NONDYNAMICAL ANALYSIS OF SPIN AMPLITUDES IN PION-PROTON ELASTIC SCATTERING IN OPTIMAL FORMALISM AT 6.0 GeV/C

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Abstract

Optimal conditions are used in nondynamical formalism to diagonalize the pion-proton reaction matrix as much as possible. Invariance laws are imposed to simplify the relationship between observables and bilinear combination of amplitudes. Transverse amplitudes are determined by using measured polarization parameter in $\pi^{\pm}p$ elastic scattering at 6.0 *GeV/C*.

Introduction

Spin polarization phenomena play a significant role in the determination of complex spin amplitudes, particularly at high energies where partial wave analysis is not possible [1]. Such determined amplitudes are used to test the validity of dynamical models such as the ReggePole model [2], the particle exchange model [3] and the Fixed-t Dispersion theory *(FTDR)* [4]. Phenomenological calculations of spin amplitudes are also used to check the reliability of different symmetries and conservation laws [5].

Pion-proton reaction is the most basic constituent reaction involved in more complicated reactions such as nucleon-nucleon scattering experiments. In such experiments, the standard factorization method is used to decompose the complex reaction into a pion-proton constituent reaction [6]. This factorization is possible as far as the structure of reaction matrix (M-matrix) is concerned.

Optimal formalism is a particular mathematical representation in which the initial and final density matrices as well as spin-momentum tensors are chosen to be Hermitian. Depending upon the direction of the quantization axis of each particle, we get two different frames, namely transversity and planar frames. The helicity frame is a special case of the planar frame.

Amplitudes are calculated in transversity frame in which the quantization axis of each particle is perpendicular to the reaction plane which is specified by the momentum of the beam and the scattered particles. Invariant laws such as Lorentz, parity and time reversal invariance reduce the number of independent amplitudes and result in a simpler relation between observables and bilinear combination of amplitudes.

Keywords: Nondynamical; Amplitude; Polarization; Scattering

Theory

The reaction matrix of pion-proton elastic scattering is denoted by [7].

$$M=\sum_{\ell}\sum_{\lambda} D(\lambda,\ell)S^{\lambda\ell} \qquad (1)$$

Where ℓ and λ are the spin components of target and recoil paticles respectively along the quantization axis. $D(\lambda,\ell)$'s are the spin amplitudes and $S^{\lambda\ell}$ are the spin-momentum tensors (set of 2×2 matrices in this case).

Initial and final polarization states of particles are described by density matrices ρI and ρF called initial and final density matrices, respectively.

$$\rho I=\overset{uv}{\rho I} \;\; with\; u,v=1,2 \qquad (2)$$

and

$$\overset{uv}{\rho F}=M\overset{uv}{\rho I}M^{\dagger} \qquad (3)$$

substituting for M and $M^{\dagger}$ we get

$$\overset{uv}{\rho F}=\sum_{\ell}\sum_{\lambda}\sum_{\ell'}\sum_{\lambda'} D(\lambda' e)\, S^{\lambda\ell}\,\overset{uv}{\rho I}\,(S^{\lambda'\ell'})^{\dagger} \qquad (4)$$

Experimental observables are given by the expectation values of spin-momentum tensors Q in the final states denoted by

$$Q = Q^{\xi w} \text{ with } \xi \text{ and } w = 1,2 \quad (5)$$

then

$$L(uv, \xi\omega) = \langle Q^{\xi\omega} \rangle = Tr(Q^{\xi\omega} \rho_E) = Tr(Q^{\xi\omega} M \overset{uv}{pI} M^{\dagger}) \quad (6)$$

where $L(uv,\xi\omega)$ denote the observables. Substitute (1) into (4) to get

$$L(uv,\xi\omega) = \sum_{\lambda}\sum_{\ell}\sum_{\lambda'}\sum_{\ell'} D(\lambda,\ell) D^*(\lambda',\ell')$$

$$\sum_{\alpha}\sum_{\beta}\sum_{a}\sum_{b} (Q^{\xi\omega})\beta\alpha(S^{\lambda\ell})\alpha a(\overset{uv}{pI})ab(S^{\lambda'\ell'})b\beta \quad (7)$$

Equation (7) gives four independent amplitudes D(++), D(+ -), D(- +) and (D- -) with 16 bilinear combinations (bicoms). Therefore (7) gives a 16×16 matrix which describes the relationship between observables and bicoms. This requires 16 different experiments from which amplitudes are found. We wish to diagonalize this matrix as much as possible. To achieve this we use an optimal condition namely

$$(S^{\lambda\ell})\alpha a = \delta_{a\ell}\delta_{\lambda\alpha} \quad (8)$$

$$(S^{\lambda'\ell'})\beta b = \delta_{\beta\ell'}\delta_{b\lambda'} \quad (9)$$

$$(\overset{uv}{pI})_{ab} = (P^{uwHp})_{ab} = \frac{1}{2}[(1+p) + (1-p)\, i]\, (\delta_{ub}\delta_{va} + p\,\delta_{ua}\delta_{vb}) \quad (10)$$

where

$$H_p = Real = R \text{ if } p = 1$$

$$H_p = I_m = I \text{ if } p = -1 \quad (11)$$

$$(Q^{\xi\omega Hq})\beta\alpha = \frac{1}{2}[(1+q) + i(1-q)]\, (\delta_{\xi\alpha}\,\delta_{\omega\beta} + q\,\delta_{\xi\beta}\,\delta_{\omega\alpha}) \quad (12)$$

$$H_q = R \text{ if } q = 1$$

$$H_q = I \text{ if } q = -1 \quad (13)$$

substitute (8)-(13) into (7) to get

$$L(uvH_p; \xi\omega H_q) = ZH_{\omega}[D(\xi,u)D^*(w,v) + pD(\xi,v)D^*(w,u)] \quad (14)$$

$$Z = 1 + pq - p + q \quad (15)$$

$$H_{\omega} = - \begin{cases} real \text{ if } w = pq = 1 \\ Im \text{ if } w = pq = -1 \end{cases}$$

In addition to the Lorentz invariance we have a time reversal invariance and parity conservation which simplify the above formula considerably in transversity frame. The imposition of time reversal invariance reduces the number of amplitudes to 3 independent ones [8], namely

$$D(++)_t, D(--)_t, D(+-)_t = D(-+)_t \quad (16)$$

Imposing parity constraint reduces the number of independent amplitudes to only 2 as follows:

$$D_t^{++}, D_t^{--}, D_t^{+-} = D_t^{-+} = 0 \quad (17)$$

With only two independent amplitudes the observable-bicom relationship is as follows:

$$L(++,++) = 4[D_t^{++}]^2 \quad (18)$$

$$L(--,--) = 4[D_t^{--}]^2 \quad (19)$$

$$L(+-R,+-R) = 2ReD_t^{++}D_t^{--*} \quad (20)$$

$$L(+-I,+-R) = 2ImD_t^{++}D_t^{--*} \quad (21)$$

Since all particles are not completely polarized we must take the average over the polarization direction denoted by A. The difference between up and down is denoted by Δ. A and Δ are defined as

$$A = (++) + (--) \quad (22)$$

$$\Delta = (++) - (--) \quad (23)$$

Therefore

$$L(A,+-) = L(++,+-) + L(--,+-) \quad (24)$$

$$L(\Delta,+-) = L(++,+-) - L(--,+-) \quad (25)$$

Numerical Determinations of Amplitudes in Transversity Frame

In order to determine the magnitudes of transverse amplitudes, only the polarization parameter Po must be measured experimentally. These measurements were done for $\pi^{\pm}p$ elastic scattering at $6GeV/C$ for extensive range of four-momentum transfer squared t from 0.05 to 2.0(GeV/C)^2 [9].

In terms of our observables Po is defined as

$$Po = \frac{L(A,\Delta)}{L(A,A)} = \frac{[D_t^{++}]^2 - [D_t^{--}]^2}{[D_t^{++}]^2 + [D_t^{--}]^2} \quad (26)$$

Normalizing the differential cross section to unity we find

$$[D_t^{++}]^2 + [D_t^{--}]^2 = Po \quad (27)$$

$$[D_t^{++}]^2 + [D_t^{--}]^2 = 1 \quad (28)$$

The experimental values of polarization parameters taken from reference [9] are tabulated in Tables 1 and 2 for π^+P and π^-P, respectively. The determined values of transverse amplitudes and their corresponding errors are also tabulated in Tables 1 and 2 for π^+P and π^-P, respectively at 6GeV/C and available range of t values. The ratio of these amplitudes (F) are plotted versus t for π^+P and π^-P reaction in Fig. 1 and Fig. 2, respectively.

Conclusion

Using optimal formalism plus symmetry constraints reduce the number of independent amplitudes to only two in transversity frame. The magnitudes of these amplitudes were calculated from available experimental data without concern about the phase between them.

Only one experimental measurement is required for a complete determination of transverse amplitudes. In π^+P scattering both amplitudes contribute almost equally for all t-values. In π^-P scattering the ratio(F) of amplitudes remains close to unity for t less than 1 but as t increases to 2 $(GeV/C)^2$, the ratio decreases to half.

T(GeV/C)^2	P	dP	$\|D_T^{++}\|$	$\|D_T^{-}\|$	F	dF
.075	.214	.040	.779	.627	1.243	.043
.125	.209	.010	.777	.629	1.236	.008
.175	.195	.008	.773	.634	1.218	.007
.225	.196	.007	.773	.634	1.220	.006
.275	.171	.010	.765	.644	1.189	.009
.325	.179	.010	.768	.641	1.198	.009
.375	.142	.010	.756	.655	1.154	.009
.425	.138	.010	.754	.657	1.149	.009
.475	.104	.020	.743	.669	1.110	.018
.525	.083	.020	.736	.677	1.087	.019
.575	.098	.020	.741	.672	1.103	.018
.625	.017	.020	.713	.701	1.017	.020
.675	.031	.030	.718	.696	1.031	.029
.725	.051	.040	.725	.689	1.052	.038
.775	.041	.040	.721	.692	1.042	.038
.825	.161	.050	.762	.648	1.176	.044
.875	.205	.060	.776	.630	1.231	.051
.925	.249	.090	.790	.613	1.290	.074
.975	.224	.090	.782	.623	1.256	.075
1.050	.467	.060	.856	.516	1.659	.046
1.150	.423	.120	.844	.537	1.570	.093
1.250	.384	.100	.832	.555	1.499	.078
1.400	.301	.090	.807	.591	1.364	.073
1.600	.360	.130	.825	.566	1.458	.102
1.800	.160	.150	.762	.648	1.175	.131
2.050	.080	.240	.735	.678	1.083	.223
2.450	-.210	.300	.628	.778	.808	.388

Table 1. The magnitudes of transverse amplitudes in π^+p scattering at 6 GeV/C.
Note: $F=|D_T^{++}|/|D_T^{-}|$.

T(GeV/C)^2	P	dP	$\|D_T^{++}\|$	$\|D_T^{-}\|$	F	dF
.075	-.157	.016	.649	.761	.854	.019
.125	-.136	.008	.657	.754	.872	.009
.175	-.136	.007	.657	.754	.872	.008
.225	-.125	.007	.661	.750	.882	.008
.275	-.105	.009	.669	.743	.900	.010
.325	-.073	.010	.681	.732	.929	.011
.375	-.069	.012	.682	.731	.933	.013
.425	-.035	.016	.695	.719	.966	.017
.475	.004	.016	.709	.706	1.004	.016
.525	.012	.020	.711	.703	1.012	.020
.575	.066	.033	.730	.683	1.068	.031
.625	-.005	.030	.705	.709	.995	.030
.675	.025	.036	.716	.698	1.025	.035
.725	-.057	.039	.687	.727	.945	.041
.800	-.067	.039	.683	.730	.935	.042
.900	-.293	.064	.595	.804	.739	.095
1.000	-.375	.069	.559	.829	.674	.119
1.100	-.460	.075	.520	.854	.608	.156
1.200	-.620	.100	.436	.900	.484	.335
1.300	-.565	.105	.466	.885	.527	.293
1.400	-.570	.150	.464	.886	.523	.425
1.500	-.400	.160	.548	.837	.657	.291
1.650	-.540	.120	.480	.877	.547	.310
1.850	-.670	.250	.406	.914	.445	1.020
2.000	-.570	.290	.464	.886	.523	.821

Table 2. The magnitudes of transverse amplitudes in π^-p scattering at 6 GeV/C.
Note: $F=|D_T^{++}|/|D_T^{-}|$.

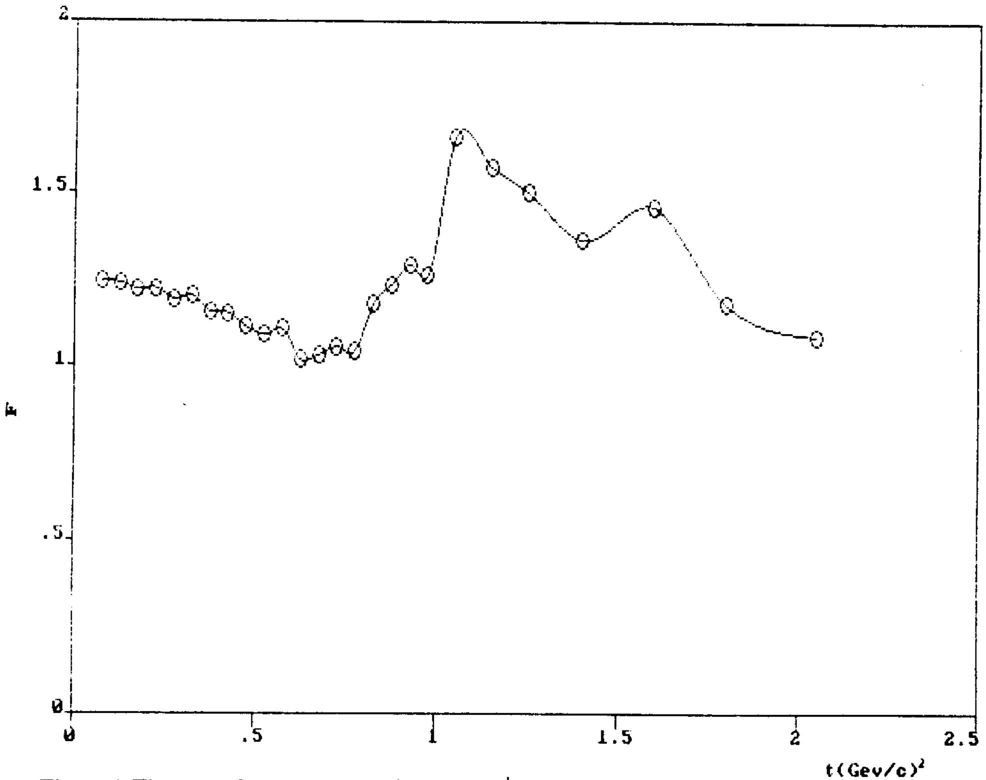

Figure 1. The ratio of transverse amplitudes for π^+p scattering at 6 GeV/C

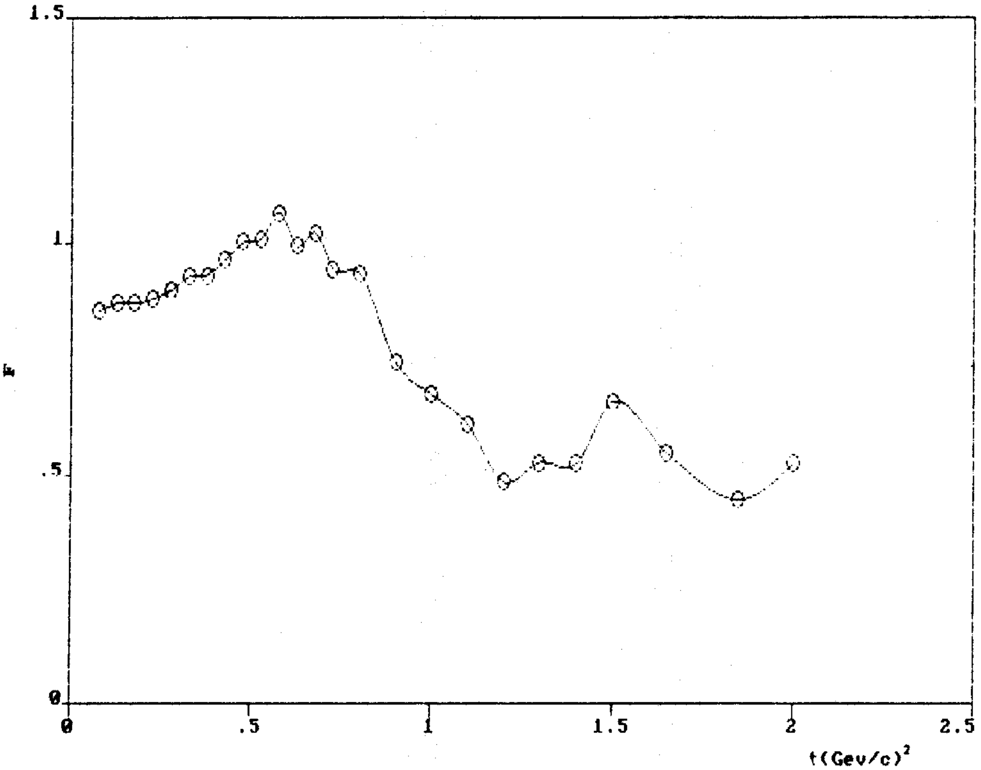

Figure 2. The ratio of transverse amplitudes for $\pi^- p$ scattering at 6 GeV/C

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