

## **An Application of the Stochastic Optimal Control Algorithm (OPTCON) to the Public Sector Economy of Iran**

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### **Abstract**

In this paper we first describe the stochastic optimal control algorithm called ((OPTCON)). The algorithm minimizes an intertemporal objective loss function subject to a nonlinear dynamic system in order to achieve optimal value of control (or instrument) variables. Second as an application, we implemented the algorithm by the statistical programming system ((GAUSS)) to determine the optimal fiscal policy for Iran during the third development plan (1383 – 1379). The obtained results show that under optimal fiscal policies, the rate of economic growth and current account balance proposed in the third development plan will be achieved. Based of the findings having found compatible results therefore the determination of optimal macroeconomic policies for the Iran's forth development plan is suggested.

**Key words:** Stochastic optimal control algorithm, OPTCON, Optimal macroeconomic policies Iran.

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## I- Introduction

During the last two decades, the application of optimal control theory has been widely developed in economic studies. In fact, policy makers in decision-making process should determine the objectives and constraints. Then, he (or she) should choose the alternative that gives the nearest outputs to objectives. For this reason mathematical models have been used. But, in practice, one is recognize the intertemporal nature of decision process, the presence of uncertainty, the imperfect information and the nonlinearity of economic models. Stochastic optimal control theory is a powerful tool for solving these problems however, because of the nonlinearity of the models, approximates solution of applied in studies. For this purpose, some stochastic optimal control algorithms were designed. Among them, the ((OPTCON)) algorithm provide the most facilities for economic planning in comparison with the others .In this paper, we first introduce the algorithm ((OPTCON)). Then, as a practical example, we use the algorithm in order to determine Iran optimal fiscal polices during third development plan.

## 2- The "OPTCON" Algorithm

An optimal control problem according to Chow is concerned with the determination of best ways to achieve a set of objectives as indexed by a criterion function when the performance is Judged over many periods and when the dynamic behavior of the system is subject to a set of constraints<sup>1</sup>. We can distinguish the deterministic optimal control stochastic optimal control problems that allows for uncertainty as well as for the presence of stochastic parameter in the system equation. In fact, when a dynamic system is subject to disturbances or parameter variation that cannot be specified ahead of time, we will have the stochastic optimal control problem.: Policy maker is concerned with a multidimensional policy decision which takes into account both current and future goals, a decision requiring some form of optimal control technique rather

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1 - For more information, see:

Chow, G. C (1975) Analysis and Control of Dynamic Economic Systems, John Wiley And sons Inc, New York.

than the standard fare of simulation results commonly provided<sup>1</sup>. In fact, the optimization is to find the one model vector of policy instruments in the model solution space that is best. In order to identify the best feasible solution, it is necessary to have a function of target variables for comparisons among the model solution alternatives. This function may be identified as the loss function.

In practice, there are three basic approaches which have been used to solve optimal control problems. The first type of solution method used by Livesey<sup>2</sup> employs a nonlinear model and a nonlinear social welfare function. Livesey applies the conjugate gradient method to the solution of his optimal control problem. This gradient procedure solves simultaneously for a joint solution to the equation system and a minimum of the loss function. The major problem with this procedure is its high cost. A second approach used by Holbrook. This is an approximation method. Holbrook starts with an initial set of assumptions for the instruments and corresponding solution to the nonlinear model for the target variables. Then, He solve for the instruments which minimize a quadratic loss function given a first-order Taylor expansion approximation to the nonlinear dynamic model. The resulting instrument solution is entered into the nonlinear model which is solved for the actual solution values for the target variables. The quadratic loss function given the first order Taylor approximation to the model. This circular iteration process is repeated until the solution values for optimal instrument values change by less than a specified epsilon value from one iteration to the next.

Third approach, which is the one employed in this paper, involves the application of dynamic programming techniques to solve the problem. Chow showed that the minimum of the quadratic loss function subject to linearized state-space model is the linear feedback function: The Chow approach assumes that the econometric model can be represented by a first-order liner difference equation system. Also, in this way, n-order linear difference equation system can be scaled down to a first-order linear difference equation system. For typical

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1- See Holbrook, R. S. (1974) A Practical Method for Controlling a Large Nonlinear Stochastic System. *Annals of Social and Economic Measurement*, vol. 3, pp 155-176.

2- See: Livesey, D.A (1971) Optimization Short Term Economic Policy. *The Economic Journal*. vol. 81, pp 525-545.

nonlinear dynamic macro econometric models, a linear approximation would have to be developed and the resulting n-order linear difference equation approximation would have to be reduced to a first-order difference equation system.

Among the algorithms used for solving Chow optimal control problems, OPTCON algorithm is modern. This algorithm can be applied to obtain approximate numerical solutions of control problems where the objective function is quadratic and the dynamic system is nonlinear. In addition, it can be used for the optimum control of dynamic model that allows for additive uncertainty and the presence of a stochastic parameter vector in the system equation. So, it can be used for solving both the stochastic and deterministic optimal control problems. The algorithm is developed by Matulka and Neck (1992). The objective function is quadratic in the deviations of the state  $X_t$  and control  $U_t$  variables from their desired values  $\bar{X}_t$  and  $\bar{U}_t$ , for  $t = 1, \dots, T$ . So that :

$$L_t(X_t, U_t) = \frac{1}{2} \begin{bmatrix} X_t - \bar{X}_t \\ U_t - \bar{U}_t \end{bmatrix} W_t \begin{bmatrix} X_t - \bar{X}_t \\ U_t - \bar{U}_t \end{bmatrix} \quad (1)$$

$X_t$  denotes an n-dimensional vector of state variables and  $U_t$  denotes an m-dimensional vector of control variables. The n-dimensional vector  $\bar{X}_t$  and the m-dimensional vector  $\bar{U}_t$  denote the given desired levels of the state and control variables, respectively. If, S denotes the initial and ((T)) the terminal period of the finite planning horizon, the matrix  $W_t$  will be defined as:

$$W_t = \begin{bmatrix} W_t^{XX} & W_t^{XU} \\ W_t^{UX} & W_t^{UU} \end{bmatrix}, t = s, \dots, T \quad (2)$$

And so,

$$W_t = \alpha^{t-1} \cdot W, t = s, \dots, T \quad (3)$$

Where, «  $\alpha$  » is the discount factor? It is assumed  $W$  is symmetric, and then:

$$\begin{bmatrix} W_t^X \\ W_t^U \end{bmatrix} = -W \begin{bmatrix} \bar{X}_t \\ \bar{U}_t \end{bmatrix} \quad (4)$$

$$W_t^c = \frac{1}{2} \begin{bmatrix} \bar{X}_t \\ \bar{U}_t \end{bmatrix} W_t \begin{bmatrix} \bar{X}_t \\ \bar{U}_t \end{bmatrix} \quad (5)$$

So, (41) can be written in the «general quadratic form »:

$$L_t(X_t, U_t) = \frac{1}{2} \begin{bmatrix} X_t \\ U_t \end{bmatrix}' W_t \begin{bmatrix} X_t \\ U_t \end{bmatrix} + \begin{bmatrix} X_t \\ U_t \end{bmatrix}' \begin{bmatrix} W_t^X \\ W_t^U \end{bmatrix} + W_t^c \quad (6)$$

The dynamic nonlinear system is given by the system of nonlinear difference equations:

$$x_t = f(x_{t-1}, x_t, u_t, \hat{\theta}, Z_t) + \varepsilon_t \quad t = s, \dots, T \quad (7)$$

Where  $\theta$  denotes a p-dimensional vector of unknown parameters  $Z_t$  denotes an l-dimensional vector of non-controlled exogenous variables and  $\varepsilon_t$  is an n-dimensional vector of additive disturbances. « OPTCON » requires as inputs:

The system function  $F(\dots)$ , the initial values of the state variables  $X_{s-1} \equiv X^o_{s-1} \equiv X^*_{s-1}$ , the tentative path of control variables  $(U_t^o)_{t=s}^T$ , the path of exogenous variables not subject to control  $(Z_t)_{t=s}^T$ , the expected values of system parameters  $\hat{\theta}$ , the covariance matrix of system parameters  $\Sigma^{\theta\theta}$ , the covariance matrix of system noise  $\Sigma \varepsilon \varepsilon$ , the weighting matrices of objective function  $W_{xx}$ ,  $W_{ux}$ ,  $W_u$ , the discount rate of objective function  $\alpha$ , the target path for state variables  $(\bar{X}_t)_{t=s}^T$  and the target path for control variables  $(\bar{U}_t)_{t=s}^T$  algorithm is implement able in the statistical programming system

“GUASS“. The optimal values of the control variables compute in an iterative fashion. So:

Step 1: Compute a tentative state path: use the gauss-seidel algorithm, the tentative control path  $\left( \begin{matrix} \circ \\ U_t \end{matrix} \right)_{t=s}^T$ , and the system equation  $F ( \dots )$  to calculate the tentative state path  $\left( \begin{matrix} \circ \\ X_t \end{matrix} \right)_{t=s}^T$ .

Step 2: Non linear loop: Repeat the (a) to (e) until convergence is reached (i.e., until the optimal control and state variables do not change more than a prespecified small number

From one iteration to the next or the number of iterations is larger than a prespecified number.

( a ) Initialization for backward recursion:

$$H_{T+1} = 0_{nn} \quad , \quad (8)$$

$$h^{xT+1} = 0_n \quad , \quad (9)$$

$$h^{cT+1} = 0 \quad , \quad (10)$$

$$h^{sT+1} = 0 \quad , \quad (11)$$

$$h^{pT+1} = 0 \quad , \quad (12)$$

(b) Backward recursion:

Repeat the following steps ( i ) to ( vii ) for  $t = T, \dots, S$ .

(i) Compute the expected values of the parameters of the linearized system

equation:<sup>1</sup>  $(X_t \cong A_t X_{t-1} + B_t U_t + C_t + e_t)$ :

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1- Showing the detailed prove and solution to the model equations will cause an excessive increase in the page numbers. Interested researchers can see The following reference:

Matulka, Josef and Neck, Reinhard (1992). OPTCON: An Algorithm For The Optimal Control Of Non Linear Stochastic Models. Annals of Operations Research. 37. PP 375-402.

$$A_t = (I - f_{xt})^{-1} f_{x-1} \quad (13)$$

$$B_t = (I - f_{xt})^{-1} f_{ut} \quad (14)$$

$$C_t = X_t^\circ - A_t X_{t-1}^\circ - B_t U_t^\circ \quad (15)$$

$$\Sigma_t^{ee} = \text{COV}_{s-1}(e_t, e_t) = (I - f_{xt})^{-1} \Sigma \varepsilon \varepsilon' [(I - f_{xt})^{-1}]' \quad (16)$$

Where all derivatives are evaluated at values  $X_{t-1}^\circ, X_t^\circ, U_t^\circ, \hat{\theta}, Z_t$  and  $\varepsilon_t^\circ = 0$ .

(ii) Compute the derivatives of the parameters of the linearized system with respect to  $\theta$ :

$$D^{A_t} = [(I - f_{x_t})^{-1} \otimes I_P] [f_{X_t, \theta} \otimes A_t + f_{X_{t-1}, \theta}] \quad (17)$$

$$D^{B_t} = [(I - f_{x_t})^{-1} \otimes I_P] [f_{X_t, \theta} \otimes B_t + f_{U_{t-1}, \theta}] \quad (18)$$

$$d^{c_t} = \text{vec}[(I - f_{x_t})^{-1} f_\theta] - D^{A_t} X_{t-1}^\circ - D^{B_t} U_t^\circ \quad (19)$$

All derivatives are evaluated at the same reference values as above.

(iii) Compute the influence of the stochastic parameters:

compute all the matrices the cells of which are defined by:

$$[\gamma_t^{AKA}]_{i,j} = \text{tr} [k_t D^{a_t,j} \Sigma \theta \theta' (D^{a_t,j})'], i=1, \dots, n \quad j=1, \dots, n \quad (20)$$

$$[\gamma_t^{BKA}]_{i,j} = \text{tr} [k_t D^{a_t,j} \Sigma \theta \theta' (D^{b_t,j})'], i=1, \dots, m \quad j=1, \dots, n \quad (21)$$

$$[\gamma_t^{BKB}]_{i,j} = \text{tr} [k_t D^{b_t,j} \Sigma \theta \theta' (D^{b_t,j})'], i=1, \dots, m \quad j=1, \dots, n \quad (22)$$

$$[V_t^{AKC}]_i = \text{tr} [k_t D^{c_t} \Sigma \theta \theta' (D^{a_t,i})'], i=1, \dots, n \quad (23)$$

$$[V_t^{BKC}]_i = \text{tr} [k_t D^{c_t} \Sigma \theta \theta' (D^{b_t,i})'], i=1, \dots, m \quad (24)$$

$$[V_t CKC] = \text{tr} [k_t D^{ct} \Sigma^{\theta\theta} (D^{ct})'] \quad (25)$$

(iv) Convert the objective function from « quadratic-tracking » to « general quadratic » format:

$$W_t^{xx} = \alpha^{t-1} W^{xx}, \quad (26)$$

$$W_t^{ux} = \alpha^{t-1} W^{ux}, \quad (27)$$

$$W_t^{uu} = \alpha^{t-1} W^{uu}, \quad (28)$$

$$W_t^X = -W_t^{XX} \bar{X}_t - U_t^{XU} \bar{U}_t, \quad (29)$$

$$W_t^U = -W_t^{UX} \bar{X}_t - U_t^{UU} \bar{U}_t, \quad (30)$$

$$W_t^I = \frac{1}{2} \bar{X}_t' W_t^{XX} \bar{X}_t + \bar{U}_t' W_t^{UX} \bar{X}_t + \frac{1}{2} \bar{U}_t' W_t^{UU} \bar{U}_t \quad (31)$$

(v) Compute the parameters of the function of expected accumulated loss:

$$K_t = W_t^{XU} + H_{t+1}, \quad (32)$$

$$K_t = W_t^X + h_{t+1}^X, \quad (33)$$

$$\rho_t^{XX} = \gamma_t A K A + A_t' k_t A_t, \quad (34)$$

$$\rho_t^{XU} = \begin{pmatrix} UX \\ \rho_t^X \end{pmatrix} \quad (35)$$

$$\rho_t^{UX} = \lambda_t^{BkB} + B_t' k_t A_t + W_t^{UX} A_t, \quad (36)$$

$$\rho_t^{UU} = \lambda_t^{BkB} + B_t' k_t A_t Z B_t' W_t^{XU} + W_t^{UU}, \quad (37)$$

$$\lambda_t^X = v_t^{AkC} + A_t' k_t C_t + A_t' k_t^X, \quad (38)$$



$$\lambda_t^U = v_t^{Bkc} + B_t' k_t c_t + B_t' k_t^X + W_t^{UX} c_t + W_t^U, \quad (39)$$

$$\lambda_t^s = \frac{1}{2} \text{tr}[k_t \Sigma_t^{ee}] + h_{t+1}^s, \quad (40)$$

$$\lambda_t^p = \frac{1}{2} v_t^{ckc} + h_{t+1}^p \quad (41)$$

$$\lambda_t^c = \frac{1}{2} c_t' k_t c_t + c_t' k_t^X + W_t^c + h_{t+1}^c, \quad (42)$$

(vi) Compute the parameters of the feed back rule:

$$G_t = -(\rho_t^{UU})^{-1} \gamma_t^{UX}, \quad (43)$$

$$g_t = -(\gamma_t^{UU})^{-1} \lambda_t^U, \quad (44)$$

(vii) Compute the parameters of the function of minimal expected accumulated loss:

$$H_t = \rho_t^{XX} - \rho_t^{XU} (\rho_t^{UU})^{-1} \rho_t^{UX}, \quad (45)$$

$$h_t^X = \lambda_t^X - \rho_t^{XU} (\rho_t^{UU})^{-1} \lambda_t^u, \quad (46)$$

$$h_t^c = \lambda_t^c - \frac{1}{2} (\lambda_t^U)' (\rho_t^{UU})^{-1} \lambda_t^U, \quad (47)$$

$$h_t^s = \lambda_t^s, \quad (48)$$

$$h_t^p = \lambda_t^p, \quad (49)$$

(c) Forward projection:

Repeat (i) and (ii) for  $t=S, \dots, T$

(i) Compute the expected optimal control variables:

$$U_t = G_t^* X_{t-1} g_t^* \quad (50)$$

(i) Compute the expected optimal state: use the Gauss-seidel algorithm to compute  $X_t^*$  such that:

$$X_t^* = f \left( X_{t-1}^*, X_t^*, U_t, \hat{\theta}, Z_t \right) \quad (51)$$

$$\left( X_t^* \right)_{t=s}^T = \left( X_t^* \right)_{t=s}^T \quad (52)$$

$$\left( U_t^* \right)_{t=s}^T = \left( U_t^* \right)_{t=s}^T \quad (53)$$

(e) Compute the expected welfare loss:

$$J_s^* = X_{s-1}' H_s X_{s-1} + X_{s-1}' + h_{s-1}^X + h_{s-1}^c + h_{s-1}^p + h_{s-1}^s \quad (54)$$

### 3-An Application of the "OPTCON" Algorithm for Iran third Development Plan.

In this section we used the "OPTCON" algorithm in order to determine the optimal fiscal policies for Iran during the third development plan. The constraint to the optimization problem is given by a macro econometric model of the Iran's economy. The list of variables is shown in appendix. The dynamic nonlinear system includes two category of equation: behavioral equations and identities. The behavioral equations of the model were estimated by O.L.S using time series data<sup>1</sup> for the period 1347-1378 behavioral equations:

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1- The real potential GDP and real capital stock data are estimated by methods used in the references:

- Arabmazar, Abbas and Abbas Bagher Klantari, (1371). The estimation of country capital stock (1367-1338). The economical and political sciences "Economics" Shahid Beheshti University. 1, No. 4, pp.42-60.
- Arabmazar, Abbas and Abbas Bagher Klantari, (1371). The estimation of country potential product (1371-1338). The economical and political sciences "Economics". 4, No. 4, pp. 55-75.

CPR=0.9	CPR <sub>t-1</sub> +0.01	YDR+728.5		$\bar{R}^2 = \%98$	D.W=2.02
(21.7)	(3.4)	(2.1)			
IMPR=0.8	IMPR <sub>t-1</sub> +0.08	GDPR-0.98	ERP	$\bar{R}^2 = \%68$	D.W=1.8
(7.2)	(2.2)	(-1.8)			
INVPR=2544+	0.1	DEMAND-50.5	INTLR	$\bar{R}^2 = \%99$	D.W=2.1
(9.04)	(13.1)	(-6.1)			
CPI=0.5	CPI <sub>t-1</sub> +0.85	AGWN+0.17	UTIL	$\bar{R}^2 = \%99$	D.W=2.1
(1.6)	(2.1)	(1.5)			
AGWN=0.32	AGWN <sub>t-1</sub> +0.5	CPI-0.7	UR+0.06	PROD	$\bar{R}^2 = \%99$ D.W=1.9
(2.6)	(7.8)	(-0.95)	(1.7)		
EMP=0.95	EMP <sub>t-1</sub> +0.01	GDPR+3.3	AGWR	$\bar{R}^2 = \%99$	D.W=1.9
(18.1)	(1.1)	(1.6)			
M=0.57	M <sub>t-1</sub> +2.7	ERN+13647.7	PRICERAT	$\bar{R}^2 = \%99$	D.W=2.1
(9.5)	(4.08)	(6.4)			
INTLN=0.89	INTLN <sub>t-1</sub> +0.000087	GDPR-0.000076	MR	$\bar{R}^2 = \%98$	D.W=1.8
(5.9)	(1.6)	(-2.1)			
GDPPOT=0.17	CAPR + 1.49	LFORCE + 20.4	TIME	$\bar{R}^2 = \%99$	D.W=1.6
(5.2)	(10.8)	(10.8)			
GDPDEF=1.05	CPI			$\bar{R}^2 = \%97$	D.W=1.8
(1.5)					
EXPORTR=0.93	EXPORTR <sub>t-1</sub> +0.54	ERR		$\bar{R}^2 = \%80$	D.W=1.9
(21.27)	(1.76)				

### Identities

$$GDPR = CPR + INVPR + GIR + GCR + EXPORTR - IMPR$$

$$YDR = GDPR - TAXRR$$

$$\text{DEMAND}=\text{GDPR}+\text{IMPR}$$

$$\text{GGDD}=\left(\frac{\text{GDPR}_t - \text{GDPR}_{t-1}}{\text{GDPR}_{t-1}}\right)*100$$

$$\text{GCPI}=\left(\frac{\text{CPI}_t - \text{CPI}_{t-1}}{\text{GDPR}_{t-1}}\right)*100$$

$$\text{INTLR}=\text{INTLN}-\text{GCPI}$$

$$\text{PROD}=\left(\frac{\text{GDPR}}{\text{EMP}}\right)*100$$

$$\text{AGWR}=\left(\frac{\text{AGWN}}{\text{CPI}}\right)*100$$

$$\text{MR}=\left(\frac{\text{M}}{\text{CPI}}\right)*100$$

$$\text{ERR}=\text{ERN}*\text{CPIF}/\text{CPI}$$

$$\text{UN}=\text{LFORCE}-\text{EMP}$$

$$\text{UR}=\text{UN}/\text{LFORCE}$$

$$\text{PRICERAT}=\text{CPI}/\text{CPIF}$$

$$\text{CAPR}=\text{CAPR}_{t-1} - \text{DEPR} + \text{INVPR} + \text{GIR}$$

$$\text{UTIL}=\left(\frac{\text{GDPR}}{\text{YPOT}}\right)*100$$

$$\text{CAD}=\left(\frac{\text{EXPORTR}-\text{IMPR}}{\text{ERN}}\right)$$

$$\text{DEF}=(\text{GC}+\text{GI}+\text{DIG})-\left(\text{NTAXRN}+\text{TAXRN}\right)$$

$$\text{DEFDAR}=\left(\frac{\text{DEF}}{\text{GDP}}\right)*100$$

$$\text{GDP}=\text{GDPR}*\text{GDPDEF}/100$$

$$\text{GCR}=\left(\frac{\text{GC}}{\text{GDPDEF}}\right)*100$$

$$\text{GIR}=\left(\frac{\text{GI}}{\text{GDPDEF}}\right)*100$$

$$\text{TAXRR}=\left(\frac{\text{TAXRN}}{\text{GDPDEF}}\right)*100$$

The model includes goods, services market and money markets from the aggregate demand side and a production function and the labor market from the aggregate supply side. The goods and services market contain private consumption function private investment function, imports and exports functions. Also, the model includes a money demand equation and wage price system. The wage - price system can be regarded as an enhanced Phillips curve. Wages are determined by the price level, labor productivity and the unemployment rate. The price level depends on wages and the capacity utilization rate. A production function is included to determine potential GDP. The labor market is modeled by specifying an employment equation, whereas the labor supply is exogenous to the model. We consider a fixed exchange rate regime in which the central bank will interest to fix the exchange rate of a \$ in 8000 rials. It should be noted that under a flexible exchange rate system, monetary policy will be active. In other words, under these circumstances variables such as volume of money supply can be considered as an endogenous (control) variables. Therefore, the distinction between the effects of monetary and fiscal policy is not possible. So, in the present paper we used a fixed exchange rate system in which the fiscal policy effects could be evaluated without worrying about the monetary policy impacts. From technical point of view, this has been shown in nominal stock of money Equation in which this variable is considered as an endogenous variables. As the stochastic model equations are estimated by OLS, no full covariance matrix of the parameters is available. In this case, we introduce stochastic parameters. Here, the covariance matrix of the parameters to be diagonal and select only some diagonal elements have the lowest t-values are regarded as stochastic. These are the coefficient of  $cpt_{t-1}$  and UTIL in the consumer price index equation, UR and PROD in the average nominal wage equation, GDPR and AGWR in the employment equation, GDPR in the interest rate equation and CPI in the GDP deflator equation.

The studies related to sensitivity analysis of optimization indicated there is not significant difference regarding the estimation methods determining the amount of control and objective variables. It should also be noted that choosing

a system method of estimation many violate the stationary and convergence conditions.<sup>1</sup>

In order to determine the approximate solutions optimum government consumption and investment expenditures as well as optimum tax revenues (the optimum fiscal policy), three main and two minor objective variables were considered in objective function<sup>2</sup>. The main variables includes: The rate of economic growth, the inflation rate, the real GDP and the rate of unemployment. Also, the minor objective variables contain: the ratio of budget deficit to GDP and the current account balance. The values of target for these variables are the values, which targeted in Iran's third development plan. So, the planning horizon for the control experiments has been chosen as  $S=1379$  to  $T=1383$ . After several experiments sensitivity analysis we have chosen a discount factor  $\alpha=1$ , the weight 1000 for main and 1 for minor objective variables. Then, in the weight matrix of the objective function, off diagonal elements were all set equal to zero. In addition, all state variables in the model not mentioned above, got the weight zero.

In order to comparison the optimal fiscal policies effects on the main and minor objective function variables with that of the proposed fiscal policies in third plan, we used MAPLE program for the simulation of the model; we used GAUSS program to determine the optimal fiscal policies. The calculated optimum government consumption and investment expenditures and optimum tax revenues are compared with those proposed valued in third development plan in table1.

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1 - For more information, see Karbuz, sohbet and Reinhard Neck (1998). On The Sensitivity of Optimal Macroeconomic Policies With Respect To Stochastic Model Parameters: A Case Study for Austria. In Second International Ec Joint Research center, Ispra. Italy. Symposium on sensitivity Analysis Of Model output. (Eds) K. Chan, S. Tarantola and F. Campolongo. Joint Research centre. Italy.

2- The Matrix of the objective variables with their weights is shown in appendix B

**Table1: The Comparison of the Optimum Fiscal Policy Variables with to those Suggested by 3 rd Development Plan. (In billons rials)**

		1379	1380	1381	1382	1383
<b>CG</b>	Optimal	83707.3	110866.2	148290	196888.9	256651.6
	Proposed In third plan	69494.8	77918.8	96924.1	109356.1	134729
<b>GI</b>	Optimal	65414.1	88544.1	122922.4	167875.1	227052.7
	Proposed In third plan	50943.3	55352.6	71352.4	80203.4	105071.9
<b>TAXRN</b>	Optimal	24868.5	29359.2	35878.5	44779	60557.1
	Proposed In third plan	32204.2	40060.3	49386.6	60130.1	72718.8

Source: Authors calculations

The table shows that, except for the last year of the 3rd development plan, the optimum government consumption and investment expenditure are above those variables in 3rd plan. Also the tax revenues are lower than the proposed in third development plan. On other hand, to achieve the targeted economic growth- under fixed exchange rate regime- there will be a big pressure on government budget .Table 2 shows the target values and the results for the most important state variables of the simulation and optimization run, respectively.

**Table 2: Target Values, Optimization and Simulation Results**

		1379	1380	1381	1382	1383
GGDP%	Targets	4.5	5.5	6.5	6.7	6.8
	Optimization results	4.2	5	5.9	5.4	5.2
	Simulation results	10.6	-0.6	2.3	-0.89	1.6
GCPI%	Targets	19.9	17.4	15.3	14	13
	Optimization results	24.8	23.1	22.2	21.5	21.1
	Simulation results	25.7	23.5	22.4	21.5	20.5
UN%	Targets	15.2	14.5	13.8	13.1	12.5
	Optimization results	14.7	17.9	20.7	23.4	26
	Simulation results	14.4	17.4	20.2	23	25.8
DEFDAR %	Targets	0.18	0.22	0.2	0.2	0.17
	Optimization results	7.7	12.5	14.6	18.9	19.6
	Simulation results	0.17	0.18	0.17	0.18	0.16
CAT (In Billions Dollars)	Targets	-0.312	-1.394	-1.234	-1.149	-0.701
	Optimization results	0.207	0.027	-0.184	-0.449	-0.713
	Simulation results	0.143	-0.059	0.273	0.504	-0.721

Source: Authors Calculations



As seen at the table, under fixed exchange rate regime the rate of economic growth is close to third plan targets with the optimum fiscal policies than those proposed by third plan. Also, under optimum fiscal policies the current account will be better. The inflation rate and unemployment rate will be the same both under optimal and proposed third plan fiscal policies. As we have shown in table1, the budget deficit is above under flexible exchange rate regime, policy makers can use the monetary instruments, since, and under this regime monetary instruments will be active. For this reason, there will be a lower pressure on government budget.

#### **4- Concluding Remarks**

In this paper we have described the ((OPTCON)) algorithm as proxy solutions to stochastic optimum control problems. Hence, the objective function is quadratic in deviations of the state and control variables from their respective desired values. Also, the constrain of optimization problem is given by a nonlinear stochastic system .We implemented the ((OPTCON)) algorithm in the programming language ((GAUSS)) and applied an econometric model to the Iran economy in order to show the feasibility of the algorithm. An empirical optimization result, show that the optimum fiscal policies may lead to a considerable stabilization of the time path of the rate of economic growth. Also, under fixed exchange rate regime, the money supply has to be adjusted so as to hold the nominal exchange rate constant .It is also shown that the executing optimal fiscal policies put pressure on government budget due to the above results. Therefore it is recommended to implement flexible exchange rate systems for the fourth development plan since in these systems; monetary tools are planning policies instruments. Based on the finding of these papers we recommend applying ((OPTCON)) algorithm to the fourth development plan of Iran.

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## Appendix A: List of Variable

State (or endogenous) variables	
GDPR	Gross domestic product, real
YDR	Disposable income, real
DEMAND	Total final demand, real
GGDP	Annual growth rate of real GDP
GCPI	Annual growth rate of CPI (rate of inflation)
INTLR	Real long run term interest rate
PROD	Labor productivity
AGWR	Average wage rate, real
MR	Money stock (liquidity), real
ERR	Real exchange rate
UN	Unemployment, 1000 persons
UR	Unemployment rate, % of the labor force
PRICERAT	Ratio Iran CPI to USA CPI
CARP	Capital stock, real
UTIL	Capacity utilization rate
EXPORTR	Exports, real
CAD	Current account
DEF	Budget deficit, nominal
DEFDAR	Budget deficit as percentage of GDP
CPR	Private consumption expenditures
IMPR	Imports, real
INVPR	Private investment, real
CPI	Consumer price index
AGWN	Average wage rate, nominal
EMP	Employment, 1000 persons
M	Nominal money stock

<b>List of variables</b>	
<b>INTLN</b>	Nominal long run term interest rate
<b>GDPPOT</b>	Potential GDP, real
<b>GDPDEF</b>	
<b>GDP</b>	Gross domestic product, nominal
<b>GCR</b>	Government consumption expenditures, real
<b>GIR</b>	Government investment expenditures, real
<b>TAXRR</b>	Tax révenues, real
<b>Exogenous variables</b>	
<b>ERN</b>	Nominal exchange rate
<b>LFORCE</b>	Labor force
<b>DEPR</b>	
<b>CPIF</b>	U.S.A CPI
<b>NTAXRN</b>	Government non tax revenues
<b>TIME</b>	Linear time trend
<b>DIG</b>	Difference between the sum of government expenditures with GC + GI
<b>Control variables</b>	
<b>GI</b>	Government investment expenditures, nominal
<b>GC</b>	Government consumption expenditures, nominal
<b>TAXRN</b>	Tax revenues, nominal

**Appendix B : The Objective Function the Following Table Show the Variables of the Objective Function .**

variable		Weight	Target Value				
			1379	1380	1381	1382	1383
<b>Main</b>	<b>GGDP%</b>	1000	4.5	5.5	6.5	6.7	6.8
	<b>GCPI%</b>	1000	19.9	17.4	15.3	14	13
	<b>UN%</b>	1000	15.2	14.5	13.8	13.1	12.5
<b>Min</b>	<b>DEFDAR%</b>	1	0.18	0.22	0.2	0.2	0.17
	<b>CAT%</b>	1	-0.312	-1.394	-1.234	-1.249	-0.701