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Numerical Studies and Simulation of the Lower Hybrid Waves Current Drive by using Fokker – Planck Equation in NSST and HT-7 Tokamaks

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Abstract

Recent experiments on the spherical tokamak have discovered the conditions to create a powerful plasma and ensure easy shaping and amplification of stability, high bootstrap current and confinement energy. The spherical tours (ST) fusion energy development path is complementary to the tokamak burning plasma experiment such as NSTX and higher toroidal beta regimes and improves the design of a power plant. To support the ST development path, one option of a Next Step Spherical Torus (NSST) device has been examined .NSST is a performance extension stage ST with a plasma current of 5-10 MA, R=1.5m, Bt<2.7T with flexible physics. The baseline heating and current device system for NSST is the 30MW NBI system and a 10 MW of ion cyclotron range of frequency (ICRF) and high harmonic fast wave (HHFW) system. In this work, we investigated lower hybrid wave interaction in the NSST reactor and found an optimal lower hybrid wave power and frequency for it. Finally, we compared the lower optimal hybrid waves and NSST spherical tokamak with HT-7 tokamak. Our results indicated that the use of lower hybrid waves in improved spherical tokamak NSST, as compared to the HT-7, was much better and more efficient.

Keywords: Lower Hybrid Waves; Tokamak; Current Drive; Fokker-Planck Equation.

Introduction

Lawson criterion is a condition for the realization of fusion reactions and is approximately expressed by $n\tau \geq 10^{20}\, s/m^3$ in 10 keV. To achieve the ignition mode in plasma and nuclear fusion reactions, various methods of current drive and heating have been used, relative to spherical and toroidal shaped tokamaks. In order to allow tokamaks to run in the steady state, some means of continuously driving the toroidal plasma current must be found. An essential requirement for reactor applications is that the power required to drive the current be only a small fraction of the fusion power output. Recently, the damping of high-phase-velocity Radio-Frequency traveling waves has been proposed as

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a way of driving the toroidal current. These currents could be efficiently generated by waves having phase velocities several times greater than the electron thermal velocity. This prediction has been confirmed by numerous experiments in which the current was driven by lower-hybrid waves. These results allow us to contemplate a steady-state tokamak reactor in which the toroidal current is driven by lower-hybrid waves [1, 2].

The feasibility of the steady-state reactor driven in this way rests crucially on the arguments concerning the question of resistivity. In essence, a new resistivity law is advanced, mainly due to factors such as the recombination and diffusion of particles in fusion plasma, in which the dissipated power was proportional to the current, as in the familiar ohmic resistivity law. It is estimated that the ratio of radio-frequency (RF) power dissipated to fusion power output is in the order of a few percent for the typical reactors. In order to conduct the analysis of electron distribution , we must use one-dimensional Fokker-Planck equation. Axial symmetry around the magnetic field allows the reduction in the complexity of the problem from three to two velocity dimensions. The reduction of velocity dimension from two to one is made under the assumption of the dependence of the electron velocity distribution function on the perpendicular velocity that supposes the electron temperature as a Maxwellian distribution. Within the framework of the one-dimensional equations, there is no easier way to check this assumption. We assess the validity of this assumption by a numerical solution for the two-dimensional effects in this paper. Note that this work is also related to the problem of wave heating in a magnetized plasma (i.e., heating a tokamak plasma with the lower hybrid waves). Our objective is to maximize the power dissipation rather than minimizing the resistivity in the instance of heating [2, 3]. The outline of the paper is as follows:

In Section 2 we express current drive theory and Landau damping coefficient. In Section 3, we write the two-dimensional Fokker-Planck equation with an additional quasi-linear diffusion term which describes the interaction of the waves with the plasma. In Section 4, we study the numerical solution method of the two-dimensional Fokker- Planck equation briefly. Finally,we simulate several parameters associated with the lower hybrid wave injection for NSST and HT-7 tokamaks and compare the results of simulation with the experimental results of HT-7 tokamak.

Materials and Methods

Current Drive Theory

Current drive theory deals with the production of toroidal electric current in tokamak, a current produced around the toroidal field of the machine. Using these currents, the fusion reactors help to operate consistently and continuously. Waves-injection causes energy exchange between waves and plasma particles by Landau mechanism and damping coefficient can be shown by the following equation:

$$\gamma = -\sqrt{\frac{\pi}{8}}\frac{\omega_{pe}}{\left|k^3\lambda_{De}^3\right|}e^{-\left(\frac{\varepsilon}{2}+\frac{1}{2k^2\lambda_{De}^2}\right)} \sim \left(\frac{\partial F_{eo}}{\partial u}\right)_{\frac{\omega_r}{|k|}} \quad (1)$$

It is proportional to the slope of the distribution function in the wave phase -point and causes pure energy-exchange between particles and waves. So waves loses energy and are damped [4]. For the current drive, waves with adequate phase velocity are injected along the toroidal magnetic field to resonant with plasma electrons and raise the energy and momentum of the electrons by the absorption of wave energy with Landau damping [5]. These resonance frequencies involve the Ion Cyclotron Resonance (ICRH), the Lower Hybrid resonance (LH), and the Electron Cyclotron Resonance (ECRH) [6]. The ICRH is the ion-ion resonance with wave frequencies in the 30 MHz to 120 MHz range, but the density limit causes the disappearance of these waves for more external areas of plasma. The Lower Hybrid waves are the combination of ion and electron cyclotron frequencies in 1 GHz to 8 GHz ranges [7]. They can penetrate into plasma center and according to their available frequencies, allow tokamak to be in a uniform mode by creating a current drive in it. Therefore, they are the best choice for current drive in the fusion plasma [8].

Fokker- Planck equation

With increasing the energy of the plasma particles, Coulomb collisions of plasma particles are increased with each other. The effect of such collisions is obtained by adding a quasi-linear term to Vlasov equation, called Fokker- Planck equation, thereby giving a general description of the distribution function changes due to successive collisions [9]. The rate of change in the distribution function 'f 'due to collisions can be written as:

$$\left(\frac{\partial f}{\partial t}\right)_{coll} = \frac{f(\vec{v},t)-f(\vec{v},t-\Delta t)}{\Delta t} = -\frac{\partial}{\partial \vec{v}}\cdot\left(\frac{d<\Delta\vec{v}>}{dt}f\right)+\frac{1}{2}\frac{\partial^2}{\partial\vec{v}\partial\vec{v}}\cdot\left(\frac{d<\Delta\vec{v}\Delta\vec{v}>}{dt}f\right) \quad (2)$$

Equation (2) for $\left(\frac{\partial f}{\partial t}\right)_{coll}$ is called the 'Fokker-Planck equation' [10]. To write Eq. (2) in the standard form, we consider a test particle moving with a speed v_T and colliding with other particles. By the calculation of diffusion coefficients in the center-of-mass and taking the average over the velocity distribution of scattering particles and its substitution in the equation(2),a standard form for the Fokker-Planck equation is obtained as follows:

$$\left(\frac{\partial f}{\partial t}\right)_{coll} = \sum_s \frac{4\pi n_s q_T^2 q_T^2}{m_T^2}\left\{-\frac{\partial}{\partial \vec{v}_i}\left(f_T \frac{\partial H_s}{\partial \vec{v}_i}\right)+\frac{1}{2}\frac{\partial^2}{\partial \vec{v}_i \partial \vec{v}_j}\left(f_T \frac{\partial^2 G_s}{\partial \vec{v}_i \partial \vec{v}_j}\right)\right\} \qquad (3)$$

Functions $G_s(\vec{v})$ & $H_s(\vec{v})$ are defined as follows:

$$G_s(\vec{v}) = \int f_s(\vec{v}')\,\vec{v}_T - \vec{v}'\, d\vec{v}' \qquad (4)$$

$$H_s(\vec{v}) = \int f_s(\vec{v}')\frac{\vec{v}_T - \vec{v}'}{\left|\vec{v}_T - \vec{v}'\right|^3} d\vec{v}' \qquad (5)$$

This describes diffusion coefficients due to velocity changes in the phase space [11].

Solving method of Fokker - Planck Equation

Initially, we consider a uniform plasma at equilibrium state. For the next *time i.*e $t > 0$, it is subject to an electric field $\vec{E}(t)$ and a wave-induced flux $\vec{S}(\vec{v},t)$. If the electric field and the wave-induced flux are weak enough, the electron distribution remains close to a Maxwellian shape for $\xi \le T$, where $\xi = \frac{1}{2}mv^2$ is the energy of an electron. Substituting $f_m + f_1$ into the Boltzmann equation for the electron distribution f and linearizing it give

$$\frac{\partial}{\partial t} f_{١} + \frac{q\vec{E}(t)}{m}\cdot\frac{\partial}{\partial \vec{v}} f_{١} - C(f) = -\frac{\partial}{\partial \vec{v}}\cdot\vec{S} - \frac{q\vec{E}(t)}{m}\cdot\frac{\partial}{\partial \vec{v}} f_m - \left[\frac{\dot{n}}{n}+\left(\frac{\xi}{T}-\frac{٣}{٢}\right)\frac{\dot{T}}{T}\right] f_m$$

(6)

where

$$f_m = n\left(m/2\pi T\right)^{3/2}\exp(-\xi/T) \qquad (7)$$

is Maxwellian distribution and $C(f) = C(f, f_m) + C(f_m f) + C(f, f_1)$ is linearized collision operator [12]. We make three assumptions:

1. We assume that f_1 is azimuthally symmetric about the ambient magnetic field.

2. We take the electric field to be constant and in the direction parallel to the magnetic field $\vec{E} = E\,\hat{v}_{\|}$.

3. We restrict our attention to those cases where S is only finite and $v >> v_t$ with $v_t^2 = T/m$ for the thermal velocity.

Linearized collision operator comes in the form of

$$C(f) = \Gamma\left(\frac{1}{v^2}\frac{\partial}{\partial v} f + \frac{1+Z}{2v^3}\frac{\partial}{\partial \mu}(1-\mu^2)\frac{\partial}{\partial \mu} f\right), \qquad (8)$$

where $\mu = \frac{v_{\|}}{v}$, $\Gamma = eq^4 \ln\Lambda / 4\pi\varepsilon_0^2 m^2$, ε_0 is the dielectric constant of free space, $\ln\Lambda$ is the Coulomb logarithm and Z is the effective ion charge state. We have included pitch-angle scattering and frictional slowing down, but ignored energy diffusion. In the case of steady- state current drive, the energy diffusion term is introduced to correct the order of $(v_t/v)^2$. Another term neglected in this approximate collision operator is the effect of the Maxwelian colliding off the perturbed distribution $C(f_m, f)$. Corrections resulting from this term lead to $(v_t/v)^3$[12]. The Fokker- Planck equation is then reduced to an equation in time and two velocity dimensions, parallel and vertical to the field [13]. We write the Fokker- Planck equation for electrons as:

$$\frac{\partial f_e}{\partial t} - \sum_s C(f_e, f_s) + \nabla\cdot S_\omega + \frac{q_e E}{m_e}\cdot\nabla f_e = 0 \qquad (9)$$

where q_s and m_s are the charge and mass of species s , $C(f_a, f_b)$ is the collision term for species *a* colliding off species *b*, the sum extends over all the species of plasma (typically electrons and ions), and S_ω is the wave induced quasi-linear flux. Because collisions in plasma are primarily due to small- angle scattering, the collision term can be written as the divergence of a flux:

$$C(f_a, f_b) = -\nabla\cdot S_c^{a/b}, \qquad (10)$$

where $S = S_c + S_w + S_e$ is the total flux in the velocity space [13]. When computing $(\partial f\ /\partial t)_{coll}$ in eq. 3, we will assume that the background distributions of both ions and electrons are of non-drifting, non-evolving Maxwellian kind, in which $(\partial f\ /\partial t)_{coll}$ is given by Trubnikov (*). This assumption has two important implications regarding the collision term. First, it is a linear equation since self-collisions among the non-Maxwellian components of the electron distribution are neglected. This linearization introduces a negligible error when the number of non Maxwellian particles remains small. Second, the background electrons represent an efficient energy sink. The evolving test electrons, being in contact with these background electrons of constant temperature, are able to lose the energy imparted to them by the RF waves. Thus, a steady-state distribution of test electrons eventually results. In an experimentally realistic situation, where heat losses eventually balance the heating by RF waves, a similar steady state will result. Of course, in the absence of a heat sink, the electron temperature would increase, leading to more particles resonant with the wave and affecting both the current and power dissipated. Our main interest is to find the resistivity for a given set of plasma parameters. Thus, the assumption of a fixed temperature background electron distribution represents not only a significant mathematical simplification, but a specific framework in which we can pose the questions of current magnitude and resistivity in the steady state [14]. The runaway velocity is the one at which collisional frictional force equals the acceleration caused by the electric field:

$$v_r \equiv -sign(qE)\sqrt{m\Gamma/|qE|}. \quad (11)$$

Notice that the sign of v_r is opposite to the direction in which electrons run away. The velocity is given by $-\sqrt{2+Z}v_r$. Similarly, we define a runaway collision frequency

$$\nu_r \equiv \Gamma/|v_r|^3 \quad (12)$$

The normalized time and velocity are given by

$$\tau = \nu_r t \quad (13)$$

and

$$\vec{u} = \vec{v}/v_r \quad (14)$$

Thus, the distribution functions of f_1 and f_m are normalized to $n/|v_r|^3$, and the RF- induced flux S to $n\nu_r v_r/|v_r|^3$, etc. Under this normalization, Eq. (1) then becomes:

$$\frac{\partial}{\partial\tau} f_1 + D(f_1) = -\frac{\partial}{\partial\vec{u}}\cdot\vec{S} \quad (15)$$

with $f_1(\vec{u},\tau=0)=0$ and the operator D being defined by:

$$D = -\frac{\partial}{\partial\vec{u}_\parallel} - \frac{1}{u^2}\frac{\partial}{\partial\vec{u}} + \frac{1+Z}{2u^3}\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial}{\partial\mu} \quad (16)$$

[12]. Then the evolution of the electron distribution function, *f*, in the presence of RF waves, is given by:

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v_\parallel} D_{RF}(v_\parallel)\frac{\partial}{\partial v_\parallel} f + \left(\frac{\partial f}{\partial t}\right)_{coll} \quad (17)$$

where $v_\parallel$ is the velocity parallel to the magnetic field, $D_{RF}(v_\parallel)$ is the quasi-linear diffusion coefficient, and $(\partial f/\partial t)_{coll}$ is the Fokker-Planck collision term. Again, by normalizing velocities to $v_{Te} = (T_e/m_e)^{1/2}$ and times $\nu_0^{-1}(\nu_0 = \log\Lambda\omega_{pe}^4/2\pi n_0 v_{Te}^3)$, eq. (17) becomes:

$$\frac{\partial f}{\partial\tau} = \frac{\partial}{\partial w} D(w)\frac{\partial}{\partial w} f + \left(\frac{\partial f}{\partial\tau}\right)_{coll} \quad (18)$$

where $\tau = \nu_0 t$, $w = v_\parallel/v_{Te}$ and $D(w) = D_{RF}(v_\parallel)/(v_{Te}^3\nu_0)$.[11]

Since driving frequency ω is small in comparison with the electron gyro frequency Ω_e, the current drive mechanism for Lower Hybrid waves is utilized only in the resonance are of parallel wave phase velocity ($\omega/k_\parallel$) and the quasi-linear diffusion tensor is reduced to the term of $v_\parallel v_\parallel$.

Results

A. Numerical Results

The wave spectrum may be of an arbitrary shape. However, we can partially solve the problem by considering very large spectrum amplitudes and anticipate that the precise wave amplitude is immaterial due to the wave saturation and so, we can ignore it. This assumption is strictly valid when $v_\perp << \Omega_e/k_\perp$. Thus:

$$D(w) = \begin{cases} D & for\, w_1 < w < w_2 \\ 0 & otherwise \end{cases} \quad (19)$$

where the constant, D, is chosen to be large enough for the solution to be insensitive to its precise magnitude. This is, in fact, what occurs in the situations of interest such as RF heating or RF-driven tokamak reactors [10, 11]. Thus, in summary, we have pinpointed the important free parameters in the problem as just two w_1 and w_2 maximum and minimum parallel phase velocities of Lower Hybrid injection waves. They were were normalized to the electron heating velocity to characterize the resonance area of Lower Hybrid injection waves . According to the above descriptions, Eq. (8) becomes:

$$\frac{\partial f}{\partial\tau} = \frac{\partial}{\partial w} D(w)\frac{\partial}{\partial w} f + \frac{Z+1}{4u^3}\frac{\partial}{\partial\mu}(1-\mu^2)\frac{\partial f}{\partial\mu} + \frac{1}{2u^2}\frac{\partial}{\partial u}(\frac{1}{u}\frac{\partial f}{\partial u} + f) \quad (20)$$

Taking distribution function as Maxwellian and integrating the Eq. (12), we obtain an equation to evaluate parallel distribution function as:

$$\frac{\partial}{\partial \tau}F(w)=\frac{\partial}{\partial w}D(w)\frac{\partial}{\partial w}F(w) + \frac{Z+2}{2}\frac{\partial}{\partial w}\times(\frac{1}{u^3}\frac{\partial}{\partial w}+\frac{1}{w^3})F(w) \quad (21)$$

That $F(w)$ is parallel distribution function. Solving the equation for the steady-state gives:

$$F(w)=C\exp(\int^{w}\frac{-w\,dw}{1+2w^3 D(w)/(2+Z)}) \quad (22)$$

where C is a constant in integration as plotted in (Fig. 1) for $D(w)=1/2$ & Z=2 i.e.

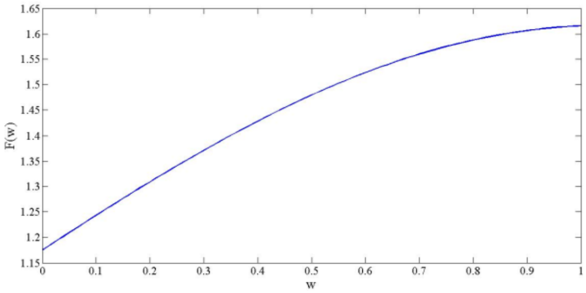

Figure 1. Distribution function F(w) of Deuterium plasma.

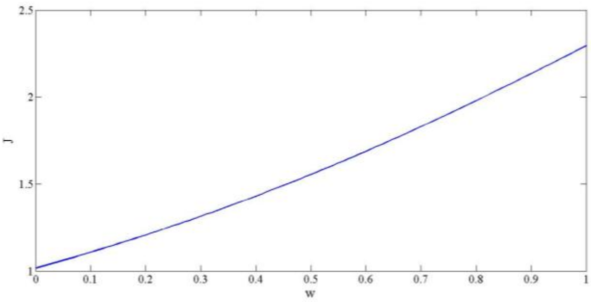

Figure 2. Current function for Deuterium plasma.

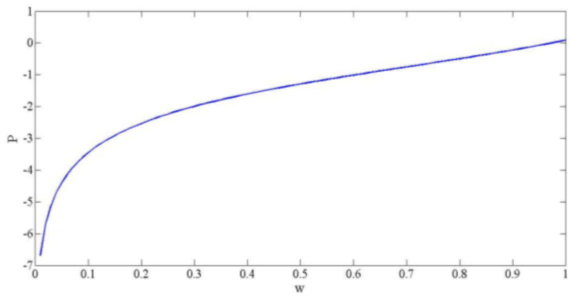

Figure 3. Power function P for Deuterium plasma.

For the numerical method of solving (Eq. 21), we first normalized this equation with a boundary condition $\vec{S}.u=0$ that could satisfy the laws of conservation of particles:

$$\frac{\partial}{\partial \tau}\int f\,d^3u=0 \quad (23)$$

and energy:

$$\frac{\partial}{\partial \tau}\int\frac{1}{2}u^2 f\,d^3u=\int u.S\,d^3u \quad (24)$$

The steady-state solution $(D(w)=0)$ is $f(u)=(2\pi)^{-3/2}e^{(-u^2/2)}$ and for the next states, the numerical solution is given by the following relation:

$$f(u=u_j+\frac{\Delta u}{2})=f(u=u_j-\frac{\Delta u}{2})\frac{1-u_j\Delta u/2}{1+u_j\Delta u/2}$$

(25)

where $u_j = j\Delta u$. Using this distribution function and its normalized velocity component, $F(w)$, we can obtain power and current as [11, 12]:

$$J=\int wF(w)\,dw=\frac{\exp(-w_1^2/2)}{(2\pi)^{1/2}}\Delta(\frac{w_1+w_2}{2}) \quad (26)$$

$$P=\int wD(w)\frac{\partial F(w)}{\partial w}dw=\frac{Z+2}{2}\frac{\exp(-w_1^2/2)}{(2\pi)^{1/2}}\log(\frac{w_2}{w_1}) \quad (27)$$

Here, we obtained equations J and P by using the numerical method for deuterium plasma and plotted them in Fig. 2 and Fig. 3 in order to characterize current drive and power transferred in the plasma environment and show the efficiency of this method.

For the Adjoint Method of solving Eq. (15), beside assuming that the total distribution function is the sum of perturbation and Maxwellian distribution functions $f_m + f_1$ and using the collision operator D, we reach the equation

$$\left(\frac{\partial}{\partial \tau}+D\right)g(\vec{u},\tau;\vec{u}')=0\,, \quad (28)$$

by which perturbation distribution function is obtained as:

$$f_1(\vec{u},\tau)=\int_0^{\tau}d\tau'\int d^3\vec{u}'\;\vec{S}(\vec{u}',\tau')\cdot\frac{\partial}{\partial \vec{u}'}g(\vec{u},\tau-\tau';\vec{u}')$$

(29)

And we use it in the simulation program [12].

B. Simulation

Nowadays, tokomaks designed and produced in

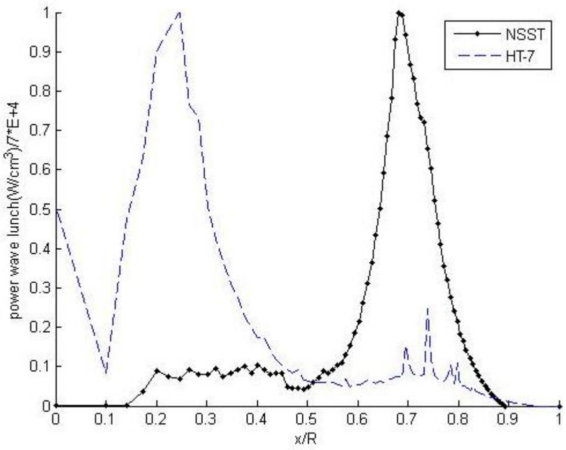

Figure 4. Relative Power of the normalized injected wave spectrum to 7×10^4 at relative distance from the plasma center.

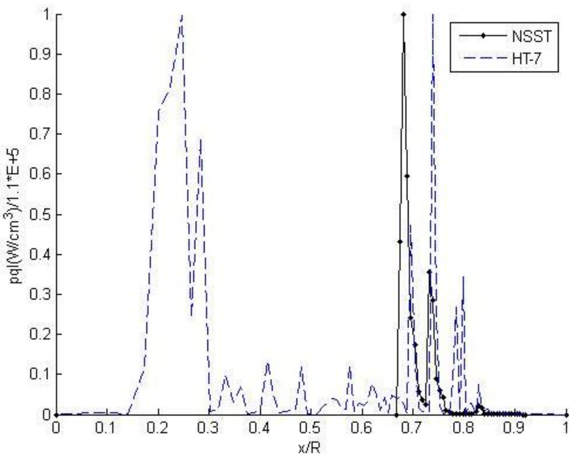

Figure 5. Quasi-linear Power of electrons normalized to 1.1×10^5 at the distance from the plasma center.

many countries are divided in to spherical and toroidal shapes, according to their parameters. These parameters, given in table.1 for HT-7 and NSST tokamaks, are used in simulation code LSC. The Lower hybrid waves Simulation Code (LSC) is a computational model for lower hybrid waves current drive based on FORTRAN programming language in which electrons and ions heating, geometric details and plasma profile are discussed and space effects of the two-dimensional phase of the wave spectrum injected in Fokker- Planck equation are approximated in order to simulate the desired parameters. In this paper, the relative power of the injected wave, electron quasi- linear power, current drive of electrons and electron power, and simulation phase velocity changes and computational data were plotted by computational software of MATLAB using LSC program outputs.. At the end, some simulation results for HT-7 tokamak were compared to the experimental information.

a. Relative Power of the injected wave spectrum at the normalized distance from the plasma center

Since the coupling of lower hybrid waves with plasma particles depends on various factors such as the distance from plasma center and the toroidal magnetic field and relative power spectrum of the injected lower hybrid wave, it has a width that provides the resonance condition and causes plasma heating. This spectral width is usually suitable for lower hybrid waves, making them suitable in tokamaks. We plotted the relative power of normalized injected wave at a relative distance from the plasma center in (Fig. 4) such that a distance was measured from the interior part of tokamak and the location of the central solenoid. As can be seen, the injected wave spectrum could penetrate in plasma center and cause plasma heating for toroidal HT-7 tokamak. But the injected wave spectrum is generally located at the plasma edges for spherical tokamak NSST and energy losses there and fusion plasma has little efficiency in the environment heating. Therefore, plasma heating by using the lower hybrid wave injection has higher output in toroidal tokamaks HT-7.

b. Quasi-linear Power of electrons at the normalized distance from the plasma center

The growth rate of the quasi -linear diffusion coefficient D_{ql}, with velocity $v_{\|}$ in *a* wave field and the wave number $k_{\|}$, is given by:

$$D_{ql}(v_{\|}) = \sum\nolimits_s \frac{\pi}{2}\left(\frac{e}{m_e}\right)^2 E_{\|}^2 \delta(\omega - k_{\|} v_{\|}) \tag{30}$$

where $E_{\|}$ is the electric field amplitude of the wave paralleled to the magnetic field. We can reach energy per time unit per volume unit of plasma from the quasi-linear term point of view using the diffusion coefficient equation (30) [16]:

$$P_{ql} = -n_e m_e \int dv_{\|}\, v_{\|}\, D_{ql}(v_{\|}) \frac{\partial f_e}{\partial v_{\|}} \tag{31}$$

This energy is absorbed by the emitted wave and plasma electrons. We plotted quasi-linear power of electrons at the relative distance from the plasma center that the distance was evaluated from the interior of tokamak and the location of the central solenoid. As shown in (Fig. 5), for toroidal tokamak HT-7 form, the main maximum peak with a larger width is usually located at the plasma center, but for the spherical tokamak of NSST, the resonance maximum peak was closer to the plasma edge and the quasi-linear power received from wave was lost at plasma edges and couldnot be much useful in transferring power and the momentum of the injected wave. This shows the advantages of toroidal tokamak HT-7 compared to the spherical tokamak of NSST.

c. Current drive of electrons at the normalized distance from plasma center

Current drive on every flux surface is given by:

$$J_{rf} = \frac{-en_e}{v_r}\int dv_{\|} D_{ql}(v_{\|}) \frac{\partial f_e(v_{\|})}{\partial v_{\|}} \cdot \frac{\partial W_s(u)}{\partial u} \qquad (32)$$

Here, $W_s(u)$ function is:

$$W_s(\vec{u} << 1, \mu) = \frac{\mu u^4}{5+Z} - \frac{(2+Z+3\mu^2)u^6}{3(3+Z)(5+Z)} \qquad (33)$$

It is calculated and used in the numerical method by expanding $W_s(u)$ function and keeping the first two terms, we have:

$$J_{rf} = \frac{-en_e}{\Gamma}\int dv_{\|} D_{ql}(v_{\|}) \times \\ \times \frac{\partial f_e(v_{\|})}{\partial v_{\|}} \frac{\text{۴}v_{\|}^{\text{۳}}}{\text{۵}+z}\left[\mu - \frac{\text{۱}+z/\text{۲}+\text{۳}\mu^{\text{۲}}/\text{۲}}{\text{۳}+z}\frac{v_{\|}^{\text{۲}}}{v_r^{\text{۲}}}\right] \qquad (34)$$

for current drive. $\mu = -1$ And $\mu = +1$ are congruent and converse current drives to the field respectively in this equation [16]. According to this equation, current drive depends on many factors such as ions effective charge, particles velocity and runaway velocity, which have different values in various places in plasma and lead to the formation of different streams. We plotted electron current drive at the distance from the plasma center in (Fig. 6), and this distance was evaluated from the interior of tokamak and the location of the central solenoid. As shown in this figure, the current drive had a positive maximum peak, indicating a maximum resonance mode and showing the optimal conditions for the coupling of waves with plasma particles. This coupling by Landau damping mechanism caused the loss of the wave energy and transference of wave momentum to plasma particles and the plasma heating. Resonance peak was about 0.72% for spherical tokamak NSST and it had various maximums for toroidal tokamakas according to their aspect ratio. It was about 0.23 for HT-7 tokamak and the lower the percentage, the higher was the tokamak performance. The normalization mechanism in the horizontal diagram has been done in LSC program and we have plotted with MATLAB software.

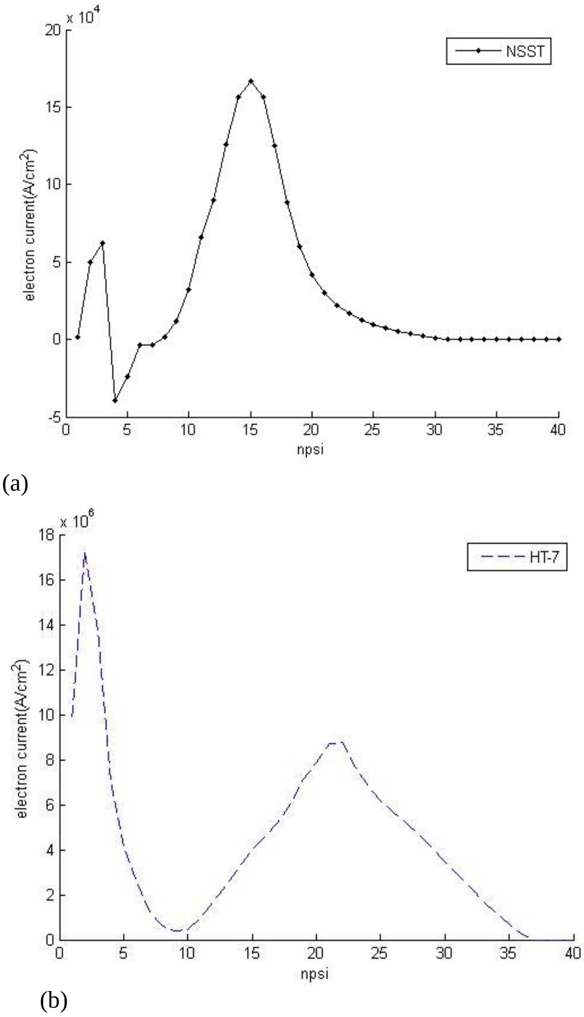

Figure 7. Electrons Current at the number of flux surfaces, (a) NSST Tokamak, (b) HT-7 Tokamak.

d. Electrons Current

Applying an electric field to the tokamak plasma environment causes the separation of charged particles by affecting them. This charge separation leads to an electric current in plasma environment: “bootstrap current”. Furthermore, by launching the wave into the plasma environment, the plasma electrons produce electron current by receiving momentum and the energy of the wave .To simulate the electron current in tokamak, we divided plasma into 40- flux surfaces of the injected wave and plotted these parameters at flux surfaces. Electron current was plotted in (Fig. 7(a), (b)) for the NSST and HT-7 tokamks. As shown in (Fig. 7), the maximum of electron current for the toroidal tokamak HT-7 was located at the plasma edge, but for the spherical tokamak, it was located near the flux surfaces of the central plasma and it had a negative maximum at the initial flux surfaces. This case could be justified since the electron current at such surfaces was in the opposite direction of bootstrap current.

e. Phase Velocity Changes

Thermal effects added to cold plasma should be considered because there are particles that move at speeds approaching the phase velocity in the heat distribution. Such particles have resonant interaction with waves and their interaction results in wave damping and instability. The waves with $k_\perp = 0$ are not affected by the magnetic field and resonance does not happen. Moreover, for such waves, we have cut-off in $\omega = \omega_p$. For waves with $k_\perp = 0$, the dispersion relation of electro dynamics waves for collision plasma along the magnetic field is presented by:

$$\frac{k^2c^2}{\omega^2} = 1 - \frac{\omega_{pe}^2}{\omega^2}\frac{1}{\left(1 \mp \frac{\omega_{ce}}{\omega} - \frac{\omega_{ce}\omega_{ci}}{\omega^2}\right)} \tag{35}$$

According to the dispersion relation, the equation of the injected wave phase velocity that is propagated along the magnetic field is changed as:

$$v_{ph} = c\left(1 - \frac{\omega_{pe}^2}{\omega^2}(1/1 \mp \frac{\omega_{ce}}{\omega})\right)^{-\frac{1}{2}} \tag{36}$$

We simulated particles phase velocity changes in order to inject the LH waves into plasma in NSST and plotted it for five beams with different frequencies in (Fig. 8). Resonant regions in which wave damping occurred and wave energy was transferred to the plasma environment have been shown in this diagram [17].

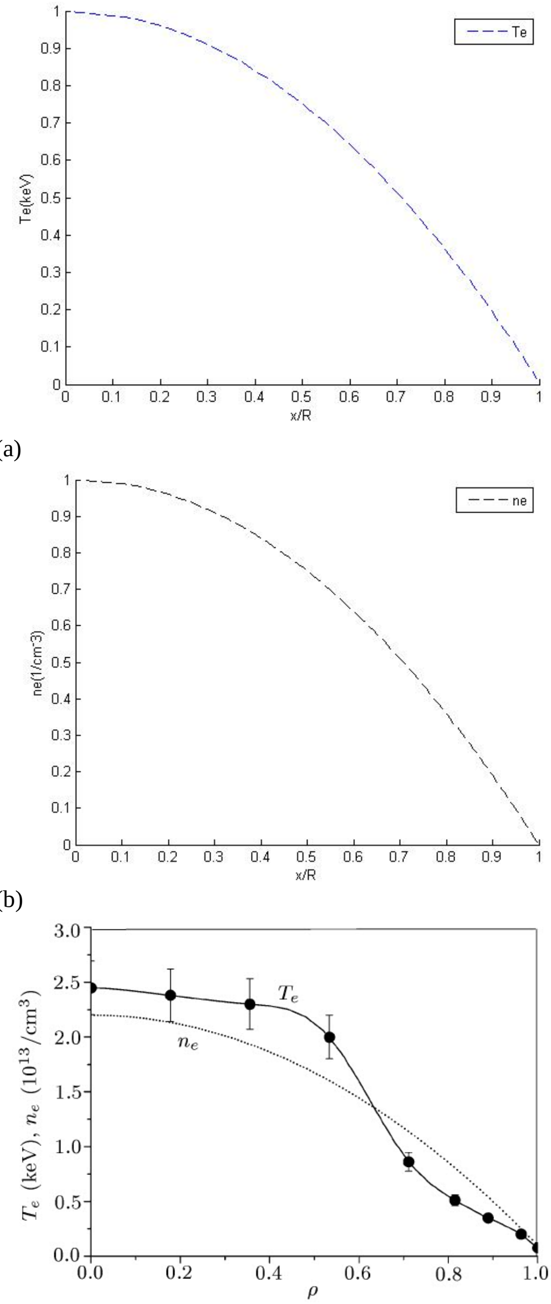

Figure 9. Comparison of simulation and experimental results for HT-7 tokamak; (a) & (b): the result of electronic temperature and density simulation. (c): experimental results [18].

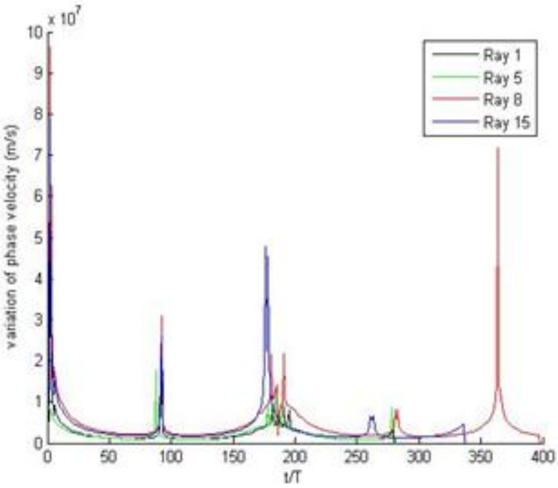

Figure 8. Phase velocity changes of the lower hybrid beams injected into plasma at time normalized to the injected wave frequency period.

C. comparison

Due to the higher degree of ionization and current at plasma center, electron temperature and density were higher, but by getting away from the plasma center, because of factors such as plasma diffusion and the recombination of particles, the temperature and density were reduced. We should keep the temperature and density high for fusion reaction in order to achieve Lawson Criterion. In (Fig. 9(a)), we simulated temperature for HT-7 tokamak. In this figure, the vertical chart is the electron temperature normalized to keV and the horizontal chart has been normalized to form the plasma center. In (Fig. 9(b)), we simulated density for HT-7 tokamak. In this figure, the vertical chart is the density normalized to $10^{13}/cm^3$ and the horizontal chart has been normalized to form the plasma center. As shown in these figures, the highest values of electrons temperature and current were in the center and these parameters were reduced by getting away from it. Thus the fusion reaction rate was higher in the center of plasma and the interaction rate was reduced by going toward the plasma edges.

A comparison between simulation and experimental results of temperature and density was plotted for HT-7 tokamak in (Fig. 9). Both cases displayed a good relative accordance in all distances normalized to the plasma center. So there was a good agreement between simulation and experimental results, indicating more the accuracy of the performed simulation work.

Conclusion

NSST tokamak had an appropriate toroidal magnetic field, and major and minor radii in the same range that changed the fusion reactor into D-shape and the spherical form. D-shape configuration of this tokamak prevented it from absorbing the power of LH wave with the maximum ratio. Due to the large β, the spherical tokamak to the tokamak torus was not very effective, but here we saw that this improved spherical tokamak with toroidal magnetic field (NSST) had the ability to get the lower hybrid wave in the distance of the plasma and the application of lower hybrid waves in this tokamak, rather than the other spherical tokamak, had better advantages.

As shown in the simulation figures, for all simulated and plotted quantities, HT-7 tokamak had quantities with more appropriate output than NSST tokamak. In other words, ST fusion reactors had a higher efficiency than spherical tokamaks, thereby resulting in the better establishment of the implement condition for the self-sustained fusion reaction, i.e., Lawson's criterion, and provided the possibility of the nuclear fusion with a higher power and efficiency.

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