Administrative rationing and multiple equilibria

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ABSTRACT

In this paper, we introduce non-interest profit/loss sharing (PLS) banks into a three-period overlapping generations model. Two types, (low- and high quality) investments are assumed and capital accumulation and level of output in the steady state is considered. Then, administrative rationing is introduced to the model. It is shown that the level of capital accumulation and output in equilibrium are lower compared to the market solution.

1. Introduction

There is a fairly large body of literature on growth and financial development indicating that the extent of financial intermediation in an economy is a significant determinant of its real growth rate (Levine, 2000). The purpose of this paper is to construct a model in which the behavior of competitive intermediaries (i.e. banks) affects resource allocation, and influences the steady-state capital intensity and per capita output. This paper will provide a general equilibrium approach to show that economies with competitive intermediaries will grow faster than the economies in which the government intervenes in the market and allocates credits through an ad.hoc. (i.e. non-market) rules. This negative consequence is indicated to come from the negative impact of saving mobilization by the intermediaries due to the government intervention. We postulate a cooperative intermediaries with a direct relationship between providers of funds and users of funds. Intermediaries in this model are assumed to accomplish two major functions: a) introducing the two parties to each other; b) provision of

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the advantages of the law of large numbers to savers.

The remainder of this article is divided into four sections. Section two sets up the basic model and Section 3 studies the effect of introducing administrative rationing into the model. Section 4 extends the model for a life time expected utility framework and section 5 provides summary and conclusion.

2. A simple model of overlapping-generations with financial intermediaries

A. Assumptions

The economy consists of three-period-lived overlapping-generations so that each individual lives for three periods. The population composition is subject to change: new individuals are continually born and old individuals are continually dying. Time is discrete so that the variables of the model are defined for \( t=1,2,... \)

\( L_t \) individuals are born in period \( t \). Young generations are identical and each contains a continuum of identical agents. Each individual supplies one unit of labor when he is young. \( c_i \) denotes age \( i \) consumption. We assume that all young period income is saved. Thus the utility of an individual born at \( t \), denoted \( U_t \), depends on \( c_{t+1} \) and \( c_{t+2} \). We assume constant-relative-risk aversion utility:

\[
u(c_1, c_2, c_3) = \ln(c_2, \phi c_3) \tag{1}\]

Where \( \phi \) is an individual-specific random variable realized at the beginning of age 2. \( \phi \) has the following probability distribution

\[
\phi = \begin{cases} 
0 & \text{with probability } 1-\pi \\
1 & \text{with Probability } \pi 
\end{cases} \tag{2}
\]

Where \( \pi \in (0,1) \). This formulation of preferences follows that of Diamond and Dybvig (1983) and Bencivenga and Smith (1991, 1992). Finally, it is assumed that only agents with \( \phi=1 \) can operate firms; (a fraction \( \pi \) of old agents are entrepreneurs.

B. Production

Production is described by the following assumptions. A non-storable consumption good is produced using capital and labor. As in OG models, all
capital is owned by a subset of old agents called "firms". There are many firms, each with a constant returns to scale production function $y_t = K_t^\theta L_t^{1-\theta}$; $\theta \in (0, 1)$. $y_t$ has constant return to scale. Markets are competitive; thus labor and capital receive their marginal products and firms earn zero profits. The real interest rate and the wage per unit of effective labor are

$$r_t = f_t' \left( k_t \right)$$  \hspace{1cm} (3)

$$w_t = f_t \left( k_t \right) - k_t f_t' \left( k_t \right)$$  \hspace{1cm} (4)

Capital is equally owned by all old individuals. Thus, in period zero, the capital owned by the old and labor supplied by the young are combined to produce output. Production takes place at the end of the period $t+2$. The old agents consume their capital income. They then die and exit the model.

C. Labor market

At each date, each age 3 whit $\phi=1$ operates a firm (Bencivenga and Smith, 1991). These firms have the same amount of capital; $k_t$. Taking this capital stock and the wage rate $w_t$ as given, each firm chooses an employment level $L_t$ to maximize $K_t^\theta L_t^{1-\theta} - w_t L_t$ which yields the profit maximization choice of $L$ as below

$$L_t = k_t \left[ \frac{(1-\theta)}{w_t} \right]^{\frac{1}{\theta}}$$ \hspace{1cm} (5)

Equation (5) gives per firm labor demand. The firm owner gets a $(\theta y_t)$ amount of the value of production. Substituting (5) into the production function gives the return to capital at $t$:

$$\theta y_t = \theta k_t \left[ \frac{(1-\theta)}{w_t} \right]^{(1-\theta)/\theta}$$ \hspace{1cm} (6)

Per firm labor supply is $1/\pi(1)$. Equating $L_t$ from (4) with $1/\pi$ gives the labor market equilibrium condition.

$$w_t = (1-\theta) \pi^{\theta} k_t^{\theta}$$ \hspace{1cm} (7)

1. Only a fraction of old agents are entrepreneurs, each of whom hires $L_t$ units of labor. For example, when $\pi=0.25$, it says that each firm can hire $1/\pi = 100/25$ of labor.
D. Portfolio decision

Young agents allocate their saving among money, capital (direct investment), and bank deposit. We assume no information cost and no information assymetry. If an individual places one unit of his saving in money, he will get \( P_t / P_{t+1} \) after one period. Where \( P_t \) and \( P_{t+1} \) are the price level at periods \( t \) and \( t+1 \), respectively. Therefore, \( P_t / P_{t+1} \) is the present value of one unit of money in period \( t+1 \). We assume \( P_t / P_{t+1} \leq 1 \).

All holding of money will be liquidated after one period.

Banks promise that for each unit of income deposited in period \( t \), \( r_1 \) will be paid if money is withdrawn after one period. If this occurs after two periods, \( r_2 \) will be paid per unit of income deposited. Using equation (6), individuals who withdraw after two periods earn the return

\[
 r_2 \theta K_t \left[ \frac{(1-\theta)}{w_{t+2}} \right]^{(1-\theta)/\theta} \text{ per unit of money deposited.}
\]

It is assumed that there are two kinds of projects that entrepreneurs can undertake. When credit is not administratively rationed, entrepreneurs will invest in normal-return projects with \( R^n \). But when administrative rationing is imposed, a fraction of the banks' loan will go to the favored projects with low return, \( R^l \), and a fraction \( (1-\alpha) \) of the loan will go to the projects with normal return; \( R^n \). In the absence of operating costs and with a competitive banking system, \( r_2 \) will be equal to the rate of return offered on deposits; \( r_2 = \left[ \alpha R^l + (1-\alpha)R^n \right] \). These payments are shown in table 1. At time \( t \), each young agent has a real income \( w_t \) all of which is saved. Let \( \lambda_1 \) be the fraction of young period saving placed in bank deposit, \( \lambda_2 \) be the fraction held in money and \( \lambda_3 = 1 - \lambda_1 - \lambda_2 \) be the fraction held in the form of capital \( \lambda_3 \in [0,1] \).

Table 1. Payments

<table>
<thead>
<tr>
<th>( \lambda_i ):</th>
<th>Saving at ( t )</th>
<th>Returns at ( t+1 )</th>
<th>Returns at ( t+2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deposit</td>
<td>1</td>
<td>( r_t, \theta )</td>
<td>( 0, r_2 \theta w_t \left[ \frac{(1-\theta)}{w_{t+2}} \right]^{(1-\theta)/\theta} )</td>
</tr>
<tr>
<td>Money</td>
<td>1</td>
<td>( \frac{P_t}{P_{t+1}} )</td>
<td>0</td>
</tr>
<tr>
<td>Capital</td>
<td>1</td>
<td>0</td>
<td>( \left[ \alpha R^l+(1-\alpha)R^n \right] \theta w_t \left[ \frac{(1-\theta)}{w_{t+2}} \right]^{(1-\theta)/\theta} )</td>
</tr>
</tbody>
</table>
When an individual holds one unit of saving in bank deposit there is a probability of $\pi$ that he will not withdraw it until the end of period $t+2$ and receive $\theta y_t$. He also may need the money meanwhile and withdraw the deposit after one period with the probability of $1-\pi$ and receive $r_t$.

When $\phi=1$, direct investment is possible, because this is the only case that a firm can be operated. If $\phi=1$, one unit of saving placed in the normal-return (low-return) investment project will give $R^a$ ($R^l$) of capital at $t+2$. Using (6), the value of this capital is:

$$
\alpha R^l \theta k_t \left[ \frac{(1-\theta)}{w_{t+2}} \right]^{(1-\theta)/\theta} + (1-\alpha) R^a \theta k_t \left[ \frac{(1-\theta)}{w_{t+2}} \right]^{(1-\theta)/\theta}
$$

(8)

The return of direct investment in period $t+1$ is assumed to be zero(1). The utility maximization problem of young individuals is:

$$
{u}_t = \left[ 1 - \pi \right] \ln (c_2) + \pi \ln (c_3)
$$

(9)

$$
c_2 = \lambda_1 r_1 w_t + \lambda_2 \frac{p_t}{p_{t+1}} w_t
$$

$$
c_3 = \lambda_1 \theta y_{t+2} + \lambda_2 \frac{p_t}{p_{t+1}} w_t + \lambda_3 \theta y_{t+2}
$$

Substituting $c_2$ and $c_3$ in (9) yields

$$
u = (1-\pi) \ln \left[ \lambda_1 r_1 w_t + \lambda_2 \frac{p_t}{p_{t+1}} w_t \right] + \pi \ln \left[ \lambda_1 \theta y_{t+2} + \lambda_2 \frac{p_t}{p_{t+1}} w_t + \lambda_3 \theta y_{t+2} \right]
$$

(10)

$\theta y_{t+2}$ is the share of firm owner from the output at $t+2$ and $\theta y_{t+2}$ can be obtained as function of rate of return and $w_{t+2}$. Using equation (6) yields.

$$
\theta y_t = \theta k_t \frac{L_t}{L_{t+1}}^{1-\theta}
$$

(6)

Substituting (5) in (6) yields,

$$
\theta y_t = \theta k_t \left[ 1 - \theta \right]^{w_t}
$$

1. The production process takes two periods. If a firm liquidates the investment project after one period, it will receive the scrap value of the investment which is assumed to be zero here.
The share of firm owner from the output at $t+2$ is:

$$\delta y_{t+2} = \delta k_{t+2} \left[ (1-\theta)/w_{t+2} \right]^\gamma$$

(11)

Where $k_{t+2} = rw_t$. Thus, we can rewrite (11) as

$$\delta y_{t+2} = \delta k_{t+2} \left[ (1-\theta)/w_{t+2} \right]^\gamma$$

(11')

Substituting (11') in (10), we have the following utility maximization problem

$$u_t = (1-\pi)\ln \left[ \lambda_1 r_1 w_t + \lambda_2 \frac{P_t}{P_{t+1}} w_t \right] + \pi \ln \left[ \lambda_1 r_2 \theta \left[ (1-\theta)/w_{t+2} \right]^\gamma w_t \right]$$

$$+ \lambda_2 \frac{P_t}{P_{t+1}} w_t + \left[ 1 - \lambda_1 - \lambda_2 \right] R^n \theta \left[ (1-\theta)/w_{t+2} \right]^\gamma w_t$$

(12)

Or,

$$u_t = \ln(w_t) + (1-\pi)\ln \left[ \lambda_1 r_1 + \lambda_2 \frac{P_t}{P_{t+1}} \right] + \pi \ln \left[ \lambda_1 r_2 \theta \left[ (1-\theta)/w_{t+2} \right]^\gamma \right]$$

$$+ \lambda_2 \frac{P_t}{P_{t+1}} + \left[ 1 - \lambda_1 - \lambda_2 \right] R^n \theta \left[ (1-\theta)/w_{t+2} \right]^\gamma$$

(13)

Where $\gamma = (1-\theta)\theta$. Assuming $r_2 \theta \left[ (1-\theta)/w_{t+2} \right]^\gamma > r_1$ and $\left( P_t / P_{t+1} \right) \leq 1$ and $\left( P_t / P_{t+1} \right) \leq r_1$ then $\lambda_2 = 0$. In this case, bank deposits dominate real cash balances as the asset of choice for young savers. In this section, we assume that the financial market is competitive and there is no administrative rationing, then $\alpha = 0$. Therefore, the utility function (12) can be written as follows:

$$u = \ln(w_t) + (1-\pi)\ln \lambda_1 r_1$$

$$+ \pi \ln \left[ \lambda_1 r_2 \theta \left[ (1-\theta)/w_{t+2} \right]^\gamma + \left[ 1 - \lambda_1 \right] R^n \theta \left[ (1-\theta)/w_{t+2} \right]^\gamma \right]$$

(14)

The solution to (14) sets:

$$\lambda_1 = \frac{\left( 1 - \pi \right) R^n}{R^n - r_2}$$

(15)
According to table 1, in a competitive credit market, one unit of direct investment yields \( R^n \theta k_t \left[ \frac{(1-\theta)}{w_{t+2}} \right]^{(1-\theta)/\theta} \) but the return of one unit of bank deposit is \( r_1 + r_2 \theta k_t \left[ \frac{(1-\theta)}{w_{t+2}} \right]^{(1-\theta)/\theta} \). Since banks are corporative entities of funds owned by young individuals, projects to the depositors in the second period. So, we can conclude that \( r_1 + r_2 \theta k_t \left[ \frac{(1-\theta)}{w_{t+2}} \right]^{(1-\theta)/\theta} > R^n \theta k_t \left[ \frac{(1-\theta)}{w_{t+2}} \right]^{(1-\theta)/\theta} \). Therefore, young agents hold all their savings as bank deposits. This means \((\lambda=1)\). This is equal to the condition \(\pi R^n = r_2\).

E. Intermediary behavior

In the context of an OLG model, intermediaries' behavior can be described as follows. Intermediaries accept deposits from young savers at \( t \) and use them to lend to the firms or keep a fraction of the deposits in money balances. Bank investment takes the form of lending to the firms. We assume a competitive profit loss sharing banking system, instances of which are Islamic and cooperative banks. The bank is viewed as a cooperative entity (like a coalition formed by young agents at \( t=1 \)) which maximizes the expected utility of a representative depositor evaluated at time \( t=1 \). The model of cooperative intermediaries provides a direct relationship between providers of funds and users of funds. Intermediaries in this model introduce two parties to each other and provide the advantages of the law of large numbers to savers.

Since Islam prohibits interest and allows profit, the model of Islamic banking and financial institutions have been built on profit/loss sharing principle. The Islamic instruments for productive cooperation between entrepreneurs and providers of capital are the various financial contracts which provide proportionately sharing of profit and loss (PLS scheme) such as Musharakah and Mudarabah. As far as banks and financial institutions are concerned, the scope for provision of Qard-al-Hassane (i.e. interest free loan) is limited. Thus, the major emphasis has to be on Musharakah and

mudarabah financing modes.

In Musharakah contract, all partners participate in profit according to the ratios that they agreed in advance. These ratios may or may not be in proportion to their capital contributions. However, if there is a loss, it is shared by the partners strictly in proportion to the capital contributions.

Mudarabah is basically a partnership between two parties: one party, called Rab-al-Mal, provides the finance and the other party called Mudarib, works or manages the business. This is the conventional Mudarabah.

It is also possible to set up a two-tier Mudarabah. In this case, Mudarib may mobilize funds from one Rab-al-Mal or more and he may use them to provide finance to other entrepreneurs on the basis of Musharakah or Mudarabah or any other financing modes permissible in Shariah as may be agreed between the provider and the user of finance.

The two-tier Mudarabah is considered to be the ideal model for setting up Islamic banks and other financial institutions whose main function is financial intermediation. They mobilize deposit from the public on the basis of Mudarabah and provide finance to entrepreneurs (Elgari, 1997). However, Islamic banks may also accept current account deposits. In this regard, the relationship between depositor and the bank is that of lender and borrower. These deposits are in the nature of Qard-al-Hassane. The principal amount is to be returned to the depositors in full and on demand. In our model of financial intermediation and growth, the amount of deposit withdrawn after one period is considered to be demand deposit.

In the case of investment deposits (deposits which stay in bank for two periods), based on Mudarabah or Musharakah, the PLS principle applies. Since the bank acts as a Mudarib under this arrangement, the loss (if only) will have to be borne by the depositors. However, the possibility of a net loss in the case of banks and other financial intermediaries is weak. Because, there will be diversified financing operations in such institutions. On the average, there will be no net loss.

The difference between cooperative banks and conventional commercial banks is that the latter separate providers of funds from users of funds. Savers lend their money to the banks. They do not bear the risk of the profit because the return is fixed. Depositors do not need to know the type of borrowers to whom banks lend money. All they need is the general information about banks’ portfolio quality. Banking laws assure depositors that banks are well supervised and thus minimize the amount of information to be produced by a single saver.
Separation of savers from producers is never an Islamic idea (Elgari, 1997). The concept of PLS on which most of the Islamic modes of finance are based is dissimilar to the concept of conventional banking. Islamic/cooperative model of inter mediation provides a direct relationship between providers of funds and the users of funds. Hence, depositors now bear the risk of profit. The model puts the bank in the position of manager and depositors bear the risk of the production directly. The model of cooperative intermediaries provides a direct relationship between providers of funds and users of funds. Intermediaries in this model introduce parties to each other and provide the advantages of the law of large numbers to savers.

Mutual Fund is an example of investment based more or less on the principle of PLS. They mobilize funds from the public by issuing certificates, without guaranteeing any return. In this case, if there is a profit on the investment projects, the mutual fund will receive its share according to the percentage agreed in advance and the rest will be distributed among the holders of certificates. When there is a loss, it will be borne entirely by the certificate holders. The loss borne by the Mutual Fund would be in the terms of the efforts put in the project and went completely un-awarded.

The funds mobilized by the mutual fund are utilized namely for equity type financing. However, they may have interest-based assets in their portfolios. If these interest-based assets are replaced by equity type financing and the other requirements of Shariah are met, the operations of Mutual Funds can be accepted in Islam.

Now, let \( \beta_i \) denote the fraction of deposits held as capital and \( 1-\beta_i \) denote the fraction of deposit held in money to prevent bank runs. So, banks choose \( \beta_i \) to maximize the utility function of a representative depositor evaluated at date \( t \). The relevant resource constraints are:

\[
(1-\pi)r_1 = (1-\beta_i) \frac{P_t}{P_{t+1}}
\]

(16)

\[
\pi r_2 = R^n \beta_i
\]

(17)

The left-hand side of (16) indicates that a fraction \((1-\pi)\) of depositors will withdraw after one period and receive \( r_1 \). On the other hand, the bank will keep \( 1-\beta_i \) of deposits in money with the return of \( P_t/P_{t+1} \). Similarly, the left-hand side of (17) says that a fraction \( \pi \) of depositors will withdraw their deposits after two periods and receive \( r_2 \) per unit deposited. As shown by the right-hand side of (17), the bank will put a \( \beta_i \) fraction of deposits in capital.
and gets $R^n$ per unit invested. So, intermediary should solve the following utility maximization problem with respect to $\beta_t$:

$$u = \ln(w_t) + (1-\pi)\ln \left( \frac{(1-\beta_t)P_t/P_{t+1}}{(1-\pi)} \right) + \pi \ln \left[ \frac{R^n}{\pi \theta} \left( \frac{(1-\theta)/w_{t+2}}{1-\pi} \right)^\gamma \right]$$

(18)

The solution to (18) sets:

$$\beta = \pi$$

(19)

F. Capital accumulation\(^{(1)}\)

We can aggregate representative individuals’ behavior to characterize the dynamics of the economy. In a closed economy, summation of individual’s assets cancels debt and claims, and hence yields the aggregate capital stock. The capital stock in period $t+2$ is the amount saved by young individuals in period $t$. Thus,

$$K_{t+2} = \lambda_1 \pi r_2 s(r_1, r_2) L_t w_t$$

(20)

Dividing both sides of (20) by $L_t$ gives us capital per unit of labor:

$$K_{t+2} = \lambda_1 \pi r_2 s(r_1, r_2) w_t$$

Here, we have $\lambda_1 = s = 1$.

$$K_{t+2} = \pi r_2 \left[ f(k_t) - k_t f(k_t) \right]$$

(21)

Assuming that production function is Cobb-Douglas, $f(k) = k^\theta_t$ and $w_t = (1-\theta) k_t^\theta_t$. Therefore, equation (20) becomes:

$$k_{t+2} = \pi R^n (1-\theta) k_t^\theta_t$$

(22)

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Figure 1: Law of Motion for $K_t$ in market solution

Equation (22) is the equation of motion for $k_t$. Figure (1) shows $k_{t+1}$ as a function of $k_t$. The fixed point of the system is shown at $E$ corresponding to the steady-state value of capital intensity. At the steady-state, the above economy has a constant saving rate, constant output per worker, and constant capital-output ratio.

3. The model with administrative rationing

In this section, we introduce the administrative rationing to the model and consider how it may impinge the average efficiency of the loan portfolio of the bank. The representative bank has to tackle two issues: deposit mobilization and portfolio optimization.

The economy is the same as in previous section and we introduce administrative rationing to the model. When administrative rationing exists, the pattern of capital formation will reflect the fact that a favored group of firms have more access to credit even if the rate of return on its investments are lower than the normal firms\(^1\). More specifically, we assume that a fraction $\alpha$ of the bank deposit is channeled to low-return investment projects and $(1-\alpha)$ of the banks' loan goes to finance normal-return projects. $\alpha$ is the degree of administrative rationing so that $0 \leq \alpha \leq 1$. When $\alpha = 0$, there is no administrative rationing. On the other extreme, when $\alpha = 1$, there is a perfect administrative rationing and all credits are channeled to the low-return projects. Presumably, we introduce government into the model. It will impose tax for current expenditure and allocate a fraction $\alpha$ (of credits.

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A. Portfolio decision

In the context of the model of previous section, young agents save their after-tax wage income and can allocate their saving between money, bank deposits, and direct investment. If an individual places one unit of his saving in money, he will receive \((P_t/P_{t+1})\leq 1\).

If a young agent places one unit of income in demand deposits, he will be paid \(r_i \geq (P_t/P_{t+1})\) if he withdraws it after one period. If he places one unit of income in deposits and waits for two periods, he will receive \(r_2\). The return to the term deposits comes from the real sector return through firms' investment in both kinds of projects. Since low-return projects cannot compete in a free financial market to get the desired amount of credit, the government channels the credit to them by administrative censure. We assume that the return of these firms per unit of capital is less than the normal per capita return in previous section; \(R^l < R^n\).

When an individual holds one unit of saving in bank deposit, there is a probability of \(\pi\) that he will not withdraw it until the end of period \(t+2\) and get \(r_{2}\); where \(r_{2} = \alpha R^l + (1-\alpha)R^n\). He also may need the money meanwhile and withdraw the deposit in the next period with the probability of \(1-\pi\) and get \(r_{1}\). Hence, a representative agent will consume at \(t+2\):

\[
c_{t+2} = \left\{ \alpha R^l \theta k_{t+2} \left[ (1-\theta)/w_{t+2} \right]^\gamma \right. \\
\left. + (1-\alpha) R^n \theta k_{t+2} \left[ (1-\theta)/w_{t+2} \right]^\gamma \right\} (1-\tau)w_t
\]  

(23)

Where \(\tau\) is the tax rate and it is assumed to be a fixed proportion of wage income and thus \((1-\tau)w_t\) is disposable income of a representative agent\(^{(1)}\). Now, a representative young agent should decide how to divide his saving among bank deposit, money balances, and direct investment. We can write the utility maximization problem of young agents as follows:

\[
u = \ln \left[ (1-\tau)w_t \right] + (1-\pi) \ln \left[ \lambda_1 r_{1} + \lambda_2 \frac{P_t}{P_{t+1}} \right] + \pi \ln \left[ \lambda_1 r_{2} \theta \left[ (1-\theta)/w_{t+2} \right]^\gamma \right] \\
+ \lambda_2 \frac{P_t}{P_{t+1}} + \left[ 1 - \lambda_1 - \lambda_2 \right] R^n \theta \left[ (1-\theta)/w_{t+2} \right]^\gamma
\]  

(24)

---

1. Since the tax rate, as shown later, does not influence consumption decision, we assume that it does not alter labor supply behavior.
Here, we have the same assumption of the last section about the relations among $r_1, r_2, R^n$ and $P_t / P_{t+1}$ so that $r_2 \theta \left[ (1 - \theta) / w_{t+2} \right] > r_1$ and $(P_t / P_{t+1}) \leq 1$ and $(P_t / P_{t+1}) \leq r_1$. Then, it is easy to see that $\lambda = 0$.

So, we have the following utility maximization problem:

$$
u = \ln(w_t) + (1 - \pi) \ln(\lambda_1 r_1) + \pi \ln \left[ \lambda_1 \bar{r}_2 \theta \left[ (1 - \theta) / w_{t+2} \right] \right]$$

$$+ \left[ 1 - \lambda_1 \right] R^n \theta \left[ (1 - \theta) / w_{t+2} \right] \right]$$

(25)

The solution to (25) sets:

$$\lambda_1 = \frac{(1 - \pi) R^n}{R^n - \bar{r}_2}$$

(26)

$\lambda_1$ will be less that unity if $\pi R^n > \bar{r}_2$.

In section 2, we conclude that $\lambda_1 = 1$ is equal to $\pi R^n = r_2$. Here, we know that $\bar{r}_2 < r_2$ because a credit-channeling program decreases the expected return paid to deposits held for two periods and prevents deposits from being too attractive relative to direct investment, and then it results $\pi R^n > \bar{r}_2$. Therefore, we conclude that

$$\lambda_1 = \frac{(1 - \pi) R^n}{R^n - \bar{r}_2} < 1$$

(27)

Equation (27) means that when administrative rationing is introduced to the model, young agents will not hold all their savings in bank deposit. As the rate of rationing increases, the share of savings placed in bank deposit will decrease and the share of direct investment will increase. This can be shown clearly by derivative of (26) with respect to $\alpha$.

$$\lambda_1 = \frac{(1 - \pi) R^n}{R^n - \alpha R^l - (1 - \alpha) R^n}$$

$$\frac{\partial \lambda_1}{\partial \alpha} = \frac{- \left[ R^n - R^l \right]}{\left[ R^n - \alpha R^l - (1 - \alpha) R^n \right]^2} < 0$$

(28)
B. Intermediary behavior

Intermediaries have the same behavior as in the last section (laissez-faire). They accept deposits from young savers at t and use them to lend firms or keep them in money balances. Let \( \beta_t \) denote the fraction of deposits held as capital and \( (1 - \beta_t) \) the fraction of deposit held in money to prevent bank running. So, banks choose \( \beta_t \) to maximize the utility of a representative depositor evaluated at date t. Relevant resource constraints are the same as (16) and (17) and we rewrite them here:

\[
(1 - \pi) r_1 = (1 - \beta_t) \frac{P_t}{P_{t+1}} \tag{29}
\]

\[
\pi \bar{T}_2 = \bar{R} \beta_t \tag{30}
\]

Where \( \bar{R} = \alpha R^1 + (1 - \alpha) R^n \). Intermediary should solve the following utility maximization problem with respect to \( \beta_t^{(1)} \):

\[
u = \ln w_t + (1 - \pi) \ln \lambda_1 r_1 + \pi \ln \left[ \lambda_1 \bar{T}_2 \left[ \frac{(1 - \theta)}{w_{t+2}} \right]^{\gamma} \right] + \left[ 1 - \lambda_1 \right] R^n \theta \left[ \frac{(1 - \theta)}{w_{t+2}} \right]^{\gamma} \tag{31}\]

Since we can not obtain a solution for the optimum value for \( \beta_t \) by solving the utility maximization of the representative agents, we proceed to find \( \beta_t \) by profit maximization of a representative firm.

B.1. Profit maximization

Total profit of a bank is total income minus total cost and can be written by the following function:

\[
\text{Profit} = \left[ (1 - \tau)w_t \right] \lambda_1 (1 - \beta) \frac{P_t}{P_{t+1}} + \left[ \lambda_1 \beta \bar{T}_2 \theta \left[ \frac{(1 - \theta)}{w_{t+2}} \right]^{\gamma} \right] + \lambda_1 (1 - \pi) r_1 - \lambda_1 \beta \bar{T}_2 \theta \left[ \frac{(1 - \theta)}{w_{t+2}} \right]^{\gamma} = 0 \tag{32}\]

Assuming constant returns to scale production function and that factors of production are paid their marginal products, the profit for a representative intermediary will be zero. Equation (32) gives the optimum

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1- For more details see Bencivenga and Smith (1992).
fraction of deposit held in capital as follows:

$$\beta_t = \pi$$

Equation (33) is the same as (19); a fraction of the deposit that bank holds in capital is the same in both cases of market solution presence of administrative rationing.

C. Capital accumulation

We can aggregate individuals’ behavior to characterize the dynamics of the economy and then compare it with that of the market solution. The capital stock in period t+2 is the amount saved by young individuals in period t. Thus,

$$k_{t+2}^* = \lambda_1 \pi \bar{R}_t \left[ f(k_t) - k_t f'(k_t) \right]$$

(34)

Figure 2: Comparing law of motion for $k_t$ in administrative rationing and in market solution:

$$k_{t+2}^* = \lambda_1 \pi \bar{R}_t \left[ f(k_t) - k_t f'(k_t) \right]$$

(35)

Assuming that production function is Cobb-Douglas, $f(k) = k^\theta$ and

$$w_t = (1-\theta) k_t^\theta$$

Therefore, equation (35) becomes:

$$k_{t+2}^* = \lambda_1 \pi \bar{R} (1-\theta) k_t^\theta$$

(36)

Equation (36) is the equation of motion for $k_t$. Figure (2) shows $k_{t+2}^*$ as a function of $k_t$. A point where the $k_{t+2}^*$ curve intersects the 45 degree line is
a point where \( k^*_{t+2} = k_t \). There is a unique equilibrium level of \( k \) at point \( E^* \), denoted by \( k^*_{t+2} \cdot k^*_{t+2} \) is stable: wherever \( k \) starts, it converges to \( k^*_{t+2} \).

It is easy to see that \( \pi \mathbf{R}^n (1 - \theta) > \lambda_1 \tilde{R}(1 - \theta) \) This induces a downward shift of the equation of motion curve in Figure 2. Therefore, introducing administrative rationing to the model will decrease the equilibrium level of capital and output. As the rate of administrative credit rationing decrease, \( \lambda_1 \) will increase and the equilibrium gradually transit to point \( E \).

4. Lifetime expected utility maximization

A. Market solution

In this subsection, we consider the role of financial intermediation in mobilizing the savings when government does not intervene in credit market. Suppose that the preferences over lifetime consumption of a young individual is represented by the following utility function:

\[
\begin{align*}
    u &= \ln w_t + (1 - \pi) \ln (\lambda_1 r_t) + \pi E \ln \left[ \lambda_1 \tilde{R} \left( \frac{(1 - \theta)}{w_{t+2}} \right)^{\gamma} \right] \\
    &+ \left[ \frac{1 - \lambda_1}{\tilde{R}^n} \right] \left( \frac{1 - \theta}{w_{t+2}} \right)^{\gamma}
\end{align*}
\]  

Equation (37) is basically the same as (14) unless we introduce the operating cost of bank for each unit deposited, \( c \), so that \( \tilde{r}_2 = \tilde{R}^n - c \). We also assume that the rate of return to normal investment project is a random variable which varies between a lower bound, \( \omega \), and an upper bound, \( \beta \). \( E \) is the expectation operator. The first order condition to (37) with respect to \( \lambda_1 \) sets:

\[
\frac{1 - \pi}{\lambda_1} + c \pi E \left[ \frac{1}{\tilde{R}^n - \lambda_1 c} \right] = 0
\]  

To find \( \lambda_1 \), we need to figure out the expected value of the second term in the right-hand side of (38). Assuming the interval \( (b - \omega) \) being continuous, we can write the expected term in the following form of integral:

\[
I_1 = \int_{\omega}^{b} \frac{1}{\tilde{R}^n - \lambda_1 c} f \left( \tilde{R}^n \right) d\tilde{R}^n
\]
where \( f \left( \tilde{R}^n \right) \) is the distribution function of \( \tilde{R}^n \). Assuming a uniform distribution function for \( \tilde{R}^n \), we can figure out the integral (39) as follows:

\[
I_1 = \frac{1}{b - \omega} \log \left( \frac{b - \lambda_1 c}{\omega - \lambda_1 c} \right)
\]  

(40)

Substituting (40) in (38), we have

\[
\frac{1 - \pi}{\lambda_1} = \frac{c_\pi}{b - \omega} \log \left( \frac{b - \lambda_1 c}{\omega - \lambda_1 c} \right)
\]

(41)

The logarithmic term in (41) can be extended in the following form:

\[
\log \left( \frac{b - \lambda_1 c}{\omega - \lambda_1 c} \right) = \log \left( 1 + \frac{b - \omega}{\omega - \lambda_1 c} \right) = \frac{b - \omega}{\omega - \lambda_1 c} - \frac{(b - \omega)^2}{2(\omega - \lambda_1 c)}^2
\]

\[
+ \frac{(b - \omega)^3}{3(\omega - \lambda_1 c)^3} - \ldots
\]

(42)

Taking the first term of the extension in (42) as an approximation for the logarithm term and substituting it in (41), we have

\[
\lambda_1 = \min \left[ \frac{(1 - \pi)\omega}{c}, 1 \right]
\]

(43)

It is clear from (43) that \((1 - \pi)\omega / c > 1\) and \(\lambda_1\) will not be less than unity. Using more sentences of (42) to figure out \(\lambda_1\) will not change the result but it will only impose a laborious computation. Therefore, when government does not intervene in credit market, young agents will hold all their savings in bank deposits.

B. Administrative rationing

Now, we consider the role of financial intermediaries in mobilizing
public savings when government allocates a fraction of deposit to favored investment projects. We assume that they have a lower grade and can not take loan in a competitive situation. Usually, socially powerful groups undertake these projects and they claim that these projects have a larger social return than the private return. But, in fact, the social return of these projects are often equal to the private return.

The economy is the same as in section 3 Here, we assume that the rates of return to investment projects, \( \tilde{R}^n \) and \( \tilde{R}^l \), are random variables so that \( e \leq \tilde{R}^l \leq f \) and \( \omega \leq \tilde{R}^n \leq b \). The co-variance between \( \tilde{R}^n \) and \( \tilde{R}^l \) is zero. Since a fraction \( \alpha \) of the banks deposit is channeled to the low-return projects and \( (1-\alpha) \) of the banks' loan goes to finance normal return projects, the weighted average of the return to the deposit is

\[
\left[ \alpha \tilde{R}^l + (1-\alpha) \tilde{R}^n - c \right] \theta y.
\]

Therefore, consumption in periods two and three are:

\[
C_3 = \lambda_1 \theta y_{t+2} + (1-\lambda_1) \theta y_{t+2}
\]

Now, a representative young agent should maximize the following utility function

\[
u = \ln w_t + (1-\pi) \ln \lambda_1 r_1 + \pi \mathbb{E} \ln \left\{ \lambda_1 \left[ \alpha \tilde{R}^l + (1-\alpha) \tilde{R}^n - c \right] \theta \left[ (1-\theta)/w_{t+2} \right]^{\gamma} + \left[ 1-\lambda_1 \right] \tilde{R}^n \theta \left[ (1-\theta)/w_{t+2} \right]^{\gamma} \right\}
\]

(44)

Here, we assume that those who put their savings in direct investment will get zero if they want to liquidate their investment after one period. In the next sub-section, we will remove this assumption for some purpose. The first order condition to (44) with respect to \( \lambda_1 \) sets.

\[
1-\pi \mathbb{E} \left\{ \frac{\tilde{R}^n}{\lambda_1 \alpha \left( \tilde{R}^l - \tilde{R}^n \right) + \tilde{R}^n - c \lambda_1} \right\} = 0
\]

(45)
We can write the expected term in the following form of integral:

\[
I_2 = \frac{\hat{R}^n}{\alpha(f-e)\lambda_1} \int_{\omega}^{b} \log \left( \frac{\dot{\lambda}_1 \alpha \tilde{R}^n + \dot{\lambda}_1 \alpha \tilde{R}^n - \tilde{R}^n + \lambda_1 c_1}{\lambda_1 \alpha e + \lambda_1 \alpha \tilde{R}^n - \tilde{R}^n + \lambda_1 c_1} \right) d\tilde{R}^n
\]

\[
I_2 = \frac{1}{\alpha(f-e)\lambda_1} \left\{ \int_{\omega}^{b} \tilde{R}^n \log \left( \frac{\dot{\lambda}_1 \alpha \tilde{R}^n + \dot{\lambda}_1 \alpha \tilde{R}^n - \tilde{R}^n + \lambda_1 c_1}{\lambda_1 \alpha e + \lambda_1 \alpha \tilde{R}^n - \tilde{R}^n + \lambda_1 c_1} \right) d\tilde{R}^n \right. \\
- \left. \int_{\omega}^{b} \tilde{R}^n \log \left( \lambda_1 \alpha e + \tilde{R}^n (\dot{\lambda}_1 \alpha - 1) + \lambda_1 c_1 \right) d\tilde{R}^n \right\}
\]  

(46)

After a very long process of handling (46) and substituting the result in (45), we have:

\[
\lambda_1^3 \left[ \frac{1}{2} \alpha^3 (f^2 + ef + e^2) (b - \omega) + \frac{3}{2} \alpha^2 c (e+f)(b-\omega) + \frac{5}{4} \alpha c (b-\omega) \right] \\
+ \frac{1}{4} \alpha^2 c (b^2 - \omega^2) - \frac{1}{4} \alpha^3 (b^3 - \omega^3) \right] + \lambda_1^2 \left[ - \frac{1}{2} \alpha^2 (f^2 + ef + e^2) (b - \omega) \\
- (e + f) (\alpha^2 + \frac{3}{2} \alpha^2 c) (b - \omega) + \left( \frac{1}{4} c^2 - 2 \alpha c \right) (b - \omega) - \frac{1}{2} \alpha c (b^2 - \omega^2) \\
+ \frac{3}{4} \alpha^3 (b^3 - \omega^3) - \frac{\alpha^2}{\pi} \right] + \lambda_1 \left[ (f^2 + ef + e^2) (b - \omega) + e (e+f)(b-\omega) \\
+ \left( \frac{3}{2} \alpha c + 2c \right) (b-\omega) + \frac{1}{4} (b^2 - \omega^2) - \frac{3}{4} \alpha (b^3 - \omega^3) + \frac{2\alpha}{\pi} \right] \\
+ \frac{1}{4} (b^3 - \omega^3) - \frac{3}{2} (b - \omega) - \frac{1}{\pi} = 0
\]  

(47)
Table 2:

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\lambda_1$</th>
<th>$\alpha$</th>
<th>$\lambda_1$</th>
<th>$\alpha$</th>
<th>$\lambda_1$</th>
<th>$\alpha$</th>
<th>$\lambda_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.50</td>
<td>4.27</td>
<td>.30</td>
<td>.85</td>
<td>.55</td>
<td>.47</td>
<td>.80</td>
<td>.32</td>
</tr>
<tr>
<td>.10</td>
<td>2.37</td>
<td>.35</td>
<td>.73</td>
<td>.60</td>
<td>.43</td>
<td>.85</td>
<td>.31</td>
</tr>
<tr>
<td>.15</td>
<td>1.64</td>
<td>.40</td>
<td>.65</td>
<td>.65</td>
<td>.40</td>
<td>.90</td>
<td>.29</td>
</tr>
<tr>
<td>.20</td>
<td>1.25</td>
<td>.45</td>
<td>.57</td>
<td>.70</td>
<td>.37</td>
<td>.95</td>
<td>.27</td>
</tr>
<tr>
<td>.25</td>
<td>1.02</td>
<td>.50</td>
<td>.52</td>
<td>.75</td>
<td>.35</td>
<td>1.00</td>
<td>.26</td>
</tr>
</tbody>
</table>

The solution to (47) in parameters is meaningless or even impossible. So, by introducing hypothetical numbers for parameters, we can solve (47) for $\lambda_1$. Assuming $\pi=.3$, $c=.02$, $\omega=1.02$, $b=1.20$, $e=.95$, $f=1.05$, and $0\leq\alpha\leq1$, we have the following result in table (2). Thus, considering expected utility, we also conclude that when we introduce administrative rationing to the model, young agents put less of their savings in banks as rationing increases. But we see in table 2 that when $\alpha=1$ then $\lambda_1=0.26$. This implies that when government allocates all deposits to favored projects, young their savings in agents will still hold. 26 of their saving in bank deposit. How can we explain this phenomenon? This is primarily explained by the scrap value of the direct investment. Here, we assumed that the scrap value of direct investment liquidated after one period was zero. This makes deposit in banks be very desirable. In the next section, this assumption will be relaxed and a different result will be obtained.

- **Positive scrap value for direct investment**

Now, we assume that the scrap value of direct investment liquidated after on period, x, is positive and can vary in a interval $0 \leq x \leq r_1 = 1$. For $x=0$, we had the results of table 2. For $0 < x \leq 1$, a representative young agent should maximize the following utility function.

\[
 u = \ln w_t + (1-\pi)\ln \left[ \lambda_1 r_1 + (1+\lambda_1) x \right] + \frac{\pi E}{\lambda_1} \left[ \alpha \tilde{R}^1 + (1-\alpha) \tilde{R}^n - c \right] + \theta \left[ \left( 1-\theta \right)/w_{t+2} \right]^\gamma + \left[ 1 - \lambda_1 \right] \tilde{R}_t^n \theta \left[ \left( 1-\theta \right)/w_{t+2} \right]^\gamma \]

\[
 (48)
\]
where \( c_2 = \left[ \lambda_1 r_1 + (1 - \lambda_1)x \right] \). This term indicates that when a young agent holds one unit of his saving in bank or it is placed in a direct investment, he may withdraw/liquidate that sum after one period with the probability of \((1-\pi)\) if he puts his saving in bank, he will get \(r_1\) in the next period. If he puts his saving in direct investment and he has to liquidate it after one period, he will get \(x\).

The first order condition to (48) with respect to the share of the individual saving placed in the bank deposit \(\lambda_1\) is given below

\[
\frac{(1-\pi)(r_1-x)}{\lambda_1 r_1 + (1-\lambda_1)x} \pi E \left\{ \frac{\alpha \left[ \hat{R}_1 - \hat{R}_n \right] - c}{\lambda_1 + \alpha \left[ \hat{R}_1 - \hat{R}_n \right]} + \hat{R}_n - \frac{\alpha c}{\lambda_1} \right\} = 0 \quad (49)
\]

The expected value of (49) is given by (50).

\[
\frac{1}{(\alpha \lambda_1 - 1)^2} \left[ \lambda_1^3 \left[ \frac{1}{2} \alpha^3 (f^2 + ef + e^2)(b-\omega) + \frac{3}{2} \alpha^2 c(e+f)(b-\omega) + \frac{5}{4} \alpha c^2 (b-\omega) + \right. \right.
\]

\[
+ \left. \frac{1}{4} \alpha^2 c(b^2 - \omega^2) - \frac{1}{4} \alpha^3 (b^3 - \omega^3) \right]
\]

\[
+ \lambda_1^2 \left[ - \frac{1}{2} \alpha^2 (f^2 + ef + e^2)(b-\omega) - (e+f)(\alpha^2 + \frac{3}{2} \alpha c)(b-\omega) \right.
\]

\[
+ \left. \left( \frac{1}{4} \alpha c^2 - 2 \alpha c \right)(b-\omega) - \frac{1}{2} \alpha c(b^2 - \omega^2) + \frac{3}{4} \alpha^2 (b^3 - \omega^3) \right] \]

\[
+ \lambda_1 \left[ \alpha (f^2 + ef + e^2)(b-\omega) + \alpha (e+f)(b-\omega) + \left( \frac{3}{2} \alpha + 2 c \right)(b-\omega) \right.
\]

\[
+ \left. \frac{1}{4} \alpha c(b^2 - \omega^2) - \frac{3}{4} \alpha (b^3 - \omega^3) \right] + \left( \frac{1}{4} \alpha c(b^3 - \omega^3) - \frac{3}{2} \alpha c(b-\omega) \right]
\]
Substituting (50) in (49), we have

\[
\lambda_1^2 \left[ \frac{1}{2} \alpha^3 (f^2 + ef + e^2)(b - \omega) + \frac{3}{2} \alpha^2 c(e + f)(b - \omega) + \frac{5}{4} \alpha c(b - \omega) + \frac{1}{4} \alpha^2 c(b^2 - \omega^2) \right]
\]

\[
- \frac{1}{4} \alpha^2 (b^3 - \omega^3) + \lambda_1^2 \left[ - \frac{1}{2} \alpha^2 (f^2 + ef + e^2)(b - \omega) - (e + f) \left( \alpha^2 + \frac{3}{2} c \right)(b - \omega) \right]
\]

\[
+ \left( \frac{1}{4} \alpha^2 - 2c \right)(b - \omega) - \frac{1}{4} \alpha c(b^2 - \omega^2) + \frac{3}{4} \alpha^2 (b^3 - \omega^3) - \frac{\alpha^2}{c^3}
\]

\[
+ \lambda_1 \left[ \alpha (f^2 + ef + e^2)(b - \omega) + \alpha (e + f)(b - \omega) + \frac{3}{2} \alpha + 2c \right](b - \omega) + \frac{1}{4} (b^2 - \omega^2)
\]

\[
- \frac{3}{4} \alpha (b^3 - \omega^3) + \frac{2 \alpha}{c^3} \right] + \frac{1}{4} (b^3 - \omega^3) - \frac{3}{2} (b - \omega) - \frac{1}{c^3}
\]

\[
\lambda_1^3 \alpha^2 \left( 1 - \pi \right) \tau_1 x - 2 \lambda_1^2 \alpha (1 - \pi) \tau_1 x + \lambda_1 (1 - \pi) \tau_1 x
\]

\[
\lambda_1 \pi (\tau_1 - x) + \pi x
\]

\[
\lambda_1 \pi (\tau_1 - x) + \pi x
\]

\[
- \lambda_1^2 \alpha^2 + 2 \lambda_1 \alpha - 1 = 0
\]

Using numerical values, we have the results shown in table 3 for 0 < x ≤ 1. So, there is a long-term negative relationship between λ_1 and α. Once again, we conclude that as the rate of administrative rationing increases, the share of savings placed in bank deposit will decrease. In table 3, we see that when α = 1 then λ_1 = .11. Table 3 enables us to make a comparison between the hypothetical values assigned for α, and the corresponding values computed for λ_1. As it is shown in the table 3, a gradual increase in the value of α form.05 to 1.00) is concurrent with a gradually declining trend in corresponding values computed for λ_1 gradually decreasing from 3.35 down to 0.11.

5. Conclusions

We used a three-period overlapping generations model of cooperative financial intermediaries and economic growth. Intermediaries operate on the basis of profit/loss sharing. Two types of (low- and high quality) investment projects were assumed.

We solved the model for the steady-state level of capital and output per worker under two distinct case: First, when the financial intermediaries operate in a free and competitive market; and second, when administrative rationing exists. The equilibrium level of capital and output per worker is unambiguously higher when administrative rationing is absent. As the degree of administrative rationing increases, the equilibrium level of bank deposits
and the level of output at steady-state falls.

In section four, we solved a similar model with a probabilistic rates of returns assuming an expected utility function. Once again, we concluded a negative relationship between the rate of administrative rationing and the level of output at steady-state decreases.

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