Civil Engineering Infrastructures Journal, 47(1): 13 – 27, June 2014
ISSN: 2322 – 2093

A Method of Function Space for Vertical Impedance Function of a Circular Rigid Foundation on a Transversely Isotropic Ground

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Received: 27 Dec. 2011; Revised: 10 Oct. 2012; Accepted: 20 Nov. 2012

ABSTRACT: This paper is concerned with investigation of vertical impedance function of a surface rigid circular foundation resting on a semi-infinite transversely isotropic alluvium. To this end, the equations of motion in cylindrical coordinate system, which because of axissymmetry are two coupled equations, are converted into one partial differential equation using a method of potential function. The governing partial differential equation for the potential function is solved via implementing Hankel integral transforms in radial direction. The vertical and radial components of displacement vector are determined with the use of transformed displacement-potential function relationships. The mixed boundary conditions at the surface are satisfied by specifying the traction between the rigid foundation and the underneath alluvium in a special function space introduced in this paper, where the vertical displacements are forced to satisfy the rigid boundary condition. Through exercising these restraints, the normal traction and then the vertical impedance function are obtained. The results are then compared with the existing results in the literature for the simpler case of isotropic half-space, which shows an excellent agreement. Eventually, the impedance functions are presented in terms of dimensionless frequency for different materials. The method presented here may be used to obtain the impedance function in any other direction as well as in buried footing in layered media.

Keywords: Circular Foundation, Function Space, Transversely Isotropic, Vertical Impedance Function.

INTRODUCTION

The interaction of rigid foundation and three-dimensional solid is an interesting subject in both applied mathematics and civil engineering. Investigation of traction-induced wave propagation, which is the base for studying soil-structure interaction, was started by the pioneering paper of Lamb (1904), where the wave propagation in a semi-infinite homogeneous elastic isotropic in both three- and two-dimensions subjected to a time harmonic surface point load was investigated in detail. Studying the static

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interaction of the foundation and the underneath soil is a prerequisite for understanding dynamic interaction. By using the technique of integral transforms, the mixed boundary value problem involved in the investigation of soil-structure interaction is transformed into some dual integral equations, which are very complicated and their solutions require precise attention (see Tichmarch, 1948; Sneddon, 1966). Avoiding the dual integral equations, Arnold et al. (1955), obtained an approximate solution by tentatively assuming that the dynamic contact stress distribution is about identical to the static distribution pattern. This assumption has been taken by a number of researchers, including Bycroft (1956). Awoboji and Grootenhuis (1965) found the vertical and torsional displacements in an isotropic half-space due to a surface circular rigid body as well as a surface rigid strip foundation under vertical and rocking motions.

On the other hand, the need of studying soil-structure-interaction, when the soil is anisotropic, is normal rather than exceptions. Eskandari-Ghadi et al. (2010a, b and 2011) have presented analytical investigations for the vertical and rocking vibrations of a rigid circular disc either rested on the surface of a transversely isotropic half-space or buried in a transversely isotropic full-space. They have changed the involved dual integral equations to the Fredholm integral equations of second kind and have solved the Fredholm integral equations in a numerical manner.

In this paper, a semi-infinite elastic transversely isotropic medium is considered in frequency domain, where a cylindrical coordinate system $\{o:(r,\theta,z)\}$ whose z-axis is depth-wise is used. This medium is affected by a time harmonic vertical vibration due to a surface rigid circular foundation. Without solving the dual integral equations and only by writing the traction in between the rigid foundation and the underneath half-space in the form of a linear combinations of some independent functions with known Hankel integral transforms, the vertical impedance functions are obtained for this foundation. Since the boundary value problem is solved via implementing the Hankel integral transforms in radial direction, writing the unknown pressure in a function space with known Hankel integral transforms enables us to find the inverse integral transforms analytically. The final results are then compared with those of Luco and Mita (1987), which have been analytically determined and numerically evaluated with great precision. It is shown that by choosing only a few functions from the mentioned space, the results approach the analytic results quickly.

STATEMENT OF THE PROBLEM

A semi-infinite space containing transversely isotropic material is considered as the domain of the boundary value problem discussed in this paper. The axis of material symmetry of the domain is considered to be depth wise as shown in Figure 1. A cylindrical coordinate system $o:(r,\theta,z)$ is attached to the half-space as depicted in Figure 1. This medium is affected by a time harmonic motion of frequency ω due to a surface rigid circular foundation of radius a. Since the foundation is rigid, every point of which has the same displacement with the amplitude Δ.

By ignoring body forces, the equations of motion in the axisymmetric case can be written in terms of displacement vector in a cylindrical coordinate system as:

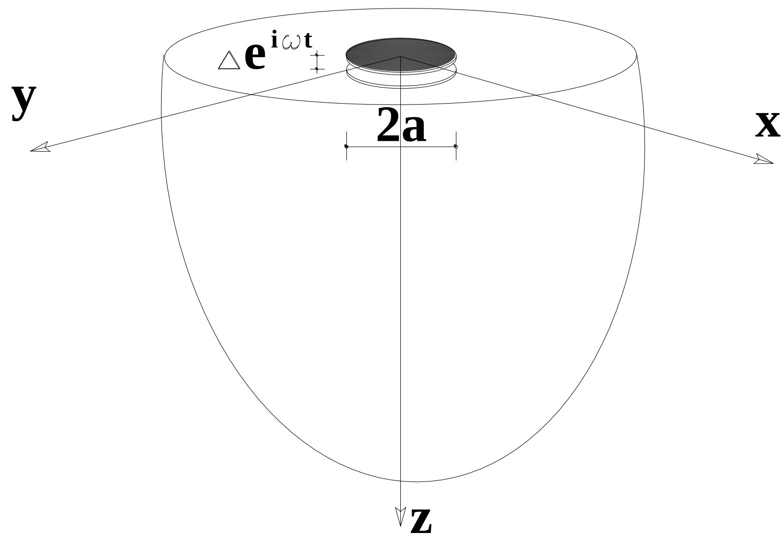

Fig. 1. Rigid foundation resting on the free surface of a semi-infinite transversely isotropic medium subjected to a harmonic vertical force.

$$A_{11}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right)+A_{44}\frac{\partial^2 u}{\partial z^2}+(A_{13}+A_{44})\frac{\partial^2 w}{\partial r\partial z}=\rho\frac{\partial^2 u}{\partial t^2},$$
$$A_{44}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)+A_{33}\frac{\partial^2 w}{\partial z^2}+(A_{13}+A_{44})\left(\frac{\partial^2 u}{\partial r\partial z}+\frac{1}{r}\frac{\partial u}{\partial z}\right)=\rho\frac{\partial^2 w}{\partial t^2} \quad (1)$$

where A_{ij} and ρ are the elasticity coefficients and mass density, and u and w are displacement components in r- and z-directions, respectively. Eqs. (1) are completely coupled, which with the use of the scalar potential function presented by Eskandari-Ghadi (2005), will be uncoupled. The displacements could be explained in terms of the potential function $F(r,z,t)$ as follows:

$$u=-\alpha_3\frac{\partial^2 F}{\partial r\partial z},$$
$$w=(1+\alpha_1)\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\gamma\frac{\partial^2}{\partial z^2}-\frac{\rho_0}{1+\alpha_1}\frac{\partial^2}{\partial t^2}\right]F \quad (2)$$

where

$$\alpha_1=\frac{A_{66}+A_{12}}{A_{66}},\ \alpha_2=\frac{A_{44}}{A_{66}},\ \alpha_3=\frac{A_{44}+A_{13}}{A_{66}},$$
$$\gamma=\frac{\alpha_2}{1+\alpha_1},\ \rho_0=\frac{\rho}{A_{66}},\ A_{66}=\frac{1}{2}(A_{11}-A_{12}) \quad (3)$$

By substituting the Eq. (2) into Eq. (1), the first equation is satisfied ideally and the second one is converted into:

$$\left[\Box_1\Box_2-\bar{\varepsilon}\frac{\partial^4}{\partial t^2\partial z^2}\right]F(r,z,t)=0 \quad (4)$$

where

$$\square_i = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{s_i^2}\frac{\partial^2}{\partial z^2} - \frac{1}{\mu_i}\rho_0\frac{\partial^2}{\partial t^2},$$
$$\mu_1 = \alpha_2, \qquad \mu_2 = 1+\alpha_1, \tag{5}$$
$$\frac{\bar{\varepsilon}}{\rho_0} = \left[\frac{A_{66}}{A_{11}}(1+\frac{A_{33}}{A_{44}}) - \frac{1}{\mu_1 s_2^2} - \frac{1}{\mu_2 s_1^2}\right]$$

and s_1^2 and s_2^2 are the roots of the following equation:

$$A_{33}A_{44}s^4 + (A_{13}{}^2 + 2A_{13}A_{44} - A_{11}A_{33})s^2 + A_{11}A_{44} = 0 \tag{6}$$

In the case of time-harmonic excitation with frequency ω, the Eq. (4) may be transformed into an ordinary differential equation in terms of depth, provided Hankel integral transforms in radial direction are used. By solving the ordinary differential equation with the use of regularity condition results in the potential function *F*, which may be used to find the displacements via displacement-potential relationships. Thus, through implementing the inverse Hankel theorem, the displacements *u* and *w* are determined as:

$$u(r,z) = -\frac{\alpha_3}{A_{33}}\int_0^\infty \frac{\xi^2}{I(\xi)}(\lambda_2(\xi)\eta_1(\xi)e^{-\lambda_2(\xi)z} - \lambda_1(\xi)\eta_2(\xi)e^{-\lambda_1(\xi)z})\tilde{P}_{z0}(\xi)J_1(\xi r)d\xi,$$
$$w(r,z) = \frac{1}{A_{33}}\int_0^\infty \frac{\xi}{I(\xi)}[(\alpha_2\lambda_1(\xi)^2 - (1+\alpha_1)\times(\xi^2 - \frac{\rho_0\omega^2}{1+\alpha_1}))(-\eta_2(\xi)e^{-\lambda_1(\xi)z}) + (\alpha_2\lambda_2(\xi)^2 - (1+\alpha_1)(\xi^2 - \frac{\rho_0\omega^2}{1+\alpha_1}))(\eta_1(\xi)e^{-\lambda_2(\xi)z})]\tilde{P}_{z0}(\xi)J_0(\xi r)d\xi \tag{7}$$

where $J_\nu(\xi r)$ is the Bessel function of the first kind and of νth order (Eskandari-Ghadi et al., 2008):

$$\lambda_1(\xi) = \sqrt{a\xi^2 + b + \frac{1}{2}\sqrt{c\xi^4 + d\xi^2 + e}},$$
$$\lambda_2(\xi) = \sqrt{a\xi^2 + b - \frac{1}{2}\sqrt{c\xi^4 + d\xi^2 + e}}$$
$$a = \frac{1}{2}(s_1^2 + s_2^2),$$
$$b = -\frac{1}{2}\rho\omega^2\left(\frac{1}{A_{33}} + \frac{1}{A_{44}}\right),$$
$$c = (s_2^2 - s_1^2)^2,$$
$$d = -2\rho\omega^2\left[\left(\frac{1}{A_{33}} + \frac{1}{A_{44}}\right)\times(s_1^2 + s_2^2) - 2\frac{A_{11}}{A_{33}}\left(\frac{1}{A_{11}} + \frac{1}{A_{44}}\right)\right], \tag{8}$$
$$e = \rho^2\omega^4\left(\frac{1}{A_{33}} - \frac{1}{A_{44}}\right)^2,$$
$$\eta_i(\xi) = (\alpha_3 - \alpha_2)\lambda_i(\xi)^2 + \xi^2(1+\alpha_1) - \rho_0\omega^2,$$
$$\nu_i(\xi) = (\eta_i(\xi) - \alpha_3\frac{A_{13}}{A_{33}}\xi^2 - \alpha_3\lambda_i(\xi)^2)\times\lambda_i(\xi),$$
$$I(\xi) = \eta_2(\xi)\nu_1(\xi) - \eta_1(\xi)\nu_2(\xi)$$

In addition, $\tilde{P}_{z0}(\xi)$ is the zero order Hankel transform of $P_z(r)$. The general form of Eq. (7) for general surface load has been obtained by Rahimian et al. (2007). $\tilde{P}_{z0}(\xi)$ should be specified in order to have the displacement components from Eq. (7), however, the distribution of the pressure, $P_z(r),\ 0 \le r \le a$, under the rigid foundation is unknown. In this paper, the pressure in between the rigid foundation and the half-space is linearly expressed in terms of a series of functions, whose Hankel integral transforms are known. Accordingly, the integrand functions in the Eq. (7) are known which allows the integrals to be evaluated. Because of rigidity of the surface foundation, the in between pressure is singular at its edge (see e.g. Eskandari-Ghadi et al., 2010a, b and 2011). The singularity is gradually smoothed by moving to the center of the foundation. Thus, the function space for expressing the in between pressure should reflect these properties. The following function space is used in this treatment:

$$\begin{aligned} \mathbf{M} = \{ f(r,\mu_n) \,|\, f(r,\mu_n) \\ = (a^2 - r^2)^{\mu_n - 1} H(a-r), \\ \mu_n > 0, a > 0 \} \end{aligned} \tag{9}$$

where $H(a-r)$ is the Heaviside step function and $a > 0$ is the radius of the rigid circular foundation (see Figure 2).

Some of the functions of the space **M** are illustrated in Figure 3. As it can be observed, for $\mu_n = 0$, the function is singular at $r = a$, which is the border of the foundation.

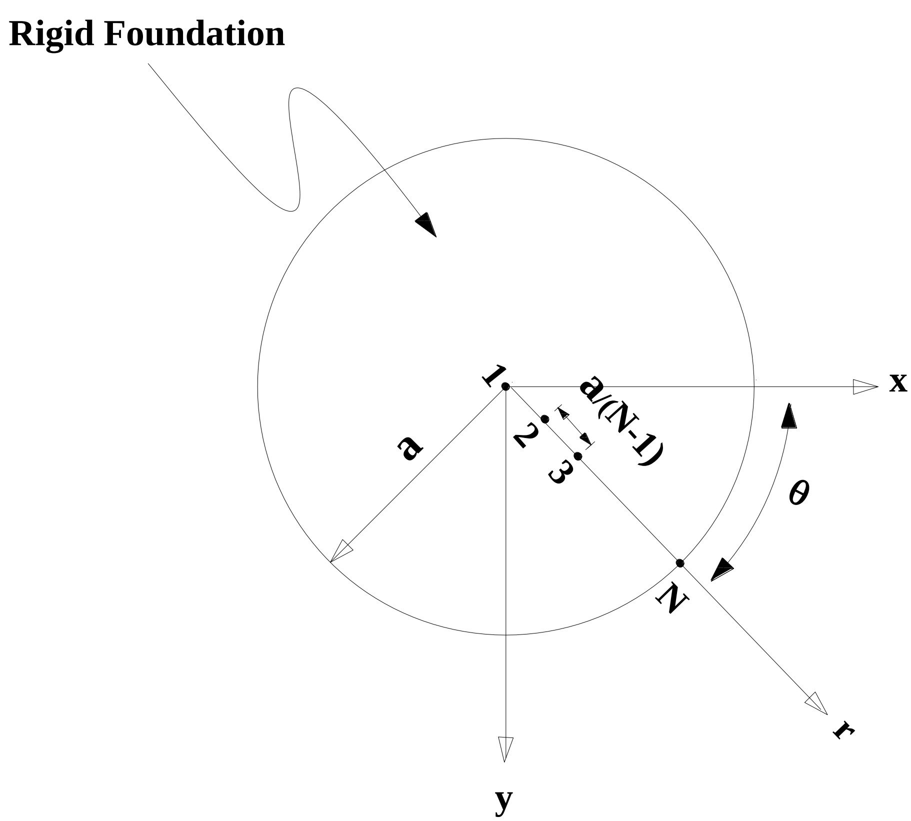

Fig. 2. N considered point at interface using to satisfy the displacement boundary condition under the disc.

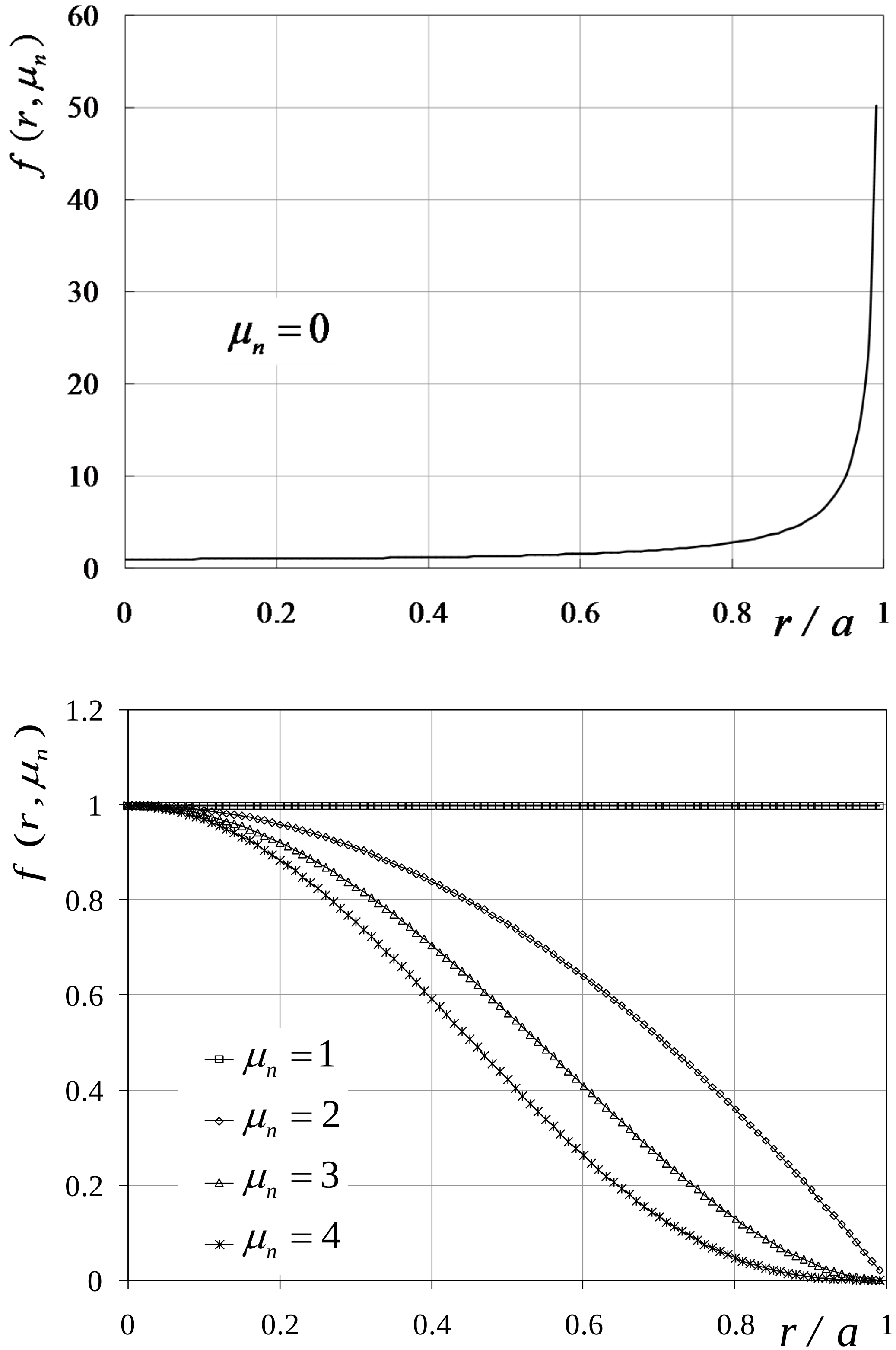

Fig. 3. Some of functions of the function space **M**.

This function is considered because of the rigidity-induced singularity of vertical stresses at the edge of the foundation especially in the static case. Also, $\mu_n = 1.0$ introduces a constant function, which describes a smooth behavior. In the Eq. (7), the zero order Hankel integral transform of P_z is needed, which means that the zero order Hankel integral transforms of all the members of **M** are needed. They are available as:

$$\tilde{\mathbf{M}}_0 = \{ \tilde{f}_0(\xi,\mu_n) \mid \tilde{f}_0(\xi,\mu_n) = 2^{\mu_n - 1}\Gamma(\mu_n) a^{\mu_n} \xi^{-\mu_n} J_{\mu_n}(\xi a), \quad \mu_n > 0, a > 0\} \tag{10}$$

The vertical component of displacement under the foundation, which implies the rigid displacement of the foundation, is written as:

$$w(r,\ z=0,\ t) = \Delta e^{i\omega t} \qquad 0 \le r \le a \tag{11}$$

As mentioned earlier, for satisfying this equation, the normal pressure under the disc is written as a linear combination of some of the functions given in the function space **M** as:

$$P_z(r;\omega) = \sum_{n=1}^{N} \beta_n(\omega)\, f(r,\mu_n) \tag{12}$$

where $\beta_n(\omega)$ is generally a complex number and N the number of functions used to express the pressure. The zero order Hankel transform of the Eq. (12) is:

$$\tilde{P}_{z0}(\xi;\omega) = \sum_{n=1}^{N} \beta_n(\omega) \tilde{f}_0(\xi,\mu_n) \tag{13}$$

By substituting $\tilde{P}_{z0}(\xi)$ from Eq. (13) into the Eq. (7), the vertical component of displacement can be written in the following form, which is a direct result of the linearity of the integral operators:

$$w(r,z;\omega) = \sum_{n=1}^{N} \beta_n(\omega)\, w_n(r,z;\omega) \tag{14}$$

In Eq. (14), $w_n(r,z;\omega)$ with the use of Eqs. (7) and (13) is:

$$\begin{aligned} w_n(r,z;\omega) = \frac{1}{A_{33}} \int_0^\infty \frac{\xi}{I(\xi)} [(\alpha_2\lambda_1(\xi)^2 \\ -(1+\alpha_1)(\xi^2 - \frac{\rho_0\omega^2}{1+\alpha_1}))(-\eta_2(\xi) e^{-\lambda_1(\xi)z}) \\ +(\alpha_2\lambda_2(\xi)^2 - (1+\alpha_1)(\xi^2 - \frac{\rho_0\omega^2}{1+\alpha_1})) \\ \times(\eta_1(\xi) e^{-\lambda_2(\xi)z})]\, \tilde{f}_0(\xi,\mu_n) J_0(\xi r) d\xi \end{aligned} \tag{15}$$

Writing $w(r,z;\omega)$ from Eq. (14) at z = 0, and substituting the result into Eq. (11), where the time harmonic term is suppressed, the equation $\sum_{n=1}^{N} \beta_n(\omega)\, w_n(r, z=0;\omega) = \Delta$ for $0 \le r \le a$ is obtained. Dividing the radius of foundation, a, into N-1 parts, results in N different points (rings), where the displacement boundary conditions would be satisfied as:

$$\sum_{n=1}^{N} \beta_n w_n(r_k, z=0) = \Delta, \qquad k = 1,2,3,\ldots,N \tag{16}$$

which gives N algebraic equations for N unknown parameter $\beta_n,\ n = 1,2,\ldots,N$. Substituting β_n as the solution of Eq. (16) into Eq. (12) results in the pressure in between the foundation and the underneath half-space, giving the total force applied on the surface of the half-space as:

$$F_z(\omega)=\int_0^a\int_0^{2\pi}P_z(r;\omega)r\,d\theta\,dr =\sum_{n=1}^{N}\beta_n(\omega)\times(\frac{\pi}{\mu_n}a^{2\mu_n}) \tag{17}$$

Eventually, the vertical impedance function is obtained as a function of frequency as:

$$K_{zz}(\omega)=F_z(\omega)/\Delta =\frac{1}{\Delta}\sum_{n=1}^{N}\beta_n(\omega)\times(\frac{\pi}{\mu_n}a^{2\mu_n}) \tag{18}$$

NUMERICAL RESULTS

As indicated in Eq. (15), the displacement components are expressed in terms of one-dimensional semi-infinite integrals and the complex coefficients, β_n, is obtained from the Eq. (16). Due to the presence of branch points, pole, exponential and Bessel functions in a complex form in the integrands, the integrals cannot be given in closed-form even in the simpler case of isotropic material. There would be, in general, two branch points at ξ_{λ_i} (*i=1, 2*), as stated before, lying on the formal path of integration. For the case of an isotropic solid, the branch points reduce to:

$$\xi_{\lambda_1}=k_d=\frac{\omega}{C_d},\quad \xi_{\lambda_2}=k_s=\frac{\omega}{C_s} \tag{19}$$

where

$$C_d=\sqrt{\frac{2\mu+\lambda}{\rho}},\ C_s=\sqrt{\frac{\mu}{\rho}} \tag{20}$$

Here, λ and μ stand for the Lame constants of the classical theory of elasticity, while C_d and C_s are the dilatational and equivoluminal wave speeds, respectively.

Some difficulties are encountered in numerical evaluations of the integrals which may be found in Eskandari-Ghadi et al. (2011). Here, the procedure accepted in Eskandari-Ghadi et al. (2011) is used for numerical evaluations. Each integral is decomposed into different intervals with limits equal to branch points and in each interval the relevant branch cut is used for λ_1 and λ_2, as shown in Figure 4.

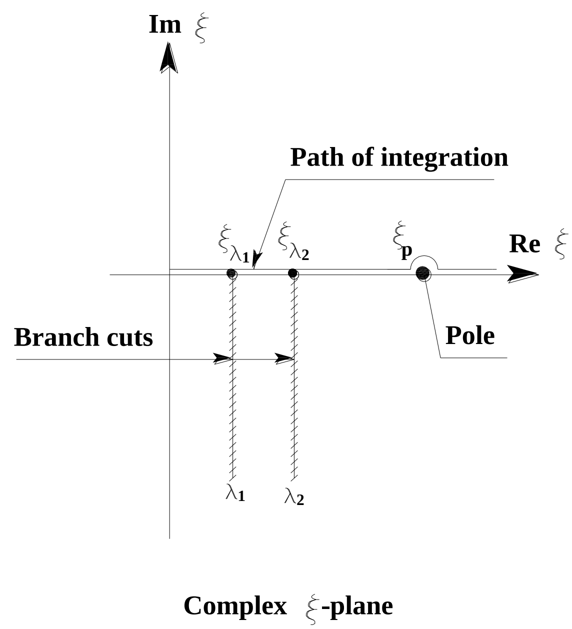

Fig. 4. Pole, branch points and branch cuts.

Besides, the function $I(\xi)$ in Eq. (8) yields poles whose positions are given by

$$I(\xi)=\eta_2\nu_1-\eta_1\nu_2=0 \tag{21}$$

Stoneley has shown that Eq. (21) has only two roots $\pm\xi_p$ along the real axis (Stoneley, 1949; Rajapakse and Wang, 1993). As for an isotropic solid, ξ_p corresponds to the root of the Rayleigh wave function (e.g., see Pak, 1987). In the case of Poisson material, it can be shown that:

$$\xi_p = \frac{1}{2}\sqrt{3+\sqrt{3}}\sqrt{\frac{\rho}{\mu}}\omega \tag{22}$$

The root ξ_p of Eq. (21) corresponding to the materials considered in the current study is given in Table 1. To deal with the singularity at the pole, since the pole at ξ_p is an interior singular point, the integral is decomposed into 3 integrals: one integral with the upper limit $\xi_p - \varepsilon$ one with a lower limit $\xi_p + \varepsilon$ and one over a small semi-circle of radius ε above the pole, ε which is a small number. The integrals from zero to $\xi_p - \varepsilon$ and from $\xi_p + \varepsilon$ to infinity are evaluated using Simpson's rule, and the integral over the small semi-circle of radius ε is evaluated using the residual method (e.g. Churchill and Brown, 1990). Since the pole is of the first order, the integrand may be written in the form of $q(\xi)/I(\xi)$, where $q(\xi)$ is analytic at ξ_p and $I(\xi)$ has been given in Eqs. (8). Thus, the integral over the limiting small semi-circle is equal to $-\pi i \,\mathrm{Res}(\xi_p)$, where $\mathrm{Res}(\xi_p) = \lim_{\xi\to\xi_p}\left[q(\xi)/(d\,I(\xi)/d\,\xi)\right]$. For the case of a branch point, the integral is broken up into the first 2 parts and evaluated with increasingly small ε until the selected error criterion is satisfied for convergence. Several numerical examples are provided to compare the present solution with the existing numerical solutions for the isotropic half-space, and to evaluate the accuracy and the efficiency of the current solutions. They are followed by a series of parametric study to explore the influence of the degree of the material anisotropy, the frequency of excitation and the type of loading on the response. It bears noting that all numerical results presented here are dimensionless, with a non-dimensional frequency defined as $\omega_0 = a\omega\sqrt{\rho / A_{44}}$. By evaluating the integrals in Eq. (15), the complex coefficients, β_n, are obtained from Eq. (16), and the vertical displacement and the vertical impedance function are evaluated from Eqs. (14) and (18), respectively. To this end, an isotropic material and three kinds of transversely isotropic materials are considered. The mechanical properties of the materials are listed in Table 1. The Poisson's ratio, υ, of the isotropic material is equal to 0.25. For numerical evaluation, the real and imaginary parts of impedance function are normalized as $Re(\bar{K}_{zz}) = Re(K_{zz}) / a A_{66}$ and $Im(\bar{K}_{zz}) = Im(K_{zz}) / \omega_0 a A_{66}$, respectively.

Table 1. Properties of tested specimens.

Material	A_{11}	A_{12}	A_{13}	A_{33}	A_{44}	A_{66}	$\frac{\xi_p\sqrt{A_{44}}}{\omega\sqrt{\rho}}$
1(Isotropic)	6.0	2.0	2.0	6.0	2.0	2.0	1.08766
2	5.5	1.5	1.8	15.9	2.0	2.0	1.03519
3	14.0	6.0	5.0	7.5	2.0	4.0	1.03800
4	26.0	14.0	10.0	10.0	2.0	6.0	1.02293

* A_{ij} in this table is in $10^4\ N / mm^2$.

In order to provide a proper perspective on the accuracy of the present method, the impedance function of the disc for vertical motion is first obtained for isotropic medium given in Table 1. The solution has been numerically evaluated for different functions given in Eq. (9) and it has been seen that the numerical results converges to those of Luco and Mita (1987) rapidly. The vertical impedance functions evaluated in this paper for different *N* (*N*=5, 7, 10 and 15) are compared with the existing results of Luco and Mita (1987) in Figure 5. As it can be observed, although there is no significant difference between the results from different N, the results of this study are highly consistent with those of Luco and Mita (1987) for dimensionless frequency smaller than $\omega_0 = 3.0$, when *N* equals 7. However, the real part of impedance function is slightly different from the results of Luco and Mita (1987) when the dimensionless frequency is larger than 3.0. It must be mentioned that for large buildings with large foundations, the dimensionless frequency rarely exceeds 3.0.

Figures 6 and 7 show the spatial variation of the vertical component of dimensionless displacement w/Δ for the four materials with respect to the radial distance r/a for different values of dimensionless frequency $\omega_0 = 0.1$ and 3.0, respectively, when N equals 7. Figures 8 and 9 demonstrates the spatial variation of the vertical component of dimensionless displacement w/Δ for the materials given in Table 1 with respect to the vertical distance z/a for different values of dimensionless frequency, ω_0. Figure 10 shows a comparison between the vertical impedance function evaluated in this study and Eskandari-Ghadi et al. (2010b), where a good agreement can be observed between the two results. Figure 11 shows the variation of the non-dimensional vertical impedance function in terms of dimensionless frequency.

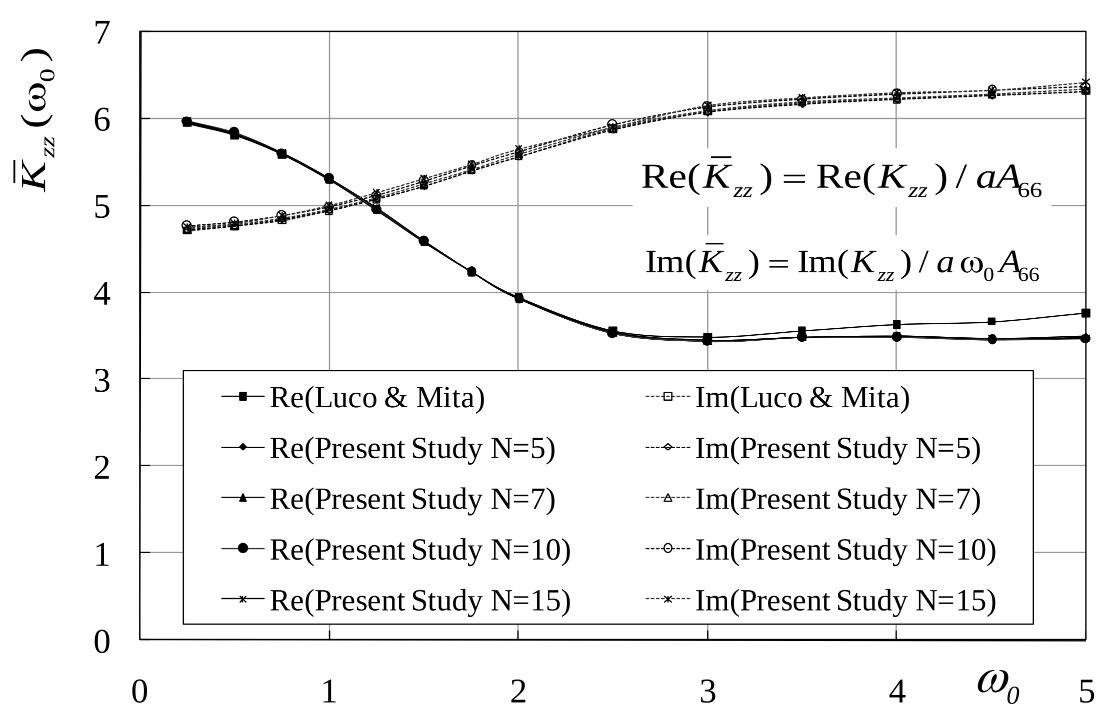

Fig. 5. Comparison of the results of this study with different number of functions of function space with that of Luco and Mita (1987).

As it is seen, the vertical displacement in different media given in Table 1 is almost the same if the frequency is small, while for large frequencies it is different in different materials. Figure 12 presents the variation of contact pressure between the rigid circular plate and the half-space for dimensionless frequency of 1.0, where a singular behavior can be seen at the vicinity of the edge of the rigid plate.

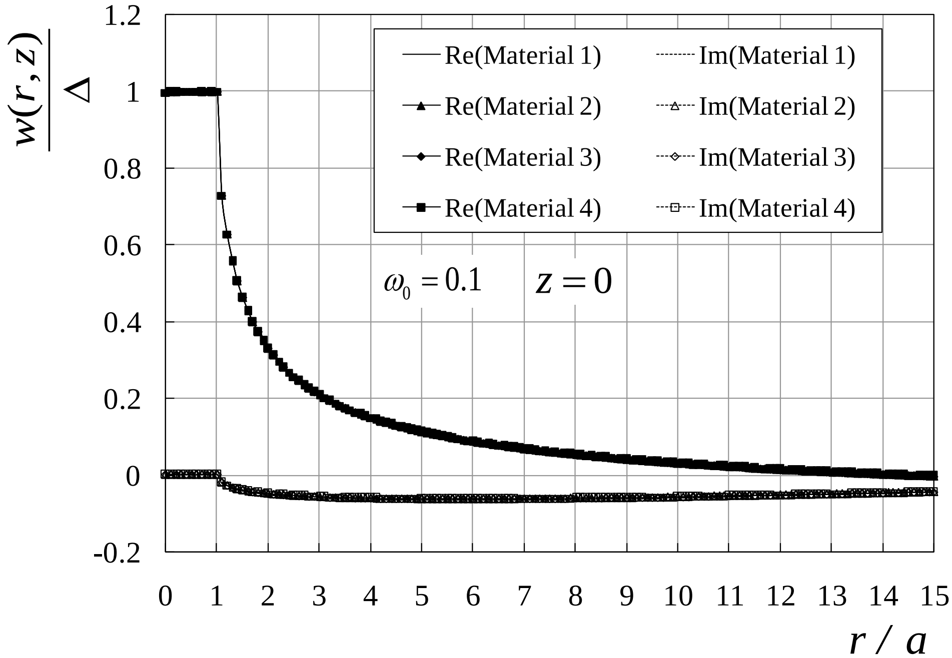

Fig. 6. Vertical component of displacement versus dimensionless radial distance for $\omega_0 = 0.1$.

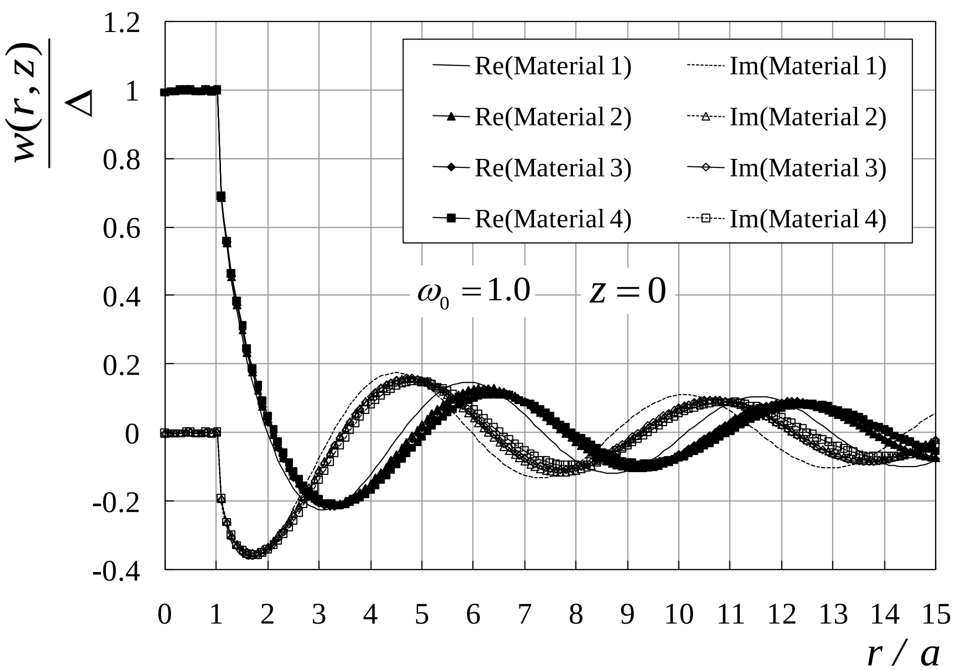

Fig. 7. Vertical component of displacement versus dimensionless radial distance for $\omega_0 = 1.0$.

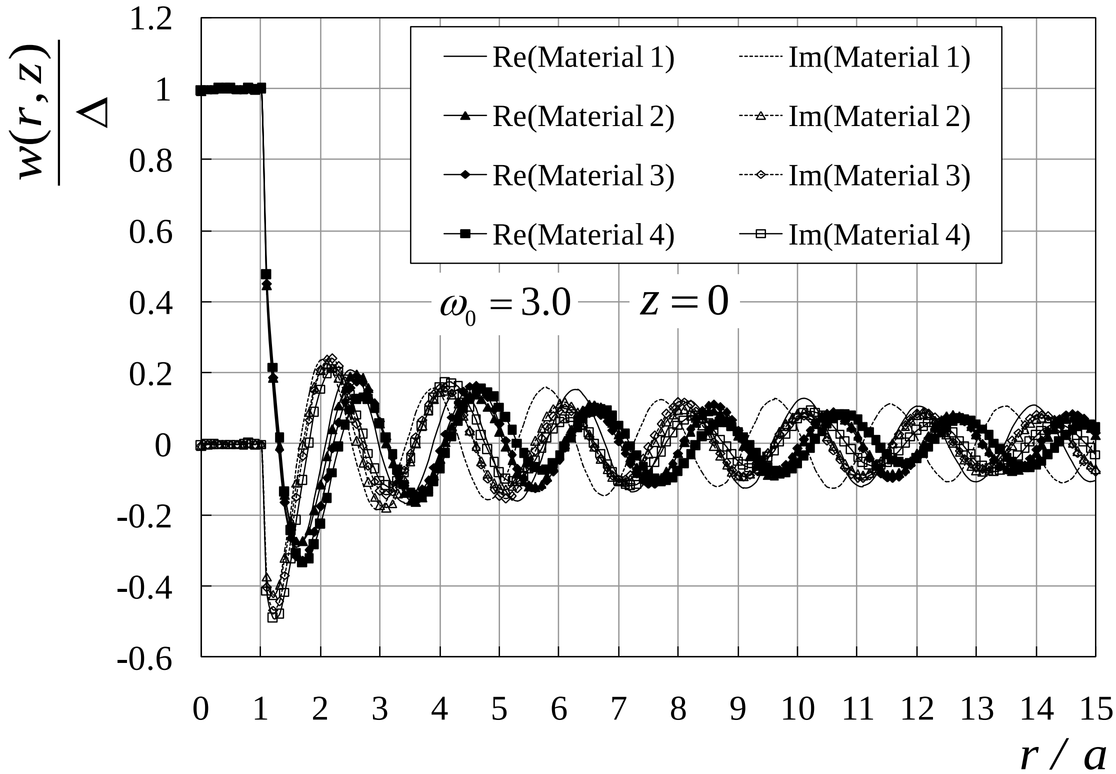

Fig. 8. Vertical component of displacement versus dimensionless radial distance for $\omega_0 = 3.0$.

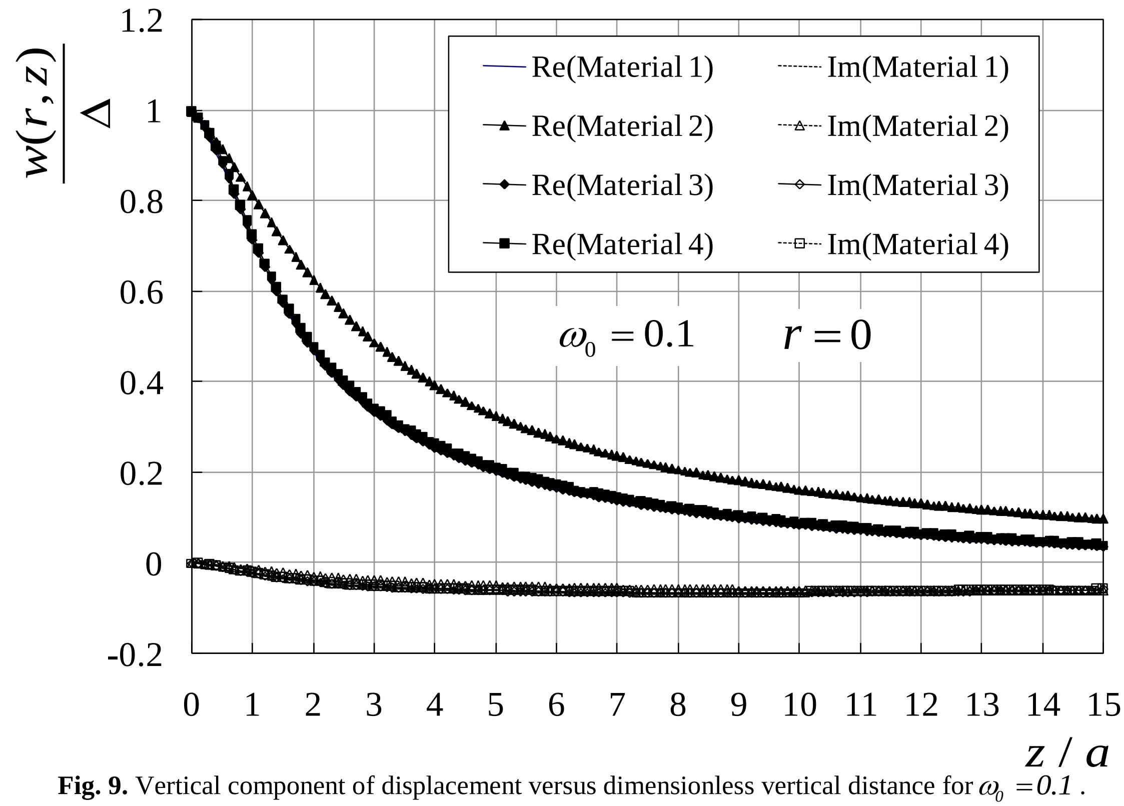

Fig. 9. Vertical component of displacement versus dimensionless vertical distance for $\omega_0 = 0.1$.

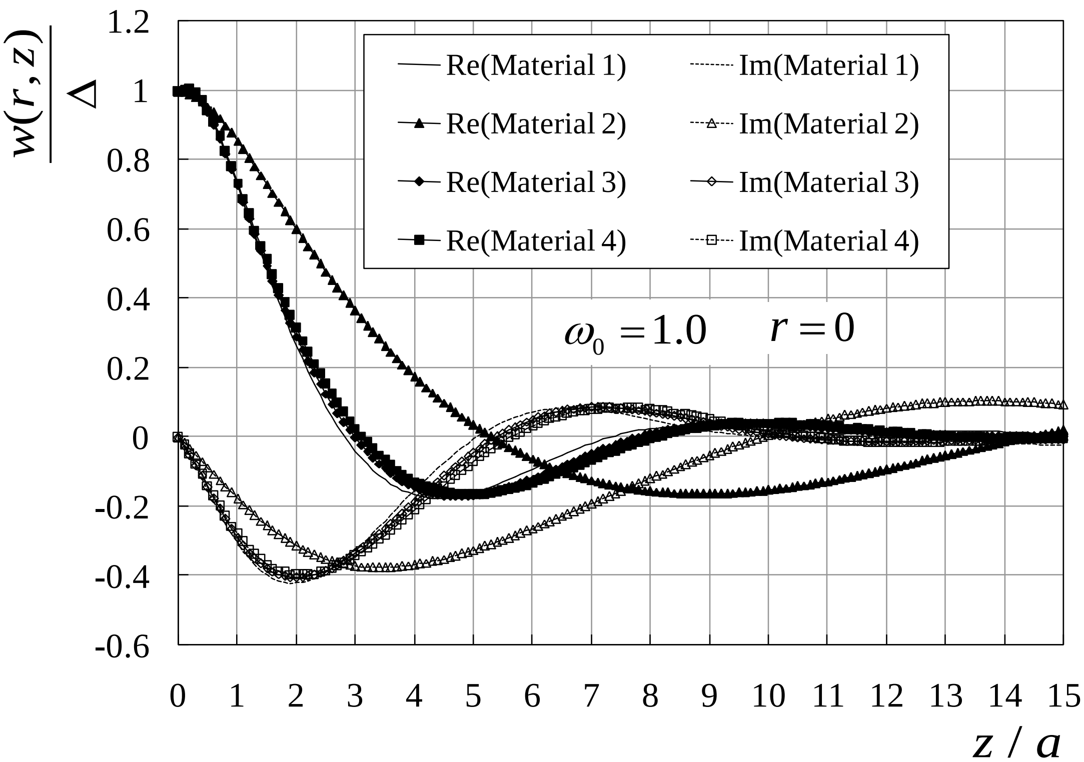

Fig. 10. Vertical component of displacement versus dimensionless vertical distance for $\omega_0 = 1.0$.

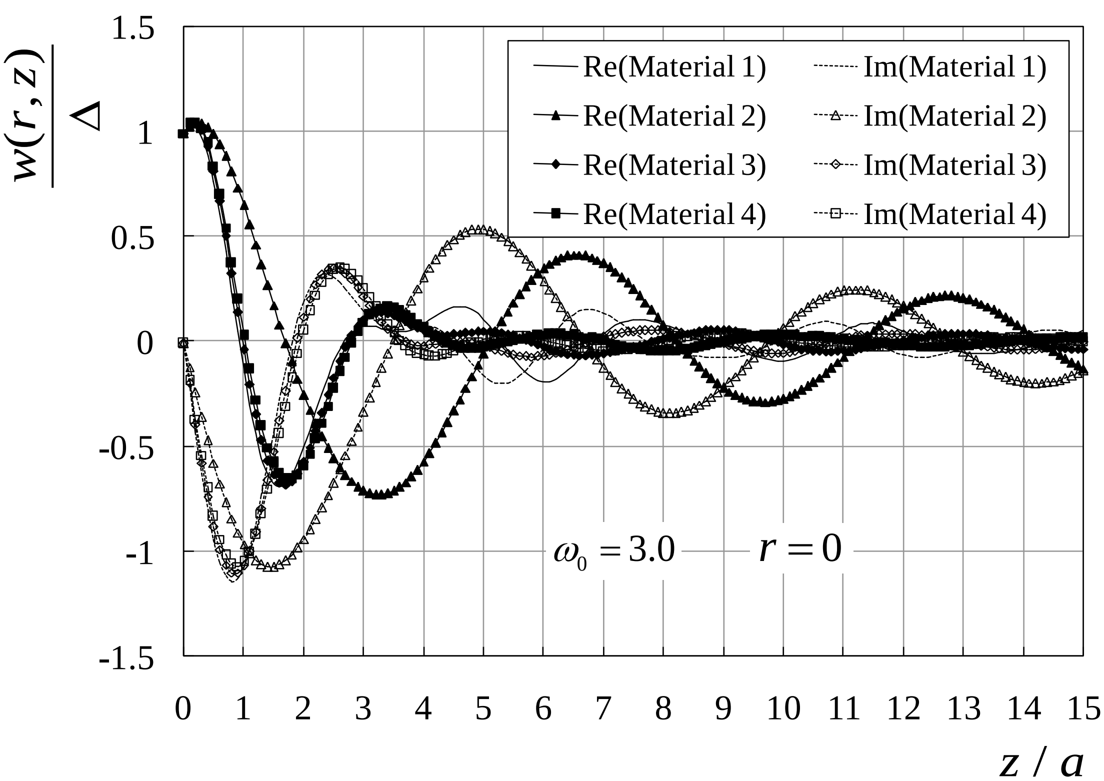

Fig. 11. Vertical component of displacement versus dimensionless vertical distance for $\omega_0 = 3.0$.

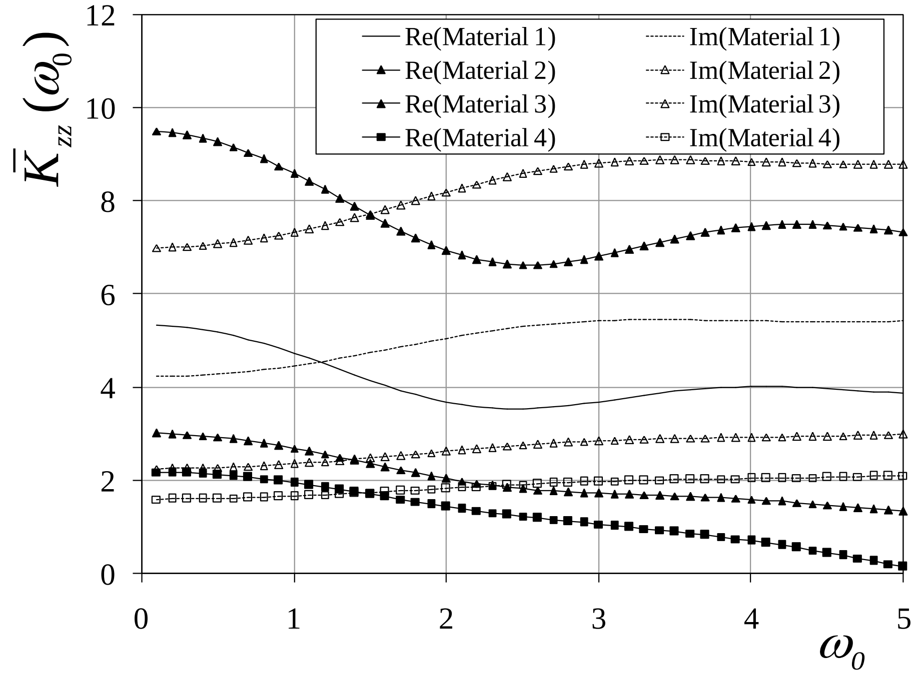

Fig. 12. Real and imaginary parts of vertical impedance function versus ω_0 .

CONCLUSIONS

In this paper, with the use of a method of function space, the vertical impedance function of a rigid circular foundation resting on a semi-infinite transversely isotropic medium and also the vertical component of displacement of the mentioned medium have been obtained in frequency domain and presented for different values of frequency. It can be observed that although the displacements of different materials given in Table 1 do not differ greatly in low frequencies, there are great differences in high frequencies. In addition, the larger the frequency, the further the wave go. The real parts of the vertical impedance functions on different transversely isotropic media are all about constant over a wide range of frequency, whereas their imaginary parts increase almost linearly with frequency. The fact indicates that any impedance function would be well approximated by a simple Voigt model with a linear spring and a dashpot arranged in parallel.

NOTATION

The following symbols are used in this paper:

A_{ij} = elasticity constants of half-space $(z \geq 0)$

a = radius of rigid circular foundation

C_d, C_s = dilatational and equivoluminal wave speeds

Δ = vertical displacement of rigid foundation

$H(a-r)$ = Heaviside step function

J_ν = Bessel function of the first kind and νth order

$K_{zz}(\omega)$ = vertical impedance function

M = function space

$P_z(r,\omega)$ = pressure in between the rigid foundation and the half-space

r = radial coordinate

t = time variable

θ = angular coordinate

u = displacement component in r-direction

υ = Poisson's ratio

w = displacement component in z-direction
z = vertical coordinate
λ = Lame's constant
λ_1, λ_2 = radicals appearing in general solutions
μ = Lame's constant
ξ = Hankel's parameter
$\xi_{\lambda_1}, \xi_{\lambda_2}, \xi_p$ = branch points and simple pole on positive real axis
ρ = material density
ω = angular frequency
ω_0 = nondimensional frequency

ACKNOWLEDGEMENT

The first author, M. E.-G., would like to acknowledge the financial support from the University of Tehran for this research under Grant No. 27840/1/04.

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