SUBSTRATUM RADIATION AND CASIMIR EFFECT

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Abstract

A heuristic way to calculate the approximate value of the Casimir force is introduced.

Introduction

In his paper "Time, Vacuum and Cosmos", McCrea [1] presents a simple way to calculate the Casimir force between two flat parallel plates. The standard quantum field theoretical value for this force (after rigorous calculations) is [2].

$$\frac{\pi hc}{480L^4} \quad (1)$$

where L is the distance between the two plates. The value derived by McCrea is

$$\frac{8\pi hc}{3k^4L^4} \quad (2)$$

where k is some number of order unity. In his derivation, McCrea assumes the existance of what he calls substratum radiation field with an intensity in wavelength range, λ, $\lambda + d\lambda$ equal to

$$I_\lambda d\lambda = 2\, hc^2 d\lambda / \lambda^5 \quad (3)$$

He also makes the assumption that radiation could be established moving across the gap if the wavelength is less than about the width of the gap ($\lambda < \lambda_L \equiv kL$).

Here I present a slightly more rigorous calculation of this force which comes out to be very near to the standard expression (1).

Consider a virtual photon with wavelength λ and momentum h/λ which moves at an angle θ with respect to two infinite parallel plates. Such a virtual photon can move at most the distance $x = \lambda/2\pi$ according to the Heisenberg uncertainty principle $x.p \sim h$. The existance of conducting walls puts a constraint on the wavelength of virtual photons:

$$x = \frac{\lambda}{2\pi} \leq \frac{L}{\sin\theta} \quad (4)$$

With this constraint, and using expression (3) for the vacuum radiation field, it is straightforward to calculate the amount of "excluded" energy between unit areas of the two parallel plates:

$$E = L.\frac{1}{C}\int_{\theta=0}^{\pi} d\omega \int_{\lambda=2\pi L/\sin\theta}^{\infty} I_\lambda d\lambda \quad (5)$$

putting $d_\omega = 2\pi \sin\theta\, d\theta$, and $I_\lambda = \frac{C}{4\pi} u_\lambda = \frac{2\,hc^2}{\lambda^5}$, this yields

$$E = \frac{16}{15}\,\frac{\pi hc.}{(2\pi)^4 L^3} \quad (6)$$

Casimir force is just the variation of this "excluded" energy with respect to changes in the separation between the two plates:

$$F = \frac{\partial E}{\partial L} = -\frac{\pi}{5\pi^4}\,\frac{hc}{L^4} \approx -\frac{\pi}{487}\,\frac{hc}{L^4} \quad (7)$$

The minus sign indicates that the force is attraction. It is very interesting that this simple derivation gives the right dependence on L, with a numerical factor $\pi/487$ which is very near although not exactly equal to the standard factor $\pi/480$.

Keywords: QED-Quantum Vacuum, Casimir Effect

References

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