Bubbles in Tehran Stock Exchange

Mohammad Ali Mani∗
Davood Zahedi∗∗

Abstract:
This paper, first, represents theoretical aspects of rational bubbles. Second, it shows one of the tests introduced to detect rational bubbles- integration/ cointegration based test.

Finally, on the empirical side, it explores existence of both explosive and periodically collapsing bubbles in Tehran Stock Exchange. The results reject the existence of explosive bubbles but fail to reject the existence of periodically collapsing ones.

Key words: Rational Bubbles, Tehran Stock Exchange.

∗ Assistant professor of economics, Shahid Beheshti University.
∗∗ PhD candidate, Department of Economics, University of Waterloo and Correspond author (davidza001@gmail.com).
1- Introduction

The Figure below shows the TEPIX/TEDIX ratio (stock price index/dividend index) in Tehran Stock Exchange (TSE) from 1378/2 to 1385/10, using monthly data. The run up in the ratio in the late 1383’s seems extraordinary, especially given the ensuing decline. Many casual commentators attributed this steep rise in stock prices to the presence of a bubble. Can such a claim be substantiated empirically?

A large and growing number of papers propose methods to detect “rational” bubbles. Equity prices contain a rational bubble if investors are willing to pay more for the stock than they know is justified by the value of the discounted dividend stream because they expect to be able to sell it at an even higher price in the future, making the current high price an equilibrium price. Importantly, the pricing of the equity is still rational, and there are no arbitrage opportunities when there are rational bubbles. Section 2 below drives the basic asset pricing relation and rational bubble from a utility maximization problem and points out the assumptions embedded in the ‘standard’ model. Section 3 is to introduce the only structural Econometric test of rational bubbles i.e. the integration / cointegration based test of Diba and Grossman (1988a,b), which, also, contains Evans’ (1991) criticism of the test. Section 4 represents empirical tests of bubbles in the TSE in which we will use two tests: one for detecting explosive bubbles and another for periodically collapsing ones. And, the final section will be concluding remarks.
2- Asset Prices and Bubbles

Consumers’ optimization problem can be used to derive the basic asset pricing relationship assuming no-arbitrage and rational expectations—standard assumptions in economics and finance. For simplicity let expected utility driven from consumption, \( u(c) \), be maximized in an endowment economy:

\[
MAX\left(E_t\{\sum_{i=0}^{\infty} \beta^i U(c_{t+i})}\right)
\]

s.t

\[
c_{t+i} = y_{t+i} + (P_{t+i} + d_{t+i})x_{t+i} - P_{t+i}x_{t+i+1}
\]

Where \( Y_t \) is the endowment, \( \beta \) is the discount rate of future consumption, \( X_t \) is the storable asset, \( P_t \) is the after-dividend price of the asset, and \( d_t \) is the pay off received from the asset. In this paper the focus is on stock prices, thus \( P_t \) is a stock price, and \( d_t \) is dividend, however, in different contexts \( P_t \) may be a house price and \( d_t \) rent, or \( P_t \) may be price of a mine and \( d_t \) the value of ore unearthed every period. The optimization problem’s first order condition is

\[
E_t\{\beta U'(c_{t+i})[P_{t+i} + d_{t+i}]\} = E_t\{U'(c_{t+i-1})P_{t+i-1}\}
\]

For asset pricing purposes, it is often implicitly or explicitly assumed that utility is linear, which implies constant marginal utility and risk neutrality. In this case, equation (1) simplifies to

\[
\beta E_t(P_{t+i} + d_{t+i}) = E_t(P_{t+i-1})
\]

Assuming further the existence of a riskless bond available in zero net supply with one period net interest rate, \( r \), no-arbitrage condition implies \( \beta = 1/(1+r) \). Hence we get to:
Equation (2) is the starting point of most empirical asset pricing tests. This first-degree deference equation can be iterated forward to get the solution

\[ P_t^* = \sum_{i=1}^{\infty} \left(1/(1+r)\right)^i E(d_{t+i}) + B_i = P_t + B_t \]

Such that

\[ E(B_{t+1}/ = (1+r)B_t \] (4)

The asset price has two components, a “market fundamental” part, which is the discounted value of expected future dividends, the first term in the left-hand-side of equation (3), and a “bubble” part, the second term. In this setup, the rational bubble is not a mispricing effect but a basic component of the asset price. Despite the potential presence of a bubble, there are no arbitrage opportunities—equation (4) rules these out.

Under the assumption that dividends grow slower than r, the market fundamental part of the asset price converges. The bubble part, in contrast, is non-stationary. The price of the asset may exceed its fundamental value as long as agents expect that they can sell the asset at an even higher price in a future date. Importantly, the path of the bubble (and consequently the asset price) is not unique. Equation (4) only restricts the law of motion of the non-fundamental part of the asset price, but it implies a different path for each possible value of the initial level of the bubble. An additional assumption about Bt is required to determine the asset price.

A special case of the solution that gives the asset price is \( B_t = 0 \), which implies that the value of the bubble is zero at all times. This is the fundamental solution that forms the basis of present value pricing approaches to equity prices. In the remainder of the paper this solution is alternatively called “the standard model,” “the present value model,” and “the market fundamentals model.” It is useful to explicitly spell out the assumptions other than the absence of bubbles that are embedded in this formulation of the present value pricing model:
1. There are no informational asymmetries.
2. The representative consumer is risk neutral. A corollary of this assumption is that there are no risk premia. This, obviously, rules out time-varying risk premia due to variation in the price or amount of risk as an explanation of volatility of stock prices.
3. The discount rate is constant. Note that this is a restriction on $r$, rather than on $\beta$, although they are not really differentiated in this model. If the discount rate is constant at $r$, and dividends grow at the constant rate $g$, $r$ must be greater than $g$ for sum of the discounted dividend stream to be finite.
4. The process that generates dividends is not expected to change. Although this is not an assumption about the model per se, it is an assumption commonly made in the econometric tests of this model. Many econometric tests need to generate an estimate of expected dividends based on history. This exercise is meaningful only if the dividend generating process is not expected to change in the future.

As stated above, the market fundamentals model is a special case of a more general model that allows for bubbles. The no bubbles special case is justified by a transversality condition in infinite horizon models. The price of the asset today is the sum of the net present value of expected dividends and the expected resale value:

$$ P_t = \sum_{i=1}^{\infty} (1/(1+r))^i E(d_{t+i}) + \lim_{t \to \infty} P_{t+i} $$

The transversality condition asserts that the second term on the right hand side is zero.

3- Integration/Cointegration based Tests of Rational Bubbles

There are variety of tests which introduced by researchers for detecting rational bubbles, for example: variance bound tests of Shiller(1981), West's(1987) two-step approach, integration/cointegration based test of Diba and Grossman(1988a,b), intrinsic bubbles of Froote and Obstefeld(1991) and so on.

Amid them, except for integration/cointegration based test the others are either ad hoc approaches or inconsistent with bubbles definition, Zahedi
Accordingly, in this section we introduce the integration/cointegration test which will be used in the next section- empirical tests.

Bubbles have certain theoretical properties that may be exploited for their detection. Diba and Grossman (1987, 1988a) observe that a rational bubble cannot start, thus if it exists now, it must always have existed. The reasoning depends on lack of arbitrage opportunities and impossibility of negative prices. Lack of arbitrage opportunities imply that there are no excess returns from holding an asset with a bubble component, i.e.

\[ E(B_{t+1}) = (1 + r)B_t \]

as in equation (4). In this case, the actual bubble process (assuming it is a stochastic bubble) follows a stochastic difference equation:

\[ B_{t+1} - (1 + r)B_t = Z_{t+1} \]

(5)

\[ E_i(Z_{t+i}) = 0, \forall i \geq 1 \]

(6)

If \( B_t \) is zero, the bubble will start with the next nonzero realization of \( Z \). If this realization is a negative number, the bubble will be negative and progressively larger in absolute value in expectation, according to its law of motion. This implies that the stock price will be negative in finite time, which is impossible. If the expected realization of \( Z \) cannot be negative when the bubble component is zero, it cannot be positive either, because it has to be zero in expectation to rule out arbitrage opportunities. Thus, when \( B_t \) is zero, all future realizations of \( Z \) must be zero with probability one, and the bubble cannot (re)start. Given this argument, Diba and Grossman conclude that, if there is a bubble it must have existed from the first day of trading. They see this as an argument to rule out rational bubbles, and propose a way to empirically test the absence of bubbles.

Their test for bubbles (1988b) allows for unobserved fundamentals, and imposes some structure on which deviations from fundamentals in data may be blamed on the presence of bubbles. Diba and Grossman specify the market fundamental price to be
\[ P_t^f = \sum_{i=1}^{\infty} (1/1+r)^i E_t (d_{t+i} + O_t) \]  \hspace{1cm} (7)

Where \( O_t \) denoting the fundamentals unobservable to the econometrician. Under the assumption that \( O_t \) is not more non-stationary than \( d_t \) (if dividends are stationary when twice differenced, \( O_t \) is assumed to be stationary when at most twice differenced, for example), the market fundamentals price will be as stationary as the dividends. In the absence of bubbles, if dividends are stationary in levels, stock prices will be equal to market fundamentals and should also be stationary in levels; if dividends are stationary in \( n \)th differences, stock prices should be stationary in \( n \)th differences.

This relationship breaks down in the presence of bubbles, which provides an intuitive bubbles test. The \( n \)th difference of the bubble process, from equation (5) is

\[ (1-L)\sum_{n} [(1-(1+r)L)B_t = (1-L)^n Z_t \]

Diba and Grossman note that with standard simple processes for \( z \) (such as white noise) the first difference of the bubble is generated by a non-stationary and non-invertible process. Indeed, the bubble process is non-stationary regardless of how many differences are taken and this is a property that can be tested econometrically.

A natural way to test for the existence of a bubble in the data, then, is to see whether stock prices are stationary when they are differenced the number of times required to make dividends stationary. They also observe that although both dividends and stock prices are integrated of order one, equation (7) imposes an equilibrium relationship between these two series. Under the null hypothesis of no bubbles in stock prices, and assuming that \( O_t \) is stationary, \textit{dividends and stock prices should be cointegrated}. Note that the assumption made about the unobserved fundamentals is essential.
this time; they should be stationary in levels although dividends only need to be stationary in deference for the test to work.

Before moving on to Evans’ (1991) criticism of these tests, it is useful to think about the interpretation of the results they had indicated that stock prices are more non-stationary than dividends, or that dividends and stock prices are both I(1) but are not cointegrated. In the case the tests do indicate the presence of a bubble, the correct interpretation is that they suggest the presence of ‘something nonstationary’ in the stock price. This could of course be because of a bubble, but it can also be that the assumption made on the unobserved fundamentals does not hold, and the $O_t$ series is, say, integrated of order two while dividends are I(1). It would of course then be an open question whether one can come up with a reasonable unobserved fundamental that would be I(2). Diba and Grossman also allude to this point and argue that although a rejection of the stationarity/cointegration conditions would not be proof of a bubble, failing to reject is proof of nonexistence of bubbles. Evans (1991) disagrees.

Evans points out that although Diba and Grossman’s argument about bubbles only starting on the initial date of trading implies a bubble cannot pop and restart, it is possible that the bubble will collapse to a small nonzero value and then continue increasing, and still follow equation (4). His example of a periodically collapsing bubble is

$$B_{t+1} = (1 + r)B_t V_{t+1}, \quad B_t \leq \alpha$$

(8)

$$B_{t+1} = \{\delta + \pi^{-1}(1 + r)\theta_t B_t - (1 + r)^{-1}\delta\}V_{t+1}, \quad B_t > \alpha$$

(9)

Where $E(V_{t+1}) = 1$, and $\theta_{t+1}$ takes the value of 1 with probability $\pi$ and 0 with probability $(1 - \pi)$. This formulation of the bubble satisfies equation (4), the expected gross return from the bubble is always $(1 + r)$. For small values of $B_t$ the bubble increases slowly, once it is larger than a threshold value, $\alpha$, it expands faster but may collapse each period with probability $(1 - \pi)$. In case of a collapse, the bubble’s value does not shrink to zero; rather, it becomes a small positive quantity, $\delta$. In this case the bubble is not subject to the Diba and Grossman criticism of restarting because it never ‘pops,’ it only gets
discretely smaller periodically. This example of bubbles exploits the fact the
bubble only has to increase at rate $r$ in expectation, but it may collapse in
realization.

Evans generates data from a model with bubbles and does Monte Carlo
experiments of the Diba and Grossman bubble detection test, using their
specification of a bubble (approximated by setting $\pi$ close to unity). He finds
that in this case the test works well, as Diba and Grossman claim. He then
uses lower values of $\pi$ so that the bubble periodically collapses. In this case,
even for values of $\pi$ as high as 0.95, the tests perform much worse, rejecting
the bubbles hypothesis more often than no-bubbles hypothesis. For $\pi$ smaller
than 0.75, the tests almost never detect bubbles.

The unit root based tests have difficulty detecting collapsing bubbles
because these behave more like stationary processes than like explosive
processes as a result of the periodic collapses involved. Of course, this does
not bode well for the Diba and Grossman testing strategy. From Evans’
study, it appears that failing to reject the no-bubbles hypothesis with these
tests may not be conclusive proof that bubbles are indeed absent from data,
either.

It is important to note that Evans does not show the existence of
bubbles in stock prices, he only shows that unit root tests are not adequate to
reject this hypothesis. However, we learn from Diba and Grossman’s unit
root tests that monotonically increasing bubbles are indeed not in stock
prices. We can at least rule out a certain class of bubbles—explosive ones.

Evans’ criticism of unit root tests of rational bubbles led to a number of
papers trying to overcome the difficulty of detecting collapsing bubbles. The
favorite method of attack was to think of collapsing periods of the bubble as
different regimes. This way of modeling the bubble leads to unit root tests
where regime shifts in the mean that follow a Markov process are allowed
for under the null.

An exception is the work of Taylor and Peel (1998). They propose a
cointegration test that is robust to skewness and kurtosis in the error term,
which will be the case for a collapsing bubble. In Monte Carlo simulations
their test is superior to Dickey-Fuller test in detecting a periodically
collapsing bubble.
4- Empirical results

Direct implications of previous discussions are that integration/cointegration based tests of bubbles can only rule out explosive bubbles. It neither can show the existence of explosive bubbles nor rule out periodically collapsing bubbles. Accordingly, to reject existence of explosive bubbles, merely, one can use the integration/cointegration based test. Fortunately, on the other hand, there is the robust test of Taylor and Peel (1998) which one can use for detecting periodically collapsing bubbles. They developed a test which can robustly detect periodically collapsing bubbles. Consequently, in this section we will use both of the tests so that we will be able to assess both explosive and periodically collapsing bubbles.

On the empirical side we will use data from TSE which are gathered monthly in the period of 1378/2-1385/10. Based on the theory explained in the second section we need two variables; price and dividend of stocks. However, since we are going to assess bubbles at the market level, we will use market price and dividend indices- TEPIX and TEDIX, respectively- as previous works used them.

Explosive bubbles

To implement integration/cointegration based test, we need to find out order of integration of both TEPIX and TEDIX. In case both of them have the same order, we can go forward to explore whether they are cointegrated. To determine order on integration of variables and if they are cointegrated, we will use ADF test. Cointegration test will be done by using ADF test on the residuals of equation (10). TEPIX and TEDIX will be cointegrated if the residuals have integration order of zero.

\[ TEPIX_t = a + \beta TEDIX_t + \varepsilon_t \]  

(10)

Results are shown in table (1) which indicates both of the variables are I(1) and they are cointegrated. Hence, the test reject the existence of explosive bubbles in the period undertaken.
Table (1): results of explosive bubbles

<table>
<thead>
<tr>
<th>variable</th>
<th>Test statistic</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level of TEPIX</td>
<td>-0.69</td>
<td>-3.50</td>
<td>-2.89</td>
<td>-2.58</td>
</tr>
<tr>
<td>First difference of TEPIX</td>
<td>-5.52</td>
<td>-4.06</td>
<td>-3.46</td>
<td>-3.15</td>
</tr>
<tr>
<td>Level of TEDIX</td>
<td>-0.56</td>
<td>-3.50</td>
<td>-2.89</td>
<td>-2.58</td>
</tr>
<tr>
<td>First difference of TEDIX</td>
<td>-8.09</td>
<td>-4.06</td>
<td>-3.46</td>
<td>-3.15</td>
</tr>
<tr>
<td>Level of $\hat{z}$</td>
<td>-1.61</td>
<td>-1.94</td>
<td>-2.59</td>
<td>-2.28</td>
</tr>
</tbody>
</table>

To determine order of integration we compare test statistic with critical values when test statistic at level of variable is algebraically less than critical value, it is $I(0)$, when test statistic at first difference is algebraically less than critical values, it is $I(1)$ and so on. And, a model to be cointegrated the residual of the model should be $I(0)$.

Periodically collapsing bubbles

Exploration of periodically collapsing bubbles will be started by estimation of following equation:

\[
\Delta \hat{e}_t = a' + \beta' \hat{e}_{t-1} + u_t
\]

Where $\hat{e}'$ residual of equation (10) and $\Delta$ is difference operator. Then we compute $\hat{W}_t$ as following:

\[
\hat{W}_t = \left[\left[\hat{u}_t^3 - 3 \hat{\sigma}^2 \hat{u}_t \hat{\sigma}^2 \right] \right]
\]

In which $\hat{u}_t$ is residuals of equation (11). In the next step, we extend equation (11) to the following equation:

\[
\Delta \hat{e}_t = a'' + \beta'' \hat{e}_{t-1} + z \hat{w}_t + \varepsilon_t
\]

After estimation of equation (13), finally, we can compute test statistic by equation (14) and compare it with critical values provided by Taylor and Peel (1998).

---

1 - see reference [5].
\[ \tau_a = \frac{\beta}{\sqrt{V(\beta)^2}} \]  

(14)

\(\tau_a\) is test statistic and \(V\) is variance.

Existence of periodically collapsing bubbles will be rejected if the test statistics is algebraically less than critical value. Table (2) shows the results of the test which indicate we can not reject existence of periodically collapsing bubbles.

**Table (2): periodically collapsing test result**

<table>
<thead>
<tr>
<th>Test statistic</th>
<th>Critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.001</td>
<td>-3.79</td>
</tr>
</tbody>
</table>

5- Concluding remarks

In this paper, we presented bubbles definition and structure, firstly, from which we understand that under rational behavior and expectations, bubbles could emerge, which are called rational bubbles.

Then we presented one of the most theoretical tests of bubble detection- integration/cointegration based test. However, with Evans critics, it has been known that the test can at most rule out explosive bubbles and it fails to show periodically collapsing bubbles.

On the empirical side, using two tests, we rejected existence of explosive bubbles in TSE, but failed to reject existence of periodically collapsing ones.

**Acknowledgment:** we are grateful of Fereydon Tafazoli, Kambize Hozhabre Kiani and anonymous referees.
References


