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Dynamics Emission for a Polluting Industry

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Abstract

This paper develops a dynamic theoretical model for a polluting industry to determine the emission dynamics, internalizing damage cost function along with production cost function, considering emission factor. It turns out that the competitive output price is composed of not only marginal production cost function but also marginal damage cost of emitting one emission unit multiplied by emission factor as well as the term indicated the pollution stock effects on the difference between damage cost function and the time derivative of implicit pollution price. Natural reductions in pollution as a proportion of decay factor lead to a fluctuations in the time paths of emission and its concentration, so that increasing marginal damage cost that it follows to strengthen social health and human welfare.

Keywords: Pollution Control, Emission Factor, Decay Factor, Value Function, Optimality Condition.

1- Introduction

A polluting industry produces private market goods along with unwanted by- products as wastes in different forms such as solid, gases, water, particulates wastes through the processes of production technology. Market goods are associated with the costs of production, whereas wastes- related goods created externality costs caused social damages for a group of citizens in any society. Therefore worse goods produce with better goods and that they are related proportionately in emitting pollution industry.

The aim of this paper is to determine the dynamics equation of the emission faced by industry in order to maximize the discounted value of

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future revenue streams, internalizing the social damages cost function as well as production cost function.

A huge amount of articles are dealing with the issues of pollution control in environmental economics literature. Some emphasized on emissions-related cost functions, minimizing discounted future social damage cost function plus total abatement cost function conditioned on time dependant emission constraint (Falk, I. and Mendelsohn, R [1993]; Gopinathe,M. J.[1999], Chern, etal. [2003]). A few explored innovation and improvements in abatement cost technology for reducing consequences and implications raised by stock pollutants (Innest, R. and Bial, J.[2002] , Downing, P [1984] , Cherp,A. etal [2003] , Innest, Etal. [2002]). A great deal of attempts attributed to pollution as a externality cost and proposed Pigovian tax for internalizing social cost of controlling for one more pollution unit (Parry, L. W. [1995], Smit, etal. [2003]).

This paper is organized as follows: In section II, theoretical model is developed in order to determine dynamic path of emission for an industry providing private market good with simultaneous producing unwanted by–products. The analyses of dynamic paths of emission and its stock are worked out using phase diagram pattern in section III. Final section deals with concluding remark.

2- Theoretical Model of Emission Dynamic

All polluting chemical industries such as nitrogen, phosphate and pesticides producers are responsible for social damages suffered citizens. A group of victimized citizens are forced to pay for their health care and security costs. A theoretical model developed here has internalized social environmental damage costs associated with production cost function. Let E_t and R_t be the aggregate level of emission and its concentration over time, respectively. If $\dot{R}_t$ is the time derivative variable, its dynamic expression can be written as:

$$\dot{R} = E - bR \ , \qquad (1)$$

where b is a decay rate and caused a natural reduction of the pollution stock, so that it is fixed coefficient. With higher R, its changes over time will be lower. It is assumed that the aggregate emission is a proportion of output produced by industry and specified as $E=\alpha q$, where q is the production level and α is its coefficient. The production cost function given by $C=C(q)$ increases with output at increasing rate that is $C_q=\dfrac{\partial C(q)}{\partial q}=>0$ and $C_{qq}=\dfrac{\partial^2 C}{\partial q^2}>0$. However the aggregate emission is a major determinant of the total and marginal production costs since its derivatives are considered as $C_e=\dfrac{C_q}{\alpha}$ and $C_{ee}=\dfrac{C_{qq}}{\alpha^2}$, respectively. Citizens in the society are the victims of the damages created by the level of aggregate emission and its stock pollution released the environment. Suppose that the damage cost function is assumed to be as $D=D(E,R)$ and rises with increasing rate according to both E and R, so that the first and second derivatives become as $D_e=\dfrac{\partial D(E,R)}{\partial E}>0$, $D_r=\dfrac{\partial D(E,R)}{\partial R}>0$, $D_{ee}=\dfrac{\partial^2 D(E,R)}{\partial E^2}>0$ and $D_{rr}=\dfrac{\partial^2 D(E,R)}{\partial R^2}>0$ .The cross derivative is assumed zero $C_{er}=\dfrac{\partial^2 D(E,R)}{\partial E\,\partial R}=0$.

Given competitive market price P and interest r, The polluting industry maximizes its sum of discounted value of future returns over the horizontal T subject to pollution constraint (1) so as to internalize the monetary equivalent of social environmental damage cost function. Its objective function can be written as:

$$\pi=\int_t^T [P\,q(E)-C(q(E))-D(E,R)]e^{rs}\,ds, \tag{2}$$

Define the value function as $J=J(R,t)$, this problem yields the fundamental optimality condition as:

$$-J_t=Max[V(E,R,t)+J_R\,(E-b\,R)] \tag{3}$$

where $V(E,R,t)=[Pq(E)-C(q(E))-D(E,R)]e^{-rt}$ is the discounted future profit, $J_R=\frac{\partial J}{\partial R}$ is the shadow value of resulting from one more unit of pollution concentration and $J_t=\frac{\partial J}{\partial t}$ is the time derivative of the value function. Differentiating (3) with respect to state and control variables R and E, the final optimal conditions are obtained as:

$$[(\frac{P-C_q}{\alpha})-D_e)e^{-rt}]+J_R=0, \tag{4}$$

$$J_{Rt}=D_r\,e^{-rt}+b\,J_{R,} \tag{5}$$

Where $J_{Rt}=\frac{\partial J^2}{\partial R\,\partial t}$ is the change in intrinsic price of accumulated pollution as time change. Condition (4) indicates that the implicit shadow value of one extra pollution unit is negative since optimal marginal competitive profit adjusted to the emission factor less of marginal damage cost discounted to the present time is positive, so that it is given by $J_R=-[(\frac{P-C_q}{\alpha})-D_e]e^{-rt}$. It is true that the change in maximized discounted future streams profit declines as one more emission added to the stock of pollution. In addition, from condition (5) it is clear that the implicit shadow price of pollution denoted by $J_R=\frac{D_re^{-rt}-J_{Rt}}{b}$ is the difference between discounted marginal pollution effect on damage cost function and time derivative of that implicit price adjusted with the natural rate of decay coefficient. Both above-mentioned expressions can yield the competitive price level at a given period of time and it is summarized as:

$$P=C_q+\alpha\,D_e+\alpha\,(\frac{D_R-J_{Rt}\,e^{rt}}{b}). \tag{6}$$

Note from expression (6) that the output competitive price is decomposed of three components. Marginal production cost of producing one extra output is a major part of price C_q, marginal damage cost of emitting an additional emission unit multiplied by emission factor is the second part of the price. The third part shows the adjusted pollution stock effect differences through its impact on total damage cost function and the rising implicit pollution stock price over time with the rate of interest.

3- Dynamic Equation of Emission

In order to determine the dynamic $\dot{E}$, differentiating (4) with respect to time, substituting for J_{Rt} from (5), and rearranging the resulted terms, it becomes as:

$$\dot{E} = \frac{D_R + \frac{\dot{P}}{\alpha} - (b+r)(\frac{P-C_q}{\alpha} - D_e)}{D_{ee} + \frac{C_{qq}}{\alpha^2}}, \qquad (7)$$

In the dynamic $\dot{E}$, the denominator is positive but the sign of the nominator is not known clearly since the third term is negative. This term indicates that adjusted competitive net price out of marginal damage cost of emitting one emission unit is a multiplication of emission factor plus the natural rate of decay pollution. If other factors hold constant, change in the rate of output price over time will be a proportion of the dynamics of emission. This coefficient is emission factor α. To analyze the optimal stationary level of E^* and R^* simultaneously, it is required to equate both differential equations (1) an (7) equal zero. As both $\dot{R}=0$ and $\dot{E}=0$, from (1) and (7) it follows that $D_R + \frac{\dot{P}}{\alpha} = (b+r)(\frac{P-C_q}{\alpha} - D_e)$ and $E = bR$, respectively. Their isoclines are depicted by phase diagram represented in

Figure 1. For $\dot{R}=0$, the isocline is positively sloped whereas for $\dot{E}=0$ it is downward sloping and their intersection determines the optimal stationary levels $R^{*}$and E^{*}. At the point M optimal emission is a fraction of optimal its stock over time. Rising both R and E, the trajectory MB moves away from optimal stationary point. This result is true for the trajectory MA as both R and E decline over time. With higher initial pollution stock above desired level, the pollution starts to decrease until to reach the point e_1, and after passing this stationary point it goes to continue its increasing until the second stationary point e_2 and from now on its trajectory bends backward and starts to reduce again. This trajectory is denoted by RR in the Figure 2. In contrast, if the initial tock of pollution is lower , it reduces until to reach its stationary level F_1 and from now on it continues its rising up to optimal stationary point F_2, and then again its reduction starts until point F_3. This trajectory is shown by RRR in figure 2and its time trend along with emission rate are drawn in Fig. 3.

As both initial emission and its accumulation are higher from optimal level R^{*} and E^{*}, decreasing in E is followed by reduction in R, when E reaches its minimum level A, R tends to be at D, lying at above level A. From point A, the path of E starts to raise up to its maximum point B at this time the time path of R will be at its highest level C which is located above point B. Both time paths of E and R begin to decline from points B and C. For the case of lower initial level of emission and its stock as compared with the higher level, their time trends are the same pattern since there are locating at the lower level as shown by doted lines in the diagram 4. Therefore, fluctuations take place over time because $E^{*}=bR^{*}$ and that the stock pollution is declined due to natural rate of decay factor.

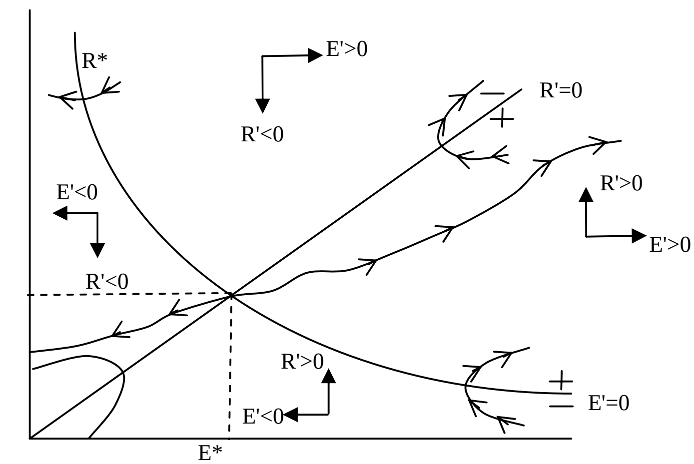

Fig 1: Phase Diagram for R and E

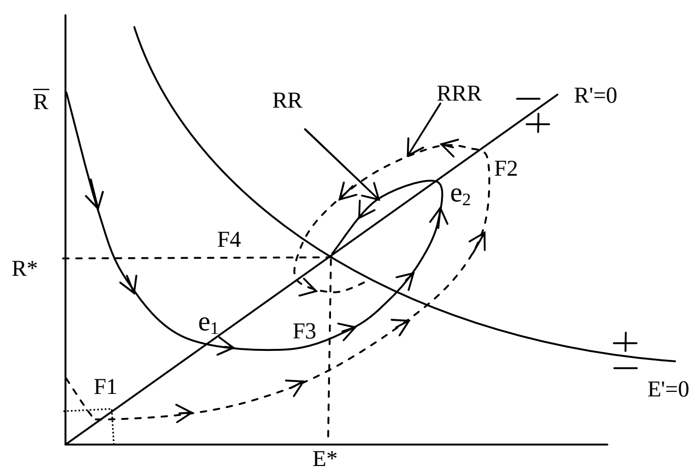

Fig 2: Trajectories for E and R

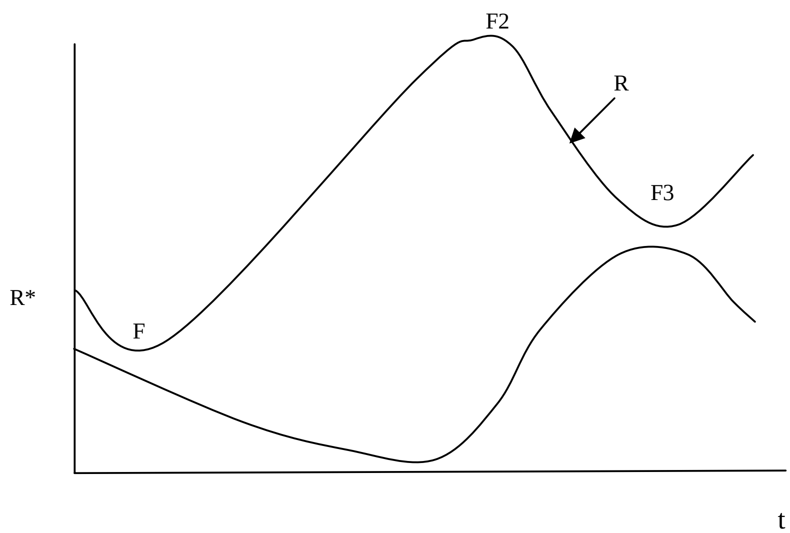

Fig 3: Time Path for E and R

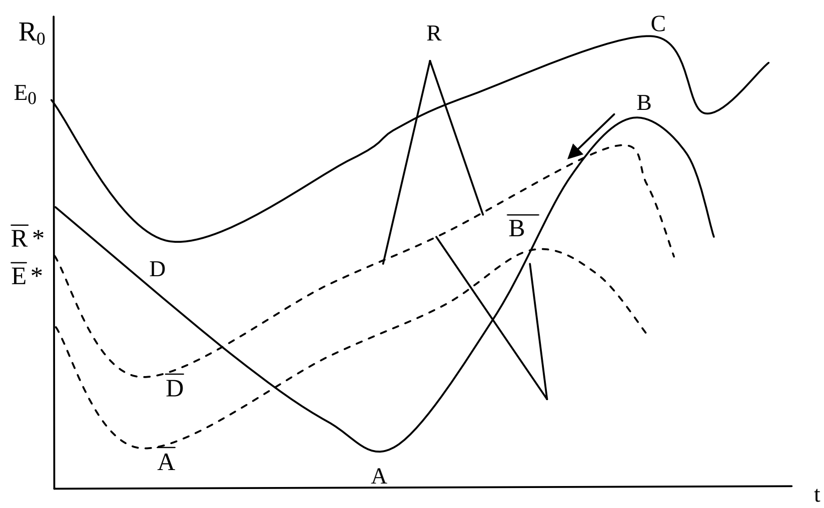

Fig 4: Time Path for E and R with initial Higher and Lower

4- Concluding Remark

This paper provides the dynamics of emission for an emitting pollution industry by maximizing the sum of discounted future streams of profits with internalizing the total damage cost as function of aggregate level of emission and its accumulation over time. The industry faces competitive output market price and the dynamic environmental constraint defined as reduction in emission due to decreased stock pollution rated with natural decay factor. The result indicates that the competitive output market price is composed of not only the marginal production cost but also marginal damage cost of increased emission multiplied by the emission factor. It is also included a term that considered as a difference between pollution stock effect on damage cost function with the exponential implicit shadow value of the sock of pollution adjusted with the emission factor- pollution decay rate ratio.

Fluctuations in emission and its concentration take place over time weather their initial value higher or lover, provide improvement in social welfare and the society's health care as marginal damage cost reduces with decreased both emission and pollution. It may be suggested that the polluting industry would plan to invest and innovate in abatement emission technology to overcome hazardous and victimized consequences resulted from emitting emission in the environment. Otherwise, other controlling tools like pollution tax and a form of permits may be adopted by the government.

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