resp. *c-convergent*) to a point x in X if for any U ∈ ?(X, x) (resp. U ∈ C(X, x)), there exists B ∈ Λ such that B ⊂ U.

Theorem 3.11. A function f: X→Y is contra-?-continuous if and only if for each point x ∈ X and each filter base Λ in X ?-converging to x, the filter base f(Λ) is c-convergent to f(x).

Proof. *Necessity*. Let x ∈ X and Λ be any filter base in X ?-converging to x. Since f is contra-?-continuous, then for any V ∈ C(Y, f(x)), there exists U ∈ ?(X, x) such that f(U) ⊂ V. Since Λ is ?-converging to x, there exists a B ∈ Λ such that B ⊂ U. This means that f(B) ⊂ V and therefore the filter base f(Λ) is c-convergent to f(x).
Sufficiency. Let x ∈ X and V ∈ C(Y, f(x)). If we take Λ to be the set of all sets U such that U∈ ?(X, x), then Λ will be a filter base which ?-converges to x. Thus, there exists U ∈ Λ such that f(U) ⊂ V.

4. Contra-?-closed graphs
We begin with the following notion:

Definition 4.1. The graph G(f) of a function f: X→Y is said to be *contra-?-closed* if for each (x, y)∈ (X×Y)- G(f), there exist U∈ ?(X, x) and V∈ C(Y, y) such that (U×V) ∩ G(f) = ∅.

Lemma 4.1. The graph G(f) of a function f: X→Y is said to be contra-?-closed in X×Y if and only if for each (x, y) ∈ (X×Y) - G(f), there exist U∈ ?(X, x) and V ∈ C(Y, y) such that f(U) ∩ V = ∅.

Theorem 4.1. If f: X→Y is contra-?-continuous and Y is Urysohn, then G(f) is contra-?-closed in X×Y.

Proof. Let (x, y)∈ (X×Y)- G(f), then y ≠ f(x) and there exist open sets V, W such that f(x)∈ V, y∈ W and Cl(V) ∩ Cl(W) = ∅. Since f is contra-?-continuous, there exists U∈ ?(X,x) such that f(U)⊂ Cl(V). Therefore, we obtain f(U) ∩ Cl(W) = ∅. This shows that G(f) is contra-?-closed

Theorem 4.2. If f: X→Y is ?ŧcontinuous and Y is T_1, then G(f) is contra-?ýclosed in X×Y.

Proof. Let (x, y)∈(X×Y)- G(f), then f(x) ≠ y and there exists an open set V of Y such that f(x)∈V and y∉V. Since f is ?Æcontinuous, there exists U∈?Ø(X, x) such that f(U) ⊂ V. Therefore, we obtain f(U) ∩ (Y-V) = ∅ and Y-V ∈ C(Y, y). This shows that G(f) is contra-?Ø closed X×Y.

Definition 4.2. A space X is said to be ?ŗ*compact* (Maheshwari & Thakur, 1985) (resp. *strongly S*-closed (Dontchev, 1996)) if every ?± open (resp. closed) cover of X has a finite subcover.

A subset A of a space X is said to be ?A*compact relative to* X (Noiri & Di Maio, 1988) if every cover of A by ?Êopen sets of X has a finite subcover. A subset A of a space X is said to be *strongly S-closed* if the subspace A is strongly S-closed.

Theorem 4.3. If f: X→Y has a contra-?(closed graph, then the inverse image of a strongly S-closed set K of Y is ?bclosed in X.

Proof. Assume that K is a strongly S-closed set of Y and x ∉$f^{-1}(K)$. For each k∈K, (x, k) ∉G(f). By Lemma 4.1, there exist U_k∈ ?ô(X, x) and V_k∈C(Y, k) such that f(U_k) ∩ V_k = ∅. Since $\{K\cap V_k \mid k\in K\}$ is a closed cover of the subspace K, there exists a finite subset K_1⊂K such that k ⊂ $\cup\{V_K \mid k\in K_1\ k\in K_1\}$. Set U=$\cap\{U_k \mid k\in K_1\}$, then U ∈?ť(X, x) and f(U)∩ K = ∅. Therefore U ∩ $f^{-1}(K)$=∅ and hence x∉ ?€l($f^{-1}(K)$). This shows that $f^{-1}(K)$ is ?Nclosed in X.

Theorem 4.4. Let Y be a strongly S-closed space. If a function f: X→Y has a contra-?çclosed graph, then f is contra-?çcontinuous.

Proof. Suppose that Y is strongly S-closed and G(f) is contra-?ǥ closed. First, we show that an open set of Y is strongly S-closed. Let V be an open set of Y and $\{H_{?‡} \mid \alpha\in\nabla\}$ be a cover of V by closed sets $H_{?‡}$ of V. For each α∈∇, there exists a closed set $K_{?‡}$ of X such that $H_{?‡}$= $K_{?‡}$∩ V. Then, the family $\{K_{?‡} \mid \alpha\in\nabla\}$ ∪ (Y-V) is a closed cover of Y. Since Y is strongly S-closed, there exists a finite subset $\nabla_\circ \subset \nabla$ such that Y = U $\{K_{?‡} \mid \alpha\in \nabla_\circ\}$ ∪ (Y-V). Therefore we obtain V = (U $\{H_{?‡} \mid \alpha\in \nabla_\circ\}$. This shows that V is strongly S-closed. For any open set V, by Theorem 4.3 $f^{-1}(V)$ is ?‡closed in X and f is contra-?‡ continuous.

5. Covering properties

Theorem 5.1. If f: X→Y is contra-?S-continuous and K is ?S-compact relative to X, then f(K) is strongly S-closed in Y.

Proof. Let $\{H_{?‡} \mid \alpha\in \nabla\}$be any cover of f(K) by closed sets of the subspace f(K). For each α∈∇, there exists a closed set $K_{?‡}$ of n Y such that $H_{?‡} = K_{?‡} \cap$ f(K). For each x∈K, there exists α(X) ∈∇ such that f(x) ∈ $K_{?‡(x)}$ and by theorem 3.1 there exists $U_x \in$ α(X, x) such that f(U_x) ⊂ $K_{?‡(x)}$. Since the family { $U_x \mid$ x∈K} is a cover of K by ?D-open sets of X, there exists a finite subset K_0 of K such that K ⊂ U{ $U_x \mid$ x∈K◦}. Therefore, we obtain f(K) ⊂U{ $f(U_x) \mid$ x∈K◦}which is a subset of ∪ { $K_{?‡(x)} \mid$ α∈K◦}. Thus, f(K)= ∪ {$H_{?‡(x)} \mid$ x∈K◦}and hence f(K) is strongly S-closed.

Corollary 5.1. If f: X→Y is a contra-?\-continuous surjection and X is ?R-compact, then Y is strongly S-closed.

Definition 5.1. A topological space X is said to be

(1) *S-closed* (Thompson, 1976) if for every semi-open cover $\{V_{?‡} \mid \alpha\in \nabla\}$of X, there exists a finite subset $\nabla_\circ$ of ∇ such that X = ∪ $\{Cl(V_{?‡}) \mid \alpha\in \nabla_\circ\}$, equivalently if every regular closed cover of X has a finite subcover,

(2) *nearly compact* (Singal & Mathur, 1969) if every regular open cover of X has finite subcover,

(3) *almost compact* (Singal & Mathur, 1969) if for every open over $\{V_{?‡} \mid \alpha\in \nabla\}$ of X, there exists a finite subset $\nabla_\circ$ of ∇ such that X = ∪ $\{Cl(V_{?‡}) \mid \alpha\in \nabla_\circ\}$,

(4) *mildly compact* (Staum, 1974) if every clopen cover f X has a finite subcover.

Remark 5.1. For the spaces defined above, we have the following implications:

?•compact ⇒ compact ⇒ nearly compact

⇓

Strongly S-closed ⇒ S-closed ⇒ almost compact ⇒ mildly compact

Theorem 5.2. If f: X→Y is contra-?ácontinuous ?ácontinuous surjection and X is an S-closed space, then Y is compact.

Proof. Let{V?‡| α∈∇ }be any open cover of Y. Then{f^{-1}(*V*?‡)| α∈∇)} is a cover of X. Since f is contra-?Acontinuous ?Acontinuous, f^{-1}(*V*?‡) *is* ?A closed and ?Îopen in X for each α∈∇. This implies that {f^{-1}(*V*?‡)| α∈∇)} is a regular closed cover of the S-closed space X. We have X=∪ {f^{-1}(*V*?‡)| α∈∇◦} for some finite ∇◦ of ∇. Since f is surjective, Y = ∪ {*V*?‡| α∈∇◦). This shows that Y is compact.

Corollary 5.2. (Dontchev, 1996). Contra-continuous ?ºcontinuous images of S-closed spaces are compact.

Theorem 5.3. If f: X→Y is contra-?-continuous precontinuous surjection and X is mildly compact, then Y is compact.

Proof. Let {V?‡| α∈∇ }be any open cover of Y. Since f is contra-?¯-continuous precontinuous, by Theorem 3.4{f^{-1}(*V*?‡)| α∈∇)} is a clopen cover of X and there exists a finite subset ∇◦ of ∇ such that shows that Y is compact.

Corollary 5.3. (Dontchev 1996). The image of an almost compact space under contra-continuous, nearly continuous (= precontinuous) function is compact.

6. Connected spaces

Theorem 6.1. Let X be connected and Y be T_1. If f: X→Y is contra-?e continuous, then f is constant.

Proof. Since Y is T_1- space, $\Omega = \{f^{-1}(\{y\}) \mid y \in Y\}$ is a disjoint ?} open partition of X. If $|\Omega| \geq 2$, then there exists a proper ?²open ?² closed set W. By Lemma 3.4, W is clopen in the connected space X. This is a contradiction. Therefore $|\Omega| = 1$ and hence f is constant.

Corollary 6.1. (Dontchev and Noiri, 1999). Let X be connected and Y be T_1. If f: X→Y is contra-continuous, then f is constant.

Theorem 6.2. If f: X→Y is a contra-?®continuous precontinuous surjection and X is connected , then Y has an indiscrete topology.

Proof. Suppose that there exists a proper open set V of Y. Then, since f is contra-?-continuous precontinuous, $f^{-1}(V)$ is ?-closed and preopen in X. Therefore, by Lemma 3.4 $f^{-1}(V)$ is clopen in X and proper. This shows that X is a connected which is a contradiction.

Theorem 6.3. If f: X→Y is contra-?gcontinuous surjection and X is connected, then Y is connected.

Proof. Suppose that Y is not connected. There exist nonempty disjoint open sets V_1 and V_2 such that $Y = V_1 \cup V_2$. Therefore, V_1 and V_2 are clopen in Y. Since f is contra-?˜continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are ?ªclosed and ?ªopen in X and hence clopen in X by Lemma 3.4. Moreover, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are nonempty disjoint and $X= f^{-1}(V_1) \cup f^{-1}(V_2)$. This shows that X is not connected.
A space (X,) is said to be *hyperconnected* (Steen & Seebach, 1970) if the closure of every open set is the entire set X. It is well-known that every hyperconnected space is connected but not conversely.

Remark 6.1. In Example 2.1, (X,) is hyperconnected and f: (X,)→ (X, σ) is a contra-?¸continuous surjection, but (X, σ) is not hyperconnected. This shows that contra-?Êcontinuous surjection do not necessarily preserve hyperconnectedness.

A function f: X→Y is said to be *weakly continuous* (Levine, 1961) if for each point x∈X and each open set V of Y containing f(x), there exists an open set U containing x such that f(U) ⊂ Cl(V). It is shown in

(Noiri, (1974), Theorem 3] that if f: X→Y is a weakly continuous surjection and X is connected , then Y is connected. However, it turns out that contra-?Œontinuity and weak continuity are independent of each other. In Example 2.1, the function f is contra-?-continuous but not weakly continuous. The following example shows that not every weakly continuous function is contra-?-continuous.

Example 6.1. Let X = {a, b, c, d} and = {∅, X, {b}, {c}, {b, c}, {a, b}, {a, b, c}, {b, c, d}}. Define a function f: (X,)→ (X,) as follows: f(a) = c, f(b) = d, f(c) = b and f(d) = a. Then f is weakly continuous (Neubrunnova, 1980). However, f is not contra-?-continuous since {a} is a closed set of (X,) and f^{-1}({a})= {d} is not ?-open in (X,).

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