Iranian Economic Review, Vol.15, No.27, Fall 2010

Weak- Form Efficiency in the German Stock Market[1]

Shahram Fattahi *

Abstract

The implications of the efficient market hypothesis are important in assessing public policy issues. This paper attempts to examine the weak-form efficiency of the DAX stock market. Five randomly chosen companies and different sub samples are used to confirm the results. The results show that the DAX stock market follows a random walk and supports the weak-form efficiency of efficient market hypothesis (EMH). However, in some models, the strict rational expectations (RE)/EMH element of 'unpredictability' is rejected, but not necessarily the view of EMH which emphasizes the impossibility of making supernormal profits.

Keywords: Stock market efficiency, German stock market, Variance Ratio Test, ARMA, GARCH.

1- Introduction

The topic of market efficiency has been hotly debated for over 30 years. Although it has been tested in the form of two related theories, that is, the random walk and the efficient market hypothesis (EMH), there is still no general agreement over the validity of these theories. The random walk theory assumes that prices are completely stochastic in nature while the EMH states that profit opportunities do not exist in perfectly efficient markets. In fact, both of these theories assert that in well-functioning markets, prices are unpredictable and fully reflect all available information.

Much of the random walk hypothesis can be traced to Louis Bachelier (1900). He came to the conclusion that "The mathematical expectation of the

---

1- The first draft of this paper has been presented at the 5th Annual International Conference 'FindEcon' 2006.

* Department of Economics, Razi University.

speculator is zero" and he described this condition as a "fair game". Many studies have been done on the hypothesis. On balance, the empirical evidence suggests that the hypothesis is at least approximately true (Beechey, Gruen and Vickery, 2000).

Samuelson (1965) asserted that the randomness in asset prices is due to large groups of investors continuously seeking ways of increasing wealth. According to this view, the movement of market prices was a direct response to unanticipated market-sensitive information. This led to the 'efficient market (Fama, 1960)'. If markets are efficient and current prices fully reflect all information, then buying and selling assets in attempt to outperform the market will effectively be a game of chance rather than skill. Agents process information effectively and immediately and incorporate this information into stock prices. Only new information or 'news' can cause changes in prices and since it is unforecastable, price changes should be unforecastable: no information at time t or earlier should help to improve the forecast of returns. Robert C. Higgins (1992) gave an interesting illustration of market efficiency: "Market efficiency is a description of how prices in competitive markets respond to new information. The arrival of new information to a competitive market can be linked to the arrival of a lamb chop to a school of flesh-eating piranha, where investors are -plausibly enough - the piranha. The instant the lamp chop hits the water; there is turmoil as the fish devour the meat. Very soon the meat is gone, leaving only the worthless bone behind, and the water returns to normal. Similarly, when new information reaches a competitive market there is much turmoil as investors buy and sell securities in response to the news, causing prices to change. Once prices adjust, all that is left of the information is worthless bone. No amount of gnawing on the bone will yield any more valuable intelligence".

Much research has been done on testing market efficiency and most of them have focused on weak-form of EMH. On the one hand, some researchers such as Cootner (1962), Osborne (1962) and Fama(1965) supported weak-form efficiency. The general result of these studies emphasized on randomness in price changes and that price changes were not useful to forecast future price changes. However, on the other hand, other researchers such as Fama and French (1988), Poterba and Summers (1988) and Fortune (1991) asserted that share price changes are predictable. Fama and French state that there are autocorrelations among returns that may imply market inefficiency or time-varying equilibrium expected returns generated by rational investor behavior. Poterba and Summers conclude that noise trading is a plausible reason for the transitory component in stock prices. Fortune presents an interesting statistical analysis of the random walk hypothesis of stock prices (a moving-average model of returns) using over 2700 daily observations on the S&P 500 share index. He concludes that previous period's forecast errors are useful in forecasting future returns, that is, a violation of informational efficiency of EMH. In fact, the tests of EMH involve the joint hypotheses rational expectations (RE) and EMH.

The rational expectations hypothesis is a building block for the efficient markets hypothesis of securities prices. In other words, an application of the concept of rational expectations is the efficient markets hypothesis of asset prices. Using the concept of rational expectations, it comes to the conclusion that stock prices follow a random walk when properly adjusted for discounting and dividends.

Finally the outcome of tests of the EMH is important in assessing public policy issues such as the desirability of mergers and takeovers, short-termism and regulation of financial institutions (Cuthbertson, 1996). This paper attempts to examine the weak-form efficiency of the DAX stock market.

The paper is divided into four sections. Following introduction in section 1 we explain the EMH in section 2. Section 3 describes the data and analyses the empirical results. Finally section 4 concludes the paper.

2- The Efficient Market Hypothesis

The efficient market hypothesis asserts that the stock price P_t already incorporates all relevant information and the only reason for prices to change between time t and time t+1 is the arrival of 'news' or unanticipated events. Forecast errors, that is, $\varepsilon_{t+1} = P_{t+1} - E_t P_{t+1}$ should therefore be zero on average and should be uncorrelated with any information Ω_t that was available at the time the forecast was made. The latter, known as orthogonality property (Sargent, 1993), is often referred to as the rational expectations (RE) element of the EMH and may be represented as:

$$P_{t+1} = E_t P_{t+1} + \varepsilon_{t+1} \qquad (1)$$

The forecast error is expected to be zero on average ($E_t\varepsilon_{t+1}= 0$) because prices only change on the arrival of 'news' which itself is a random variable, sometimes 'good' sometimes 'bad'.

Rational expectations place restrictions only on the behavior of the first moment (i.e. expected value) of ε_t[1]. As a result, one can test the rational expectations hypothesis regarding these restrictions. If ε_t is serially correlated then the orthogonality property is violated, that is, the rational expectations element of the EMH is violated. For example, suppose a serially correlated error term is the first-order autoregressive process, AR (1):

$$\varepsilon_{t+1} = \rho\varepsilon_t + v_t \tag{2}$$

where v_t is a random term (white noise). The forecast error ($\varepsilon_t=P_t-E_{t1}P_t$) is known at time t and hence is a part of information at time t, Ω_t. According to equation (2), the forecast error at

time t (ε_t) has a predictable effect on the forecast error at time t+1 (ε_{t+1}) but ε_{t+1} would be useful in forecasting future prices regarding equation (1). This violates the EMH since information known at time t, ε_t, helps forecast future prices[2].

The efficient markets hypothesis is often applied to the return on stocks, R_t, and implies that one cannot earn supernormal profits by buying and selling stocks. Thus an equation similar to (1) applies to stock returns, that is,

$$\varepsilon_{t+1}= R_{t+1}-E_tR_{t+1} \tag{3}$$

$$E_t\, \varepsilon_{t+1}=0$$

where ε_{t+1} is considered as 'forecast error'. To test EMH, we need a model of how investors form their expectations about the returns. For

---

1- There are no restrictions on the form of second and higher moments of the distribution of ε_t if we assume the EMH/RE. For example, consider an ARCH process that variance of $\varepsilon_{t+1}(\sigma^2_{t+1})$ may be related to its past value (σ^2_t), without violating RE.

2- There are no restrictions on the form of second and higher moments of the distribution of ε t if we assume the EMH/RE. For example, consider an ARCH process that variance of ε_{t+1} (σ²t+1) may be related to its past value (σ²t), without violating RE.

example, assume that: (i) Stocks pay no dividends, so that the expected return is the expected capital gain due to price changes, (ii) Investors are willing to hold stocks as long as expected or required returns are constant, hence:

$$E_t R_{t+1} = k \tag{4}$$

Substituting in (4) in (3)

$$R_{t+1} = k + \varepsilon_{t+1} \tag{5}$$

where ε_{t+1} is white noise and independent of Ω_t. We may consider the expected or required rate of return k on the risky asset as consisting of a risk-free rate r and a risk premium rp (i.e. k=r+rp) and equation (4) assumes both of these are constant over time. Since for a non-dividend paying stock, $R_{t+1}=(P_{t+1}-P_t)/\ P_t \approx \ln\ (P_{t+1}/\ P_t)$ equation (5) implies that ex post the proportionate change in the stock price will equal a constant plus a random error, or equivalently:

$$\ln P_{t+1} = k + \ln P_t + \varepsilon_{t+1} \tag{6}$$

Equation (6) is a random walk in the logarithm of P with drift term k. It should be noted that (the logarithm of) stock prices will only follow a random walk under the EMH if the risk-free rate r and the risk premium rp are constant and dividends are zero. The EMH assumes that excess returns (or forecast errors) only change in response to news so that these errors are innovations with respect to the information available. To test EMH, a definition is needed what constitutes 'relevant information'. There are three forms of the efficient market

hypothesis : (1) Weak-Form: the current price (return) is considered to incorporate all the information in past prices (returns) (2) Semi-strong-Form: the current price (return) incorporates all publicly available information (including past prices or returns)

(3) Strong-Form: prices reflect all information that can possibly be known, including 'insider information'.

To make the tests of the efficient market hypothesis (EMH) operational we assume expected equilibrium returns are constant and rational expectations holds (i.e. $R_{t+1} = E_t R_{t+1} + \varepsilon_{t+1}$). We can write this as: $R_{t+1}=k+\varepsilon_{t+1}$. Consider the regression:

$$R_{t+1} = k + \gamma' \Omega_t + \varepsilon_{t+1} \tag{7}$$

Where Ω_t= information available at time t. A test of $\gamma' = 0$ provides evidence on the 'informational efficiency' element of the EMH/RE. In fact, tests of EMH usually involve the joint hypotheses RE and EMH, that is, (i) that agents use information rationally, (ii) that they all use the same equilibrium model for asset pricing which happens to be the 'true model'.

The regression tests vary, depending on the information assumed which is usually of the following type:

a) data on past returns R_{t-j} (j= 0,1,2,…,m) – that is, weak form efficiency,

b) data on scale variables such as the dividend price ratio, the earning price ratio or interest rates at time t or earlier,

c) data on past forecast errors ε_{t-j}, (j=0,1,2,…,m).

If (a) and (c) are examined together this gives rise to ARMA models. A general ARMA (p, q) model for returns may be represented as follows:

$$R_{t+1} = k + \gamma(L)R_t + \theta(L)\varepsilon_{t+1} \tag{8}$$

Where γ(L) and θ(L) are polynomials in the lag operator such that

$$\gamma(L) = 1 + \gamma_1 L + \gamma_2 L^2 + \gamma_3 L^3 + \ldots + \gamma_P L^P, \quad L^n R_t = R_{t-n} \tag{9}$$

$$\theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \ldots + \theta_q L^q \tag{10}$$

Under the EMH we expect all parameters in γ (L) and θ (L) to be zero. Consider, for example, the ARMA (1, 1) model:

$$R_{t+1} = k + \gamma_1 R_t + \varepsilon_{t+1} + \gamma_2 \varepsilon_t \tag{11}$$

In order to test weak form efficiency, the autocorrelation coefficients between R_{t+1} and R_{t-j} (j=0,1,…,m) can be examined to see if they are non-zero. If the result shows that informational efficiency does not hold, information at time t can help to predict future returns, then it will be worth to examine the ability to make supernormal profits after taking account of transaction costs and possible borrowing constraints.

3- Data and Empirical Results

3-1- Data

The study uses daily market return of the DAX stock market for the period of 2nd January 2004 to 14th March 2005.It includes 308 daily observations for the period. It also considers the 5 randomly selected companies including BMW, LHA, BASF, RWE and TUI. To confirm the results of the analysis, we also examine the first sub-sample (2nd January 2004- 6th August 2004) and the second sub-sample (9th August 2004-14th March 2005).

This research utilizes natural log of market returns and natural log of individual share return in the following way:

$R_t = \ln P_t - \ln P_{t-1}$

R_t = market return (or daily individual share return) in period t

P_t = price index (or daily price per share) at period t

P_{t-1}= price index (or daily price per share) at period t-1

The paper examines some tests including autocorrelation, Dicky-Fuller test and variance ratios. Also, auto-regressive (AR) and auto-regressive-moving average (ARMA) models will be analyzed. Finally, Theil inequality coefficient and Wilcoxon test are used to compare the models.

The aim of this study is to examine whether the DAX stock market follows a random walk or the market is weak-form efficient. If the random walk hypothesis holds, the weak-form of the efficient market hypothesis must hold, but not vice versa. Thus, evidence supporting the random walk model is the evidence of the market efficiency. But violation of the random walk model needs not to be evidence of market inefficiency in the weak form. (Ko and Lee, 1991)

3-2- Empirical Results

3-2-1- Auto-Correlation Test and Q-Statistic

The most obvious test for the weak form of the random walk hypothesis is to directly test the null hypothesis that the autocorrelation coefficients of the returns are zero. One problem with testing for IID (Independent Identically Distributed) returns using autocorrelations is that it is not clear what lags to use to test for zero autocorrelation. If returns are IID, then all autocorrelations should be zero. One solution is to use a statistic that summarizes many autocorrelations known as “portmanteau” statistics, which are joint tests over the set of individual correlation coefficients. The Q-Statistic, developed by Box and Pierce (1970), is a portmanteau statistics that strongly tests for the random walk hypothesis. The Q-statistic at lag k is a test statistic for the null hypothesis that there is no autocorrelation up to order k and simply sums the squares of autocorrelation statistics:

$$Q_k = T \sum_{i=1}^{k} \hat{\rho}_i^2$$

where $\hat{\rho}_i$ is the sample autocorrelation at lag i and k is the number of lags. This statistic tests for zero autocorrelation at all of k lags, giving power to test against a broad variety of alternative hypotheses for return dynamics. It has a chi-squared distribution with k degrees of freedom equal to the number of autocorrelations. This distribution can be used to determine whether or not the statistic is significantly different from zero. In short, the Q-statistic is a reliable measure and a more powerful test because predictability held within a number of lags may not be identified by examining the correlation at one particular lag.

The auto-correlation coefficients that have been computed for the log of the market return series (DAX) in table 1.1 show no significant autocorrelation at different lags for the whole sample period. The Q-statistic and its respective probability for whole lags confirm that there is no significant autocorrelation. The results are similar to the findings of Fama and French (1988).

To confirm the results, the auto-correlation coefficients of the return series for the two different sub-samples have been calculated. The results in table 1.1 confirm that there is no significant auto-correlation of daily DAX market returns for the whole sample period and the sub-sample periods. Thus no significant auto-correlation of the series suggests that the DAX return series follow a random walk model. Our findings also show no significant autocorrelation of BMW, LHA, RWE and TUI returns. However, for BASF returns, there is significant autocorrelations at 1 and 2 lags.

Table1-1: Results of Auto-Correlation (log of DAX return)

	Ac (total)	Q-Stat	Ac (sub sample 1)	Q-Stat	Ac (sub sample 2)	Q-Stat
1	-.034	3.0	-.073	.837	-.002	.001
2	.092	3.0	.175	5.668	-.083	1.01
3	-.056	4.0	-.108	7.513	-.020	1.16
4	-.026	4.2	-.057	8.032	.018	1.21
5	-.003	4.2	-.006	8.038	-.060	1.79
6	.058	5.3	.097	9.572	.004	1.79
7	.012	5.3	.042	9.857	-.062	2.42
8	-.079	7.3	-.098	11.420	-.046	2.77
9	-.042	7.9	-.015	11.458	-.086	4.00
10	-.133	13.6	-.191	17.476	-.038	4.24
11	.024	13.7	.034	17.673	.064	4.92
12	-.017	13.8	-.058	18.245	-.017	4.97

3-2-2- Dicky-Fuller Test

The results of Dicky-Fuller tests for the daily observations are presented in table 1.2 for the no constant & no trend model (case 1), the constant & no trend model (case 2) and the constant & trend model (case 3). The unit root tests support the random walk hypothesis for log levels of DAX [I (1), denoted as 'integrated' series and there is one unit root] and all companies, except BMW, and they also are I (0), a stationary series, in the first differences (i.e. returns).

Our findings are similar to the results of Cooray (2003). He examined the random walk behavior of some stock markets including DAX using unit root tests and spectral analysis and came to conclusion that the DAX stock market follows a random walk. It should be noted that a unit root test is only a necessary (but not sufficient) condition for a random walk process. Although the random walk hypotheses are contained in the unit root null hypothesis, it is the permanent/temporary nature of shocks to the series that concern unit root tests. These tests are clearly not designed to detect predictability and have no bearing on the random walk hypothesis (Campbell et all 1997).

Table 1-2: Results of Dicky-Fuller Test (log levels (L-L), log first differences (L-F-D))

	ADF test statistic			ADF test statistic			ADF test statistic	
DAX	L-L	L-F-D	LHA	L-L	L-F-D	RWE	L-L	L-F-D
Case 1	.49	-18.06 ***	Case1	-.55	-18.07***	Case1	1.32	-17.46***
Case 2	-1.42	-18.04***	Case2	-1.33	-18.06***	Case2	-1.89	-17.55***
Case 3	-2.11	-18.06 ***	Case3	-.98	-18.08***	Case3	-3.03	-17.55***
BMW			BASF			TUI		
Case1	-.38	-16.91***	Case1	1.17	-19.65***	Case1	.70	-18.33***
Case2	-3.34**	-16.89***	Case2	-.07	-19.72***	Case2	-.67	-18.32***
Case3	-3.63 **	-16.90***	Case3	-2.51	-19.83***	Case3	-.57	-18.44***

significant levels for no concept & no trend model (case 1): 1%, -2.57: 5%, -1.94: 10%, -1.61; constant & no trend model (case 2): 1%, -3.45: 5%, -2.87:10%, -2.57; constant & trend model (case 3): 1%, -3.98: 5%, -3.42:10%, -3.13 .*, **, *** significant at the 10%,5% and 1% levels respectively.

3-2-3- Variance Ratio Test

In this section, the random walk hypothesis will be examined by applying the variance-ratio test.

Random Walk 1 (IID Increments): the strongest version of the random walk hypothesis is the independently and identically distributed (IID) increments case that the dynamics of {Pt} are given by the following equation:

$$P_t = \mu + P_{t-1} + \epsilon_t \qquad \epsilon_t \sim \text{IID}\ (0, \sigma^2) \tag{12}$$

where μ is the expected price change or drift. Independence of increments $\{\epsilon_t\}$ implies that the random walk is also a fair game. In fact, the assumption of independence implies that increments are uncorrelated and non-linear functions of the increments are also uncorrelated. The mean and variance of P_t conditional on some initial value P_0 at date t=0 can be expressed as follows

$$E\ [P_t |\ P_0] = P_0 + \mu t \tag{13}$$

$$\text{Var}[P_t |\ P_0] = \sigma^2 t \tag{14}$$

It follows that the random walk is non-stationary and conditional mean and variance are both linear in time.

Random walk 2 (Independent Increments): the second form of random walk hypothesis assumes that increments are independent but identically distributed. In this case, there is unconditional heteroskedasticity in ϵ_t's. It is difficult to test for independence without assuming identical distributions. The lack of powerful tests for this version of the random walk hypothesis has led to much empirical research to develop "economic" test of predictability (e.g., filter rules and technical analysis).

Random walk 3 (Uncorrelated Increments): the weakest version of the random walk hypothesis assumes that asset prices may have dependent but uncorrelated increments at all leads and lags. In this case, there is a process with Cov $[\epsilon_t, \epsilon_{t-k}] = 0$ for all $k \neq 0$, but where Cov $[\epsilon^2_t, \epsilon^2_{t-k}] \neq 0$ for some $k \neq 0$. This form of process allows for conditional heteroskedasticity.

A general feature of the three random walk hypotheses is that the variance of random walk increments must be a linear function of the time interval. For example, variance of $r_t + r_{t-1}$ must be twice the variance of r_t. Thus the variance of random walk increments must result in a ratio close to one. For a time-scale q, the variance ratio statistic, VR (q), takes the form:

$$VR(q) = \frac{Var(r_t(q))}{qVar(r_t(1))} \tag{15}$$

We follow Lo and Mackinlay (1988) to estimate variance ratios. The variance ratio is defined as

$$VR(q) = \frac{\sigma_c^2(q)}{\sigma_a^2} \tag{16}$$

Where σ_a^2 and $\sigma_c^2(q)$ are an unbiased single period and q period variances and can be estimated using:

$$\hat{\sigma}_a^2 = \frac{1}{nq-1}\sum_{k=1}^{nq}(p_k - p_{k-1} - \hat{\mu})^2 \tag{17}$$

$$\hat{\sigma}_c^2(q) = \frac{1}{m}\sum_{k=q}^{nq}(p_k - p_{k-q} - q\hat{\mu})^2 \tag{18}$$

$$m = q(nq - q + 1)(1 - \frac{q}{nq}) \tag{19}$$

$$\hat{\mu} = (p_{nq} - p_0)/nq \tag{20}$$

The following test statistic, Z (q), has an asymptotically standard normal distribution under the assumption of homoscedasticity of returns and can be used to test Random Walk 1.

$$Z(q) = [VR(q) - 1]/\sqrt{\hat{\phi}(q)} \tag{21}$$

$$\hat{\phi}(q) = [2(2q-1)(q-1)]/[3q(nq)] \tag{22}$$

As return volatilities change over time and deviate from normality, the heteroscedasticity-robust standard normal test statistics, Z*(q), which is used to test Random Walk 3, can be written as

$$Z^*(q) = [VR(q) - 1]/\sqrt{\hat{\theta}(q)} \tag{23}$$

$$\hat{\theta}(q) = \sum_{k=1}^{q-1}\left(\frac{2(q-k)}{q}\right)^2 \hat{\delta}(k) \tag{24}$$

$$\hat{\delta}(k) = [nq\sum_{j=k+1}^{nq}(p_j - p_{j-1} - \hat{\mu})^2(p_{j-k} - p_{j-k-1} - \hat{\mu})^2]/[\sum_{j=1}^{nq}(p_j - p_{j-1} - \hat{\mu})^2]^2 \tag{25}$$

Table1-3 presents the results of variance ratio tests of returns for sampling intervals of 2, 4, 6, 8, and 10 days. In testing random walk, both homoscedasticity test statistic, Z (q), and heteroscedasticity-robust test statistic, Z*(q), are calculated for various q's. By using 1-day as our base observation interval, Z (q) and Z*(q) statistics are calculated for each q by comparing the variance of the base interval with that of 2-day, 4-day, 6-day, 8-day, and 10-day observation intervals. The values reported in the main rows are the actual variance ratios, the entries below the variance ratios are Z (q) and the values below Z (q) are Z*(q) values. The test for random walk hypothesis assumes that the variance ratio of any period should be close to one. Trending behavior is detected in the time series if the variance ratio is significantly more than 1 while mean-reverting behavior is identified if VR (q) is significantly less than 1.

The null hypothesis that the DAX stock returns follow a homoscedastic random walk can not be rejected for different q's. In fact, the variance ratio of any period is significantly close to 1. Also, the results from calculating a heteroscedastic-consistent statistic shows that Z*(q) is not significant, confirming that the DAX stock market follow a random walk. The VR (q), Z (q), and Z*(q) have also been computed for all companies. The random walk null hypothesis under homoscedasticity and heteroscedasticity is not rejected in the most intervals. The random walk hypothesis can be rejected when the test statistics are rejected for all q.

Table 1-3: Estimates of variance-ratios VR (q), Z (q) and Z*(q)

	q=2	q=4	q=6	q=8	q=10
DAX	1.09 (1.69) (0.87)	1.03 (0.28) (0.16)	1.01 (0.12) (0.05)	1.04 (0.28) (0.07)	1.03 (0.15) (0.08)
BMW	2.67 (29.27)** (20.44)**	1.03 (0.29) (0.20)	0.96 (-0.24) (0.03)	0.87 (-0.73) (0.03)	0.84 (-0.80) (0.04)
LHA	0.85 (-2.64)* (-2.45)*	1.05 (0.49) (0.47)	1.05 (0.35) (0.02)	1.09 (0.53) (0.02)	1.10 (0.54) (0.02)
BASF	0.97 (-0.52) (-0.31)	0.89 (-1.06) (-0.67)	0.84 (-1.11) (0.04)	0.86 (-0.82) (0.05)	0.84 (-0.83) (0.05)
RWE	2.06 (18.63)** (14.21)**	1.03 (0.33) (0.26)	1.03 (0.22) (0.02)	1.04 (0.28) (0.03)	1.04 (0.21) (0.03)
TUI	0.04 (-16.80)** (-15.90)**	1.02 (0.22) (0.22)	1.00 (0.02) (0.01)	1.02 (0.12) (0.02)	1.00 (0.00) (0.02)

*,** Indicate significance at the 5% and 1% levels respectively. The estimates of variance ratios are shown in the main row. The figures below the variance ratios are Z (q) and the values below Z (q) are Z*(q) values.

3-2-4- Auto-Regressive (AR) model

To test weak form efficiency, first, different AR models over different time horizons are estimated then, for diagnostic checking, the correlogram (autocorrelations) of returns from the regression tests is examined. An autocorrelation coefficient significantly different from zero indicates the predictability of share returns from the past information. Table 1.4 reports the results of the AR (5) model. Such a model is used to examine the impact of the past returns on the future returns and is, as mentioned before, a test of weak form efficiency (Bei & Zhong-ying, 2006). Since the results were the same for the different AR models, only the AR (5) model has been reported. For DAX returns, it is clear that there is no significant coefficient of AR terms. This imply that the past returns can't help predict for future returns and hence weak-form efficiency hold. In the last sections we concluded that the DAX stock returns follow a random walk. Thus the result is consistent with the findings of Ko and Lee (1991).

The AR (5) model is not significant for daily returns of all companies. Examining BASF returns shows that if there is only the AR (1) term in the regression (i.e. the AR (1) model), then the regression is acceptable.

A logistic map equation for possible non-linearity in returns was also considered, but the estimates were not significant.

Table 1-4: The AR (5) Model for DAX Returns and Daily Returns of Companies

	Constant	AR(1)	AR(2)	AR(3)	AR(4)	AR(5)
DAX (t-Statistic)	.00027 (.5228)	-.02259 (-.3889)	.09202 (1.5875)	-.05695 (-.9794)	-.04021 (-.6929)	.00698 (.1202)
BMW (t-Statistic)	-.00008 (-.1357)	.01963 (.3402)	-.01517 (-.2620)	.00343 (.0588)	-.02863 (-.4993)	-.11286 (-1.96)
LHA (t-Statistic)	-.00062 (-.6333)	-.01307 (-.2247)	.13329 (2.29)*	-.05526 (-.9673)	-.03437 (-.6062)	-.00041 (-.0073)
BASF (t-Statistic)	.00075 (1.3791)	-.11614 (-1.99)*	.06001 (1.0277)	-.02830 (-.4829)	-.05195 (-.8855)	-.01533 (-.2617)
RWE (t-Statistic)	.00126 (1.5125)	-.01583 (-.2725)	.07426 (1.2765)	-.04720 (-.8149)	-.02903 (-.5011)	.04865 (.8382)
TUI (t-Statistic)	.00054 (.5573)	-.03854 (-.6631)	.09458 (1.6334)	.00582 (.1002)	-.08716 (-1.505)	-.00685 (-.1189)

*significant at 5% level of significance

3-2-5 Auto-Regressive Moving Average (ARMA) model

Regressions based on ARMA models are often used to test the informational efficiency assumption of the EMH. A collection of ARMA (p, q) models, for different orders of p and q, have been estimated and then the best model was selected according to Akaike information criterion (AIC) and Schwarz information criterion (SIC). Examining ARMA models for DAX stock returns show that ARMA (5, 3) is the best-fitting model. The regression is as follows with t-values in parentheses:

$$R_{dax} = .0003 + .51AR(1) - .28AR(2) - .56AR(3) + .002AR(4) + .07AR(5)$$
(t) (.65) (5.7) (-3.5) (-7.3) (.03) (1.32)

$$-.55MA(1) + .43MA(2) + .49MA(3) \qquad \bar{R}^2 = 0.04 \qquad F=2.6$$
(t) (-7.1) (7.9) (9.1) Prob=0.009

It is clear that the MA (1), MA (2) and MA (3) terms are statistically significant. Since previous periods' forecast errors are known (at time t) this may be considered as a violation of informational efficiency. However, only 4 percent ($\bar{R}^2 = .04$) of the variability in daily stock returns is explained by the regression. As a result, potential profitable arbitrage possibilities are likely to involve substantial risk. This regression may reject the strict RE/EMH element of 'unpredictability', but not necessarily the view of the EMH which emphasizes the impossibility of making supernormal profits.

The regression results show that there are some significant ARMA models only for BASF, LHA and RWE returns. For LHA returns, the ARMA (1, 1) model is acceptable, and the best-fitting regressions for BASF and RWE returns are ARMA (2, 4) and ARMA (5, 5) models respectively.

An autoregressive integrated moving average model (ARIMA) is also used to test whether the DAX stock returns follow a random walk. To examine the random walk model we need to fit the model ARIMA (0, 1, 0) for the price index. If the coefficient is significantly different from zero, then the assumption of the random walk model and weak-form efficiency will be violated. The results in Table 1.5 show that the ARIMA (0, 1, 0) model for the DAX index series and for the whole companies support the random walk model. Diagnostic checking confirms the results. It should be noted that there is significant residual autocorrelation at lag 10, 11, 12, and 19 for LHA returns and at lag 1 and 2 for BASF returns.

Table 1-5: ARIMA (0, 1, 0) Model for DAX Index and Daily Share Price of Companies

	Coefficient	Std. Error	t-Statistic	P-value
DAX	.000271	.000534	.507423	.6122
BMW	-.000245	.000701	-.349070	.7273
LHA	-.000460	.000967	-.475810	.6345
BASF	.000731	.000619	1.181369	.2384
RWE	.001102	.000802	1.374363	.1703
TUI	.000733	.001010	.726162	.4683

3-2-6- Out-of-Sample Forecasting

To examine the forecasting performance of various models, we use the first difference of the logarithm of stock prices (returns) instead of a raw price series. The reasons behind the issue are simple. First, as the stock prices often include a trend, any prediction using such a variable is problematic. Second, taking logarithm of the data compacts the dynamic range of the series and reduces the effect of outliers.

Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and the Theil inequality coefficient (Theil's U) are usually used to evaluate ex post forecasts. The first two forecast error criteria depend on the scale of the dependent variable and thus are not perfect measures to compare forecasts for the same series across different models. Instead, Theil's U is scale invariant and is useful to examine returns. It lies between zero and one, where zero indicates a perfect fit. All models including Random Walk, ARMA and Generalized Auto Regressive Conditional Heteroscedasticity (GARCH) are used to generate a dynamic 5-day-ahead, 10-day-ahead and 15-day-ahead forecast of the DAX stock returns. According to table 1.6, Theil's U-statistic is very close to 1, implying a poor out-of-sample forecast for all models.

Table 1-6: Out-Of-Sample Forecasting Performance based on Theil's U

Day-Ahead	Random Walk	ARMA(5,3)	GARCH(1,1)
5	.97	.96	.95
10	.95	.90	.91
15	.95	.88	.93

Although the forecasting performance of ARMA and GARCH models is a little better than that of the random walk model, this criterion can't determine whether they are in fact significantly better. A solution to this is to perform a Wilcoxon test between two alternative models. We use this test to compare the square errors of a random walk model and a rival model. The performance of the ARMA and GARCH models appeared to not differ significantly from a random walk model (with p-values more than .44). More specifically, the p-values of testing the ARMA model against the random walk was p=.72 and that of testing the GARCH model against the random walk was p=.44. Thus, there seems to be no significant difference between the models' performance. The similar results are obtained for different sub samples and different rival models.

4- Conclusions

The results of autocorrelation, variance ratio and autoregressive tests show that the DAX stock market follows a random walk and supports the weak-form efficiency of efficient market hypothesis (EMH). Our findings are consistent with the results of Ko and Lee (1991), who asserted that if the random walk hypothesis holds, then the weak-form efficiency must also hold, but not vice versa. Thus, evidence supporting the random walk model is the evidence of market efficiency. But violation of the random walk model need not be evidence of market inefficiency in the weak form[1].

Some ARMA models may be considered as a violation of informational efficiency, but not necessarily of the view of EMH which emphasizes the impossibility of making supernormal profits. Finally Theil's inequality coefficient indicates a poor forecasting power for all models and there is no significant difference between the forecasting performance of rival models and that of random walk model based on Wilcoxon test.

References

1- Beechey M., Gruen D., and Vickery J. (2000-1) The Efficient Market Hypothesis: A Survey, Economic Research Department Reserve Bank of Australia, Research Discussion Paper.

2- Campbell JY, Lo AW, and MacKinlay, AC (1997), The Econometrics of Financial Markets, Princeton University Press, Princeton, New Jersey.

3- Cooray, A. (2003-05), The Random Walk Behaviour of Stock Prices: A Comparative Study, School of Economics, University of Tasmania, Australia, Discussion Paper.

4- Cuthbertson K. (1996), Quantitative Financial Economics: stock, bonds, and foreign exchange, John Willy & Sons Inc.

5- Darrat AF, Zhong M. (2000), On Testing the Random-Walk Hypothesis: A Model- Comparison Approach, The Financial Review, Vol. 35.

1- In fact, the nature of linear and non-linear serial dependencies should be examined. The weak-form efficiency holds when the random walk hypothesis cannot be rejected by a non-linearity test such as the BDS test. However, when the random walk hypothesis is instead rejected due to the presence of certain dependency structures, it can be concluded that the market is inefficient.

6- Fama EF (1965), Random Walks in Stock Market Prices, Selected Papers, No. 16, Graduate School of Business University of Chicago.
7- Ko KS, Lee SB (1991), “A Comparative Analysis of Daily Behavior of Stock Returns: Japan, The US and Asian NICs”, Journal of Business Finance and Accounting, Vol.18 (2), pp.219-234.
8- Lo AW, Mackinlay AC (1988) Stock market prices do not follow random walks: evidence from a simple specification test, Review of Financial Studies, Vol. 1, pp.41-66.
9- Mills TC (2000), The Econometric Modeling of Financial Time Series, 2. ed, Cambridge University Press.
10- Higgins RC (1992), Analysis for Financial Management, McGraw-Hill.
11- Sargent, T J (1993), Bounded Rationality in Macroeconomics: The Arne Ryde Memorial Lectures, Oxford University Press.
12- Shadbolt J., Taylor JG (2002), Neural Networks and the Financial Markets, Springer.
13- Tsay RS (2002), Analysis of Financial Time Series: Financial Econometrics, John Wiley & Sons, Inc.
14- Z Bei, Q Zhong-ying (2006), Empirical Research on Efficiency of Chinese Future Markets based on GARCH Model, International Conference on Management Science and Engineering, ICMSE '06.