Iraninan Economic Review, Vol. 7, No.7, Fall 2002

Measuring Productivity and Efficiency Via the Production Function

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Abstract

This paper is a survey of recent contributions to and developments in the relationship between efficiency and productivity via the production function. The objective is to provide a brief discussion on data and on the methods of measuring efficiency and productivity. First, we introduce the reader to the measurement of partial and total factor productivity in the context of static and firm – specific modeling. Second, we survey the econometric approach to efficiency analysis. Third, the relationship between productivity growth is reviewed.

Key words: Production, Return to Scale, Total Factor Productivity , Technical Change, Panel Data, Stochastic Frontier.

1- Introduction

For several decades, productivity efficient and growth in production have been popular topics of both theoretical and empirical research. The empirical evidence is often based on aggregate data at country or industry

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level mainly applicable to country and regional studies; the productivity and efficiency studies are mainly micro oriented. These issues are frequently used in performance studies concentrated on agriculture, manufacturing and are relatively scarce in services, especially in view of the major role of the service sector in our modern developed economies.

This paper does not intend to provide a comprehensive review of the voluminous literature on the subject, but instead it seeks to briefly overview the literature on performance measurement. It will illustrate the progress achieved in this class of problems namely the issues of data, models, functional forms, estimation methods and results. It will also present a list of production sector relevant research topics and methods. The main focus is on the empirical analysis of productivity, efficiency and growth in manufacturing at the micro level. Section 4 summarizes the progress and usefulness of these techniques[1].

2 – Productivity Measurement

2 – 1 – Measurement of TFP Growth

Production function modeling is a crucial tool in analyzing returns to scale, technical change and productivity growth. Annual change in productivity has influence on organization and influence on regulatory structures, privatization, deregulation, etc.

Measurement of productivity is based on the ratio of some function of outputs (Y_m) to some function of inputs (X_j). The subscripts m and j denote types of outputs and inputs, respectively. In cases with single output, partial factor productivity (PFP) indices describing the average product of input factor j (j = capital, labor, material, energy, land and service) are constructed as:

$$PFP_j = Y/X_j \quad (1)$$

Partial factor productivity is often referred to as 'single factor productivity'. However, this can be misleading and consequently should be avoided. Productivity is negatively related to the factor intensity and changes in shares of production factors. In order to account for changes in input mixes a total factor productivity (TFP) measure is defined as the ratio of output to the weighted sum of inputs:

1- Kind assistance from Almas Heshmati is appreciated .

$$TFP = Y / \Sigma_j \alpha_j X_j. \quad (2)$$

The TFP is measured as changes over time or relative to other firms in a given period where changes are compared to some reference time or firm as:

$$TFP_{t,t-1} = (Y_{it} / X_{it}) / (Y_{it-1} / X_{it-1}) \text{ and } TFP_{i,j} = (Y_i / X_i) / (Y_j / X_j). \quad (3)$$

The first productivity measure is credited to Tinbergen (1942). Following Tinbergen it has been modified by Solow (1956, 1957)[1], Kendrick (1961), and others. The productivity growth, (TFP), over two points in time (0 and 1) following Kendrick (1961) is measured as the ratio of the TFP measures:

$$\overset{\circ}{TFP} = \Delta TFP / TFP = [((Y_1 / Y_0) / (\Sigma_j W_j X_{j0} / \Sigma_j W_j X_{j1})) - 1] \quad (4)$$
$$= [(Y_1 / Y_0) / (\Sigma_j \alpha j (Xj1 / X_{j0})]$$

where $\Delta TFP = TFP_{t+1} - TFP_t$ is change in TFP, W is input price, α_j is the cost minimizing expenditure share for inputs j and 0 denote the reference time period. TFP growth can be decomposed into technical change and scale components.

2-2- Decomposition of TFP Growth

Diewert (1981) classified the various measures of technical change into four groups: (i) econometric estimation of production and cost functions, (ii) Divisia indices, (iii) exact index numbers, and (iv) non-parametric methods using linear programming. In this survey we focus on the first approach. In the econometric approach, technical change has generally been represented by a simple time trend. Estimates of rate of technical change are then calculated as the percentage change in production or cost over time. With the advent of flexible functional forms (Christensen, Jorgenson and Lau) (1973), the simple time trend representation of technical change has been modified to include time squared and interactions between

1- Solow (1957) assumed Cobb Douglas as functional form and constant returns to scale.

time and the other explanatory variables. A time trend approach is attractive in the analysis of manufacturing or industrial production, where long–run technical change is mainly determined by capital equipment; while short–run changes in productivity are caused by cyclical factors.

Access to panel data allows a much more detailed evaluation of the relative performance of micro units and, therefore, a richer specification of technical change. Time trend representation of the rate of technical change is quite restrictive. The rate of technical change is smooth and slowly increasing, constant or decreasing over time. This model cannot capture the erratic patterns of technical change such as sudden switches in productivity growth from progress to regress and back to progress. A general index of technical change was introduced by Baltagi and Griffin (1988) where the time trend is replaced by a vector of time dummies to overcome the limitations of time trend. They argue for the advantages of the general index over the time trend model in measuring technical change. In the following only the econometric and Divisia index approaches are discussed. Let the production function be characterized by:

$$Y = f\,(X, t) \tag{5}$$

where Y is output, X is a vector of J input variables, and t is a time trend variable. Taking the total differential of (1) we get:

$$\overset{\circ}{Y} = \Sigma_j\,(f_j\,X_j\,/\,Y)\,X_j + (f_t\,/\,Y) \tag{6}$$

where a dot indicates growth rate and f_j is the marginal product of the jth input. The relationship can be rewritten as:

$$\overset{\circ}{Y} - \Sigma_j\,S_j\,X_j = (RTS - 1)\;\Sigma_j\,S_j\,X_j + (f_t\,/\,Y) \tag{7}$$

where S_j is the cost share of input j and RTS is returned to scale, respectively. The left–hand side is the Divisia index of total factor productivity growth expressed as:

$$TF\overset{\circ}{P} = \overset{\circ}{Y} - \Sigma_j\,S_j\,X_j \tag{8}$$

where only the growth rates in inputs and outputs and the cost shares are required for the calculation of the TFP growth index. Constant return to scale is assumed. In the absence of prices, the TFP growth estimates can be obtained by estimating a production function using econometric methods. The main advantage of using a parametric approach over the non–parametric approach of Divisia index is that by allowing for variable returns to scale one can decompose TFP growth into technical change, scale components (and in certain cases assuming flexible functional forms input and scale biases components) as:

$$\overset{\circ}{TFP} = TC + (RTS - 1)\,\Sigma_j\, \beta_j\, X_j \qquad (9)$$

where TC denotes the rate of technical change obtained from the log derivative of output with respect to time and β is a vector of parameter estimates of the production function. A positive (negative) rate of TC in production (cost) function approach indicates technical progress manifested by a positive (negative) shift in the production (cost) function over time. Details on a more general decomposition of TFP into contributions from technological change, changes in technical and allocative efficiency, effects of non–marginal cost pricing and effects of non–constant returns to scale are found in Balk (1998).

Assuming a cost function approach where the firms minimize the cost for given output quantity, input prices and technology, C = g (Y, W, t) the TFP growth rate is written as:

$$\overset{\circ}{TFP} = - TC + (1-RTS^{-1})\,\overset{\circ}{Y}\,. \qquad (10)$$

2-3 - Selected TFP Applications

In general the components of TFP growth are constant across firms and over time. The parametric time trend approach is generalized by Baltagi, Griffin and Rich (1995), Kumbhakar, Heshmati and Hjalmarsson (1999) and Kumbhakar, Nakamura and Heshmati (2000) to incorporate firm–and time–specific rate of technical change and scale components of TFP growth. For other approaches incorporating the issues of capacity utilization and dynamics, see Good, Nadiri and Sickle (1997). Crepon, Duguet and Mairesse (1998) examine the relationship between research, innovation and productivity in French manufacturing. The issues of selection, simultaneity, specification and estimation are discussed as well. They find that using the more widespread methods, the more usual data and model specifications, may lead to sensibly different estimates. One specific finding is that simultaneity and selectivity tend to bias the results.

In a recent essay Hulten (2000), explains the origins of the growth accounting and productivity methods. He discusses the importance of TFP in the process of economic growth and the controversy about measurement methods and underlying assumptions. TFP is estimated as a residual measuring of our ignorance, possible measurement errors, unmeasured gains in product quality and environmental costs of growth. Hulten classifies the recent developments as: (i) the growing preference for econometric modeling of the factors causing productivity change, (ii) the shift in aggregate and industry level productivity studies to firm and plant level, (iii) a shift in emphasis from competitive models to non – competitive models of industrial organization.

Bartelsman & Doms (2000) present another excellent review of research using longitudinal micro-data to measure productivity changes and to examine factors causing growth. The authors discuss the issues of dispersion of productivity growth, persistence of productivity differentials, the consequences of exit, entry and resource allocation on productivity growth, and finally how factors are correlated and their causality with productivity growth are explored. Possible areas for future work in productivity research include: reasons for heterogeneity, non–manufacturing sectors, linking data on workers skills with their place of work, data quality, errors in variables, statistical properties of linked data, market structure, cross country productivity comparisons from micro-data, and increased micro-macro linkage.

3- Efficiency in industry

3-1- Measurement of Efficiency

In empirical studies, production function have been traditionally described as average function estimating the mean output rather than the maximum output. However, the maximum possible output is relevant in measuring the performance of firms. Farrell (1957) provided a definition of frontier production function which embodies the idea of maximality. The measurement of efficiency has been the main motivation for study of frontier functions. The frontier is used to measure the efficiency of production units by comparing observed and potential outputs. Potential output is obtained using best practice technology from a given vector of inputs.

The literature on the estimation of frontier functions to measure economic efficiency of firms has been developed in different directions. The different approaches of production, cost and profit frontiers are used to estimate the components of economic efficiency, i.e. technical and allocative efficiencies. The former is a measure of possible reduction in inputs to produce a given level of output or alternatively potential increase in output for given level of input usage, while the latter is a measure of the possible reduction in the cost of using the correct input proportions.

3-2- Stochastic Frontier Functions

Frontier functions can be classified according to the way the frontier is specified and estimated. The classification might be based on the parametric /non–parametric, deterministic/stochastic and cross–section / panel data specifications of the frontier functions. Schmidt (1986), Greene (1997) and Kumbhakar and Lovell (2000) present an overview of the concept, modeling, estimation of models and methods involved in making efficiency comparisons. In addition to this, they also survey some of the empirical applications of frontier functions.

This section focuses on the parametric stochastic frontiers. The stochastic production frontier model introduced by Aignar, Lovell and Schmidt (1977) is defined as:

$$\ln Y_i = \beta_0 + \Sigma_j \beta_j \ln X_{ji} + \varepsilon_i , \qquad \varepsilon_i = v_i - u_i \qquad (11)$$

wherein y_i is logarithm of output of firm i, X_j is a vector of logarithm of J inputs, and β is vector of unknown parameters to be estimated. The error term ε_i is composed of two components, a symmetric random component (v_i # 0), and a one–sided component (u_i ? 0) representing technical inefficiency. The frontier is stochastic allowing for variation of frontier across firms. The stochastic frontier models can be estimated by corrected ordinary least square, methods of moments, generalized least square or maximum likelihood methods. The random component is assumed to be independently and identically normally distributed while the inefficiency component assumed to be distributed as either exponential, half – normal, truncated normal or gamma. The estimated model gives an aggregate fitted value of the two components. Measures of the firm–specific rate of technical inefficiency require decomposition of the error term. Jondrow, Lovell, Materov and Schmidt (1982) have suggested a decomposition method to obtain point estimates of û using the mean or mode of the conditional distribution $E(u_i \mid v_i - u_i)$. Technical efficiency is then obtained from the following relation:

$$TE_i = \exp(-\hat{u}_i). \qquad (12)$$

Technical efficiency lies in the interval 0 and 1, 0 ? TE ? 1, where 1 indicates full efficiency. The previously defined frontier (11) was set up in the case where cross–sectional data, i.e. when data on a number of firms observed for a single period is available. If each firm is observed across a number of time periods, the data is referred to as panel data. The stochastic frontier production model (11) in a panel data context is rewritten as:

$$\ln Y_{it} = \beta_0 + \Sigma_j \beta_j \ln X_{jit} + \varepsilon_{it}, \qquad \varepsilon_{it} = v_{it} - u_{it} \qquad (13)$$

where i (i =1, 2, 3, ..., N), t(t = 1,2, ..., T) and j indexes firm, period and inputs. Panel data models in the stochastic production / cost frontier literature can be divided into two main groups. The first group assumes technical inefficiency to be time–invariant, ($u_{it} = u_i$) for $\forall i$. The second group allows technical inefficiency to be time–varying. Each of these two groups can further be classified into two sub–groups depending on whether any distributional assumptions are imposed on the error components or not.

4- Conclusion

The main focus of this paper is on the empirical analysis of the relationship between productivity and efficiency at the micro level. At first, different parametric and non-parametric approaches to the productivity measurement are discussed. Second, the econometric approach to efficiency analysis is surveyed followed by a discussion of issues related to modeling and estimation methods. Analyses should preferably be performed at micro level and based on panel data for producers. Panel data has the advantage of analyzing the unobserved heterogeneity and temporal patterns of performance. Using various methods, data and model specifications may lead to sensibly different estimates.

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