An Application of the Stochastic Optimal Control Algorithm (OPTCON) to the Public Sector Economy of Iran

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Abstract
In this paper we first describe the stochastic optimal control algorithm called ((OPTCON)). The algorithm minimizes an intertemporal objective loss function subject to a nonlinear dynamic system in order to achieve optimal value of control (or instrument) variables. Second as an application, we implemented the algorithm by the statistical programming system ((GAUSS)) to determine the optimal fiscal policy for Iran during the third development plan (1383 – 1379). The obtained results show that under optimal fiscal policies, the rate of economic growth and current account balance proposed in the third development plan will be achieved. Based of the findings having found compatible results therefore the determination of optimal macroeconomic policies for the Iran’s forth development plan is suggested.

Key words: Stochastic optimal control algorithm, OPTCON, Optimal macroeconomic policies Iran.

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1- Introduction

During the last two decades, the application of optimal control theory has been widely developed in economic studies. In fact, policy makers in decision-making process should determine the objectives and constraints. Then, he (or she) should choose the alternative that gives the nearest outputs to objectives. For this reason mathematical models have been used. But, in practice, one is recognize the intertemporal nature of decision process, the presence of uncertainty, the imperfect information and the nonlinearity of economic models. Stochastic optimal control theory is a powerful tool for solving these problems however, because of the nonlinearity of the models, approximates solution of applied in studies. For this purpose, some stochastic optimal control algorithms were designed. Among them, the ((OPTCON)) algorithm provide the most facilities for economic planning in comparison with the others .In this paper, we first introduce the algorithm ((OPTCON)). Then, as a practical example, we use the algorithm in order to determine Iran optimal fiscal polices during third development plan.

2- The "OPTCON" Algorithm

An optimal control problem according to Chow is concerned with the determination of best ways to achieve a set of objectives as indexed by a criterion function when the performance is Judged over many periods and when the dynamic behavior of the system is subject to a set of constraints¹. We can distinguish the deterministic optimal control stochastic optimal control problems that allows for uncertainty as well as for the presence of stochastic parameter in the system equation. In fact, when a dynamic system is subject to disturbances or parameter variation that cannot be specified ahead of time, we will have the stochastic optimal control problem.: Policy maker is concerned with a multidimensional policy decision which takes into account both current and future goals, a decision requiring some form of optimal control technique rather

¹ - For more information, see:
than the standard fare of simulation results commonly provided. In fact, the optimization is to find the one model vector of policy instruments in the model solution space that is best. In order to identify the best feasible solution, it is necessary to have a function of target variables for comparisons among the model solution alternatives. This function may be identified as the loss function.

In practice, there are three basic approaches which have been used to solve optimal control problems. The first type of solution method used by Livesey employs a nonlinear model and a nonlinear social welfare function. Livesey applies the conjugate gradient method to the solution of his optimal control problem. This gradient procedure solves simultaneously for a joint solution to the equation system and a minimum of the loss function. The major problem with this procedure is its high cost. A second approach used by Holbrook. This is an approximation method. Holbrook starts with an initial set of assumptions for the instruments and corresponding solution to the nonlinear model for the target variables. Then, He solve for the instruments which minimize a quadratic loss function given a first-order Taylor expansion approximation to the nonlinear dynamic model. The resulting instrument solution is entered into the nonlinear model which is solved for the actual solution values for the target variables. The quadratic loss function given the first order Taylor approximation to the model. This circular iteration process is repeated until the solution values for optimal instrument values change by less than a specified epsilon value from one iteration to the next.

Third approach, which is the one employed in this paper, involves the application of dynamic programming techniques to solve the problem. Chow showed that the minimum of the quadratic loss function subject to linearized state-space model is the linear feedback function. The Chow approach assumes that the econometric model can be represented by a first-order liner difference equation system. Also, in this way, n-order linear difference equation system can be scaled down to a first-order linear difference equation system. For typical

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nonlinear dynamic macro econometric models, a linear approximation would have to be developed and the resulting n-order linear difference equation approximation would have to be reduced to a first-order difference equation system.

Among the algorithms used for solving Chow optimal control problems, OPTCON algorithm is modern. This algorithm can be applied to obtain approximate numerical solutions of control problems where the objective function is quadratic and the dynamic system is nonlinear. In addition, it can be used for the optimum control of dynamic model that allows for additive uncertainty and the presence of a stochastic parameter vector in the system equation. So, it can be used for salving both the stochastic and deterministic optimal control problems. The algorithm is developed by Matulka and Neck (1992). The objective function is quadratic in the deviations of the state $X_t$ and control $U_t$ variables from their desired values $\overline{X}_t$ and $\overline{U}_t$, for $t = 1, \ldots, T$. So that:

$$L_t(X_t, U_t) = \frac{1}{2} \begin{bmatrix} X_t - \overline{X}_t \\ U_t - \overline{U}_t \end{bmatrix} W_t \begin{bmatrix} X_t - \overline{X}_t \\ U_t - \overline{U}_t \end{bmatrix}$$

(1)

$X_t$ denotes an n-dimensional vector of state variables and $U_t$ denotes an m-dimensional vector of control variables. The n-dimensional vector $\overline{X}_t$ and the m-dimensional vector $\overline{U}_t$ denote the given desired levels of the state and control variables, respectively. If, S denotes the initial and $((T))$ the terminal period of the finite planning horizon, the matrix $W_t$ will be defined as:

$$W_t = \begin{bmatrix} W_{XX} & W_{XU} \\ W_{UX} & W_{UU} \end{bmatrix}$$

(2)

And so,

$$W_t = a^{t-1} \cdot W$$

(3)

Where, « $\alpha$ » is the discount factor? It is assumed $W$ is symmetric, and then:
\[
\begin{bmatrix}
W_t^X \\
W_t^U
\end{bmatrix} = -W_t \begin{bmatrix}
X_t \\
U_t
\end{bmatrix}
\] (4)

\[W_t^c = \frac{1}{2} \begin{bmatrix}
X_t \\
U_t
\end{bmatrix} W_t \begin{bmatrix}
X_t \\
U_t
\end{bmatrix}
\] (5)

So, (41) can be written in the «general quadratic form»:

\[
L_t(X_t, U_t) = \frac{1}{2} \begin{bmatrix}
X_t \\
U_t
\end{bmatrix}' W_t \begin{bmatrix}
X_t \\
U_t
\end{bmatrix} + \begin{bmatrix}
X_t \\
U_t
\end{bmatrix}' \begin{bmatrix}
W_t^X \\
W_t^U
\end{bmatrix} + W_t^c
\] (6)

The dynamic nonlinear system is given by the system of nonlinear difference equations:

\[
x_t = f(x_{t-1}, x_t, u_t, \hat{\Theta}, Z_t) + \varepsilon_t \quad t = s, ..., T
\] (7)

Where \( \Theta \) denotes a \( p \)-dimensional vector of unknown parameters \( Z_t \) denotes an \( l \)-dimensional vector of non-controlled exogenous variables and \( \varepsilon_t \) is an \( n \)-dimensional vector of additive disturbances. « OPTCON » requires as inputs:

The system function \( F(...) \), the initial values of the state variables \( X_{s-1} \equiv X*'s-1 \equiv X*'s-1 \), the tentative path of control variables \( (U_t^*)'_{t=s} \), the path of exogenous variables not subject to control \( (Z_t)'_{t=s} \), the expected values of system parameters \( \hat{\Theta} \), the covariance matrix of system parameters \( \Sigma_{\Theta} \), the covariance matrix of system noise \( \Sigma_{\varepsilon} \), the weighting matrices of objective function \( W_{xx}, W_{ux}, W_{u} \), the discount rate of objective function \( \alpha \), the target path for state variables \( (\bar{X}_t)'_{t=s} \) and the target path for control variables \( (\bar{U}_t)'_{t=s} \) algorithm is implementable in the statistical programming system
"GUASS". The optimal values of the control variables compute in an iterative fashion. So:

Step 1: Compute a tentative state path: use the gauss-seidel algorithm, the tentative control path \( (\overset{0}{U_t})^T_{t=S} \), and the system equation \( F(\ldots) \) to calculated the tentative state path \( (\overset{0}{X_t})^T_{t=S} \).

Step 2: Non linearity loop: Repeat the (a) to (e) until convergence is reached (i.e., until the optimal control and state variables do not change more than a prescribed small number.

From one iteration to the next or the number of iterations is larger than a prespecified number.

(a) Initialization for backward recursion:

\[
\begin{align*}
H_{T+1} &= 0_{nn} \quad , \\
h^{xT+1} &= 0_{n} \quad , \\
h^{cT+1} &= 0 \quad , \\
h^{sT+1} &= 0 \quad , \\
h^{pT+1} &= 0 \quad , \\
\end{align*}
\]

(b) Backward recursion:

Repeat the following steps (i) to (vii) for \( t = T, \ldots, S \).

(i) Compute the expected values of the parameters of the linearized system equation: \( X_t = A_{t} X_{t-1} + B_{t} U_{t} + C_{t} + e_{t} \):

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1- Showing the detailed prove and solution to the model equations will cause an excessive increase in the page numbers. Interested researchers can see the following reference:

\[ A_t = (I - f_{xt})^{-1} f_{x-1} \]  

(13)

\[ B_t = (I - f_{xt})^{-1} f_{ut} \]  

(14)

\[ C_t = X^*_t - A_t X^*_{t-1} - B_t U^*_t \]  

(15)

\[ \Sigma_t^{ee} = \text{COV}_{s-1}(e_t, e_t) = (I - f_{xt})^{-1} \Sigma^{ee} [(I - f_{xt})^{-1}]' \]  

(16)

Where all derivatives are evaluated at values \( X^*_t, X^*_t, \ldots, U^*_t, \hat{\theta}, Z_t \) and \( \varepsilon_t = 0 \).

(ii) Compute the derivatives of the parameters of the linearized system with respect to \( \theta \):

\[ D^{A_t} = \left[(I - f_{xt})^{-1} \otimes I_p \right] [f_{x,t,\theta} \otimes A_t + f_{x_{t-1},\theta}] \]  

(17)

\[ D^{B_t} = \left[(I - f_{xt})^{-1} \otimes I_p \right] [f_{x,t,\theta} \otimes B_t + f_{u_{t-1},\theta}] \]  

(18)

\[ d^{ct} = \text{vec} \left[(I - f_{xt})^{-1} f_0' \right] - D^{A_t} X^*_{t-1} - D^{B_t} U^*_t \]  

(19)

All derivatives are evaluated at the same reference values as above.

(iii) Compute the influence of the stochastic parameters: compute all the matrices the cells of which are defined by:

\[ [\gamma^{\text{AKA}}]_{i,j} = \text{tr} \left[ k_t D^{a_{t,j}} \sum_{\theta} \left( D^{a_{t,j}} \right)' \right], \quad i=1,\ldots,n \quad j=1,\ldots,n \]  

(20)

\[ [\gamma^{\text{BKA}}]_{i,j} = \text{tr} \left[ k_t D^{a_{t,j}} \sum_{\theta} \left( D^{b_{t,j}} \right)' \right], \quad i=1,\ldots,m \quad j=1,\ldots,n \]  

(21)

\[ [\gamma^{\text{BKB}}]_{i,j} = \text{tr} \left[ k_t D^{b_{t,j}} \sum_{\theta} \left( D^{b_{t,j}} \right)' \right], \quad i=1,\ldots,m \quad j=1,\ldots,n \]  

(22)

\[ [V^{\text{AKC}}]_i = \text{tr} \left[ k_t D^{ct} \sum_{\theta} \left( D^{a_{i,j}} \right)' \right], \quad i=1,\ldots,n \]  

(23)

\[ [V^{\text{BKC}}]_i = \text{tr} \left[ k_t D^{ct} \sum_{\theta} \left( D^{b_{i,j}} \right)' \right], \quad i=1,\ldots,m \]  

(24)
\[
[V_t^{CKC}] = \text{tr} \left[ k_t \ D^c t \ \Sigma^\theta \Psi ( D^c t )' \right] 
\] (25)

( iv ) Convert the objective function from « quadratic-tracking » to « general quadratic » format:

\[
W_t^{xx} = \alpha t^{-1} W^{xx}, 
\] (26)

\[
W_t^{ux} = \alpha t^{-1} W^{ux}, 
\] (27)

\[
W_t^{uu} = \alpha t^{-1} W^{uu}, 
\] (28)

\[
W_t^X = -W_t^{XX} \overline{x}_t - U_t^{XX} \overline{U}_t, 
\] (29)

\[
W_t^U = -W_t^{UX} \overline{x}_t - U_t^{UU} \overline{U}_t, 
\] (30)

\[
W_t^l = \frac{1}{2} x_t W_t^{XX} x_t + \overline{U}_t W_t^{UX} x_t + \frac{1}{2} \overline{U}_t W_t^{UU} \overline{U}_t 
\] (31)

(v) Compute the parameters of the function of expected accumulated loss:

\[
K_t = W_t^{XU} + H_{t+1}, 
\] (32)

\[
K_t = W_t^X + h_{t+1}^X, 
\] (33)

\[
\rho_{tXX} = \gamma_t \kappa \kappa + A_t' k_t A_t, 
\] (34)

\[
\rho_{tUX} = \rho_{tUX}' 
\] (35)

\[
\rho_{tUX} = \lambda_t^{BkB} + B_t' k_t A_t + W_t^{UX} A_t, 
\] (36)

\[
\rho_{tUU} = \lambda_t^{BkB} + B_t' k_t A_t ZB_t W_t^{UX} + W_t^{UU}, 
\] (37)

\[
\lambda_t^X = \nu_t^{AkC} + A_t' k_t C_t + A_t' k_t^X, 
\] (38)
\[ \lambda_t^{U} = v_t^{B_k} + B_k k_t c_t + B^e c_t + W_t^{UX} c_t + W_t^{U}, \]  
(39)

\[ \lambda_t^{S} = \frac{1}{2} \text{tr}(k_t \Sigma_t^{ee}) + h_t^{S}, \]  
(40)

\[ \lambda_t^{P} = \frac{1}{2} \gamma_t^{C_k} + h_t^{P} \]  
(41)

\[ \lambda_t^{C} = \frac{1}{2} c_t k_t c_t + c_t^X c_t + W_t^{C} + h_t^{C}, \]  
(42)

(vi) Compute the parameters of the feedback rule:

\[ G_t = -\left( \rho_t^{UU} \right)^{-1} \gamma_t^{UX}, \]  
(43)

\[ g_t = -\left( \gamma_t^{UU} \right)^{-1} \lambda_t^{U}, \]  
(44)

(vii) Compute the parameters of the function of minimal expected accumulated loss:

\[ h_t = \rho_t^{XX} - \rho_t^{XU} \left( \rho_t^{UU} \right) \rho_t^{UX}, \]  
(45)

\[ h_t^{X} = \lambda_t^{X} - \rho_t^{XU} \left( \rho_t^{UU} \right)^{-1} \lambda_t^{U}, \]  
(46)

\[ h_t^{C} = \lambda_t^{C} - \frac{1}{2} \left( \lambda_t^{U} \right)^{Y} \left( \rho_t^{UU} \right)^{-1} \lambda_t^{U}, \]  
(47)

\[ h_t^{S} = \lambda_t^{S}, \]  
(48)

\[ h_t^{P} = \lambda_t^{P}, \]  
(49)

(c) Forward projection:
Repeat (i) and (ii) for t=S, ..., T

(i) Compute the expected optimal control variables:

\[ U_t = G_t^* X_{t-1} g_t \]  
(50)
(i) Compute the expected optimal state: use the Gauss-seidel algorithm to compute $X_t$, such that:

$$X_t = f(\begin{vmatrix} X_{t-1}, X_t, U_t, \hat{\theta}, Z_t \end{vmatrix})$$  \hspace{1cm} (51)$$

$$\begin{pmatrix} X_t \end{pmatrix}_{t=s}^T = \begin{pmatrix} X_t \end{pmatrix}_{t=s}^T$$  \hspace{1cm} (52)$$

$$\begin{pmatrix} U_t \end{pmatrix}_{t=s}^T = \begin{pmatrix} U_t \end{pmatrix}_{t=s}^T$$  \hspace{1cm} (53)$$

(e) Compute the expected welfare loss:

$$J_s = X_{s-1}^t H_s X_{s-1}^t + X_{s-1}^t + h_{s-1}^X + h_{s-1}^c + h_{s-1}^p + h_{s-1}^s$$  \hspace{1cm} (54)$$

3-An Application of the "OPTCON" Algorithm for Iran third Development Plan.

In this section we used the "OPTCON" algorithm in order to determine the optimal fiscal policies for Iran during the third development plan. The constraint to the optimization problem is given by a macro econometric model of the Iran’s economy. The list of variables is shown in appendix. The dynamic nonlinear system includes two category of equation: behavioral equations and identities. The behavioral equations of the model were estimated by O.L.S using time series data\(^1\) for the period 1347-1378 behavioral equations:

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\(^1\) The real potential GDP and real capital stock data are estimated by methods used in the references:

\[ \text{CPR} = 0.9 \text{ CPR}_{t-1} + 0.01 \text{ YDR} + 728.5 \]
\[
\begin{array}{ccc}
21.7 & 3.4 & 2.1
\end{array}
\]
\[ \overline{R}^2 = 98 \quad \text{D.W.} = 2.02 \]

\[ \text{IMPR} = 0.8 \text{ IMPR}_{t-1} + 0.08 \text{ GDPR} - 0.98 \text{ ERP} \]
\[
\begin{array}{ccc}
7.2 & 2.2 & -1.8
\end{array}
\]
\[ \overline{R}^2 = 68 \quad \text{D.W.} = 1.8 \]

\[ \text{INVPR} = 2544 + 0.1 \text{ DEMAND} - 50.5 \text{ INTLR} \]
\[
\begin{array}{ccc}
9.04 & 13.1 & -6.1
\end{array}
\]
\[ \overline{R}^2 = 99 \quad \text{D.W.} = 2.1 \]

\[ \text{CPI} = 0.5 \text{ CPI}_{t-1} + 0.85 \text{ AGWN} + 0.17 \text{ UTIL} \]
\[
\begin{array}{ccc}
1.6 & 2.1 & 1.5
\end{array}
\]
\[ \overline{R}^2 = 99 \quad \text{D.W.} = 2.1 \]

\[ \text{AGWN} = 0.32 \text{ AGWN}_{t-1} + 0.5 \text{ CPI} - 0.7 \text{ UR} + 0.06 \text{ PROD} \]
\[
\begin{array}{ccc}
2.6 & 7.8 & -0.95
\end{array}
\]
\[ \overline{R}^2 = 99 \quad \text{D.W.} = 1.9 \]

\[ \text{EMP} = 0.95 \text{ EMP}_{t-1} + 0.01 \text{ GDPR} + 3.3 \text{ AGWR} \]
\[
\begin{array}{ccc}
18.1 & 1.1 & 1.6
\end{array}
\]
\[ \overline{R}^2 = 99 \quad \text{D.W.} = 1.9 \]

\[ \text{M} = 0.57 \text{ M}_{t-1} + 2.7 \text{ ERN} + 13647.7 \text{ PRICERAT} \]
\[
\begin{array}{ccc}
9.5 & 4.08 & 6.4
\end{array}
\]
\[ \overline{R}^2 = 99 \quad \text{D.W.} = 2.1 \]

\[ \text{INTLN} = 0.89 \text{ INTLN}_{t-1} + 0.000087 \text{ GDPR} - 0.000076 \text{ MR} \]
\[
\begin{array}{ccc}
5.9 & 1.6 & -2.1
\end{array}
\]
\[ \overline{R}^2 = 98 \quad \text{D.W.} = 1.8 \]

\[ \text{GDPPOT} = 0.17 \text{ CAPR} + 1.49 \text{ LFORCE} + 20.4 \text{ TIME} \]
\[
\begin{array}{ccc}
5.2 & 10.8 & 10.8
\end{array}
\]
\[ \overline{R}^2 = 99 \quad \text{D.W.} = 1.6 \]

\[ \text{GDPDEF} = 1.05 \text{ CPI} \]
\[
\begin{array}{ccc}
1.5
\end{array}
\]
\[ \overline{R}^2 = 97 \quad \text{D.W.} = 1.8 \]

\[ \text{EXPORTR} = 0.93 \text{ EXPORTR}_{t-1} + 0.54 \text{ ERR} \]
\[
\begin{array}{ccc}
21.27 & 1.76
\end{array}
\]
\[ \overline{R}^2 = 80 \quad \text{D.W.} = 1.9 \]

**Identities**

\[ \text{GDPR} = \text{CPR} + \text{INVPR} + \text{GIR} + \text{GCR} + \text{EXPORTR} \cdot \text{IMPR} \]

\[ \text{YDR} = \text{GDPR} - \text{TAXRR} \]
DEMAND=GDPR+IMPR
GGDD=((GDPR_t - GDPR_{t-1})/GDPR_{t-1})*100
GCPI=((CPI_t-CPI_{t-1})/GDPR_{t-1})*100
INTLR=INTLN-GCPI
PROD=(GDPR/EMP)*100
AGWR=(AGWN/CPI)*100
MR=(M/CPI)*100
ERR=ERN*CPIF/CPI
UN=LFORCE-EMP
UR=UN/LFORCE
PRICERAT=CPI/CPIF
CAPR=CAPR_{t-1} - DEPR + INVPR + GIR
UTIL=(GDPR/YPOT)*100
CAD=(EXPORTR-IMPR)/ERN
DEF=(GC+GI+DIG)-((NTAXRN)+(TAXRN))
DEFDAR=(DEF/GDP)*100
GDP=GDPR*GDPDEF/100
GCR=(GC/GDPDEF)*100
GIR=(GI/GDPDEF)*100
TAXRR=(TAXRN/GDPDEF)*100
The model includes goods, services market and money markets from the aggregate demand side and a production function and the labor market from the aggregate supply side. The goods and services market contain private consumption function private investment function, imports and exports functions. Also, the model includes a money demand equation and wage price system. The wage - price system can be regarded as an enhanced Phillips curve. Wages are determined by the price level, labor productivity and the unemployment rate. The price level depends on wages and the capacity utilization rate. A production function is included to determine potential GDP.

The labor market is modeled by specifying an employment equation, whereas the labor supply is exogenous to the model. We consider a fixed exchange rate regime in which the central bank will interest to fix the exchange rate of a $ in 8000 rials. It should be noted that under a flexible exchange rate system, monetary policy will be active. In other words, under these circumstances variables such as volume of money supply can be considered as an endogenous (control) variables. Therefore, the distinction between the effects of monetary and fiscal policy is not possible. So, in the present paper we used a fixed exchange rate system in which the fiscal policy effects could be evaluated without worrying about the monetary policy impacts. From technical point of view, this has been shown in nominal stock of money Equation in which this variable is considered as an endogenous variables. As the stochastic model equations are estimated by OLS, no full covariance matrix of the parameters is available. In this case, we introduce stochastic parameters. Here, the covariance matrix of the parameters to be diagonal and select only some diagonal elements have the lowest t-values are regarded as stochastic. These are the coefficient of \( c_{pt_{t-1}} \) and \( \text{UTIL} \) in the consumer price index equation, \( UR \) and \( \text{PROD} \) in the average nominal wage equation, \( \text{GDPR} \) and \( \text{AGWR} \) in the employment equation, \( \text{GDPR} \) in the interest rate equation and \( \text{CPI} \) in the GDP deflator equation.

The studies related to sensitivity analysis of optimization indicated there is not significant difference regarding the estimation methods determining the amount of control and objective variables. It should also be noted that choosing
a system method of estimation many violate the stationary and convergence conditions.\(^1\)

In order to determine the approximate solutions optimum government consumption and investment expenditures as well as optimum tax revenues (the optimum fiscal policy), three main and two minor objective variables were considered in objective function\(^2\). The main variables includes: The rate of economic growth, the inflation rate, the real GDP and the rate of unemployment. Also, the minor objective variables contain: the ratio of budget deficit to GDP and the current account balance. The values of target for these variables are the values, which targeted in Iran's third development plan. So, the planning horizon for the control experiments has been chosen as \(S=1379\) to \(T=1383\). After several experiments sensitivity analysis we have chosen a discount factor \(\alpha=1\), the weight 1000 for main and 1 for minor objective variables. Then, in the weight matrix of the objective function, off diagonal elements were all set equal to zero. In addition, all state variables in the model not mentioned above, got the weight zero.

In order to comparison the optimal fiscal policies effects on the main and minor objective function variables with that of the proposed fiscal policies in third plan, we used MAPLE program for the simulation of the model; we used GAUSS program to determine the optimal fiscal policies. The calculated optimum government consumption and investment expenditures and optimum tax revenues are compared with those proposed valued in third development plan in table1.

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\(^2\) The Matrix of the objective variables with their weights is shown in appendix B
Table 1: The Comparison of the Optimum Fiscal Policy Variables with to those Suggested by 3rd Development Plan. (In billions rials)

<table>
<thead>
<tr>
<th></th>
<th>1379</th>
<th>1380</th>
<th>1381</th>
<th>1382</th>
<th>1383</th>
</tr>
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<tbody>
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<td><strong>CG</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimal</td>
<td>83707.3</td>
<td>110866.2</td>
<td>148290</td>
<td>196888.9</td>
<td>256651.6</td>
</tr>
<tr>
<td>Proposed In third plan</td>
<td>69494.8</td>
<td>77918.8</td>
<td>96924.1</td>
<td>109356.1</td>
<td>134729</td>
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<td><strong>GI</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Optimal</td>
<td>65414.1</td>
<td>88544.1</td>
<td>122922.4</td>
<td>167875.1</td>
<td>227052.7</td>
</tr>
<tr>
<td>Proposed In third plan</td>
<td>50943.3</td>
<td>55352.6</td>
<td>71352.4</td>
<td>80203.4</td>
<td>105071.9</td>
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<tr>
<td><strong>TAXRN</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Optimal</td>
<td>24868.5</td>
<td>29359.2</td>
<td>35878.5</td>
<td>44779</td>
<td>60557.1</td>
</tr>
<tr>
<td>Proposed In third plan</td>
<td>32204.2</td>
<td>40060.3</td>
<td>49386.6</td>
<td>60130.1</td>
<td>72718.8</td>
</tr>
</tbody>
</table>

Source: Authors' calculations

The table shows that, except for the last year of the 3rd development plan, the optimum government consumption and investment expenditure are above those variables in 3rd plan. Also, the tax revenues are lower than the proposed in third development plan. On other hand, to achieve the targeted economic growth under fixed exchange rate regime, there will be a big pressure on government budget. Table 2 shows the target values and the results for the most important state variables of the simulation and optimization run, respectively.
<table>
<thead>
<tr>
<th></th>
<th>1379</th>
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<th>1381</th>
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<th>1383</th>
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<tr>
<td><strong>GGDP%</strong></td>
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<tr>
<td>Targets</td>
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<td>5.5</td>
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<td>5.0</td>
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<td>5.2</td>
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<td>2.3</td>
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<td><strong>GCPI%</strong></td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Targets</td>
<td>19.9</td>
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<td>15.3</td>
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<td>13</td>
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<td>23.1</td>
<td>22.2</td>
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<td>21.1</td>
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<tr>
<td>Simulation results</td>
<td>25.7</td>
<td>23.5</td>
<td>22.4</td>
<td>21.5</td>
<td>20.5</td>
</tr>
<tr>
<td><strong>UN%</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Targets</td>
<td>15.2</td>
<td>14.5</td>
<td>13.8</td>
<td>13.1</td>
<td>12.5</td>
</tr>
<tr>
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<td>14.7</td>
<td>17.9</td>
<td>20.7</td>
<td>23.4</td>
<td>26</td>
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<tr>
<td>Simulation results</td>
<td>14.4</td>
<td>17.4</td>
<td>20.2</td>
<td>23</td>
<td>25.8</td>
</tr>
<tr>
<td><strong>DEFDAR %</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Targets</td>
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<td>0.22</td>
<td>0.2</td>
<td>0.2</td>
<td>0.17</td>
</tr>
<tr>
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<td>7.7</td>
<td>12.5</td>
<td>14.6</td>
<td>18.9</td>
<td>19.6</td>
</tr>
<tr>
<td>Simulation results</td>
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<td>0.18</td>
<td>0.17</td>
<td>0.18</td>
<td>0.16</td>
</tr>
<tr>
<td><strong>CAT (In Billons Dollars)</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Targets</td>
<td>-0.312</td>
<td>-1.394</td>
<td>-1.234</td>
<td>-1.149</td>
<td>-0.701</td>
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<tr>
<td>Optimization results</td>
<td>0.207</td>
<td>0.027</td>
<td>-0.184</td>
<td>-0.449</td>
<td>-0.713</td>
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<tr>
<td>Simulation results</td>
<td>0.143</td>
<td>-0.059</td>
<td>0.273</td>
<td>0.504</td>
<td>-0.721</td>
</tr>
</tbody>
</table>

Source: Authors Calculations
As seen at the table, under fixed exchange rate regime the rate of economic growth is close to third plan targets with the optimum fiscal policies than those proposed by third plan. Also, under optimum fiscal policies the current account will be better. The inflation rate and unemployment rate will be the same both under optimal and proposed third plan fiscal policies. As we have shown in table1, the budget deficit is above under flexible exchange rate regime, policy makers can use the monetary instruments, since, and under this regime monetary instruments will be active. For this reason, there will be a lower pressure on government budget.

4- Concluding Remarks

In this paper we have described the ((OPTCON)) algorithm as proxy solutions to stochastic optimum control problems. Hence, the objective function is quadratic in deviations of the state and control variables from their respective desired values. Also, the constrain of optimization problem is given by a nonlinear stochastic system .We implemented the ((OPTCON)) algorithm in the programming language ((GAUSS)) and applied an econometric model to the Iran economy in order to show the feasibility of the algorithm. An empirical optimization result, show that the optimum fiscal policies may lead to a considerable stabilization of the time path of the rate of economic growth. Also, under fixed exchange rate regime, the money supply has to be adjusted so as to hold the nominal exchange rate constant .It is also shown that the executing optimal fiscal policies put pressure on government budget due to the above results. Therefore it is recommended to implement flexible exchange rate systems for the fourth development plan since in these systems; monetary tools are planning policies instruments. Based on the finding of these papers we recommend applying ((OPTCON)) algorithm to the fourth development plan of Iran.
References


## Appendix A: List of Variable

<table>
<thead>
<tr>
<th>State (or endogenous) variables</th>
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</thead>
<tbody>
<tr>
<td>GDPR</td>
</tr>
<tr>
<td>YDR</td>
</tr>
<tr>
<td>DEMAND</td>
</tr>
<tr>
<td>GGDP</td>
</tr>
<tr>
<td>GCPI</td>
</tr>
<tr>
<td>INTLR</td>
</tr>
<tr>
<td>PROD</td>
</tr>
<tr>
<td>AGWR</td>
</tr>
<tr>
<td>MR</td>
</tr>
<tr>
<td>ERR</td>
</tr>
<tr>
<td>UN</td>
</tr>
<tr>
<td>UR</td>
</tr>
<tr>
<td>PRICERAT</td>
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<tr>
<td>CARP</td>
</tr>
<tr>
<td>UTIL</td>
</tr>
<tr>
<td>EXPORTR</td>
</tr>
<tr>
<td>CAD</td>
</tr>
<tr>
<td>DEF</td>
</tr>
<tr>
<td>DEFDAR</td>
</tr>
<tr>
<td>CPR</td>
</tr>
<tr>
<td>IMPR</td>
</tr>
<tr>
<td>INVPR</td>
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<tr>
<td>CPI</td>
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<td>AGWN</td>
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<tr>
<td>EMP</td>
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<tr>
<td>M</td>
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</table>
### List of variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTLN</td>
<td>Nominal long run term interest rate</td>
</tr>
<tr>
<td>GDPPOT</td>
<td>Potential GDP, real</td>
</tr>
<tr>
<td>GDPDEF</td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>Gross domestic product, nominal</td>
</tr>
<tr>
<td>GCR</td>
<td>Government consumption expenditures, real</td>
</tr>
<tr>
<td>GIR</td>
<td>Government investment expenditures, real</td>
</tr>
<tr>
<td>TAXRR</td>
<td>Tax revenues, real</td>
</tr>
</tbody>
</table>

### Exogenous variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
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</thead>
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<tr>
<td>ERN</td>
<td>Nominal exchange rate</td>
</tr>
<tr>
<td>LFORCE</td>
<td>Labor force</td>
</tr>
<tr>
<td>DEPR</td>
<td></td>
</tr>
<tr>
<td>CPIF</td>
<td>U.S.A CPI</td>
</tr>
<tr>
<td>NTAXRN</td>
<td>Government non tax revenues</td>
</tr>
<tr>
<td>TIME</td>
<td>Linear time trend</td>
</tr>
<tr>
<td>DIG</td>
<td>Difference between the sum of government expenditures with GC + GI</td>
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</table>

### Control variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>GI</td>
<td>Government investment expenditures, nominal</td>
</tr>
<tr>
<td>GC</td>
<td>Government consumption expenditures, nominal</td>
</tr>
<tr>
<td>TAXRN</td>
<td>Tax revenues, nominal</td>
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</table>
Appendix B: The Objective Function the Following Table Show the Variables of the Objective Function.

<table>
<thead>
<tr>
<th>variable</th>
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<td>GGDP%</td>
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<td>4.5</td>
</tr>
<tr>
<td>GCPI%</td>
<td>1000</td>
<td>19.9</td>
</tr>
<tr>
<td>UN%</td>
<td>1000</td>
<td>15.2</td>
</tr>
<tr>
<td><strong>Min</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DEFDAR%</td>
<td>1</td>
<td>0.18</td>
</tr>
<tr>
<td>CAT%</td>
<td>1</td>
<td>-0.312</td>
</tr>
</tbody>
</table>