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Hypercentral Constituent Quark Model and Isospin for the Baryon Static Properties

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Abstract

So far the static properties of hadrons have been introduced in various models. The static properties of hadrons (charge radius, magnetic moment, *etc*.) are useful for understanding the quark structure of hadron. In this work we have introduced the hypercentral constituent quark and isospin dependent potentials. Here constituent quarks interact with each other *via* a potential in which we have taken into account the three body force effect and standard two-body potential contributions. According to our model the static properties of hadrons containing u, d, and s quarks are better than the other models and closer to the experiment. The two key ingredients of this improvement are the effective quark-gluon hypercentral potentials, the hyperfine interaction and isospin-dependence potential.

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Introduction

The Constituent Quark Model (CQM) has been extensively applied to the description of baryon properties. There are many approaches where the three-quark problem is solved numerically [1].

The idea of multiquark forces has been already considered in the early days of the quark model. The main ingredient of this model is the interquark potential, which contains a spin-independent and spin dependent terms characterized by the presence of a long range part giving rise to confinement. The 3q-interactions are more easily introduced and treated within the hypercentral interaction.

The internal three quark motion is described by the Jacobi coordinates ρ and λ [2]. In order to describe the three-quark dynamics it is convenient to introduce the hyperspherical coordinates, which are obtained by substituting the absolute value of ρ and λ in $x = \sqrt{\rho^2 + \lambda^2}$, where x is the hyperradius.

The spin independed potential is hypercentral and hence depends only on hyperradius x. In this model there are 3 hypercentral interacting potentials. First, the six-dimensional hyper Coulomb potential [2,3] which is attractive for small separations, originating from the color charge:

$$V_{hyc}(x) = \frac{k\alpha_s}{x} = -\frac{c}{x} . \tag{1}$$

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while at large separations a hyper linear term gives rise to quark confinement [4]

$$V_{con}(x) = bx \ . \tag{2}$$

However there have been some interesting attempts to interpolate between $V_{hyc}(x)$ and $V_{con}(x)$ [5-10].

From Equations 1 and 2, the interaction potential can be taken as Colomb term plus confining term $(bx - \frac{c}{x})$ as suggested by the lattice QCD calculations [11,12]. In this article we have added the six-dimension harmonic oscillator (h.o.) potential, which has a two-body character, and turns out to be exactly hypercentral since

$$V_{h.o.} = \sum_{i<j}^{i=3} \frac{1}{2} k (r_i - r_j)^2 = \frac{3}{2} kx^2 = ax^2 \tag{3}$$

Here the interaction potential is assumed (from Eqs. 1, 2 and 3) as below:

$$A(x) = ax^2 + bx - \frac{c}{x} \ . \tag{4}$$

The hypercentral interaction potential (4) acts as nonperturbative potential. In This article the quark interacting potential also contains hyperfine spin-isospin interaction form [2] and we use this as a perturbation potential which improves the results. In section (2) we calculated the relativistic nonpertubative wave function for valence quarks.

In section (3) we obtained pertubative wave function $\psi_\gamma(x)$ using nonconfining hyperfine potential. The magnetic moments in section (4) and the charge radius in section (5) were found for different quark masses. The results indicate that this potential is useful for quarks having masses in the range used in the phenomenological analysis of quark model. By determining the magnetic moment and charge radius in our model it is concluded that there is a reasonable consistency between the calculated values and the experimental results.

Hypercentral Relativistic Wave Function for Three Quarks in a Nucleon

If we denote the quark wave function satisfying the Dirac equation by $\psi(\vec{r})$, then

$$[\gamma_0 \varepsilon + i \vec{\gamma}.\vec{\nabla} - (m + U(r))]\psi_\alpha(\vec{r}) = 0 \tag{5}$$

where $\alpha \in \{1,2,3,\}$. Summing the three equations in (5) we obtain the hypercentral constituent quark equation.

The hypercentral potential $U(x)$, which leads to analytical solution in our model, would be

$$U(x) = \frac{1}{2}(1 + e\gamma_0)A(x) \tag{6}$$

with the potentials $A(x)$ given by (4)

The parameter e is arbitrary [13-15], so we take in to be 1.

This potential has interesting properties and yields reasonable physical results and the solution of Dirac equation can be worked out analytically. The quark potential $U(x)$ is assumed to depend on the hyperradius x only. The eigenspinor of (5) denoted by $\psi^0_{jj_3}$ is rewritten as

$$\psi^0_{jj_3}(x) = \begin{pmatrix} \varphi \\ \chi \end{pmatrix} \tag{7}$$

Now combining Equations 4-7, we get:

$$\begin{aligned} &(\sigma.P)\chi + (m + U_0(x) + V_0(x))\varphi = \varepsilon\varphi \\ &(\sigma.P)\varphi - (m + U_0(x) - V_0(x))\chi = \varepsilon\chi \end{aligned} \tag{8}$$

where $\varphi = g_k(x) y^{j_3}_{j_l}(x)$ and $\chi = if_k(x) y^{j_3}_{j'_l}(x)$.

Here $U_0(x)$ and $V_0(x)$ are the scalar hypercentral and the vector hypercentral potentials, respectively. For Dirac upper component we combine two equations in (8) and use Equations 4 and 6 to obtain

$$\frac{P^2 g(x)}{m + \varepsilon} + (m - \varepsilon + A(x))g(x) = 0 \tag{9}$$

The internal quark motion is usually described by means of the Jacobi relative coordinates. By separating the common motion, the P^2 operator of a quark in the 3q system becomes $(\hbar = c = 1)$ [2]

$$P^2 = -(\nabla^2_\rho + \nabla^2_\lambda) = -(\frac{d^2}{dx^2} + \frac{5}{x}\frac{d}{dx} + \frac{L^2(\Omega)}{x^2}) \tag{10}$$

Hence

$$\begin{aligned} &g''_\gamma(x) + \frac{5}{x} g'(x) + \frac{L^2 g(x)}{x^2} + \\ &(\varepsilon^2 - m^2 - (\varepsilon + m)A(x))g(x) = 0 \end{aligned} \tag{11}$$

with $A(x)$ given by (4), and $L^2(\Omega) = -\gamma(\gamma + 4)$ is the grand orbital operator and γ is the grand angular quantum number given by $\gamma = 2n + l_\rho + l_\lambda$.

Following the method used by Znojil [16,17], we make an ansatz

$$g_\gamma(x) = \exp(h(x)) \tag{12}$$

with $h(x)$ as

$$h(x) = -\frac{1}{2}\alpha x^2 + \beta x + \delta \ln x \tag{13}$$

[18-21].

This implies

$$g''_\gamma(x) + \frac{5}{x} g'_\gamma(x) = \left[h''(x) + h'^2(x) + \frac{5h'}{x} \right] g_\gamma(x) \tag{14}$$

Equations (14) and (11) yield α, β, γ and the constraints between the potential parameters a, b, and c. These read

$$\begin{cases} \alpha = (a(\varepsilon + m))^{\frac{1}{2}} & (15\text{-}1) \\ \beta = -\left[2\alpha(3+\gamma) - (\varepsilon^2 - m^2)^{\frac{1}{2}} \right] & (15\text{-}2) \\ \beta(5+2\gamma) = -(\varepsilon + m)c = -\dfrac{b(\varepsilon + m)}{2\alpha} & (15\text{-}3) \\ \delta = \gamma, -\gamma - 4 \end{cases}$$

Taking $\delta = \gamma$, leads to a wavefunction which is well behaved at the origin.

We try to solve this problem by taking into account the center of mass correction. Using Jacobian coordinates, the distance between particles would separate into three equations for ρ, λ, and R, where R is the center of mass of the three quarks system with equal mass (m),

$$\vec{R} = \frac{1}{3}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3) \tag{16}$$

and the two other equations, ρ and λ, were combined to give the hypercentral equations which we discussed previously. For three quarks with energy ε and mass m, from Equation 9, we have

$$\left[\sum_{i=1}^{3} \frac{d^2}{dr_i^2} + \sum_{i=1}^{3} (\varepsilon + m) A(r_i) - 3(\varepsilon^2 - m^2) \right] \prod_{i=1}^{3} \varphi_i = 0 \tag{17}$$

Let $\eta = \sqrt{3}R$, then

$$\left[-\frac{d^2}{d\eta^2} + A_1(\eta) - (\varepsilon^2 - m^2) \right] \varphi(\eta) = 0 \tag{18}$$

Now it is obvious that the center of mass energy is

$$E_{cm} = (\varepsilon^2 - m^2)^{\frac{1}{2}}. \tag{19}$$

Finally we have shown that for three quarks, with energy ε and mass m, with the potential $U(x)$ the center of mass energy is $(\varepsilon^2 - m^2)^{1/2}$.

As is well known Bogoliuabv's assumption is

$$M' = M + E_{cm} = 3\varepsilon \tag{20}$$

In which M' and E_{cm} are corrected nucleon mass and center of mass energy, respectively.

From Equations 20 and 21, assuming

$$\xi = \frac{m}{\varepsilon} \tag{21}$$

we get

$$\varepsilon = \frac{M}{3 - \sqrt{1-\xi^2}} \quad \text{and} \quad \xi = \frac{m}{M} \left(\frac{3 + \sqrt{1 - 8\dfrac{m^2}{M^2}}}{1 + \dfrac{m^2}{M^2}} \right) \tag{22}$$

then E_{cm} and the strength of *h.o.* potential are as follows

$$E_{cm} = \frac{M(1-\xi^2)^{\frac{1}{2}}}{3 - \sqrt{1-\xi^2}}, \tag{23}$$

$$\alpha = \left[\frac{aM(1+\xi)}{3 - \sqrt{1-\xi^2}} \right]^{\frac{1}{2}}. \tag{24}$$

From Equations 12, 14, 15, 15-1, 15-2, and 15-3, the upper component of Dirac spinor of the nucleon is as below:

$$g_\gamma(x) = x^\gamma \exp\left\{ -\frac{1}{2}\alpha x^2 - \left[2\alpha(3+\gamma) - (\varepsilon^2 - m^2)^{\frac{1}{2}} \right] x \right\} \tag{25}$$

Equations 15-2, 19, and 23 are used to summarized $g_\gamma(x)$ which reads

$$g_\gamma(x) = x^\gamma e^{E_{cm}x} \exp\left[-\frac{1}{2}\alpha x^2 - 2\alpha(3+\gamma)x \right] \tag{26}$$

The lower component $f_\gamma(x)$ of the Dirac hyper-central spinor can be found from (8,26). The normalized spin $\frac{1}{2}$ positive parity solution of the quark under standard hyperspherical potential (4 and 6) is introduced by the following form.

$$\psi^0{}_\gamma(x) = \begin{bmatrix} g_\gamma(x) \\ \dfrac{-i\,\vec{\sigma}\cdot\hat{x}\,(3-\sqrt{1-\xi^2})}{M\,(1+\xi)}\left(g'_\gamma(x) - \dfrac{J^2 - L^2 - \frac{3}{4}}{x^2}\, g_\gamma(x)\right) \end{bmatrix} \quad (27)$$

By using this wavefunction and introducing hyperfine interaction of the standard form as the pertubertive, we can treat this function.

Hamiltonian with the Hyperfine Interaction (Depending on Spin and Isospin)

The standard hyperfine interaction is used in order to reproduce the splitting within the SU (6) multiplets. It contains a δ-like term which is an illegal operator [2]:

$$H_{Spin} = B_S \left(\frac{1}{\Lambda_S} e^{-(\frac{x}{\Lambda_S})} \right) \sum_{i<j}^{i=3} (\vec{s}_i \cdot \vec{s}_j) \quad (28)$$

where s_i is the spin operator of the i-th quark and $x = r_{ij}$ is the relative quark pair coordinate. The strength of the hyperfine interaction is determined by the $\Delta - N$ mass difference. The fitted parameters are [2]

$B_s = 196.4\, fm^2$, $\Lambda_s = 1.6$ fm

To introduce the isospin nonconfining potential we have the chiral Constituent Quark Model (CQM) [22,23]. The nonconfining part of the potential is provided by the interaction with the Goldstone bosons, giving rise to a spin- and isospin dependent part, which is crucial for the description of the spectrum for energies lower than 1.7 GeV. It has been also pointed out quite recently that an isospin dependence of the quark potential can be obtained by means of quark exchange [24]. More generally, one can expect that the quark-antiquark pair production can lead to an effective quark interaction containing an isospin (or flavour)-dependent term. On the other hand, the fact that the constituent quark model does not contain explicitly this mechanism, may account for the low Q^2 behaviour of the electromagnetic transition in which form factors are not reproduced [24,25].With these motivations in mind, we have introduced isospin-dependent terms in the hCQM Hamiltonian. To this end we have added two terms in the three-quark Hamiltonian with the hyperfine interaction. The first one depends on the isospin only and has the form:

$$H_I = A_I \sum_{i<j} \frac{1}{(\sqrt{\pi}\sigma_I)} e^{-(\frac{r_{ij}^2}{\sigma_I^2})} (t_i \cdot t_j) \quad (29)$$

where t_i is the isospin operator of the quark and $x = r_{ij}$ is the relative quark pair coordinate. The second one is a spin-isospin interaction, given by

$$H_{sI} = A_{sI} \sum_{i<j} \frac{1}{(\sqrt{\pi}\sigma_{sI})} e^{-(\frac{r_{ij}^2}{\sigma_{sI}^2})} (s_i \cdot s_j)(t_i \cdot t_j) \quad (30)$$

where s_i and t_i are respectively the spin and isospin operators of the $i-th$ quark and $x = r_{ij}$ is the relative quark pair coordinate. The fitted parameters in Equation 30 again, can be fitted with the $\Delta - N$ mass difference [2] $A_I = 51.7 fm^2$, $\sigma_I = 3.45\ fm$, $A_{SI} = -106.2\, fm^2$ $\sigma_{SI} = 2.31\ fm$. If the nucleon mass M and the phenomenological quark mass $(100 \le m_q \le 350)$ MeV are used as input then the Equation 27 contain unknown parameter a only. In order to find this parameter a for different values of γ (γ = 0, 1, 2,…), another constraint is introduced $\frac{g_A}{g_V} = 1.26$ which was performed by Golowich [26] for the first time.

$$\frac{g_A}{g_V} = \frac{5}{3}(1-2\delta) = \frac{5}{3}\left[1 - 2 < \psi_\gamma \left| l_z \right| \psi_\gamma >\right] \quad (31)$$

where ψ_γ is the perturbed wave function and we write it as

$$\psi_\gamma = \psi'_\gamma + \sum_{\gamma' \pm \gamma} \frac{\langle \psi_{\gamma'}^{0'} \left| H_{in} \right| \psi_\gamma^0 \rangle \psi'^0_{\gamma'}}{E_\gamma^0 - E_{\gamma'}^0} \quad (32)$$

in which $\psi'^0_\gamma = \psi^0_\gamma \chi_s \chi_I$ and $H_{in} = H_s + H_I + H_{SI}$.

By using the wavefunction (32) and Equation 31 the parameter a can be found.

We first assume $\gamma = 0$. The potential parameters can be extracted from Equations 31 and 15 for proton with $M = 938$ MeV and $m_q = 100$ MeV as follows: $a = 0.511 f_m^{-3}$, $b = 2.294 f_m^{-2}$ and $c = 0.885$.

Calculations for different values of γ = 1, 2,… in the mentioned range for the quark can be done in the same way and are tabulated in Table 1. For proton and other hadrons such as Λ, N,… the calculations are similar. Taking Λ as another example, the quark masses of s and u in Λ can be calculated from $M'_\Lambda = 2\varepsilon_u + \varepsilon_s$.

Parameters for quarks in Λ that were obtained using the above method, are shown in Table 2. For calculating the difference between *u-quark* and *s-quark* masses one must use the ratio $\frac{m_s}{m_u} = 1.46$ of Chiral symmetry [21].

Table 1. Static properties of proton for different quark masses a, b, and c are the strength parameters of potential. This table shows that as the quark mass increases the potential parameters a, b, and c decrease and for $m_q = M/3$ the results are close to the naïve quark model (NQM)

m_q (MeV)	$(<r_{em}^2>_p^{1/2})$ (fm)	(μ_p) (n.m)	(a)(fm^{-3})	(b)(fm^{-2})	(c)
100	0.782	2.732	0.511	2.294	0.885
125	0.789	2.786	0.431	1.190	0.791
150	0.794	2.794	0.392	1.177	0.762
175	0.803	2.801	0.271	1.121	0.618
200	0.812	2.808	0.183	0.810	0.501
225	0.835	2.812	0.136	0.616	0.442
250	0.849	2.814	0.102	0.482	0.403
275	0.869	2.818	0.077	0.389	0.379
300	0.885	2.821	0.048	0.266	0.335
312	0.894	2.824	0.015	0.089	0.205

Table 2. The static properties of Λ for different quark masses. a, b, and c are the strength parameters of potential. This table shows that the quark mass increases the potentials parameters a, b, and c decrease for $m_q = M/3$ the results are close to the naïve quark model (NQM)

m_u (MeV)	m_s (MeV)	$(<r_{em}^2>_\Lambda^{1/2})$ (fm)	(μ_Λ) (n.m)	(a)(fm^{-3})	(b)(fm^{-2})	(c)
100	146	0.453	−0.632	0.831	2.324	0.987
125	182.5	0.453	−0.630	0.721	1.993	0.892
150	219	0.453	−0.624	0.511	1.714	0.831
175	255.5	0.453	−0.620	0.428	1.382	0.718
200	292	0.453	−0.617	0.393	0.931	0.602
250	328.5	0.482	−0.614	0.427	0.780	0.512
250	365	0.494	−0.612	0.258	0731	0438
275	401.5	0.508	−0.611	0.175	0.634	0.394
300	438	0.519	−0.610	0.131	0.321	0.249
320	467.2	0.527	−0.609	0.069	0.181	0.201

Based on the Table 2 for Λ quarks it follows that $0.112 \le (\alpha_s)_u \le 1.110$.

Baryon Charge Radius

Let's take proton and Λ charge-radius. The charge-radius $<r_{em}^2>_p^{\frac{1}{2}}$ is defined as

$$<r_{em}^2>_p = \sum e_q <r^2>_q \tag{33}$$

where

$$\langle r^2 \rangle_q = \int_0^\infty x^2 \ \psi_\gamma(x)\psi_\gamma(x) d^3x \tag{34}$$

Here $\psi_{(\gamma)}(\vec{r})$ is the quark wave function given by (32). Charge radius for different quark masses were calculated and tabulated in Tables 1 and 2. Using the potential parameters in these tables, the results fall in the expected ranges for the charge radius of proton and Λ. That is

$$0.782 fm \le (r_{em}{}^2 >_p{}^{\frac{1}{2}}) \le 0.894 fm \quad \text{for} \quad P \tag{35}$$

$$0.436 \ fm \le (<r_{em}{}^2 >_\Lambda{}^{\frac{1}{2}})^b \le 0.527 \ fm \quad \text{for} \quad \Lambda \tag{36}$$

The charge radius proton surprisingly agrees with experiment

Table 3. Magnetic moment of several baryons of $J^p = \frac{1}{2}^+$ octed and decupled $J^p = \frac{3}{2}^+$ for different quark masses as derived in our model. The quark masses are in the range 100 MeV ≤ $m_q = m_d = m_u$ ≤ 350 MeV and the strange quark mass is $m_s = 1.46\ m_u$

Baryons	Magnetic momentum our model (n.m)	Magnetic Momentum Observed (n.m)
$P(\frac{1}{2}^+)$	$2.732 \le \mu_p \le 2.824$	2.793
$N(\frac{1}{2}^+)$	$-1.993 \le \mu_N \le -1.849$	−1.913
$\Lambda(\frac{1}{2}^+)$	$-0.632 \le \mu_\Lambda \le -0.609$	−0.614±0.005
$\Sigma^+(\frac{1}{2}^+)$	$2.256 \le \mu_{\Sigma^+} \le 2.427$	2.38±0.02
$\Sigma^-(\frac{1}{2}^+)$	$-1.048 \le \mu_{\Sigma^-} \le -0.942$	−100±0.12
$\Xi^0(\frac{1}{2}^+)$	$-1.291 \le \mu_{\Xi^-} \le -1.215$	−1.25±0.014
$\Delta^{++}(\frac{1}{2}^+)$	$3.753 \le \mu_{\Delta^{++}} \le 4.165$	?
$\Delta^+(\frac{1}{2}^+)$	$3.869 \le \mu_{\Delta^+} \le 4.282$	?
$\Sigma^+(\frac{3}{2}^+)$	$-2.131 \le \mu_{\Sigma^+} \le 2.373$	?
$\Sigma^-(\frac{3}{2}^+)$	$-1.179 \le \mu_{\Sigma^-} \le -0.974$	?

The agreement for the magnetic moments are obvious according to the above results which is a consequence of using the (hCQM) and isospin dependent potentials for the baryon magnetic moment for $P, N, \Lambda, \sum^+, \sum^-, \ldots$

Baryon Magnetic Moments

Taking proton and Λ as examples and using the standard definitions of magnetic moment, it can be shown that the general expression for the magnetic moment of a quark in its ground state is [10]:

$$\mu_q = -\frac{2}{3} e_q N^2 \int_0^\infty x^3 f_\gamma(x) g_\gamma(x) dx \tag{37}$$

Using the upper and lower components of the spinor (32) and the potential parameters a_1, b_1, and c_1 from Table 1 and 2, the magnetic moment for different quark masses can be calculated. These are also tabulated in Tables 1 and 2. Based on these tables, proton and Λ magnetic moment vary as:

$$2.712 \text{ n.m} \le (\mu_p) \le 2.872\text{n.m} \tag{38}$$

$$-0.632 \text{ n.m} \le (\mu_\Lambda) \le -0.609\text{n.m} \tag{39}$$

respectively (n.m. = nuclear magnetons), which are well consistent and comparable with the measured value $\mu_p^{\exp} = 2.792$ n.m., $\mu_\Lambda^{\exp} = -0.614 \pm 0.005$ n.m.

In addition based on our method the magnetic moment of other hadrons $(N(\frac{1}{2}^+)\sum^+(\frac{1}{2}^+)$, $\sum^+(\frac{1}{2}^+),\ldots)$ are in good agreement with the experimental results (Table 3).

Conclusion

A considerable improvement in the description of the static properties of nucleon is obtained with an isospin-dependent potential. As quoted in the previous section, a possible motivation of the isospin-dependent terms of the quark interaction is given by quark-antiquark pair production mechanisms would improve theoretical results. In this article we have shown the complete interaction including spin and isospin terms reproduces the position of the quark. The hypercentral potential is a good starting point for construction of an unperturbed states and leads to realistic quark states, which shows the static properties of nucleon which are sensitive to the corrected wave functions. The higer-order correction will give better results.

Since this model gives reasonable results, it would lead us to determine the kind of modification which yields the observable static properties of a nucleon that is super singly close to the experiment. In Table 1 we have shown the relative modification of the axial charge $\frac{g_A}{g_V} = 1.26$, magnetic moment (μ) and root mean square radius (RMS) which are comparable well to the experimental results . The consistency for the magnetic moments is surprisingly good for p, N, Λ,… The deviations are very small and probably undetectable and give uncertainties. The results are applied to all of the baryons as well.

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