An Approach for Investment Allocation among

Alternative Economic Sectors

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Abstract

This paper proposes a procedure for an effective investment allocation: a Linear Programming Model using a Social Accounting Matrix (SAM). The Gross Regional Products (GRP) is the objective function. Several constraints, such as job creation for different levels of human force, income distribution inequality, supply and demand constraints for products, are considered in the model. The economic sectors of the region for a certain proportion of products are ranked with respect to the above objective function and constraints for any level of investment. The model is examined with the SAM of the Golestan Province in Iran for the year 1993/1994. The study shows that the model introduced can lead to a maximum level of GRP with respect to other goals and constraints for any level of investment.

Keywords: Social Accounting Matrix, Linear Programming, Investment Allocation, Iran, Golestan Province

1- Introduction

Investment is often considered as a barrier for progress in developing countries. There have been numerous plans that were not implemented properly due to insufficient investment in these countries. In other words, the shortage of investment has been recognised as one of the problems of developing countries.

In addition, almost all development theories, e.g., those of Smith (ed. 1904), Keynes (1936), Myrdal (1957), and Solow (1956), consider capital as the

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main factor for economic growth and development. Besides, in view of experimentally speaking, there are a number of recent studies, e.g., Yue (1999) and Gripaios *et al.* (1997), that confirm the role of investment in economic growth and development. Hence, an adequate procedure for investment allocation seems be the solution to the problem.

A great amount of research has been carried out so far to allocate investment in different regions in a country or in different sectors of a region or a country. Aninur (1963) proposed a model for regional allocation of investment, using a linear programming model, which maximised national income subject to some constraints such as equality total investment with total saving in the whole country for a given year and regional income disparity to demonstrate that the rate of growth of national income is not necessarily maximised by concentrating on investment in the most productive region of a country if regional rates of saving are not identical. Also, Reiner (1963), Sakashita (1967) and Yamano and Ohkawara (2000) allocated investment among regions, based on some criteria such as Gross National Product (GNP), national income and the marginal productivity of public capital, respectively. A regional or national investment was allocated among some sectors by Hurter *et. al* (1964), Fakin (1997), Araujo and Teixeira (2002), Chan (2004) and Garvin and Cheah (2004) subject to some criteria such as expenditures on fixed unit items.

To this end, different procedures have been used for investment allocation. A geographic partitioning analysis in organizing regional investment criteria was used by Reiner (1963). Hurter et al. (1964) applied a linear programming model for a regional investment allocation. Likewise, a linear dynamic programming model was proposed by Sakashita (1967) for public productive presumption investment allocation between two regions. A dynamic goal programming model was also applied by Watanabe et al. (1981) for a similar purpose, i.e., water resources allocation to meet an increase in regional activity and water quality conservation of the rivers as two objectives.

Furthermore, Azis (1992) presented a simultaneous model to evaluate the impacts of selected allocation criteria associated with government transfers. Similarity, a multi-attribute decision model that accounts for both tangibles and intangibles was presented by Chan (2004) as a tool for more effective allocation of capital budgeting decisions in municipal governments. The marginal productivity of public capital was also applied by Yamano and Ohkawara (2000) to compare alternative policies of allocation of public investment. The marginal rate of substitution between investment and labour framework was developed as

an investment decision by Fakin (1997) using a dynamic and static effects. And finally, to allocate the investment between capital and consumption goods sectors, Araujo and Teixeira (2002) analysed the Pasinetti's approach in a multi-sector framework.

This paper intends to propose a technique for optimum allocation at any level of investment on alternatives production sectors. For this purpose, different groups of production sectors with certain levels of production are ranked with respect to GRP maximisation and constraints of the planning model. By estimating the necessary investment (through a relation between outputs of sectors and the required capital asset) for different levels of the optimum GRP, the required investment for all collection of sectors have been estimated. To demonstrate the result of considering investment effectiveness in investment allocation, the results of allocation are compared in two opposite conditions, i.e., considering or ignoring the investment effectiveness. Finally, it was shown that it is possible to find an optimum procedure for any level of investment allocation with respect to the aims of planning.

As to the contribution of the model, it can be said that it has several contributions. First of all, using SAM as a data base for the model, i.e., allows the researcher to calculate GRP, employment of labour, income distribution for new employment and investment for planning according to the sectors' production activities. Another contribution of the model is that it leads to a maximum GRP as objective function with respect to other goals and constraints by allowing the planner to use the investment in an effective situation. Finally, flexibility of the model for any level of investment is another contribution that makes it useful for different situation.

The linear programming model including of the objective function and all constraints is introduced in the next section. Then the collections of economic sectors are ranked by the proposed model through a discussion to achieve the highest GRP with respect to or irrespective of the required investment. Finally, the results of discussion are classified as the conclusion of the study.

2- Linear Programming Model

A Linear Programming Model linked to a Social Accounting Model is used in this paper. The GRP of the region is taken into account as the objective function. The paper considers several constraints in terms of job creation for different groups of human force, income distribution inequality, supply and

demand constraints for products. The social accounting matrix of Golestan Province in Iran for the year 1993/1994 is used to estimate the related coefficients as shown in Table 4.

The GRP of the region can be divided into two devices. The first part includes the value added concerning private or public production factors that are generated in the region and are examined relationship **Error! Reference source not found.**. The $\mathbf{M^{v1}}$ is a row vector in which $\mathbf{M^{v1}_j}$ concerns the vertical sum of the block $\mathbf{M^{v1}}$ a submatrix of $\mathbf{M}$ located ^{between} $m_{1,10}$ and $m_{9,36}$, associated with the production factors' income in production activities, in matrix $\mathbf{M_n}$ the Leontief matrix's inverse of the SAM as shown in relationship **Error! Reference source not found.**. Y^* is a subvector of $\mathbf{Y}$ associated with the exogenous final demand for products of production sectors including Y_{10}, Y_{11}, Y_{12}, …, Y_{36}. Hence, GRP_1 explores changes in the total income generated as a result of responding to the exogenous final demand for products of the region:

$$
\mathbf{M_n} = \begin{bmatrix}
m_{1,1} & \cdots & m_{1,9} & m_{1,10} & \cdots & m_{1,36} & m_{1,37} & \cdots & m_{1,52} \\
\vdots & & & \vdots & & \vdots & \vdots & & \vdots \\
m_{8,1} & & m_{8,9} & m_{8,10} & & m_{8,36} & m_{8,37} & & m_{8,52} \\
m_{9,1} & \cdots & m_{9,9} & m_{9,10} & \cdots & m_{9,36} & m_{9,37} & \cdots & m_{9,52} \\
m_{10,1} & \cdots & m_{10,9} & m_{10,10} & \cdots & m_{10,36} & m_{10,37} & \cdots & m_{10,52} \\
\vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
m_{36,1} & \cdots & m_{36,9} & m_{36,10} & \cdots & m_{36,36} & m_{36,37} & \cdots & m_{36,46} \\
\vdots & & \vdots & \vdots & & \vdots & \vdots & & \\
m_{52,1} & & m_{52,9} & m_{52,10} & & m_{52,36} & m_{52,37} & & m_{52,52}
\end{bmatrix} \quad (1)
$$

$$
GRP_1 = \mathbf{M^{v1}} \times \mathbf{Y^*} = \sum_{i=1}^{9} m_{i,10} \times Y_{10} + \sum_{i=1}^{9} m_{i,11} \times Y_{11} + \sum_{i=1}^{9} m_{i,12} \times Y_{12} \cdots + \sum_{i=1}^{9} m_{i,36} \times Y_{36} = \quad (2)
$$

$$
M_1^{v1} \times Y_{10} + M_2^{v1} \times Y_{11} + M_3^{v1} \times Y_{12} + \cdots + M_{27}^{v1} 1 \times Y_{36}
$$

The second part of *GRP* concerns the net indirect taxes received by the government in the region, denoting GRP_2. Since the net indirect tax depends on the level of the products of production sectors, it can be formulated with respect to the level of these products:

$$GRP_2 = \mathbf{C} \times \mathbf{Y}^*$$ (3)

Where $\mathbf{C}$ is a row vector, i.e., C_1, C_2, C_3... C_{27}, denoted as the total net indirect tax receivable in the region from a unit increase in Y_{10}, Y_{11}, Y_{12},..., Y_{36}, respectively.

Finally, *GRP* of the region can be derived by summation of increasing GRP_1 and GRP_2 that are examined through equation **Error! Reference source not found.**.

$$GRP = GRP_1 + GRP_2 = (M_1^{v1} + C_1) \times Y_{10} + (M_2^{v1} + C_2) \times Y_{11} + (M_3^{v1} + C_3) \times Y_{12} + \cdots$$
$$+ (M_{27}^{v1} + C_{27}) \times Y_{36} = g_1 Y_{10} + g_2 Y_{11} + g_3 Y_{12} + \cdots + g_n Y_{27}$$ (4)

$\mathbf{G}$ is a raw vector in which g_j is the place of $M^{v1}{}_j + C_j$.

The relationships concerning the job creation for human force are considered as constraints of this model. These relationships pertain under high school (HS) diploma, HS diploma, undergraduate and postgraduate employment.

$$N_{1,1}Y_1 + N_{1,2}Y_2 + \cdots + N_{1,n}Y_n \leq A_1$$ (5)

$$N_{2,1}Y_1 + N_{2,2}Y_2 + \cdots + N_{2,n}Y_n \leq A_2$$ (6)

$$N_{3,1}Y_1 + N_{3,2}Y_2 + \cdots + N_{3,n}Y_n \leq A_3$$ (7)

$$N_{4,1}Y_1 + N_{4,2}Y_2 + \cdots + N_{4,n}Y_n \leq A_4$$ (8)

where N_{ij}`s shows total i^{th} group of human force that would be employed for a unit increase in the exogenous final demand for j^{th} sector's products. A_1, A_2, A_3 and A_4 refer to the maximum size of different educational groups of human force's supply in this wage level. The smaller and equal signs of relationships enable us to prevent extra employment with respect to labour supply of the region that may lead to certain problems for the region due to immigration, of cource, if assumed necessary.

To prevent an undesired level of unemployment for different groups of human force, the relationships **Error! Reference source not found.** to **Error! Reference source not found.** are used in the model. E_1, E_2, E_3 and E_4 refer to the level of minimum desired level of job creation for different groups of human force. The left hand side of the relationships **Error! Reference source not**

found. to **Error! Reference source not found.** and **Error! Reference source not found.** to **Error! Reference source not found.** measure the size of demand for different groups of human force which are exactly the same.

$$N_{1,1}Y_1 + N_{1,2}Y_2 + \cdots + N_{1,n}Y_n \geq E_1 \tag{9}$$

$$N_{2,1}Y_1 + N_{2,2}Y_2 + \cdots + N_{2,n}Y_n \geq E_2 \tag{10}$$

$$N_{3,1}Y_1 + N_{3,2}Y_2 + \cdots + N_{3,n}Y_n \geq E_3 \tag{11}$$

$$N_{4,1}Y_1 + N_{4,2}Y_2 + \cdots + N_{4,n}Y_n \geq E_4 \tag{12}$$

The mean income level for the new human force of the region is another constraint that will be considered in this model. This constraint is shown as relationship **Error! Reference source not found.**. b_{1j}, is the vertical sum of $\mathbf{M^{v2}}$, a submatrix of $\mathbf{M}$ associated with the private production factors' income in production activities, located between $m_{1,10}$ and $m_{8,36}$. Thus, b_{1j} measures the role of a unit exogenous final demand for products in sector j on the mean income for the new employed human force of the region. B_1 denotes the minimum desired level that is specified as constraint for the mean income of the new human force that is employed in the region. In addition, the greater or equal signs of the relationship let the mean income of private production factors holders of the region turn out more than or at least equal to a minimum desired level in the resource allocation process.

$$\mu = b_{1,1}Y_{10} + b_{1,2}Y_{11} + \cdots + b_{1,n}Y_{36} \geq B_1 \tag{13}$$

Relationship **Error! Reference source not found.** measures the new employment income distribution inequality. The relative mean deviation index, I, that can take a linear form is applied, which takes the following form (Kakwani 1980).

$$I = (\frac{1}{2 \times N \times \mu})M^{pc} = (\frac{1}{2 \times N \times \mu})(M_1^{pc} \times Y_{10} + M_2^{pc} \times Y_{11} + \cdots + M_{27}^{pc} \times Y_{36}) =$$
$$b_{2,1} \times Y_{10} + b_{2,2} \times Y_{11} + b_{2,3}Y_{12} + \ldots + b_{2,n} \times Y_{36} \leq B_2 \tag{14}$$

N refers to the number of the new employment and $\mathbf{M^{pc}}$ components exhibit the total difference of per capita income of the sectors' production factors from the average level per capita income of the region due to a unit exogenous final

demand for goods and services produced in these sectors. Thus, $b_{2,j}$ reveals the impact of a unit increase in exogenous final demand for goods or services that is produced in the jth production sector on income distribution inequality of the newly employed of the region. In addition, B_2 shows a maximum acceptable income distribution inequality for the human force that will be employed in the region.

Relationship **Error! Reference source not found.** is used as a constraint of the model to consider the supply and demand for the products of production sectors in the region. Hence, relationship **Error! Reference source not found.** is representative of 27 constraints for products of 27 production sectors in which d_i, the maximum possible products of sector i, is shown in Table 4. Thus, relationship **Error! Reference source not found.** is considered as another constraint of the model.

$$X^* = M^p \times Y^* \leq d_i \qquad , i = 1, \cdots, n \qquad (15)$$

Finally, since all the decision variables are the exogenous final demand for the sectors' products, Y_is would be greater than or at least equal to zero, as shown in **Error! Reference source not found.**.

$$Y_i^* \geq 0 \qquad , i = 1, \cdots, n \qquad (16)$$

3- Ranking of economic sectors

The linear programming model is applied in different stages. The above model is solved in the first stage excluding relationship **Error! Reference source not found.**. In other words, there is no supply and demand constraint in the first stage. The optimum solutions of the model include both zero and non-zero solutions, requiring the optimum level of products of sectors that leads to the optimum value of GRP. In fact, these sectors are considered as the first highest rank sectors for GRP maximisation in comparison with other sectors.

In the second stage, a new solution can be obtained for the highest rank sectors by adding the supply or demand constraints on products from relationship **Error! Reference source not found.** associated with these sectors to the linear programming model of the previous stage. Since any constraint such as a barrier prevents the maximisation of the objective function, the optimum GRP obtained in the second stage is smaller than or equal to that of the

first stage. Consequently, the results for the sectors are considered as the second highest rank (see Appendix 1).

In the above model, the objective function and all constraints except the supply and demand of sectors were solved. In the first stage, sectors 16, 21, 24, 25 and 26 that are associated to Metal Products, Communication, Education, Health and Public Services were selected as the highest rank sectors in which the value of products of these sectors (as mentioned above) should also be considered. For instance, based on above calculations, a collection of 17820, 93579, 15160, 2085 and 14215 million rials worth production in Metal Products, Communication, Education, Health and Public Services sectors, in that order, will maximise the GRP of the region considering the above constraints. In the second stage, considering the supply or demand constraints of the region on products of these sectors (as shown in Table 4) these five sectors' supply or demand constraints were added to the first stage model. To consider the constraints mentioned above, a collection of sectors, i.e., 6, 16, 21, 24, 25 and 26, were selected as the second highest rank. Similarly, all eligible sectors were specified as the non-zero solution set in a total of 13 stages.

Table 1: Ranking of sectors based on objective function and constraints

Rank	Sectors No.	entered	Dropped	The Maximum GRP (1000 rials)
1	16,21,24,25,26	16,21,24,25,26	-	246679642
2	6,16,21,24,25,26	6	-	235225805
3	5,6,16,21,23,24,26	5,23	25	232564759
4	6,8,16,21,23,24,26	8	5	231558224
5	6,8,16,18,21,23,24,26	18	-	231261749
6	5,6,7,8,16,18,21,23,24,26	5,7	-	230816799
7	4,5,6,7,8,16,18,21,23,24,25,26	4,25	-	225097617
8	4,5,6,7,8,12,16,18,21,22,23,24,25,26	12,22	-	224269886
9	4,5,6,7,8,12,15,16,18,21,22,23,24,25,26	15	-	224110824

10	4,6,8,12,13,16,21,23,24,25,26	13	5,7,15,18,22	221355073
11	3,4,6,7,8,12,13,14,16,18,21,22,23,24,26	3,7,14,18,22	25	220790636
12	3,4,6,7,8,12,13,14,16,18,21,22,23,24,26,27	27	-	220574601
13	3,4,6,7,8,12,13,14,16,18,21,22,23,24,26,27	-	-	220574601

To conclude, according to Fig. 2, each stage leads to a higher (or at least an equal) optimum value for GRP in comparison with the next one (position of *B* in compare to *C*) if there is no constraint for its sectors' supply or demand and it can be fully implemented. Thus, the related GRPs can be calculated for Table 1.

However, the above results were obtained irrespective of any of the constraints on the new non-zero solutions at any stage. Since these constraints are determined independent from the role of these sectors in the model, the available result may change the sectors' priority in Table 1. Thus the maximum available GRP at different stages are calculated for the Table 2.

Table 2:The maximum available GRP, required investment[#] and GRP/Investment relating different ranks

stage	GRP[*]	Required Investment[**]	GRP / Investment[***]	stage	GRP	Required Investment	GRP / Investment
1	22143932	534811	41.41	8	19446105	334368	58.16
2	13602542	325342	41.81	9	19320363	360084	53.66
3	19649913	288975	68	10	19796539	287270	68.91
4	21055602	342011	61.56	11	19491326	311121	62.65
5	1654568	29381	56.31	12	19465842	312072	62.38
6	9013006	174598	51.62	13	19465842	312072	62.38
7	19292341	319557	60.37				

* (1000 rials)   ** (1,000,000 rials)   ***(thousands rials GRP for per 1,000,000 rials Investment)

[#]Source: Calculations based on the Golestan Province Budget and Planning Organisation Archive

As to the classification of economic sectors, based on the maximum effectiveness of investment, it is necessary to consider the required investment for different collections of sectors, as displayed in Table 2. For this purpose, it is necessary to calculate the required investment associated to the collection of sectors, which categorise different stages. In addition, dividing the maximum available GRP associated to any stage by their related required investment, the values of GRP resulting from one million rials investment for different collections of sectors were obtained (see Table 2).

The ratio of GRP to investment was applied in order to have a maximum effectiveness for different levels of investment. To this end, first different rows of Table 2 were sorted in descending order with respect to the ratio of GRP to investment. The collection of sectors concern the highest GRP to investment ratio is specified as the first rank collection that has the maximum effectiveness for investment(See Appendix 2). In the second stage, with respect to the related GRP, the rows whose GRPs were less than or equal to that relating the maximum GRP/Investment ratio were ignored. This is due to the possibility of achieving this value of GRP by less investment or allocating the investment with a higher effectiveness. Thus, the second stage obtained with respect to the second best GRP/Investment ratio, its GRP being greater than that of the first rank among the remaining rows. By continuing this procedure, other collections of sectors were specified as displayed in Table 3.

Table 3: Ranking the collection of sectors with respect to higher effectiveness of investment (1000 rials)

Current Rank	Rank in Table 1	Sectors' No.	GRP[*]	Required Investment[+]	GRP / Investment[^]
1	10	4,6,8,12,13,16,21,23,24,25,26	19796539	287270	68.91
2	4	6,8,16,21,23,24,26	21055602	342011	61.56
3	1	16,21,24,25,26	22143932	534811	41.41

*(1000 rials)   + (1,000,000 rials)   ^(thousands rials GRP for per 1,000,000 rials Investment)

A comparison of Table 1 and Table 3 reveals that there are differences among the results of these tables. For example, the collection of sectors denoted as rank 10 in Table 1 is specified as rank 1 in Table 3 to maximise the GRP of the region for investment funds up to 287,270 millions rials with respect to the same constraints considered for Table 1 collections. Using the first rank collection of sectors in Table 1, it can be demonstrated that 478,062 million rials investment is required to reach 19,796,539 thousand rials value of GRP. But, 287,270 million rials investment will be enough to reach the same level of GRP by selecting the first rank collection of sectors in Table 3 that is about 66 percent less than the above collection. In addition, because of the characteristics of Leontief function, the results of Table 3 can be worked out by calculating the required investment for a certain level of GRP for any collection of sectors denoted as the first, second and third ranks in this table.

Therefore, the initial funds of investment are advised to be made on the sectors denoted as rank 1 in Table 3. Thus, consideration of the determined values as outputs of these sectors leads to the highest efficiency for investment. Obviously, when higher funds of investment is available, the collection of sectors relating the second or third rank will lead to higher levels of GRP, respectively, though being less effective for investment.

Moreover, it can also be proved that any proportion of a linear programming solution will be optimum when compared with similar feasible solutions. This can be proved through multiplying all the right hands side constraints' by a positive value, which leads to the same proportion of change on the optimum solutions of the model (See Appendix 3). For example, in the case of Fig. 1, when due to insufficient investment, allocation is less than that determined at the first stage, the optimum solutions of the model will lie on *OE*. But the optimum solutions of different levels of the second stage will lie on *GC* so that, if continued, the line will pass *O*, i.e., the intersection of axes. Consequently, in the case of two or three variables, the sets of optimum solutions lie on a straight line that passes through the intersection of axes.

Therefore, any proportion of the first stage products leads to a maximum GRP in the region in comparison with any other collections of products. For example, when the investment funds are less than 287,270 million rials, the most effective investment will be obtained by multiplying the share of each sector's investment by 287,270 million rials associated with the maximum GRP by the ratio of the proposed investment funds to 287,270 million rials .

In addition, in cases when a fund more than that specified as the required investment associated to maximum GRP of the first rank is available, a similar procedure should be followed in allocation of this investment funds. Obviously, when a higher level of funds for investment is available, the planner should allocate them to obtain the highest value of GRP, the objective of the model. Hence, the extension of this procedure leads to the allocation any level of investment funds up to 534,811 millions rials, i.e., the maximum required investment (public or private) for the region with respect to other constraints specified in Table 3, based on an effectiveness for investment. As a result, the model is flexible to be applied for any level of investment.
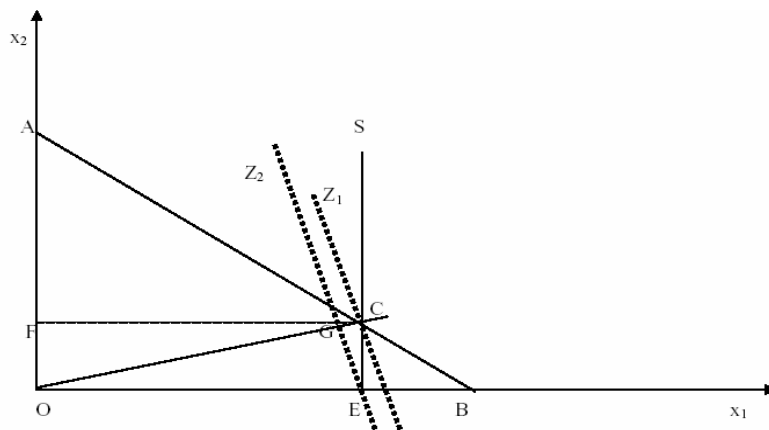

Fig. 1.: Optimum solutions of the model in the case of insufficient investment

4- Conclusion

To rank economic sectors, a Linear Programming Model linked to a Social Accounting Model has been applied. The GRP of the region is taken into account as the objective function. The job creation for different groups of human

force, income distribution inequality as well as supply and demand constraints for products are considered as constraints of the model. Thus, the economic sectors were ranked based on maximum GRP value in two separate conditions, i.e., with respect to and irrespective of the required investment. The results of the two cases were compared with each other. It was found that when the sectors are selected with respect to the required investment, this will lead to an effective investment achieving the highest GRP.

Based on the achieved results of the study, the collection of sectors denoted as NO.s 4, 6, 8, 12, 13, 16, 21, 23, 24, 25 and 26 with a certain level of outputs are recommended to achieve the highest effectiveness for investment. One of the advantages of this procedure is finding an effective allocation for investment that will be quite valuable for developing countries, which generally suffer from its shortage especially when the can affect investment through certain economic or administrative instruments such as finance facilities and establishment licences. In addition, this technique considers other relevant constraints that are important in the planning process. Finally, this technique can be applied to other production factors like water, land, and environment.

Appendix 1

The role of constraints in a Linear Programming model can be illustrated through a very basic model in Fig. 2. In this figure, x_1 and x_2 are two decision variables that display the level of the outputs of sectors *I* and *II*, respectively. The line *AB* runs as the constraint of the model. Thus, triangle *OAB* specifies the feasible area of the model. *Z* is assumed to have the slope of the objective function. In this simple linear programming model, the intersection of the feasible area with the highest available level of the objective function slope line, i.e., *B*, shows the optimal solution of the model. Hence, the optimum solution for the model is determined as $x_1 = OB$ and $x_2 = 0$. In other words, sector *I* is recognised as the first rank sector.

In the second stage, it is assumed that due to the supply or demand constraint of the sector *I*, it is not possible to produce more than *OE* units in this sector. Hence, the *ES* is another constraint of the model and the feasible area for the solution of the model changes into trapezium *OACE*. Therefore, C, the intersection of the feasible area with the highest level of the objective function slope shows the position of the optimum solution of the model. As a result, the

$x_1 = OE$ and $x_2 = OF$ are considered as the optimum solutions of the model. In fact, the latter group of solutions are obtained when, due to supply or demand constraint, it is not possible to produce on the B position ($x_1 = OB$ and $x_2 = 0$) that is associated to the Z the first highest rank. Hence, the C position can be considered as the second highest rank.
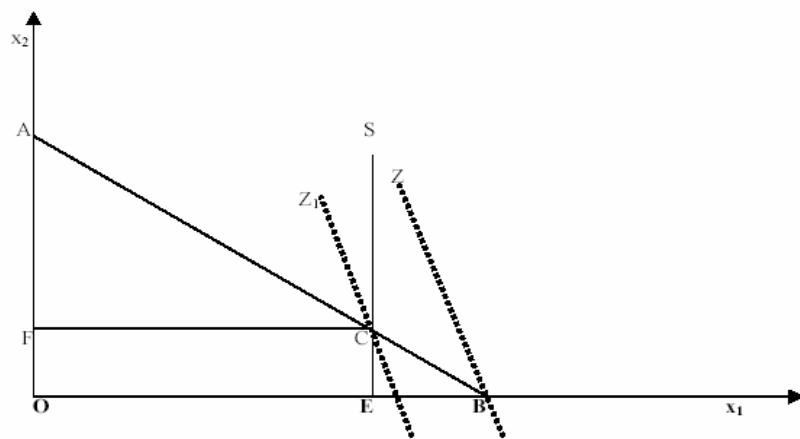

Fig 2: The effects of consideration of supply and demand on maximum solution

Appendix 2

It is notable that in the case of the two sectors, when B is representative of more GRP in comparison with A (Fig. 3), A is representative of the greatest GRP/investment ratio among infinite positions between B and A, if it has not a greater ratio of GRP/investment. Since it is assumed that there is a position like C which is representative of a greater GRP/investment ratio in comparison with that of B, it can be proved that the position D is representative of a greater GRP/investment ratio. This is due to fix investment required for a unit product of sectors and proportionate to changes in GRP due to changes in sectors products (See appendix 3). Similarly, it can be proved that movement from D to A leads to an increase in the value of GRP/investment ratio.
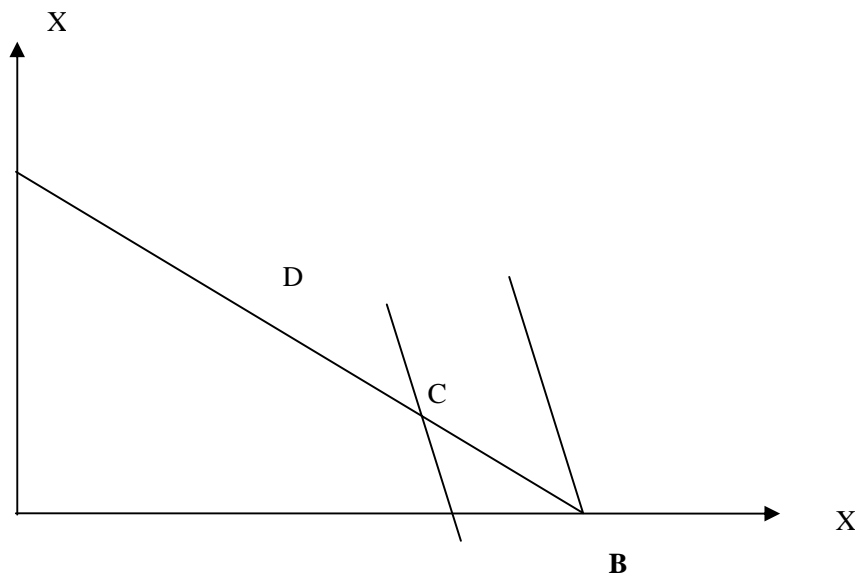
Fig. 3. Consideration of Investment on maximum solution

Appendix 3

In a general linear programming maximisation form in which:

$$\text{max} \qquad Z = \mathbf{C} \times \mathbf{X} \tag{17}$$

$$s.t. \qquad \mathbf{A} \times \mathbf{X} \leq \mathbf{b} \tag{18}$$

$$\mathbf{X} \geq 0 \tag{19}$$

Z is the objective function, $\mathbf{C}$ is a $1 \times n$ decision variable coefficient vector, and $\mathbf{X}$ the $n \times 1$ decision variables vector. In addition, $\mathbf{A}$, an $m \times n$ matrix, represents the resource required for X_i. Finally, $\mathbf{b}$, the available resources, is an $m \times 1$ column vector. Because of the Proportionality Assumption of the linear programming model in which there is no initial income or cost, it can be proven that if all the resources increase t times, the optimum solution will equally increase.

Proof: In the optimisation process in any iteration, $\mathbf{b}^\sim$ shows the new value of decision variable, i.e., $\mathbf{X}^\sim$, and can be written as:

$$\mathbf{b}^{\sim} = \mathbf{B}^{-1} \times \mathbf{b} \tag{20}$$

in which **B** is the basic matrix of the relevant iteration with $m{\times}n$ dimensions. Substituting equation **Error! Reference source not found.** in equation **Error! Reference source not found.** in any iteration, Z can be written as:

$$Z = \mathbf{C} \times \mathbf{B}^{-1} \times \mathbf{b} \tag{21}$$

Consequently, if **b** is multiplied by t:

$$\mathbf{b_t} = \mathbf{t} \times \mathbf{b} \tag{22}$$

the new optimum solution will be as follows:

$$Z_t = \mathbf{C} \times \mathbf{B}^{-1} \times \mathbf{b_t} = t \times \mathbf{C} \times \mathbf{B}^{-1} \times \mathbf{b} = t \times Z \tag{23}$$

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Table 4: Estimation of coefficients of the model for the Golestan Province of Iran for the year 1993/1994

Title of sectors	Sec.no	g_{j*}	N_{1j**}	N_{2j**}	N_{3j**}	N_{4j**}	b_{1j**}	B_{2j***}	$^{\#}d_{j*}$
Farming	1	1.833	47.2	0.11	0.48	0.05	3.64	3.58	31855796
Traditional Livestock	2	1.912	46.69	0.14	0.48	0.06	2.56	4.17	1667870
Modern Husbandry	3	1.354	27.96	0.54	0.54	0.12	1.51	3.53	41229000
Modern Hen-breeding	4	0.868	11.75	0.13	0.31	0.07	1.51	2.12	10233650
Fish-breeding	5	0.853	16.75	1.26	1.49	0.03	2.11	1.63	32252361
Forestry	6	1.808	26.58	0.78	0.73	0.04	1.79	3.09	2772049
Fishery	7	1.498	20.43	0.21	1.41	0.10	2.62	2.73	1268760
Mining	8	1.787	30.77	0.66	0.58	0.05	2.93	3.07	13494300
Food Processing Industries	9	1.372	33.10	0.18	0.71	0.08	2.39	3.92	15925520
Textile industries	10	1.351	27.67	0.23	0.62	0.07	2.98	2.85	5585530
Carpets	11	1.567	82.82	0.13	0.39	0.05	2.66	4.81	1949786
Wood Products	12	1.576	29.77	0.49	1.09	0.11	2.87	2.93	6641249
Publication & Paper	13	1.516	26.82	2.83	0.70	0.06	2.87	3.73	3585400
Chemical Products	14	1.557	25.28	2.64	0.63	0.06	2.70	4.00	3153700
Non-metals Products	15	1.486	27.71	0.38	0.81	0.08	3.02	3.18	2925760
Metal Products	16	1.633	29.83	6.60	0.46	0.05	2.61	4.04	32678000
Machinery Products	17	1.440	28.67	0.31	0.70	0.07	1.78	3.00	48729000
Other Industry	18	1.288	17.97	0.36	0.38	0.09	1.96	2.34	272617
Water, Electricity and Gas	19	1.060	17.87	0.39	1.37	0.13	2.60	2.39	6184642
Construction	20	1.371	31.86	0.15	0.434	0.05	2.51	2.60	27866900
Communication	21	1.729	25.56	0.93	2.68	0.05	3.18	2.74	9363500
Transportation	22	1.683	31.85	0.16	0.54	0.05	2.62	3.59	17571970
Bank and Insurance	23	1.770	22.71	0.37	1.33	0.04	3.37	3.17	3159090
Education	24	1.770	18.89	0.16	15.05	0.51	3.25	3.16	56177580
Health	25	1.671	28.89	0.33	4.98	2.23	3.40	2.91	1192096
Public Services	26	1.790	38.35	0.18	4.90	0.14	3.44	2.65	86468900
Personal Services	27	1.782	28.68	0.13	0.48	0.09	0	4.43	56570550

*: for 1993/1994 1000 rials **: for1993/1994 100,000,000 rials
***: for 1993/1994 $10^{(12)}$ rials
Source: Collected from different organsation