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Final State Interaction Effects in $B^+ \to J/\psi\, \pi^+$ Decay

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Abstract

In this research the exclusive decay of $B^+ \to J/\psi\, \pi^+$ is calculated by QCD factorization (QCDF) method and final state interaction (FSI). First, the $B^+ \to J/\psi\, \pi^+$ decay is calculated via QCDF method. The result that is found by using the QCDF method is less than the experimental result. So FSI is considered to solve the $B^+ \to J/\psi\, \pi^+$ decay. For this decay, $D^+\bar{D}^{0^*}$ via the exchange of D^- and D^{-^*} mesons are chosen for the intermediate state. The above intermediate state is calculated by using the QCDF method. In the FSI effects, the results of our calculations depend on η as the phenomenological parameter. The range of this parameter is selected from 2 to 3. It is found that if $\eta = 2.8$ is selected, the numbers of the branching ratio are placed in the experimental range. The experimental branching ratio of $B^+ \to J/\psi\, \pi^+$ decay is 4.9×10^{-5} and our results calculated by QCDF and FSI are 0.56×10^{-5} and 3.9×10^{-5} respectively.

Keywords: B meson; QCD factorization; Final state interaction; Intermediate state; Branching ratio.

Introduction

B meson non-leptonic decays are significant for testing theoretical frameworks and searching new physics beyond the standard model. The next-to-leading order low-energy effective Hamiltonian is used for the weak interaction matrix elements and (FSI). The importance of the FSI in hadronic processes has been identified for a long time. Recently, its applications in D and B decays have attracted extensive interests and attentions of theorists. Since the hadronic matrix elements are fully controlled by non-perturbative QCD, the most important problem is how to evaluate them properly. The factorization method enables one to separate the non-perturbative QCD effects from the perturbative parts and to calculate the latter in terms of the field theory order by order. Several factorization approaches have been proposed to analyze B meson decays, such as the naive factorization approach, QCDF approach, the perturbative QCD approach and soft-collinear-effective theory, but none provided an estimate of the FSI at the hadronic level. These approaches successfully explain many phenomenons; however, there are still some problems which are not easy to describe within this framework. These may be some hints for the need of FSI in B decays. FSI effects are non-perturbative in nature [1]. In many decay modes, FSI may play a crucial role [2]. In this way, the CKM matrix elements and the color factor are

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suppressed and the CKM's most favored two-body intermediate states are the only ones that have been taken into consideration [3]. The FSI can be considered as a soft re-scattering style for certain intermediate two-body hadronic channel $B^+ \to D^+\bar{D}^{0^*}$decay [4]. Therefore, FSI are estimated via the one particle exchange processes at the hadron loop level (HLL) as explained in section 4. We calculated the $B^+ \to J/\psi\ \pi^+$ decay according to QCDF method and obtained the $BR(B^+ \to J/\psi\ \pi^+) = 0.56 \times 10^{-5}$. The FSI can give sizable corrections. Re-scattering amplitude can be derived by calculating the absorptive part of triangle diagrams. In this decay, intermediate state is $D^+\bar{D}^{0^*}$. Then we calculated the $B^+ \to J/\psi\ \pi^+$ decay according to HLL method. By FSI method we obtain the branching ratio of $B^+ \to J/\psi\ \pi^+$ decay, 3.9×10^{-5} and the experimental result of this decay is 4.9×10^{-5} [5].

We present the calculation of QCDF for $B^+ \to J/\psi\ \pi^+$ decay in Sec. 2. In Sec. 3, we calculate the amplitudes of the intermediate states. Then we present the calculation of HLL for $B^+ \to J/\psi\ \pi^+$ decay in Sec. 4. In Sec. 5, we give the numerical results, and in Sec. 6, we have conclusion.

1. QCD Factorization of $B^+ \to J/\psi\ \pi^+$ Decay

To compare QCDF with FSI, we explore QCDF

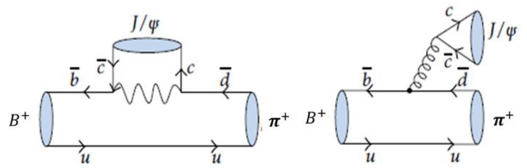

Figure. 1. The Feynman diagrams contributing to the $B^+ \to J/\psi\pi^+$ decay.

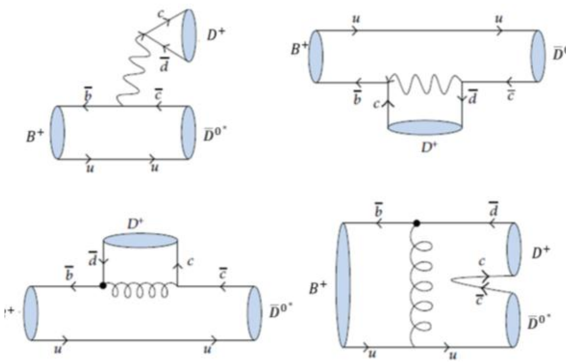

Figure. 2. $B^+ \to D^+\bar{D}^{0^*}$ decay diagrams.

analysis. In this section, we obtain the amplitude of $B^+ \to J/\psi\ \pi^+$ decay using QCDF method. In factorization approach, there are color-suppressed tree and allowed penguin diagrams to $B^+ \to J/\psi\ \pi^+$ decay. We adopt leading order Wilson coefficients at the scale m_b for QCDF approach. The diagrams describing this decay are shown in Fig.1.

According to the QCDF the amplitude of $B^+ \to J/\psi\ \pi^+$ decay is given by

$$A_{QCD}(B^+ \to J/\psi\ \pi^+) = -i\sqrt{2}G_F m_{J/\psi}(\varepsilon_{J/\psi}.p_B)f_\pi A_0^{B\to J/\psi}\left\{V_{cb}V_{cd}^* a_2 + \lambda_p\left[a_3 + r_\chi^{J/\psi}\ (a_5 + a_7 + a_9)\right]\right\}. \tag{1}$$

Where λ_p are the products of elements of the quark mixing matrix. Using the unitarity relation $\lambda_u + \lambda_c + \lambda_t = 0$, we write

$$\lambda_p = \sum_{p=u,c} V_{pb}V_{pd}^*. \tag{2}$$

And for vector meson J/ψ, the ratios $r_\chi^{J/\psi}$ is defined as

$$r_\chi^{J/\psi} = \frac{2m_{J/\psi}}{m_b}\frac{f_{J/\psi}^\perp}{f_{J/\psi}}. \tag{3}$$

The effective coefficients a_i, which are specific to the factorization approach, and defined as

$$\begin{aligned} a_i &= c_i^{eff} + \frac{1}{N_c}c_{i+1}^{eff} \qquad (i = odd), \\ a_i &= c_i^{eff} + \frac{1}{N_c}c_{i-1}^{eff} \qquad (i = even), \end{aligned} \tag{4}$$

where the quantities of c_i^{eff} are effective Wilson coefficients at the renormalization scale μ for the $\bar{b} \to \bar{d}$ transition. In the above amplitude the determination of a_2 in the $b \to c$ current-current transitions has received a lot of attention, the quantities of a_3, a_5 and a_7, a_9 are the QCD-penguin and electroweak-penguin coefficients, respectively. Numerical values of a_i $(i = 1, \ldots, 10)$ for representative value of the phenomenological parameter N_c are displayed in Section 5.

2. Amplitudes of Intermediate States

Before analyzing FSI in the $B^+ \to J/\psi\ \pi^+$ decay, we introduce the factorization approach in detail. In FSI effects $D^+\bar{D}^{0^*}$ is chosen for the intermediate state. For $B^+ \to D^+\bar{D}^{0^*}$ decay, Feynman diagrams are shown in Fig.2. And the amplitude comes

$$A\,(B^+ \to D^+\bar{D}^{0^*}) = -i\sqrt{2}G_F m_{D^*}(\varepsilon_{D^*}.p_B) f_D A_0^{BD^*}\{(a_1+a_2)V_{cb}V_{cd}^* +\lambda_p[a_4 + r_\chi^D(a_6+a_8)+a_{10}]\} - i\frac{G_F}{\sqrt{2}} f_B f_D f_{D^*}\left[b_3 + \frac{1}{2} b_{3,EW}\right]\lambda_p, \quad (5)$$

$$b_3 = \frac{C_F}{N_c^2}[c_3 A_1^i + c_5(A_3^i + A_3^f) + N_c c_6 A_3^f],$$

$$b_{3,EW} = \frac{C_F}{N_c^2}[c_9 A_1^i + c_7(A_3^i + A_3^f) + N_c c_8 A_3^f]. \quad (6)$$

Where c_i are the Wilson coefficients, N_c is the color number and

$$A_1^i \approx 2\pi\alpha_s[9(X_A - 4 + \frac{\pi^2}{3}) + r_\chi^{D^+} r_\chi^{\bar{D}^{0^*}} X_A^2],$$
$$A_3^i = 0, \quad (7)$$

$$A_3^f = 2\pi\alpha_s(r_\chi^{D^+} + r_\chi^{\bar{D}^{0^*}})(2X_A^2 - X_A),$$

$$C_F = \frac{N_C^2-1}{2N_C}. \quad (8)$$

There are large theoretical uncertainties related to the modeling of power corrections corresponding to weak annihilation effects, we parametrize these effects in terms of the divergent integrals X_A (weak annihilation)

$$X_A = (1+\rho e^{i\phi})\, ln\frac{m_B}{\Lambda_{QCD}},$$
$$\rho \le 1\,, \quad \Lambda_{QCD} = 0.225\ GeV. \quad (9)$$

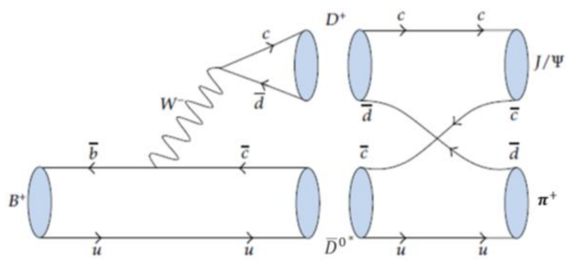

Figure. 3. Quark level diagrams for $B^+ \to D^+\,\bar{D}^{0^*} \to J/\psi\pi^+$ decay.

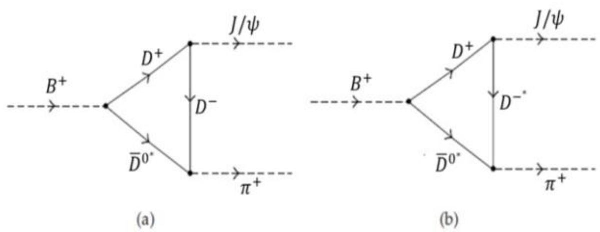

Figure. 4. The hadronic level diagrams for $B^+ \to D^+\,\bar{D}^{0^*} \to J/\psi\pi^+$decay

3. *Final State Interaction of $B^+ \to J/\psi\,\pi^+$ Decay*

For $B^+ \to J/\psi\,\pi^+$ decay, two-body intermediate state such as $D^+\bar{D}^{0^*}$is produced. We can write out the decay amplitude involving HLL contributions with the following formula

$$\text{Abs M (B(p}_B) \to \text{M(p}_1)\text{ M(p}_2)\to \text{M(p}_3)\text{ M(p}_4))$$
$$= \frac{1}{2}\int \frac{d^3\overrightarrow{p_1}}{2E_1(2\pi)^3}\frac{d^3\overrightarrow{p_2}}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_B - p_1 - p_2)M(B\to M_1M_2)\,G(M_1M_2 \to M_3M_4), \quad (10)$$

for which both intermediate mesons (M_1, M_2) are pseudoscalar. And the absorptive part of the HLL diagrams for VP case can be calculated by

$$\text{Abs M (B(p}_B) \to \text{M(p}_1)\text{ M(p}_2)\to \text{M(p}_3)\text{ M(p}_4))$$
$$= \frac{1}{2}\int \frac{d^3\overrightarrow{p_1}}{2E_1(2\pi)^3}\frac{d^3\overrightarrow{p_2}}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_B - p_1 - p_2)\,V_{CKM} \times\{2a_i m_V(\varepsilon_V^*.p_B)(f_p A_0^{B\to V} + f_V F_1^{B\to P}) + f_B f_P f_V b_i\}G(M_1M_2 \to M_3M_4), \quad (11)$$

where $M(B\to M_1M_2)$ is the amplitude of $B\to M_1M_2$ decay that calculated via QCDF method. and $G(M_1M_2\to M_3M_4)$ involves hadronic vertices factor defined as

$$\langle D(p_3)\psi(\epsilon_2,p_2)|i\pounds|D(p_1)\rangle = -ig_{\psi DD}\epsilon_2.(p_1+p_3),$$

$$\langle D^*(\epsilon_3,p_3)\psi(\epsilon_2,p_2)|i\pounds|D(p_1)\rangle = -i\sqrt{2}g_{\psi D^*D}\varepsilon_{\mu\nu\alpha\beta}\varepsilon_2^\mu\varepsilon_3^{*\nu}p_1^\alpha p_2^\beta, \quad (12)$$

$$\langle D^*(\varepsilon_3,p_3)\psi(\epsilon_2,p_2)|i\pounds|D^*(\varepsilon_1,p_1)\rangle = -i\varepsilon_1^\beta\varepsilon_2^\nu\varepsilon_3^\lambda[2p_{1\nu}g_{\alpha\beta} - (p_1+p_2)_\alpha g_{\nu\beta} + 2p_{2\beta}g_{\alpha\nu}].$$

The dispersive part of the rescattering amplitude can be obtained from the absorptive part via the dispersion relation [6, 7]:

$$\text{Dis M}(m_B^2) = \frac{1}{\pi}\int_s^\infty \frac{Abs\,M(s')}{s'-m_B^2}ds'. \quad (13)$$

Where s' is the square of the momentum carried by the exchanged particle and s is the threshold of intermediate states, in this case $s\sim m_B^2$. Unlike the absorptive part, the dispersive contribution suffers from the large uncertainties arising from the complicated integration.

3.1. Final State Interaction In $B^+ \to D^+\overline{D}^{0^*} \to J/\psi\, \pi^+$ Decay

The quark model diagram for $B^+ \to D^+\overline{D}^{0^*} \to J/\psi\, \pi^+$ decay is shown in Fig.3. And the hadronic level diagrams are shown in Fig. 4.

The amplitude of the mode $B^+ \to D^+(p_1)\overline{D}^{0^*}(\varepsilon_2, p_2) \to J/\psi(\varepsilon_3, p_3)\pi^+(p_4)$ are given by

$$\text{Abs}(4a) = \frac{-iG_F}{2\sqrt{2}}\int_{-1}^{1}\frac{d^3\vec{P}_1}{2E_1(2\pi)^3}\frac{d^3\vec{P}_2}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_B - p_1 - p_2)$$

$$\times\left(-\mathrm{i}g_{\psi DD}\right)\varepsilon_3.(p_1 + q)(-ig_{DD^*\pi})\,\varepsilon_2.(-q)\{2m_{D^*}(\varepsilon_2.p_1)f_D A_0^{BD^*}$$

$$T_1 = (p_1 - p_3)^2 - m_D^2 = p_1^2 + p_3^2 - 2p_1^0p_3^0 + 2\overrightarrow{p_1}.\overrightarrow{p_3} - m_D^2, \quad (15)$$

$$q^2 = m_1^2 + m_3^2 - 2E_1E_3 + 2|\overrightarrow{p_1}||\overrightarrow{p_3}|\cos\theta$$

θ is the angel between $\overrightarrow{\mathrm{p}_1}$ and $\overrightarrow{\mathrm{p}_3}$, q is the momentum of the exchange D^0 meson, and $F(q^2, m_D^2)$ is the form factor defined to take care of the off–shell of the exchange particles, which introduced as [8]

$$F(q^2, m_D^2) = (\frac{\Lambda^2 - \mathrm{m}_\mathrm{D}^2}{\Lambda^2 - \mathrm{q}^2})^n. \quad (16)$$

The form factor (i.e.n=1) normalized to unity at $q^2 = m_D^2$. m_D and q are the physical parameters of the exchange particle and Λ is phenomenological parameter.

It is obvious that for $q^2 \to 0, F(q^2, m_D^2)$ becomes a number. If $\Lambda \gg m_D$ then $F(q^2, m_D^2)$ turns to be unity, whereas, as $q^2 \to \infty$ the form factor appraochs to zero and the distance becomes small and the hadron interaction is no longer valid. Since Λ shoud not be far from m_K and q, we choose

$$\Lambda = m_D + \eta\Lambda_{\mathrm{QCD}}. \quad (17)$$

Where the η is the phenomenological parameter that its value in the form factor is expected to be of the order of unity and can be determined from the measured rates, and

$$\text{Abs}(4b) = \frac{-iG_F}{2\sqrt{2}}\int_{-1}^{1}\frac{d^3\vec{P}_1}{2E_1(2\pi)^3}\frac{d^3\vec{P}_2}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_B - p_1 - p_2)$$

$$\times\left(-i\sqrt{2}g_{DD^*\psi}\right)\varepsilon_{\mu\nu\alpha\beta}\varepsilon_3^\mu\varepsilon_{D^*}^\nu p_1^\alpha p_3^\beta\left(-i\sqrt{2}g_{D^*D^*\pi}\right)\varepsilon_{\rho\sigma\lambda\eta}\varepsilon_2^\rho\varepsilon_{D^*}^\sigma p_2^\lambda p_4^\eta$$

$$\times\{2m_{D^*}(\varepsilon_2.p_1)f_D A_0^{BD^*}\left[a_4 + r_\chi^D(a_6 + a_8) + a_{10}\right](V_{tb}V_{td}^*)$$

$$-f_Bf_Df_{D^*}\left[b_3 + \frac{1}{2}b_{3ew}\right](V_{tb}V_{td}^*)\}\,\frac{F^2(q^2, m_{D^*}^2)}{T_2}$$

$$= \frac{iG_F}{8\sqrt{2}\pi m_B}g_{\psi D^*D}g_{D^*D^*\pi}\int_{-1}^{1}|P_1|\,d(\cos\theta)\,\{2H_4m_{D^*}(\varepsilon_2.p_1)f_DA_0^{BD^*}$$

$$\times[a_4 + r_\chi^D(a_6 + a_8) + a_{10}](\,V_{tb}V_{td}^*)$$

$$-f_Bf_Df_{D^*}\times[b_3 + \frac{1}{2}b_{3ew}](V_{tb}V_{td}^*)H_3\}\frac{F^2(q^2, m_{D^*}^2)}{T_2}, \quad (18)$$

$$H_3 = \varepsilon_{\mu\nu\alpha\beta}\varepsilon_{\rho\sigma\lambda\eta}\varepsilon_3^\mu\varepsilon_{D^*}^\nu p_1^\alpha p_3^\beta\varepsilon_2^\rho\varepsilon_{D^*}^\sigma p_2^\lambda p_4^\eta,$$

$$H_4 = m_3^2(p_1.p_2) - (p_1.p_3)(p_2.p_3) + \left(\frac{E_2|p_3| - E_3|p_2|\cos\theta}{m_B|p_3|}\right)\times[(p_B.p_1)(p_3.p_4) - (p_B.p_3)(p_1.p_4)], \quad (19)$$

$$T_2 = (p_1 - p_3)^2 - m_{D^*}^2 = m_D^2 + m_{J/\psi}^2 - 2p_1^0p_3^0 + 2|\overrightarrow{p_1}||\overrightarrow{p_3}$$

$$= p_1^2 + p_3^2 - 2p_1^0p_3^0 + 2\overrightarrow{p_1}.\overrightarrow{p_3} - m_{D^*}^2,$$

$$q^2 = m_1^2 + m_3^2 - 2E_1E_3 + 2|\overrightarrow{p_1}||\overrightarrow{p_3}|\cos\theta$$

$$= m_D^2 + m_{\frac{J}{\psi}}^2 - 2p_1^0p_3^0 + 2|\overrightarrow{p_1}||\overrightarrow{p_3}|\cos\theta - m_{D^*}^2.$$

The dispersion relation is

$$\text{Dis4}(m_B^2) = \frac{1}{\pi}\int_s^\infty\frac{Abs\ 4a(s') + Abs\ 4b(s')}{s' - m_B^2}ds'. \quad (20)$$

The decay amplitude of $B^+ \to J/\psi\,\pi^+$ via the HLL diagrams is

$$\mathrm{A}(B^+ \to J/\psi\,\pi^+) = Abs\ (4a) + Abs(4b) + Dis4(m_B^2). \quad (21)$$

4. Numerical Results

Numerical values of effective coefficients a_i for $\bar{b} \to \bar{d}$ transition at $N_c = 3$ are given by [9]

$$a_1 = 1.05, \quad a_2 = 0.053,$$
$$a_3 = 0.0048, \quad a_4 = -0.046 - 0.012i,$$
$$a_5 = -0.0045, \quad a_6 = -0.059 - 0.012i, \quad (22)$$
$$a_7 = 0.00003 - 0.00018i, \quad a_8 = 0.0004 - 0.00006i,$$

Table 1. The branching ratio of $B^+ \to J/\psi\pi^+$ decay with η=2~3 and experimental data (in units of 10^{-5}).

η	2	2.2	2.4	2.6	2.8	3	EXP
BR	1.1	1.6	2.2	3	3.9	5.1	4.9

$a_9 = -0.009 - 0.00018i, \quad a_{10} = -0.0014 - 0.00006i,$

The relevant input parameters used as follows [1, 10, 11]: (The values of the masses and decay constants are in units of GeV)

$m_b = 4.2 \quad m_u = 0.0024 \quad m_d = 0.0047$

$m_c = 1.29 \quad m_B = 5.279 \quad m_\pi = 0.139 \quad m_D = 1.87$

$m_{D^*} = 2.01 \quad m_{J/\psi} = 3.096 \quad f_B = 0.176$

$f_D = 0.22 \quad f_{D^*} = 0.23 \quad V_{ub} = 0.004 \quad V_{ud} = 0.974$

$V_{cb} = 0.042 \quad V_{cd} = 0.230$

$$A_1^{BD^*}(m_{D^*}^2)=1.1 \qquad A_2^{BD^*}(m_{D^*}^2)=1.82 \tag{23}$$

$A_0^{BD^*}(m_{D^*}^2)=2.5 \quad r_\chi^{D^+} = 1.88 \quad r_\chi^{\bar{D}^{0^*}} = 0.82$

$\phi = -55^\circ\,(PP) \quad \phi = -70^\circ\,(VP) \quad \phi = -20^\circ(PV)$

$\rho = 0.5 \quad \Lambda_{QCD} = 0.225 \quad G_F = 1.166 \times 10^{-5}$

$g_{\psi DD} = 7.71\; g_{\psi D^* D} = 8.64 \quad g_{DD^*\pi} = \sqrt{\frac{m_D}{m_{D_s}}}\, g_{D^* D_s K} = 18.22$

By using the input parameters and according to the QCDF method of $B^+ \to J/\psi\,\pi^+$ decay, we get

$$BR(B^+ \to J/\psi\,\pi^+) = 0.56 \times 10^{-5}. \tag{24}$$

We note that our estimate of branching ratio of $B^+ \to J/\psi\,\pi^+$ decay according to QCDF method seems less than the experimental result. Before calculating the $B^+ \to J/\psi\,\pi^+$ decay amplitude via FSI, we have to compute the intermediate state amplitude, for the $B^+ \to D^+\bar{D}^{0^*}$ decay amplitude we get

$$\mathrm{A}(B^+ \to D^+\bar{D}^{0^*}) = 8.17 \times 10^{-7}. \tag{25}$$

We are able to calculate the branching ratio of $B^+ \to J/\psi\,\pi^+$ decay with different values of η which are shown in Table 1.

Results

In this work, we have calculated the contribution of the t-channel FSI, that is, inelastic re-scattering processes to the branching ratio of $B^+ \to J/\psi\,\pi^+$ decay. For evaluating the FSI effects, we have only considered the absorptive part of the HLL because both hadrons which produced via the weak interaction are on their mass shells. We have calculated the branching ratio of $B^+ \to J/\psi\,\pi^+$ decay by using QCDF and FSI. The experimental result of this decay is BR($B^+ \to J/\psi\,\pi^+$)= 4.9×10^{-5}. According to QCDF and FSI, our results are BR($B^+ \to J/\psi\,\pi^+$)= 0.56×10^{-5} and 3.9×10^{-5}, respectively.

We have introduced the phenomenological parameter η that its value in the form factor is expected to be of the order of unity and can be determined from the measured rates. For a given exchanged particle, we have used η=2∼3. If η= 2.8 is selected, the branching ratio of the $B^+ \to J/\psi\,\pi^+$ decay approaches to the experimental bound.

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