

# Coupling Geomechanics and Transport in Naturally Fractured Reservoirs

M.R. Jalali\* and M.B. Dusseault

University of Waterloo, Waterloo, Ontario, Canada

Received 24 December 2011; Received in revised form 3 June 2012; accepted 1 December 2012

## Abstract

Large amounts of hydrocarbon reserves are trapped in naturally fractured reservoirs which are challenging in terms of accurate recovery prediction because of their joint fabric complexity and lithological heterogeneity. Canada, for example, has over 400 billion barrels of crude oil in fractured carbonates in Alberta, most of this being bitumen of viscosity greater than  $10^6$  cP in the Grosmont Formation, which has an average porosity of about 13-15%. Thermal methods are the most common exploitation approaches in such viscous oil reservoirs which, in the case of steam injection, are associated with up to 275-300°C temperature changes, leading to considerable thermoelastic expansion. This temperature change, combined with pore pressure changes from injection and production processes, leads to massive effective stress variations in the reservoir and surrounding rocks. The thermally-induced (thermoelastic) stress changes can easily be an order of magnitude greater than the pore pressure effects because of the high intrinsic stiffness of the low porosity limestone and bounding strata. Study of these stress-pressure-temperature effects requires a thermo-hydro-mechanical (THM) coupling approach which considers the simultaneous variation of effective stress, pore pressure, and temperature and their interactions. For example, thermal expansion can lead to significant joint dilation, increasing the macroscopic, joint-dominated transmissivity by an order of magnitude in front of and normal to the thermal front, while reducing it in the direction tangential to the heating front. This leads to strong induced anisotropy of transport processes, which in turn affects the spatial distribution of the heating arising from advective heat transfer.

**Keywords:** THM Coupling, Fractured Reservoirs, Geomechanics, Numerical Methods, Thermo-poroelasticity, Dual Porosity

## 1-Introduction

Petroleum geomechanics has become more-and-more part of oil industry analysis approaches to explain and evaluate phenomena such as wellbore stability in shale, reservoir compaction and surface subsidence during depletion, sand production during well drawdown, hydraulic fracture stimulation, and so on [Dusseault, 2011]. These issues require simultaneous consideration of transport (i.e. fluid, heat, chemistry) and geomechanics (i.e. deformation, stress), which is known as coupling (THMC).

The word coupling refers to combined analysis of interacting physical processes that have in the past been treated separately. For example, in the case of fluid flow in fractured media, input of hot or cold fluids changes the stresses, altering the fracture apertures, which affects the permeability and thus the flow rate and temperature changes. This type of feed-

back loop is characteristic of coupled processes.

In reality almost all co-temporal, co-spatial processes are coupled, although uncoupled models can be used in situations where one phenomenon is strongly dominant, such as heat flow in hot, dry, unfractured rocks (heat conduction dominates, stress change is irrelevant), or fluid flow in shallow aquifers with a moderate matrix compressibility [Dusseault, 2008]. In oil and gas reservoir exploitation we deal with great complexity (and heterogeneity), and coupled modeling generally helps to understand and predict reservoir behavior. Good recovery predictions in Naturally Fractured Reservoirs (NFRs) are challenging because fracture behavior dominates production and injection activities, and fracture flux is affected strongly by changes in pore pressure, temperature, saturation, and effective

\*Corresponding author, Email Address: [mrj1982@gmail.com](mailto:mrj1982@gmail.com) (M.R. Jalali)

stress. Pore pressure changes ( $\Delta p$ ) arise as a result of production or injection of fluids from various wells, and temperature changes ( $\Delta T$ ) arise from injection of fluids that may be colder or hotter than the reservoir temperature. During enhanced oil recovery using steam injection, fluid temperatures may be 250-300°C greater than the initial reservoir temperature.  $\Delta T$  leads to thermoelastic strains, which in turn cause effective stress changes ( $\Delta\sigma'$ ). Hence, coupled  $\Delta p$ ,  $\Delta T$  and  $\Delta\sigma'$  effects must be considered for fractured reservoirs, and the magnitude of the stresses induced by large  $\Delta T$  values is huge, far higher than the initial *in situ* effective stresses at the typical depths involved (300-800 m in Alberta).

Moreover, the conductivity of a fracture is a strong function of fracture aperture ( $v \propto a^3$ ) and the aperture is highly sensitive to the normal effective stress across the fracture. Depending on the fracture orientation with respect to the heated zone, this stress can increase, decrease, or the shear stress can increase, which may cause dilatant behavior (aperture increase) if the rock is strong and the pore pressures are elevated. Fracture conductivity changes of a factor of two to ten are expected in naturally fractured carbonate reservoirs being subjected to thermal stimulation. Unless these changes are understood and analyzed, predictive modeling based on physical processes is not possible (excepting curve-fitting to history, which is not fully physical modeling).

Conventional non-coupled reservoir simulators are not appropriate for modeling these phenomena, as they consider pore compressibility as the only geomechanical parameter for simulation and assume permeability and porosity as static or pressure-dependent variables. These assumptions are insufficient because permeability and fracture conductivity are strong functions of effective stress and temperature as well as pressure. Parameter impacts on both reservoir characterization and simulation processes should be

considered via a thermo-hydro-mechanically (THM) coupled approach for a more precise simulation.

## 2-Thermo-Hydro-Mechanical Coupling

Coupling of a reservoir simulator with a geomechanics module has increasingly wide application in the petroleum industry. In a conventional simulator, surface subsidence is often estimated only by a simple formula without knowing the complete geomechanical response. The only geomechanical parameter considered may be pore compressibility, which is insufficient to reproduce pore volume changes induced by complex pressure and temperature variations [Settari and Mourits, 1998; Tortike and Farouq Ali, 1993]. In some problems, such as primary production and linear-elastic reservoir response, subsidence computed by a reservoir simulator alone may give results comparable to coupled solutions, but when nonlinear material response is strong; the results from the two approaches will diverge. In a coupled simulator, flow can be strongly affected by the stress and strain distributions that give changes in porosity and permeability, but in conventional simulation  $\Delta\sigma'$ -dependence is ignored. Such approaches cannot give appropriate predictions if a stress-sensitive reservoir (*e.g.* naturally fractured reservoir or poorly compacted reservoir) is considered [Mainguy and Longuemare, 2002].

According to Settari and Mourits (1998), there are two main coupling components:

**Volume coupling:** The pore volume changes as a result of stress, pressure or temperature variations are considered in this case. For convergence purposes, the calculated pore volume changes should be equal in both fluid flow and geomechanics models. The pore volume changes from the geomechanics model are usually more accurate than those of the fluid-flow model because it is computed by volumetric strain through a complex and hopefully more realistic material constitutive model.

This coupling is more suitable for problems that deal with the large porosity

changes resulting from shear or plastic deformation. These problems are common in unconsolidated heavy oils and oil sands, North Sea chalk, California diatomite and perhaps some other materials.

**Coupling through flow properties:** In this approach to coupling, the changes in permeability and relative permeability are related to the changes in stress, shear stress, or compaction. When the shear failure condition is satisfied, the nature of the medium is changed, and permeability, relative permeability, compressibility and other parameters are altered.

This is important in some reservoirs where the compressibility effects do not have a significant role in the volumetric behavior, such as gas reservoirs in which volume coupling is not important. Another example is a waterflood or solid waste injection process with pressures close to or above fracturing pressure; shear dilation under low confining stress will lead to permeability enhancement. A third example is cold water injection that leads to a thermally induced drop in horizontal stress until  $\sigma_h < p_{inj}$ , with hydraulic fracture propagation that massively improves injectivity [Perkins & Gonzalez 1985].

Also, there are different types of coupling between fluid-flow and geomechanics processes, some of which are described below:

### 2-1-Pseudo-Coupling

Pseudo-coupling is based on an empirical model of the absolute permeability and porosity as functions of pressure (not stress). During this process, a conventional reservoir simulator will

compute some geomechanical parameters such as compaction (via a relationship between compressibility and  $\Delta p$ ) and horizontal stress changes (using the relationship between porosity, stress and  $\Delta p$ ). Usually, the empirical model is a table of the porosity and absolute permeability versus pressure which is then introduced to the simulator [Tran *et al.*, 2005]. The permeability may then be altered for the next time-step in the numerical simulation.

### 2-2-Explicit Coupling

In this approach, which is also called one-way coupling method, the information from a reservoir simulator is sent to a geomechanics model, but the results from the geomechanics calculations are not fed back to the reservoir simulator. In this case, the reservoir fluid flow is not affected explicitly by the geomechanical responses calculated by the geomechanics module. However, the change in reservoir flow variables will affect the geomechanics variables (**Error! Reference source not found.**) [Tran *et al.*, 2005].

This coupling is a useful and time-saving approach for subsidence problems because the geomechanical calculations can be performed on a different time scale than the fluid-flow calculations. The fluid-flow usually propagates in a short time-step frame within the flow simulation, in comparison with the deformation (subsidence) calculation, which can be done when needed (especially for low compressibility reservoirs).

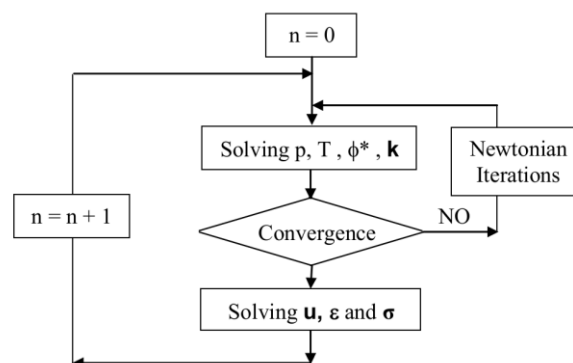


Figure 1. Explicitly coupled approach [Tran *et al.*, 2005].

So, by using different time scales for the fluid-flow and geomechanical simulation, the performance of the simulation will be increased [Dean *et al.*, 2006]. This method is a flexible and straightforward technique for coupling that can use an existing fluid-flow simulator and an existing geomechanics simulator, simultaneously [Settari and Walters, 1999].

On the other hand, one of the big concerns in this technique is its stability and accuracy that imposes some time step restrictions on the calculations. However, for many subsidence problems, the fluid-flow calculations require time steps that are smaller than those imposed by the explicit coupling calculations [Dean *et al.*, 2006].

### 2-3-Iterative Coupling

In this coupling method, which is also known as two-way coupling, the information computed in the reservoir simulator and in the geomechanics model is exchanged back-and-forth through nonlinear iterations for each time step. Therefore, the reservoir flow is affected by the geomechanical responses as calculated by the geomechanics model [Tran *et al.*, 2005]. During each nonlinear iteration, a simulator performs computations sequentially for multiphase porous flow and for displacements. The flow and displacement calculations are then coupled through calculations of pore volumes (or

reservoir porosity) at the end of each nonlinear iteration (Figure 2).

The main advantage of this coupling is its flexibility, i.e., the two systems can be solved by different numerical methods. Usually, the fluid-flow simulator uses a finite difference volume-based grid, whereas the geomechanics simulator uses a finite-element node-based grid. In addition, a conventional reservoir simulator can be coupled with a suitable geomechanics module with modest modification in both codes [Dean *et al.*, 2006]. The simulation domains for fluid-flow and geomechanics can be substantially different (even within the reservoir). There is no need to simulate fluid flow in the non-reservoir rocks if they are essentially impermeable [Minkoff *et al.*, 2003], although if it is a temperature problem, conductive heat transport calculations for the rock surrounding the reservoir may be necessary. The convergence of fluid-flow variables  $P$  and  $T$  is often much slower than for the displacements  $u$ , especially for complex multi-component multi-phase processes, so separate solution of the two problems allows for optimal CPU usage. For example, the fluid-flow solution might require a short time step, but the geomechanics calculations need be done only every 10<sup>th</sup> or 20<sup>th</sup> time step.

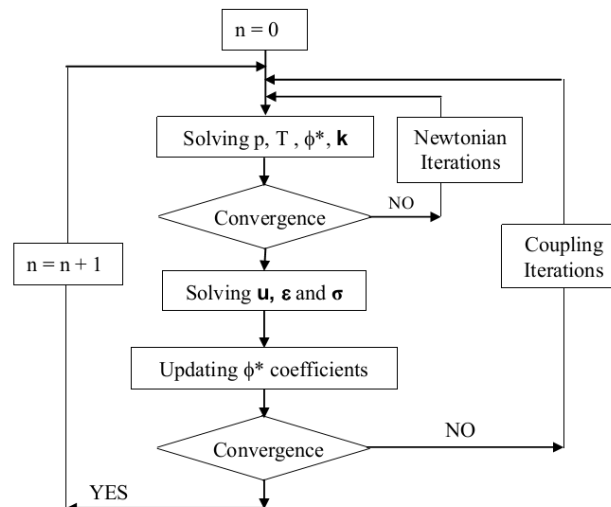


Figure 2. Iteratively coupled approach [Tran *et al.*, 2005].

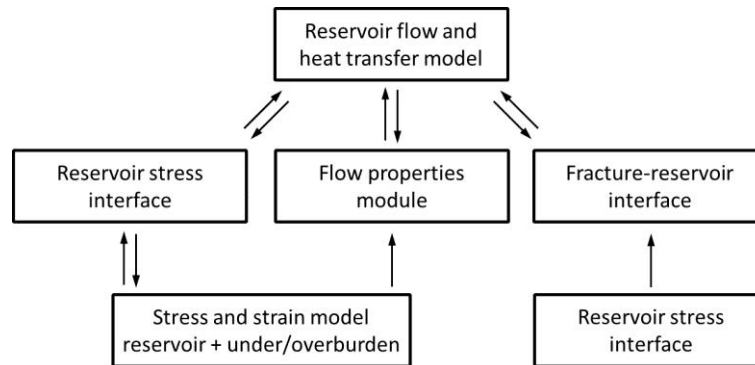


Figure 3. Fully coupled approach [Settari *et al.*, 2001].

Table 1. Advantages and disadvantages of coupling techniques.

Approach	Advantage	Disadvantage
<b>Explicit Coupling</b>	<ul style="list-style-type: none"> <li>Effective and time-saving for subsidence problems</li> <li>Flexible and straightforward</li> </ul>	<ul style="list-style-type: none"> <li>One-way coupling for geomechanics</li> <li>Stability and accuracy</li> </ul>
<b>Iterative Coupling</b>	<ul style="list-style-type: none"> <li>Flexible in the case of numerical methods</li> <li>It is possible to couple conventional reservoir simulators (e.g. ECLIPSE, TOUGH2, STARS) with geomechanical modules (e.g. FLAC, UDEC, VISAGE).</li> </ul>	<ul style="list-style-type: none"> <li>Require large number of iterations (1<sup>st</sup> order convergence)</li> <li>Relatively small jumps in pore volume can be handled</li> </ul>
<b>Full Coupling</b>	<ul style="list-style-type: none"> <li>The most stable approach</li> <li>Preserves 2<sup>nd</sup> order convergence</li> <li>Reliable and a benchmark for other coupling methods</li> </ul>	<ul style="list-style-type: none"> <li>Difficult to couple existing modules and softwares</li> <li>Require more code development</li> <li>Slower than other coupling techniques</li> </ul>

This method will be challenging for difficult problems as it may require a large number of iterations due to a first-order convergence rate in the nonlinear iterations [Dean *et al.*, 2006]. Another bottleneck to this technique is that only relatively small jumps in pore volume (or the reservoir porosity) can be handled due to the large volume of fluids which must move to the wells to conserve mass when compaction occurs in the field [Minkoff *et al.*, 2003]. An iteratively coupled approach will produce the same results as a fully coupled approach if both techniques use sufficiently tight convergence tolerances for iterations [Settari and Walters, 1999].

#### 2-4-Full Coupling

In this approach fluid-flow and displacement calculations are performed together, and the program's linear equation

solver must handle both fluid-flow variables and displacement variables (Figure 3). The primary attraction of the fully coupled approach is that it is the most stable approach of the three techniques and preserves second-order convergence of nonlinear iterations. The solution is reliable and can be used as a benchmark for other coupling approaches. Drawbacks to the fully coupled approach include the following: it may be difficult to couple existing porous-flow simulators and geomechanics simulators, it requires more code development than other techniques, and it can be slower than the explicit and iterative techniques on some problems [Dean *et al.*, 2006].

Advantages and disadvantages of different coupling approaches can be easily summarized in **Error! Reference source not found.**

Dusseault (2008) summarized a list of different type of coupling as below:

**Hydro-Mechanical Coupling (HM)** accounts for the simultaneous effect of effective stress and fluid pressure variation and their effects on each other [Minkoff et al., 2003].

**Static-Dynamic Hydro-Mechanical Coupling** considers the effect of inertial process (i.e. fluids and solid matrix acceleration) on the fluid diffusion [Spanos et al., 2002].

**Thermo-Mechanical Coupling (TM)** accounts for the simultaneous effect of effective stress and temperature variation and their effects on each other [Minkoff et al., 2003].

**Thermo-Hydro-Mechanical Coupling (THM)** involves a combined analysis of pore pressure, temperature and deformation and their effects. This type of coupling is important in stress sensitive reservoirs (e.g. naturally fractured reservoirs and poorly compacted reservoirs) [Mainguy and Longuemare, 2002].

**Hydro-Chemo-Mechanical Coupling (HCM)** is usually encountered in fine-grained formations (i.e. shales and clays) with high surface contacts or in soluble materials such as various salts. In these materials, fluid chemistry variations are associated with volume changes, permeability changes and effective stress variations, simultaneously [Di Miao et al., 2002].

**Electro-Hydro-Mechanical Coupling (EHM)** occurs in fine-grained materials where electrical potential fields affect the physical flow and volume changes, and in turn potential fields are influenced by physical properties variation [Huyghe et al., 2005].

**Thermo-Hydro-Mechanical and Chemical Coupling (THMC)** consists of the coupling process of deformation, effective stress, pressure, temperature and chemistry of material [Lanru et al., 2003]. This coupling is one of the complete approaches to coupling but the complexity of these

processes and the “cross-coupling effects” generally require many assumptions and simplifications.

### 3-Fractured Reservoirs

Geomechanics play a key role in management of naturally fractured reservoirs. Production from fractured reservoirs relies on permeability of fractures; this permeability is affected by  $\Delta\sigma'$  arising from fluid injection or production ( $\Delta T$ ,  $\Delta p$ ), so the transport properties of the fractures change, associated with opening, closure or shear dilation of natural fractures in different locations and orientations, changing the physical flow parameters of the reservoir. As a definition, Narr et al. (2006) proposed that “all reservoirs should be considered fractured until proven otherwise...”, but this seems extreme in the case of high permeability unconsolidated sandstones where, if fractures exist, their impact on flow is negligible.

Nelson (2001) proposed the following classification for naturally fractured reservoirs based on the positive effect of fractures on reservoir transport properties (

Figure 4):

**Type I** - Fractures provide the essential porosity (i.e. storage capacity) and flow capacity in a reservoir where matrix porosity and permeability are low. In this type of reservoir, fracture characteristics are the dominant parameters for reservoir evaluation and few producing wells are required to deplete the reservoir.

**Type II** - Rock matrix has higher porosity whereas fractures provide the essential flow capacity in a reservoir. In this case, cross-flow between fractures and matrix and rate control are the key parameters, with production rate controlled by fractures. Monitoring fracture behavior is important during water flooding processes and to assess the effect of large drawdowns

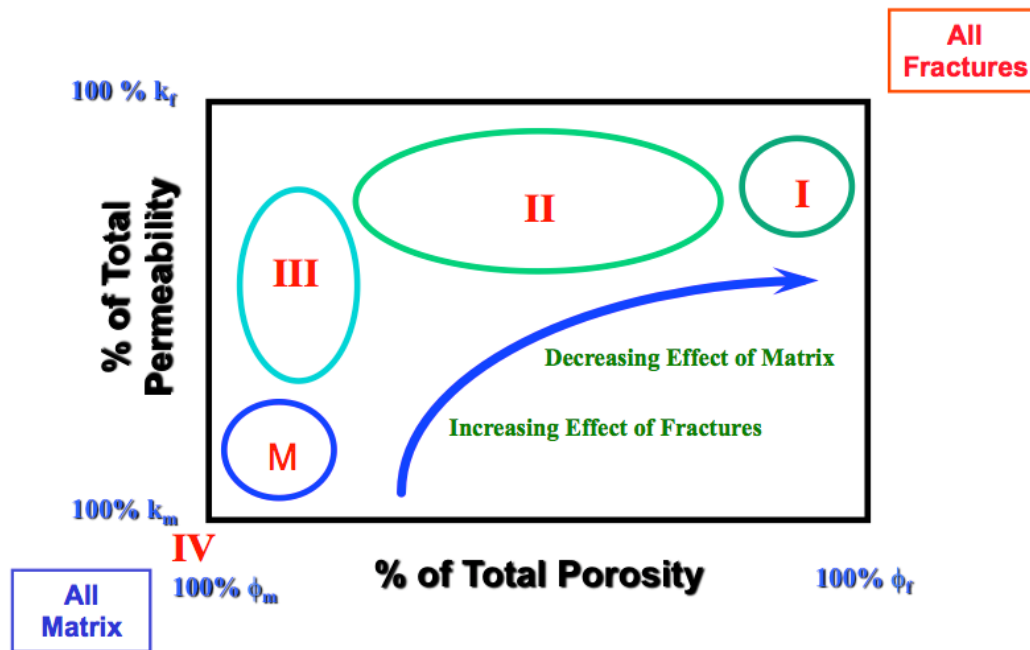


Figure 4. Naturally fractured reservoir classification [Nelson, 2001]

which may lead to greater fracture closure and flow reduction.

Type III - The fractured reservoir is already economically producible (high porosity and permeability in the matrix) and the fractures provide an assist and tend to define the reservoir’s flow property anisotropy.

Type IV – Fractures, perhaps partially filled with cementing agents, act as baffles and barriers to flow in an already producible reservoir and reduce the drainage and sweep efficiency.

**3-1-Dual Porosity / Dual Permeability Model**

The theory of fluid flow in fractured media was developed by Barenblatt et al. in the 1960’s. Warren and Root (1963) introduced the dual-porosity concept into a petroleum reservoir model, and Kazemi et al. (1976) used the dual-porosity concept in a numerical model (Finite Difference Method) of a fractured reservoir at a large scale.

Dual-porosity models consist of two contiguous (superposed) continua (

Figure 5): matrix-blocks (primary pores) and fractures (secondary pores) [Barenblatt et al., 1960]. Each continuum has its own fluid pressure system, and during production a gradient is generated between the fluid in the matrix pores and the adjacent fractures. This causes fluid within the matrix continuum to flow into the fracture continuum, where it is removed through the wells connecting to the fracture continuum.

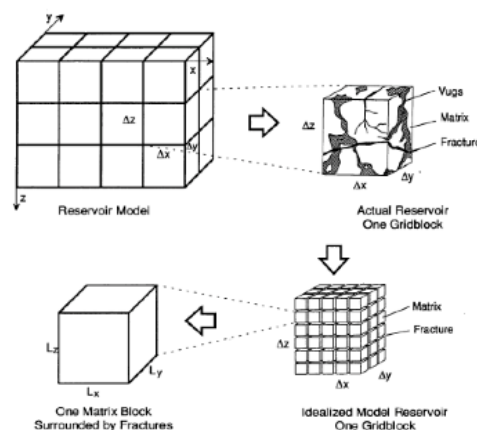


Figure 5. Idealization of a fractured system with a dual-porosity model [Warren and Root, 1963]

In fractured porous media, bulk volume is defined as

$$V_b = V_s + V_{p1} + V_{p2} \quad (1)$$

Here,  $V_b$  is the bulk volume of the fractured porous media,  $V_s$  is the solid volume and  $V_{p1}$  is the pore volume of matrix-blocks and  $V_{p2}$  is the pore volume of fractures. Equation (1) can be written in term of porosities:

$$1 = \phi_s + \phi_1 + \phi_2 \quad (2)$$

Where  $\phi_s = V_s/V_b$  (solid volume fraction),  $\phi_1 = V_{p1}/V_b$  and  $\phi_2 = V_{p2}/V_b$ . In the following, subscript 1 represents the matrix blocks (primary pores) and subscript 2 denotes the fractures (secondary pores).

### 3-2-Single Porosity / Single Permeability Model

Why a single porosity/single permeability model?

Single porosity-single permeability (SPSP) models of fractured reservoirs require half the number of grid cells required for dual porosity-dual permeability (DPDP) ones, and approximately five times faster running time (Narr et al., 2006). This is reasonable motivation to evaluate SPSP models as a fast and economical alternative for fractured reservoir simulations, but under what conditions would SPSP models be realistic representations of fractured reservoirs? Factors to be considered include fracture-to-matrix permeability contrast, matrix productivity and the effect of fractures on reservoir production [Abdel-Ghani, 2009]. These detailed studies in fractured reservoirs, which included evaluation of image logs, static and dynamic data, well tests, production logs, etc., have shown that Type III fractured reservoirs, where the fracture network enhances the production of an already productive reservoir, do not behave significantly as DPDP reservoirs and a SPSP model can be used for simulation (with caution).

The main challenge in real cases using SPSP modeling appears to be that the produced oil volume prior to water breakthrough in a waterflooding process is

usually over-estimated (high oil-water ratio) because the matrix blocks contain far more fluid than fracture networks and in a single porosity model, water is more likely to stably displace the oil front toward production wells. In reality, in most cases with aggressive drawdown, water breakthrough occurs early in the fractures and matrix block oil displacement is small. Different solutions have been proposed to “slow down” the oil movement and “speed up” the water movement in grid blocks where fractures are present. One of the typical solutions is the use of Local Grid Refinement (LGR) where fractures are represented explicitly via thinner lines of grid blocks. An additional permeability is added to the thinner block to account for the fractures [Henn et al., 2000]. LGR requires more computational time due to the increase in the number of grid blocks and is impractical for complex fracture patterns. Numerical difficulties may arise as a result of large flow rates in thinner grid blocks with small pore volume such as when a water tongue develops rapidly in fractures due to gravity segregation. Henn et al. (2000) addressed this problem by applying vertical lumping to thinner grid blocks containing fractures.

An alternative single porosity method for fractured reservoir simulation consists of oil and water relative permeability curves modification (“pseudo-curves”) to slow oil flow and speed water flow in the single porosity model to mimic the behavior of the fractured reservoir. Van Lingen et al. (2001) proposed an analytical formulation based on weighted averaging to modify pseudo-permeability curves. Fracture and matrix relative permeability curves are combined to generate a single pseudo-curve (



Figure 6). This method is suitable for oil-wet fractured reservoirs with high matrix permeability and large fracture spacing (i.e. Type III).

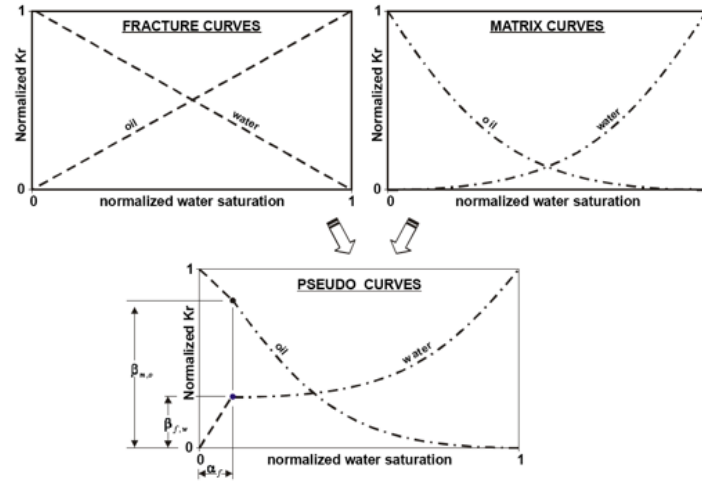


Figure 6. Generation of pseudo relative permeability curves [Van Lingen et al., 2001].

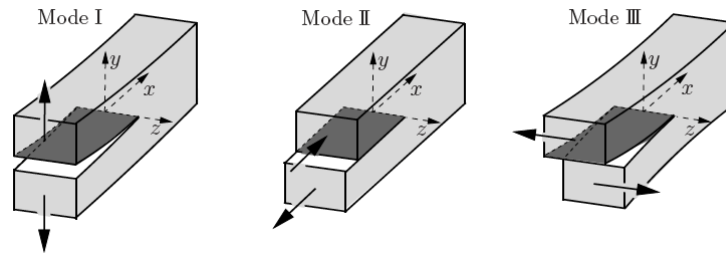


Figure 7. Three different crack opening types [Gross & Seelig, 2006]

Abdel-Ghani (2009) modified Van Lingen et al.'s method to improve the water breakthrough time and water-cut prediction where the fractured reservoirs have a low to medium fracture-to-matrix permeability contrast (i.e. same magnitude order of fracture and matrix permeability). Based on his results, the modified pseudo-permeability model can be implemented in giant fractured carbonate reservoirs with a long production history, such as Middle East reservoirs, with reasonable precision and computational time.

### 3-3-Fracture Mechanics

Mechanical behavior of fractured rocks is complex and usually influenced by a variety of factors such as rock elastic properties, interface friction, surface adhesion, surface roughness, and presence of fluids and debris at interfaces.

Based on the stress state, three different types of fracture behavior are noted (Figure 7). Mode I describes a symmetric fracture opening (with respect to x-z plane) under a normal tensile stress. Mode II corresponds to fracture slip (in x-direction normal to the fracture front) under in-plane (co-directional) shear stress. Mode III denotes the fracture tearing (in z-direction tangential to fracture front) as a result of out-of-plane shear stress.

Various empirical relationships between normal and shear stress and displacements of the fracture plane have been proposed. Goodman (1976) proposed a hyperbolic relation between normal stress ( $\sigma_n$ ) and fracture normal displacement ( $\Delta v_n$ ):

$$\sigma_n = \sigma_{ni} + R\sigma_{ni} \left( \frac{\Delta v_n}{v_{\max} - \Delta v_n} \right)^t \quad \Delta v_n < v_{\max}$$

$$(3)$$

where  $\sigma_{ni}$  is the initial normal stress,  $v_{max}$  is the maximum closure and parameters  $R$  and  $t$  are experimentally determined (Figure 8). Bandis *et al.* (1983) proposed an alternative form of equation (3) for a small initial stress condition.

$$\sigma_n - \sigma_{ni} = \frac{\Delta v_n}{a_n - b_n \Delta v_n} \quad (4)$$

where  $a_n$  and  $b_n$  are constant parameters which are defined based on the limiting values of normal stress as below

$$\begin{aligned} \sigma_n \rightarrow \infty &\Rightarrow \frac{a_n}{b_n} \\ &= \text{vertical asymptote to the hyperbola} \\ &= v_{max} \end{aligned}$$

$$\sigma_n \rightarrow 0 \Rightarrow \Delta v_n \rightarrow 0 \Rightarrow K_n = \frac{1}{a_n} = K_{ni}$$

Equation (4) can be re-written based on maximum closure and initial normal stiffness:

$$\sigma_n - \sigma_{ni} = \frac{v_{max} \cdot K_{ni} \cdot \Delta v_n}{v_{max} - \Delta v_n} \quad (5)$$

For shear displacement, non-linear behavior is usually expressed via hyperbolic functions. Kulhaway (1975) proposed the following relationship between shear stress ( $\tau$ ) and displacement ( $\Delta u_s$ ) for the pre-peak range of the shearing phase (Figure 9):

$$\tau = \frac{\Delta u_s}{a_s + b_s \Delta u_s} \quad (6)$$

where constants  $a_s$  and  $b_s$  are defined as below

$$\begin{aligned} \Delta u_s \rightarrow \infty &\Rightarrow \frac{1}{b_s} \\ &= \text{horizontal asymptote to the hyperbola} \\ &= \tau_{ult} \\ \tau \rightarrow 0 &\Rightarrow \Delta u_s \rightarrow 0 \Rightarrow K_s = \frac{1}{a_s} = K_{si} \end{aligned}$$

In the above formulation,  $\tau_{ult}$  is the peak shear stress and  $K_{si}$  is the initial shear stiffness of a fracture. Barton *et al.* (1985) used the concept of mobilized roughness to describe the shear behavior of a fracture. To quantify shear stress, a mobilized roughness coefficient ( $JRC_{mob}$ ) is used which is a function of joint properties such as normal load, fracture length, current shear displacement and shear displacement history [UDEC Manual]. For the mobilized shear strength ( $\tau_{mob}$ ) for any shear displacement they suggested:

$$\tau_{mob} = \sigma_n \tan [JRC_{mob} \log_{10} (JCS / \sigma_n) + \phi_r] \quad (7)$$

where  $JCS$  is the joint compressive strength and  $\phi_r$  is the residual friction angle.

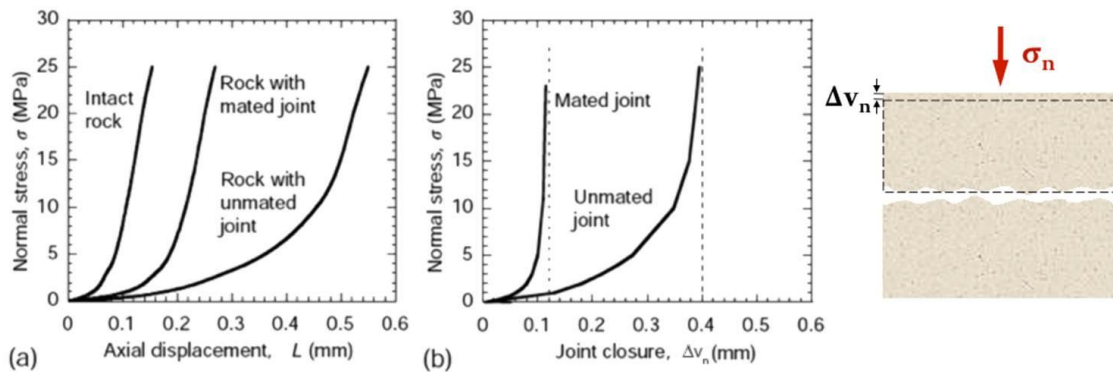


Figure 8. Normal stress vs normal deformation relation of intact and fractured rocks [Bandis *et al.*, 1983]

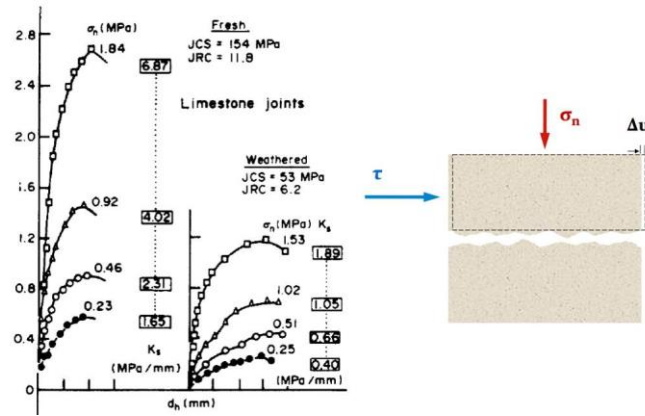


Figure 9. Shear stress vs shear deformation relation of fresh and weathered joints [Bandis et al., 1983].

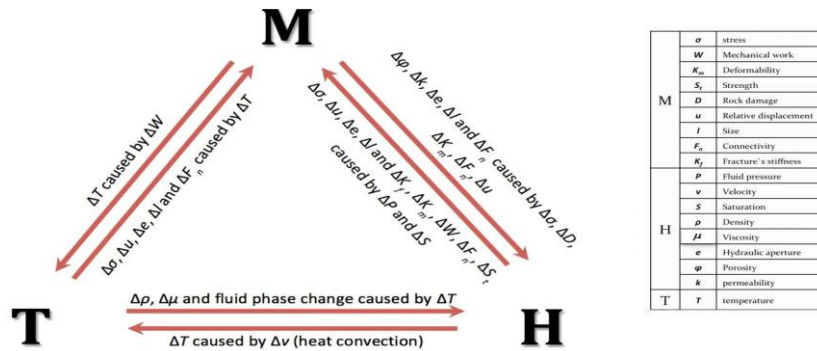
Clearly, the complexity of fracture displacement in real cases requires the acceptance of a great degree of empiricism, and calibration of real cases will always be necessary because of this complexity and the uncertainty of the geometrical disposition and individual properties of an array of fractures.

**4-THM Coupling in Fractured Reservoirs**

**4-1-Why THM Coupling?**

The coupled response of geomaterials to man-made perturbations (e.g. cold water injection, steam injection, hydraulic

fracturing) cannot be predicted by considering each process separately, although this is part of the process of understanding coupling. A THM model is needed to study the two-way interactions among temperature (T), pressure (H) and deformation (M) (Figure 10, Table 2), a problem rendered far more complex when fractures are included because many ambiguities remain associated with the thermal, hydraulic and mechanical response of fractures under coupled circumstances.



$\sigma$	stress
$W$	Mechanical work
$K_e$	Deformability
$S_r$	Strength
$D$	Rock damage
$u$	Relative displacement
$l$	Size
$F_c$	Connectivity
$K_f$	Fracture's stiffness
$p$	Fluid pressure
$v$	Velocity
$S$	Saturation
$\rho$	Density
$\mu$	Viscosity
$e$	Hydraulic aperture
$\phi$	Porosity
$k$	permeability
$T$	temperature

Figure 10. Basic mechanisms of coupled THM processes [Lanru & Xiating, 2003].

Table 2. Coupled THM processes [Lanru & Xiating, 2003].

	<b>Mechanics</b>	<b>Hydraulics</b>	<b>Thermal</b>
<b>Mechanics</b>	<p><u>Mechanical process</u></p> <p>Stress, deformation, damage, strength and failure in matrix; Initiation, growth, coalescence, damage and displacement of fractures. Source: in-situ stress, tectonic movement, gravity, and excavation.</p>	<p><u>M-H coupling</u></p> <p>Stress-deformation-damage effects on matrix porosity and permeability, and fracture transmissivity and network connectivity.</p>	<p><u>M-T coupling</u></p> <p>Mechanical work conversion into heat increment. (Coupling effect is not well defined and usually negligible).</p>
<b>Hydraulics</b>	<p><u>H-M coupling</u></p> <p>Effective stress of matrix; Aperture-pressure-stiffness function of fractures; Capillary and swelling pressure-relative saturation.</p>	<p><u>Hydraulic process</u></p> <p>Darcian or non-Darcian fluid flow in matrix and fractures. Source: surface water infiltration (recharge), groundwater movement, seawater intrusion, fluid flow in hydrocarbon reservoirs, hot/cool water pumping and injection in geothermal fields.</p>	<p><u>H-T coupling</u></p> <p>Heat convection by fluid velocity field.</p>
<b>Thermal</b>	<p><u>T-M coupling</u></p> <p>Thermal stress and expansion of matrix; Closure, opening, damage and/or irreversible deformation of fractures.</p>	<p><u>T-H coupling</u></p> <p>Fluid buoyancy and viscosity change; Fluid phase change (evaporation and condensation); Thermal diffusion of moisture flow.</p>	<p><u>Thermal process</u></p> <p>Heat conduction, convection &amp; radiation due to man-made or natural heat sources. Source: radioactive waste decay, geothermal gradients, hot/cool water injection and production, cooling by natural gas storage.</p>

THM coupling is based on the two basic coupling principles of thermoelasticity (TM) and poroelasticity (HM).

In the beginning of 18<sup>th</sup> century, Gough introduced thermoelasticity through the concept of material temperature changes due to stretching and Weber (1830) formularized the thermoelastic effect. Schiffman (1971) introduced heat transfer into Biot's poroelastic concepts, and the governing equation of thermoelastic consolidation was provided by Booker and Savvidou (1985). They considered only conductive flux, not convection, which is the mechanism of heat transfer via fluid flow. In 1978, Aktan and Farouq Ali worked on induced thermal stresses due to hot water injection and introduced the thermoelastic stress-strain relationship. Hojka et al. (1993) solved the convection and conduction-coupling problem for a plane strain borehole in analytical form. They showed the stress and temperature distributions around the borehole for some steady-state flow cases.

The fundamentals of poroelasticity are based on the original concept of effective

stress and one-dimensional consolidation for incompressible solid grains formulated by Terzaghi in 1923. Thereafter, Biot investigated the coupling between stresses and pore pressure in a porous medium and developed a generalized three-dimensional theory of consolidation with the basic principles of continuum mechanics, the "Theory of Poroelasticity" [Geertsma, 1966]. Biot's theory and published applications are oriented more toward rock mechanics than fluid flow so it is less compatible with conventional fluid-flow models (without geomechanics consideration) in terms of concept understanding, physical interpretation of parameters (e.g., rock compressibilities), and computer code implementation.

Skempton (1954) derived a relationship between the total stress and fluid pore pressure under undrained initial loading through the so-called Skempton pore pressure parameters A and B. Geerstma (1957) gave a better insight of the relationship among pressure, stress and volume, clarifying the concept of compressibility in a porous medium and

Van der Knaap (1959) extended his work to nonlinear elastic geomaterials such as dense but uncemented sands. Geertsma (1966) applied Biot's theory to subsidence problems in petroleum engineering, perhaps the first loosely coupled flow-geomechanics analysis published.

Nur and Byerlee (1971) proved that the effective stress law proposed by Biot is more general and physically sensible than that proposed by Terzaghi, although Terzaghi understood clearly the limitations of the assumptions he had to make in the 1920's to solve practical engineering problems in clay consolidation (one-dimensional analysis, ignoring fluid and mineral grain compressibility, etc.). In other developments relevant to coupled flow-stress problems, Ghaboussi and Wilson (1973) introduced fluid compressibility into classic soil mechanics consolidation theory, and Rice and Cleary (1976) showed how to solve poroelasticity problems by assuming pore pressure and stress as primary variables instead of displacements as employed by Biot.

THM coupled models are based on three fundamental laws, i.e. Hooke's law of elasticity, Fourier's law of heat conduction, and Darcy's law of fluid flow in porous media, which are governed by three coupled partial differential equations, which are described below. Note that in the following formulations, subscript  $i = 1$  represents the matrix-blocks (primary pores),  $i = 2$  denotes the fractures (secondary pores), and subscripts  $s$  and  $b$  refer to solid matrix block and bulk fracture rock, respectively.

- **Conservation of momentum**

$$G\nabla^2 \mathbf{u} + (\lambda + G)\nabla(\nabla \cdot \mathbf{u}) = \sum_{i=1}^2 (\alpha_i \nabla \mathbf{p}_i + \beta_i \nabla \mathbf{T}_i) \quad (8)$$

where  $\lambda$  and  $G$  are the Lamé constants,  $\mathbf{u}$ ,  $\mathbf{p}$  and  $\mathbf{T}$  are displacement, fluid pressure and temperature, respectively, and  $\alpha_i$  and  $\beta_i$  are fluid pressure and thermal ratio factors. Bai and Roegiers (1994) proposed a relationship for fluid pressure ratio factors as a function of porosity, which is

$$\alpha_i = 1 - (1 - \phi_i) K_b / K_s \quad (9)$$

where  $\phi$  is porosity and  $K_b$  and  $K_s$  are the bulk modulus of fractured rock and solid grains, respectively. Thermal ratio factors are defined as

$$\beta_i = (3\lambda + 2G)\alpha_{Ti} = 3K_b\alpha_{Ti} \quad (10)$$

where  $\alpha_{Ti}$  is the linear thermal expansion coefficient.

- **Conservation of mass**

$$\frac{k_i}{\mu} \nabla^2 \mathbf{p}_i + \sum_{j=1}^2 [(-1)^{i+j} a_{ij} \frac{\partial \mathbf{p}_j}{\partial t} + b_i \alpha_{Ti} \frac{\partial \mathbf{T}_i}{\partial t}] \pm \Gamma(\Delta \mathbf{p}) = 0 \quad (11)$$

where  $k$  is intrinsic permeability and  $\mu$  is dynamic viscosity.  $\Gamma$  is the fluid transfer coefficient, usually assumed to be a linear function of pressure difference between the matrix block and fracture network for quasi-steady state flow [Warren and Root, 1963]:

$$\Gamma = \gamma \frac{k_1}{\mu} (\mathbf{p}_1 - \mathbf{p}_2) \quad (12)$$

where  $\gamma$  represents the characteristic of fractured rock. Warren and Root (1963) defined  $\gamma$  as below:

$$\gamma = \frac{4n(n+2)}{d^2} \quad (13)$$

where  $n$  is the number of normal sets of fractures and  $d$  is the average dimension of a porous matrix block. Kazemi et al. (1976) used another formula to consider the shape factor in finite-difference representation of a dual-porosity fractured reservoir. They proposed the following relationship for a three-dimensional case:

$$\gamma = 4 \left( \frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2} \right) \quad (14)$$

where  $L_x$ ,  $L_y$  and  $L_z$  are the matrix block dimensions in each direction. In equation (11) the coefficient  $a_{ij}$  and  $b_i$  are defined as [Master et al., 2000]:

$$a_{ij} = \left[ \frac{1 - \phi_i}{K_i} + \frac{\phi_i}{K_f} \right] \delta_{ij} + \left[ \frac{2\phi_j^*}{K_j + K_f} \right] (1 - \delta_{ij}) + \frac{(-1)^{i+j}}{H} \left[ 1 - (1 - \phi_i) \frac{K_b}{K_s} \right] \left[ 1 - (1 - \phi_j) \frac{K_b}{K_s} \right] b_i = \frac{3K_b}{H} \left[ 1 - (1 - \phi_i) \frac{K_b}{K_s} \right] \quad (15)$$

where  $\phi_j^*$  is the effective porosities considering an average compressibility,  $K_1(=K_s)$ ,  $K_2(=K_{ns}^*)$  and  $K_f$  are the bulk modulus of solid matrix block, fracture and pore fluid, respectively,  $K_n$  is the fracture normal stiffness and  $s^*$  is the fracture spacing.  $H (= \lambda + 2G)$  is the uniaxial compaction modulus or oedometer modulus and  $\delta_{ij}$  is the Kronecker delta.

- **Conservation of energy**

$$-\nabla^T K_T \nabla T + \rho_f c_f (\mathbf{v}_1 + \mathbf{v}_2) \nabla T + Q_h = \frac{\partial}{\partial t} \{ [(1 - \phi_T) \rho_s c_s + \phi_T \rho_f c_f] T \} \quad (16)$$

where  $K_T$  is the thermal conductivity,  $c_f$  and  $c_s$  are the isobaric heat capacity of fluid and solid, respectively,  $Q_h$  is the heat source/sink,  $\mathbf{v}$  is the fictitious velocity of fluid, and  $\phi_T (= \phi_1 + \phi_2)$  is the total porosity of fractured material. The first term on the left hand side represents heat conduction in the rock medium and the second term is the heat convection in both fracture and matrix blocks. The right hand side of equation 16 represents the transient temperature changes. It should be noted that the effect of volumetric strain (i.e. geomechanics effects) on temperature variation is negligible and is usually neglected for computational convenience [Booker & Savvidou, 1985].

#### 4-2-Analytical or Semi-Analytical Methods

Analytical or semi-analytical solutions are the preferred initial approach in any engineering analysis. Analytical solutions permit direct physical insight of problems and quick estimation of unknowns. Although there are some closed-form solutions for coupled stress-flow problems in the literature, THM coupled problems cannot be evaluated properly using analytical or semi-analytical solutions. Dusseault (2008) summarized some of the drawbacks of analytical or semi-analytical solutions of coupled problem as below:

- These type of solution can only consider a limited degree of coupling (e.g. Wang and Dusseault coupled solution for tangential stress calculation around a wellbore (2003))

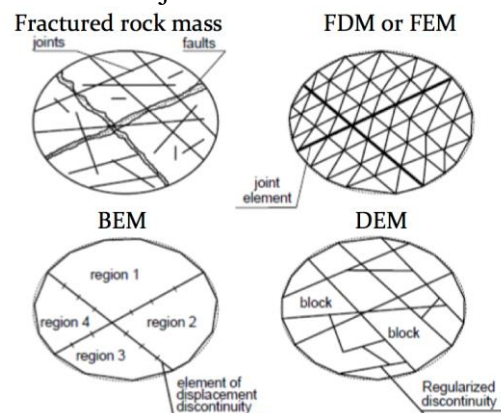
- A linear constitutive law is usually implemented (e.g. elasticity)
- Simple loading behavior is used (e.g. uniform pressure changes, single well)
- Simple geometry (e.g. plane strain, plane stress, axisymmetric)
- Homogeneous and isotropic materials
- Constant boundary condition (e.g. constant far field stresses, constant flux or pressure at wellbore face)

Closed form solutions are usually a good benchmark for numerical solutions and can be used for model verification.

#### 4-3-Numerical Methods

The main idea of any numerical method is to replace the problem with an approximate problem which is easier solved, with the solution as close as possible to the original solution. A variety of numerical methods have been used to model THM coupled problems (Figure 11). In numerical methods, a continuum is usually subdivided into a finite number of domains (elements, block-averaged nodes...) with finite degrees of freedom and simplified mathematical behavior. To solve the discretized problem numerically, the following criteria should be satisfied properly:

- The physical statement of the problem as expressed by the governing partial differential equations, and,
- The continuity condition at interfaces between adjacent elements



**Figure 11. Fractured rock mass representation using different numerical methods [Jing, 2003].**

In the case of fractured reservoir modeling, two different approaches have been used in



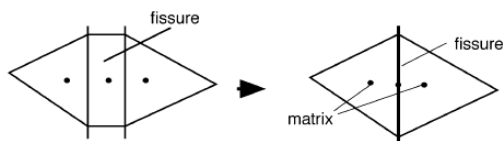
the literature, continuous and discontinuous (discrete) methods. In the following some of the numerical techniques used in each category are described:

#### 4-3-1-Continuous Methods

- Finite Difference Method (FDM)

The Finite Difference Method (FDM) is based on finding an approximate solution for partial differential equations (PDEs), using a finite number of points (i.e. grid points, mesh points or net points). In this case, each derivative is replaced with an approximate differential formula based on the Taylor's series expansion and then the PDE is converted to a set of algebraic system equations which relate the values of unknown variables (i.e. pressure, temperature, displacement) at each grid point [Aziz & Settari, 1979]. The solution of this algebraic system equation by considering the applied boundary conditions of the problem will satisfy the governing PDEs as well as the specified boundary conditions.

Fractures cannot be modeled explicitly in FDM, as it requires continuity of the governing equations between neighboring grid points. However it is possible to introduce weakness zones with a certain amount of thickness that cannot have an opening and must be attached to the neighboring nodes [Jing, 2003]. Also, Caillabet et al. (2000) and Granet et al. (2001) implemented special elements, known as "fracture elements", to model fluid flow (Figure 12). In their approach, fracture thickness is taken into account for the fluid flow calculation but not in the geometrical representation of the problem, as the fracture thickness is negligible compared to the matrix block size.



**Figure 12. Real and geometrical fracture element representation [Granet et al., 2001]**

- Finite Element Method (FEM)

The Finite Element Method (FEM) is based on a piecewise representation of the solution in terms of specified basis functions. FEM consists of three fundamental steps, which are: domain discretization, local approximation, and global matrix assembly and solution [Jing, 2003]. The problem's domain is discretized into a finite number of subdomains (finite elements) with a regular shape and fixed number of nodes. The field variables are then written as a trial function of its nodal value in a polynomial form (i.e. weak form). Appropriate test functions are multiplied by the weak form of the governing equations, and then integrated over each element. The results are then assembled into a global matrix, and by solving the linear system of equations therein, the value of field variables at each integration point is determined.

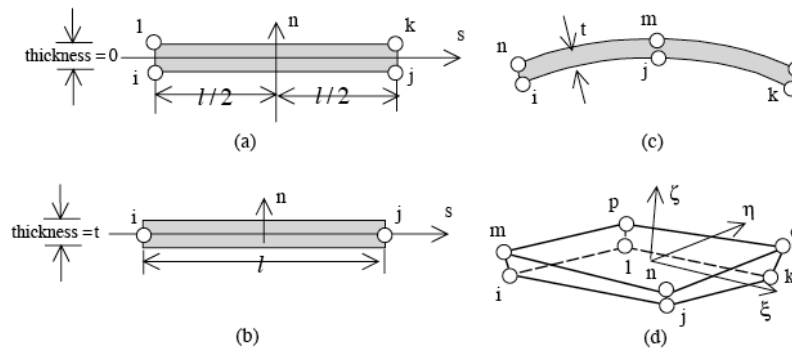
Much work has been done during the last 40 years to represent rock fractures in the FEM. Goodman et al. (1968) proposed a zero thickness "joint element" in which the normal and shear stresses and the deformation normal to and along the fractures are related through constant normal and shear stiffness values ( $K_n$  and  $K_s$ ). The zero thickness assumption (i.e. large aspect ratio) may lead to some numerical ill-conditioning.

Zienkiewicz et al. (1970) introduced a six-point small thickness fracture element by adding two more nodes in the middle of element. Adding more nodes allows the element to be curved, increasing the efficiency of FEM to model problems with complex geometries. Ghaboussi and Wilson (1973) implemented plasticity theory in a finite thickness FEM fracture element and Desai et al. (1984) proposed a "thin-layer" element that used a special constitutive law for contact and frictional

sliding. Buczkowski and Kleiber (1997) implemented an interface element model in contact mechanics with an orthotropic friction based on the theory of plasticity. Nevertheless, these methods cannot be

used to model large-scale fracture opening, sliding and complete detachment (opening) because of the basic continuum assumption of FEM (

Figure 13).



**Figure 13. FEM fracture elements by (a) Goodman *et al.* (1968), (b) Ghaboussi and Wilson (1973), (c) Zienkiewicz *et al.* (1970) and (d) Buczkowski and Kleiber (1970) [Jing, 2003].**

#### • Boundary Element Method (BEM)

Unlike finite difference and finite element methods, which require discretizing the whole region of the problem, in the Boundary Element Method (BEM) only the boundary of region is discretized. A known or calculated solution of a simple singular problem is used to build up the numerical solution for the whole mass by satisfying the boundary condition at each boundary element. BEM solutions can be summarized as:

1. Boundary discretization with a finite number of elements
2. Approximation of the local solution at boundary elements via shape functions
3. Evaluation of boundary influence coefficients
4. Application of boundary conditions and solution of the linear system of algebraic equations
5. Evaluation of field variables inside the domain

BEM methods can be classified into direct and indirect methods. In the direct formulation, calculated deformation and stresses have a clear physical meaning, whereas there is no explicit physical

meaning for the displacements and tractions in the indirect formulation, they are expressed as fictitious source densities [Jing, 2003].

In direct BEM, fractures are modeled by assuming two opposite surfaces along the fracture plane; expect at the fracture tips where special singular tip elements have to be used. An alternative for fracture modeling is the Displacement Discontinuity Method (DDM), which is an indirect method. This method proposed by Salamon (1963, 1968) and developed by Crouch and Starfield (1983) is based on integrating the analytical solution of a constant displacement discontinuity over a finite line segment embedded within an infinite or semi-infinite elastic solid that can be orthotropic in properties.

#### 1.1.1. Discontinuous Methods

##### • Discrete Element Method (DEM)

In the Discrete Element Method (DEM) two types of mechanical behavior are considered: those of the discontinuities and of the solid materials. The analysis domain is treated as a combination of rigid or deformable (using FDM or FEM) matrix



blocks and the contacts among them which are identified and updated during the simulation. As a result, the main difference between DEM and continuum methods is that in DEM the contacts' pattern changes continuously whereas in continuum methods the pattern is fixed [Jing, 2003].

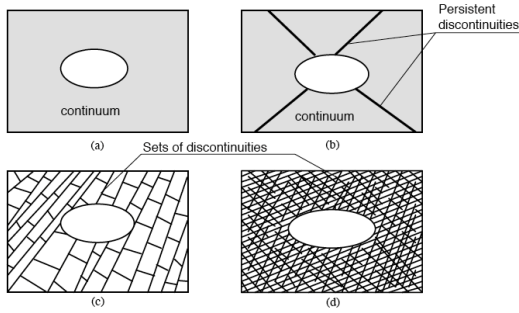
Cundall and Hart (1989) identified five main classes of DEM codes:

- Distinct Element Programs assume rigid or deformable matrix blocks as well as deformable contacts among blocks. Time-marching is done explicitly in each time step to calculate the motion in the model to define the new set of contacts between blocks. Some of the representative codes are UDEC, 3DEC, PFC (ITASCA solution) and DIBS (Walton, 1980).
- Modal Methods use a modal superposition to deal with deformable blocks, which is suitable for loosely packed discontinua and dense packing simulation. Hocking et al. (1985) implemented this method in their code, CICE.
- Discontinuous Deformation Analysis (DDA) is similar to DEM for the block and contact deformation, except it assumes an iterative scheme of time-marching as well as superposition of strain modes. A representative code is DDA by Shi (1989).
- Momentum-exchange Methods assume rigid contacts and matrix blocks. During a collision, momentum is exchanged between two contacting bodies, and it also is possible to model frictional sliding [Hahn, 1988]
- Limiting Equilibrium Methods consider a rigid system of blocks and use vector analysis to evaluate the movement of blocks in the fractured system. Some examples include work done by Goodman and Shi (1985) and Warburton (1981).
- Discrete Fracture Network (DFN) Method  
Discrete Fracture Network (DFN) considers fluid flow and transport

phenomena in a discrete connected fractured media. The two key factors in DFN models are fracture geometry and transmissivity. It is difficult to model heat flow and mechanical deformation using DFN, so these should be approximated with another method. DFN is suitable for the fluid flow and transport simulation in areas which thermal and mechanical processes are not significant such as in shallow fractured aquifers. Most applications of DFN models focus on fracture permeability characterization, fracture influence on flow, and near-field studies such as around a wellbore or tunnel [Jing, 2003].

#### 4-3-2-Hybrid Methods

Hybrid models are defined as a combination of different numerical methods (i.e. continuous and discrete fracture models) such as FEM/BEM, DEM/BEM and FDM/BEM. As BEM discretizes only the boundary of the problem and is suitable for infinite or semi-infinite problems, it is mostly used for far-field rock simulation as an equivalent elastic continuum. FEM and FDM methods are suitable for near-field regions where no fracture exists or where the density of fractures is so high that an equivalent continuum model could be used to represent the fractured area. Also, continuum methods with fracture elements are an alternative for modeling of a low fracture density area where the deformation of fractures is negligible and there is no fracture opening or complete detachment. DEM is suitable for moderately fractured regions as well as large-scale displacement (Figure 14) [Jing, 2003].



**Figure 14. Suitability of different numerical methods in fractured rocks: (a) continuum method, (b) discrete method or continuum method with fracture elements, (c) discrete method, (d) equivalent continuum method [Jing, 2003]**

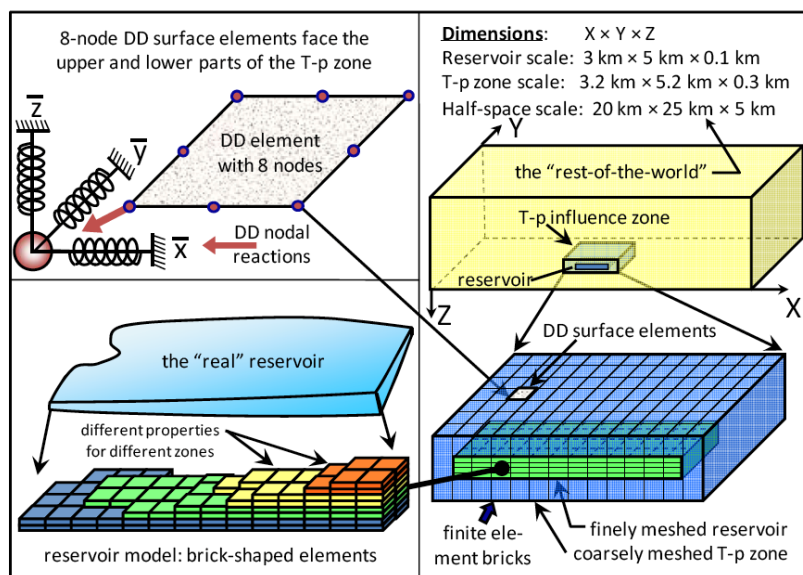
In the following, three different hybrid methods are briefly described:

- Hybrid FEM/BEM Models

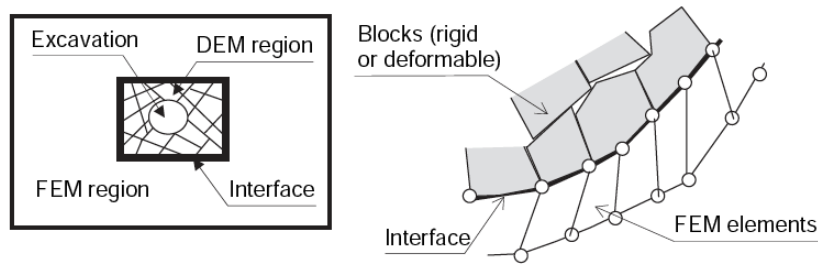
Hybrid FEM/BEM models were introduced by Zienkiewicz et al. (1977) for the first time as a general stress analysis technique and have been implemented by others, especially for the simulation of underground excavation in the mining industry (e.g. Varadarajan et al. (1985), Ohkami et al. (1985) and Von Estorff and Firuziaan (2000)). Yin et al. (2007a, b) implemented the concept of hybrid

FEM/DDM model from seam mining simulation (Salamon, 1963, 1968) to a compacting conventional oil reservoir to simulate the behavior of the reservoir under thermal processes considering the effect of overburden, under-burden and side-burdens. Three zones were assumed: the reservoir, a T-p reservoir influence zone which only considers conduction (not convection) and the “rest-of-the-world”, represented by Displacement Discontinuity elements (

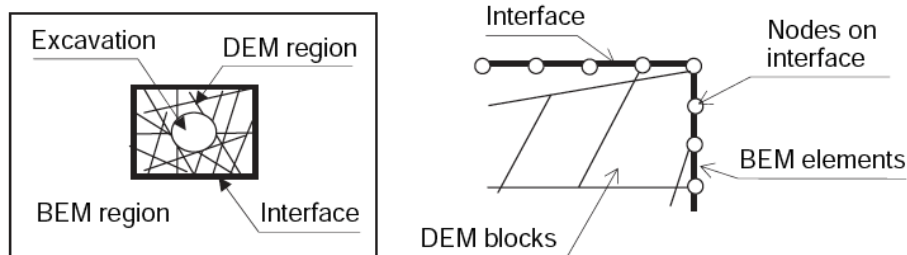
Figure 15). Their results showed that the FEM/DDM hybrid model can integrate the merits of FEM, implemented only for the reservoir and the T-p zone, and the DDM, representing the elastic “rest-of-the-world”. The degrees of freedom (number of equations) in the FEM/DDM hybrid approach are reduced by a factor of 5 or so compared to a full FEM discretization.



**Figure 15. FEM/DDM scheme for reservoir engineering analysis of tabular reservoirs [Dusseault et al., 2007].**



**Figure 16. Hybrid DEM/FEM model and interface representation [Jing & Stephansson, 2007].**



**Figure 17. Hybrid DEM/BEM model and interface representation [Jing & Stephansson, 2007].**

- **Hybrid DEM/FEM Models**

This hybrid model is usually used for deformation analysis where the fracture network and matrix blocks in the smaller near-field region are represented with a discrete and explicit method (i.e. DEM) and an equivalent continuum method (i.e. FEM) is chosen to model the large-field area (

Figure 16).

The important issue to consider in a hybrid model is to ensure displacement continuity at the interface between FEM and DEM. An iterative procedure is used for the coupling between the two different numerical methods. In each iteration, the induced nodal forces at the interface are determined by DEM and are sent to FEM to calculate the nodal displacement vector of the interface. The calculated nodal displacements are fed back to DEM to

update the induced normal forces. This procedure is continued until the results converge [Jing & Stephansson, 2007].

- **Hybrid DEM/BEM Models**

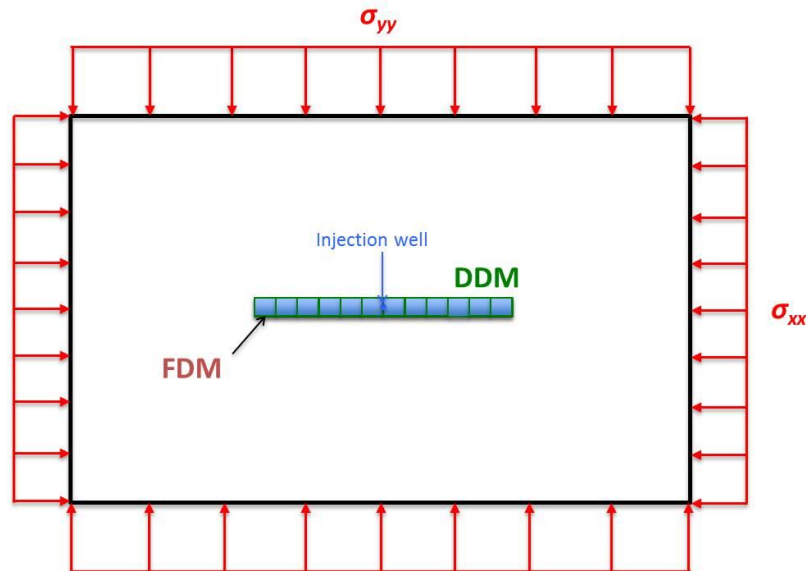
In DEM/BEM approaches, the near-field area is much larger than the near-field region in DEM/FEM because the far-field region is assumed to be linearly elastic as in DEM/BEM and is simulated by a boundary element method (BEM) (

Figure 17). The method is suitable for mechanical and hydro-mechanical analyses and has been implemented into UDEC by Lemos (1987). The advantage of DEM/BEM over DEM/FEM is that only the interface between DEM and BEM regions must be discretized with boundary elements. The key issues in DEM/BEM hybrid models are:

- Displacement continuity and stress equilibrium along the interface between DEM and BEM
- Similar elastic properties near the interface of two region
- No separation and slipping between DEM and BEM regions are allowed.
- Hybrid FDM/BEM Models

In this method, fluid and heat flow in a fracture is modeled via FDM and the effect of pressure and temperature variation on the fracture and surrounding rocks are estimated using a boundary element method, of which the DDM is the more favorable approach. Fracture deformation can be used to update the fracture aperture as well as fracture permeability (

Figure 18).

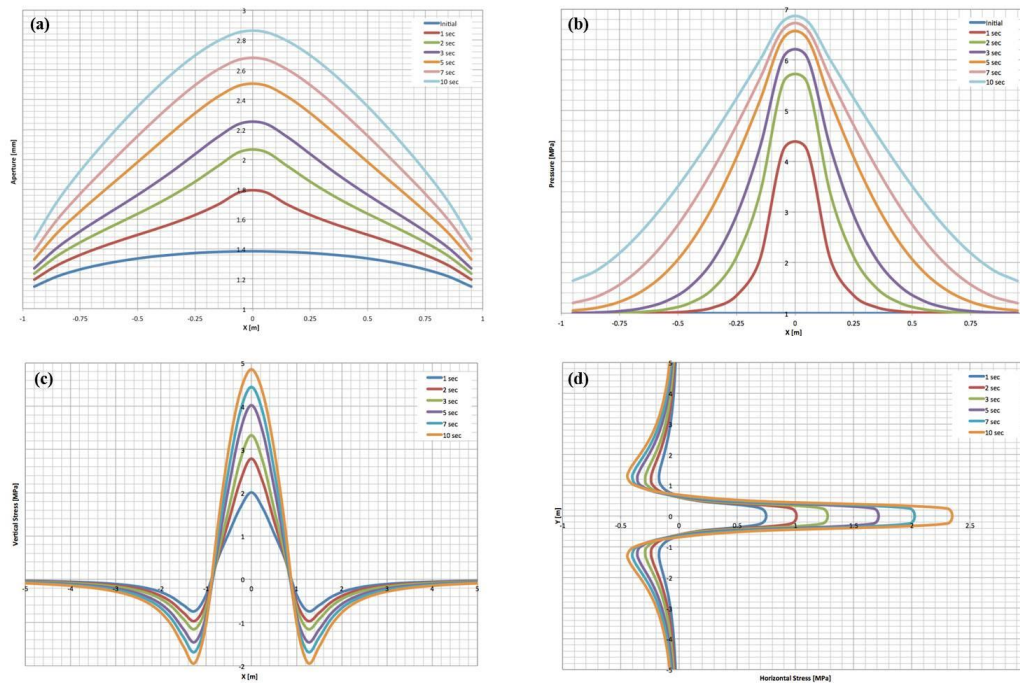


**Figure 18. Schematic representation of FDM/DDM method for a single fracture in an impermeable matrix block.**

This method is suitable for the reservoirs with a low density of fractures as well as simulation of single fracture behavior at the laboratory scale. Also, it should be noted that the surrounding rock is assumed as linear isotropic to implement the boundary elements to estimate deformation and stress variations.

Figure 19 depicts the effects of fluid injection at a rate of  $10^{-5} \text{ m}^3/\text{s}$  into a 2 m long fracture with an initial aperture of 1 mm. In situ stresses were assumed to be equal to zero in this example, and injection was simulated for 10 seconds. The pore pressure evolution during the injection process is shown in

Figure 19 (b).



**Figure 19.** The results of 2D hydro-mechanical FDM/DDM model during an injection process. (a) aperture, (b) pore pressure, (c) vertical stress @  $y=0.5$  m and (d) horizontal stress @  $x=0$ .

As expected, the pressure in the fracture increases symmetrically around the injection source. This pressure increase is associated with an increase in fracture aperture that is shown in

Figure 19 (a). There is a good correlation between pressure and aperture variations as a linear elastic relationship has been assumed for the fracture deformation.

Figure 19 (c) and (d) depict the vertical and horizontal stress variation during the injection process into the fracture. Note that the compressive and tensile stresses are assumed to be positive and negative, respectively. Compressive stress increases in the fracture, symmetrically around the injection point, when the pore pressure increases. Due to the stress redistribution in the fracture, tensile stresses are generated near the fracture tips, interpreted as a possible driving force for fracture propagation during fluid injection or thermal processes.

Table 3 summarizes the advantages and disadvantages of possible numerical methods which can be implemented to simulate the THM coupling in fractured reservoirs.

### 5-Summary & Conclusion

In this study, the importance of geomechanics in the oil industry was reviewed. There are some problems such as reservoir subsidence and compaction which require thermo-hydro-mechanical coupling. There are different level of THM coupling, and it was shown that full coupling is the tightest and most reliable and stable technique in comparison with other techniques if all conditions are identical. This technique needs code development for general coupling achievement, as at the present time the formulation is based on some simplifying assumptions such as linear processes and fixed parameters. Such assumptions can be addressed in a numerical formulation, but many questions such as code stability and efficacy will undoubtedly arise. The governing equations of THM coupling in the case of a non-isothermal dual porosity reservoir were derived. These



equations are partial differential equations which need numerical techniques to be solved. A brief review on the existing numerical methods to solve these governing equations was given. Among them, hybrid models, which are defined as a combination of different numerical

methods, are quite popular among modelers as they benefit from all numerical methods that are involved. However, the computational load is heavy and displacement continuity and stress equilibrium at the interfaces are concerns.

**Table 3. Advantages and disadvantages of possible numerical methods for THM coupling of naturally fractured reservoirs.**

Numerical Method		Advantages	Disadvantages
Continuous	FDM	<ul style="list-style-type: none"> <li>• Easy to formulate.</li> <li>• No local trial (interpolation) functions required to approximate PDEs.</li> <li>• The most direct and intuitive technique.</li> </ul>	<ul style="list-style-type: none"> <li>• Cannot handle complex geometries, material inhomogeneities.</li> <li>• Neumann boundary condition can be only approximated, not exactly enforced.</li> <li>• Inflexible in dealing with explicit fractures, due to the necessity of continuity of functions between neighboring grid points.</li> </ul>
	FEM	<ul style="list-style-type: none"> <li>• Flexible in handling material inhomogeneity and anisotropy.</li> <li>• No coordinate transformation is required for complex geometries.</li> <li>• Can handle complex boundary conditions.</li> <li>• Suitable for dynamic problems.</li> <li>• Neumann boundary conditions are enforced exactly.</li> </ul>	<ul style="list-style-type: none"> <li>• Large-scale opening, sliding and complete detachment are not permitted due to continuum assumptions.</li> <li>• Numerical ill-conditioning will occur due to large aspect ratios of fracture elements in the explicit representation of a large number of fractures.</li> <li>• Cannot be used for fracture growth, however there are some special algorithms to overcome this problem, e.g., enriched FEM.</li> </ul>
	BEM	<ul style="list-style-type: none"> <li>• Discretization of the boundary only (reduction of model dimension by one).</li> <li>• Simplified pre-processing.</li> <li>• Improved accuracy in stress concentration problems.</li> <li>• Simple and accurate modeling of problems involving infinite and semi-infinite domains.</li> <li>• Simplified treatment of symmetrical problems (no discretization needed in the plane of symmetry).</li> </ul>	<ul style="list-style-type: none"> <li>• Non-symmetric, fully populated system of equations in collocation BEM.</li> <li>• Treatment of inhomogeneous and non-linear problems is extremely difficult.</li> <li>• Requires the knowledge of a suitable fundamental solution, hence usually an elastic solution.</li> <li>• Practical application relatively recent, not as well-known as FEM among users.</li> </ul>
Discontinuous	DEM	<ul style="list-style-type: none"> <li>• Explicit representation of fractures.</li> <li>• Flexible to handle large amount of fractures.</li> </ul>	<ul style="list-style-type: none"> <li>• Lack of knowledge of the geometry of fractures.</li> <li>• Large amount of computation required.</li> <li>• Matrix-fracture flow interaction cannot handled adequately.</li> </ul>
	DFN	<ul style="list-style-type: none"> <li>• Requires fewer degrees of freedom in comparison with FEM.</li> <li>• Suitable for generic quantitative studies of fracture impacts.</li> </ul>	<ul style="list-style-type: none"> <li>• Lack of knowledge of the geometry of fractures and hydraulic properties of fractures.</li> <li>• Large amount of computation required.</li> <li>• Can't handle heat flow and mechanical process.</li> </ul>
Hybrid Methods		<ul style="list-style-type: none"> <li>• Benefits from both continuous and discrete methods.</li> </ul>	<ul style="list-style-type: none"> <li>• Large amount of computation required.</li> <li>• Displacement continuity and stress equilibrium at the interface is a big concern.</li> </ul>

## 6-References

- 1- Abdel-Ghani, R. (2009). Single Porosity Simulation of Fractures with Low to Medium Fracture-to-Matrix Permeability Contrast. SPE/EAGE Reservoir Characterization and Simulation Conference. Abu Dhabi, UAE. SPE 125565.
- 2- Aktan, T., & Farouq-Ali, S. (1978). Finite Element Analysis of Temperature and Thermal Stresses Induced by Hot Water Injection. SPE Journal, SPE 5765 , 18, pp.457-469.
- 3- Aziz, K., & Settari, A. (1979). Petroleum Reservoir Simulation. London: Elsevier.

- 4- Bagheri, M., & Settari, A. (2008). Modeling Fluid Flow in Deformable Fractured Reservoirs using Full Tensor Permeability. SPE Europec/EAGE Annual Conference and Exhibition. Rome, Italy. SPE 113319.
  - 5- Bagheri, M., & Settari, A. (2008). Modeling of Geomechanics in Naturally Fractured Reservoirs. SPE Reservoir Evaluation & Engineering , 11, pp.108-118.
  - 6- Bai, M., & Roegiers, J. (1994). Fluid Flow and Heat Flow in Deformable Fractured Porous Media. International Journal of Engineering Science , 32 (10), pp.1615-1633.
  - 7- Bandis, S.C., Lumsden, A.C., & Barton, N.R. (1983). Fundamentals of rock Joint Deformation. International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts, 20 (6), pp.249-268.
  - 8- Barenblatt, G., Zehltov, I., & Kochina, I. (1960). Basic Concepts in the Theory of Seepage of Homogeneous Liquids in Fissured Rocks. Journal of Applied Mathematics and Mechanics, USSR , 24, pp.1286-1303.
  - 9- Barton, N., Bandis, S., & Bakhtar, K. (1985). Strength, Deformation and Conductivity Coupling of Rock Joints. International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts, 22 (3), pp.121-140.
  - 10- Biot, M. (1941). General Theory of Three Dimensional Consolidation. Journal of Applied Physics , 12, pp.155-164.
  - 11- Booker, J., & Savvidou, C. (1985). Consolidation around a Point Heat Source. International Journal for Numerical and Analytical Methods in Geomechanics , 9, pp.173-184.
  - 12- Buczkowski, R., & Kleiber, M. (1997). Elasto-Plastic Interface Model for 3D Frictional Orthotropic Contact Problems. International Journal for Numerical Methods in Engineering, 40. pp.599-619.
  - 13- Caillabet, Y., Fabrie, P., Landerau, P., Noetinger, B., & Quintard, M. (2000). Implementation of a Finite-Volume Method for the Determination of Effective parameters in fissured porous media. Numerical Methods of Partial Differential Equations, 16. Pp.237-63.
  - 14- Chin, L., & Boade, R. (1990). Full-Field, 3-D Finite Element Subsidence Model for Ekofisk. 3rd North Sea Chalk Symposium. Copenhagen, Denmark.
  - 15- Crouch, S.L., & Starfield, A.M. (1983) Boundary Element Methods in Solid Mechanics. London. George Allen & Unwin.
  - 16- Cundall, P.A., & Hart, R.D. (1989). Numerical Modelling of Discontinua. Proceedings of the 1<sup>st</sup> U.S. Conference on Discrete Element Methods. Golden, Colorado. pp.1-17.
  - 17- Dean, R., Gai, X., Stone, C., & Minkoff, S. (2006). A Comparison of Techniques for Coupling Porous Flow and Geomechanics. Reservoir Simulation Symposium. Houston, Texas. SPE 79709.
  - 18- Desai, C.S., Zamman, M.M., Lightner, J.G., & Siriwardane, H.J. (1984). Thin Layer Element for Interfaces and Joints. International Journal for Numerical and Analytical Methods in Geomechanics, 8. pp.19-43.
  - 19- Di Maio, C., Hueckel, T., & Loret, B. (2002). Proceedings Workshop on Chemo-Mechanical Coupling in Clays. Balkema, Rotterdam. From Nano-scale to Engineering Applications - Proceedings of the Workshop, Maratea, 28-30 June 2001
  - 20- Dusseault, M.B., Yin, S., Rothenburg, L., & Han, H. (2007). Seismic Monitoring and Geomechanics Simulation, The Leading Edge, 26, pp.610-620.
  - 21- Dusseault, M.B. (2008). Coupling Geomechanics and Transport in Petroleum Engineering. Proceedings 1<sup>st</sup> Southern Hemisphere International Rock Mechanics Symposium, Perth, Western Australia.
  - 22- Dusseault, M.B. (2011). Geomechanical challenges in petroleum reservoir exploitation. Korean Journal of Civil Engineering, Special Issue on Energy Geomechanics, in press
-

- 23- Fredrich, J., Arguello, J., Thorne, B., Wawersik, W., Deitrick, G., de Rouffignac, E. (1996). Three-Dimensional Geomechanical Simulation of Reservoir Compaction and Implications for Well Failures in the Belridge Diatomite. SPE Annual Technical Conference and Exhibition, (pp. 195-210). Denver, Colorado. SPE 36698.
  - 24- Fung, L., Buchanan, L., & Wan, R. (1994). Coupled Geomechanical-Thermal Simulation for Deforming Heavy-Oil Reservoirs. *Journal of Canadian Petroleum Technology*, 33, pp.22-28.
  - 25- Gai, X., Dean, R., Wheeler, M., & Liu, R. (2003). Coupled Geomechanical and Reservoir Modeling on Parallel Computers. SPE Reservoir Simulation Symposium. Houston, Texas. SPE 79700.
  - 26- Geertsma, J. (1957). The Effect of Pressure Decline on Volumetric Changes of Porous Rocks. *Trans. AIME*, 201, pp.331-340.
  - 27- Geertsma, J. (1966). Problems of Rock Mechanics in Petroleum Production Engineering. First Congress on International Society of Rock Mechanics, 1, pp.585-594, Lisbon.
  - 28- Ghaboussi, J., & Wilson, E. (1973). Flow of Compressible Fluid in Porous Elastic Media. *International Journal for Numerical Methods in Engineering*, 5 (3), pp.419-442.
  - 29- Ghafouri, H., & Lewis, R. (1996). A Finite Element Double Porosity Model for Heterogeneous Deformable Porous Media. *International Journal for Numerical and Analytical Methods in Geomechanics*, 20 (11), pp.831-844.
  - 30- Goodman, R.E., Taylor, R.L., & Brekke, T.L. (1968). A Model for the Mechanics of Jointed Rock. *Journal of the Soil Mechanics and Foundations Division, ASCE* 94, SM3, pp.637-659.
  - 31- Goodman, R.E. (1976). *Methods of Geological Engineering in Discontinuous Rock*. New York. 472 p.
  - 32- Goodman, R.E., & Shi, G.H. (1985). *Block Theory and Its Application to Rock Engineering*. New Jersey, Prentice Hall.
  - 33- Granet, S., Fabrie, P., Lemonnier, P., & Quintard, M. (2001). A Two-Phase Flow Simulation of a Fractured Reservoir Using a New Fissure Element Method. *Journal of Petroleum Science and Engineering*, 32 (1), pp.35-52.
  - 34- Gross, J., & Seelig, T. (2006). *Fracture Mechanics with an Introduction to Micromechanics*. The Netherlands: Springer.
  - 35- Gutierrez, M., & Lewis, R. (1998). The Role of Geomechanics in Reservoir Simulation. SPE/ISRM Eurock-98 Symposium, (pp. 439-448). Trondheim, Norway. SPE 47392.
  - 36- Hahn, J.K. (1988). Realistic Animation of Rigid Bodies. *Computer Graphics*, 24 (4), pp.299-308.
  - 37- Henn, N., Bourbiaux, B., Quintard, M., & Sakthikumar, S. (2000). Modeling Fluid Flow in Reservoir Crossed by Multiscale Fractures: A New Approach. 7<sup>th</sup> European Conference on Mathematic Oil Recovery. Baveno, Italy.
  - 38- Hocking, G., Mustoe, G.G.W., & Williams, J.R. (1985). *CICE Discrete Element Code - Theoretical Manual*. Lakewood, Colorado: Applied Mechanics Inc.
  - 39- Hojka, K., Dusseault, M., & Bogobowicz, A. (1993). Analytical Solutions for Transient Thermoelastic Stress Fields around a Borehole during Fluid Injection into Permeable Media. *Journal of Canadian Petroleum Technology*, 32 (4), pp.49-57.
  - 40- Huyghe, J.M., Raats, P.A.C., & Cowin, C. (eds.) (2005). Section 3 on Poromechanics, Geomechanics and Porous Media in IUTAM Symposium on Physicochemical and Electromechanical Interactions in Porous Media. *Solid Mechanics and its Applications Series XXVIII*. Berlin, Springer, 373 p.
  - 41- Jing, L. (2003). A Review of Techniques, Advances and Outstanding Issues in Numerical Modelling for Rock Mechanics and Rock Engineering. *International Journal of Rock Mechanics & Mining Sciences*, 40, pp.283-353.
-



- 
- 42- Jing, L., & Stephansson, O. (2007). *Fundamentals of Discrete Element Methods for Rock Engineering – Theory and Applications*. Stockholm: Elsevier.
  - 43- Kazemi, H., Merrill, L., Porterfield, K., & Zeman, P. (1976). Numerical Simulation of Water-Oil Flow in Naturally Fractured Reservoirs. *SPE Journal*, SPE 5719 , PP.317-326.
  - 44- Kulhaway, F.H. (1975). Stress-Deformation Properties of Rock and Rock Discontinuities. *Engineering Geology*, 8, pp.327-350.
  - 45- Lanru, J., & Xiating, F. (2003). Numerical Modeling for Coupled Thermo-Hydro-Mechanical and Chemical Processes (THMC) of Geological Media – International and Chinese Experiences. *Chinese Journal of Rock Mechanics and Engineering*, 22 (10), pp.1704-1715.
  - 46- Lemos, J.V. (1987). A Hybrid Distinct Element Computational Model for the Half-Plane. MSc thesis, Department of Civil Engineering, University of Minnesota.
  - 47- Lewis, R., & Schrefler, B. (1998). *The Finite Element Method in the Static and Dynamic Deformation and Consolidation of Porous Media* (2nd ed.). New York: Wiley.
  - 48- Lewis, R., & Sukirman, Y. (1994). Finite Element Modelling for Simulating the Surface Subsidence above a Compacting Hydrocarbon Reservoir. *International Journal for Numerical and Analytical Methods in Geomechanics* , 18, pp.619-639.
  - 49- Mainguy, M., & Longuemare, P. (2002). Coupling Fluid Flow and Rock Mechanics: Formulation of the Partial Coupling between Reservoir and Geomechanical Simulators. *Oil & Gas Science and Technology* , 57 (4), pp.355-367.
  - 50- Masters, I., Pao, W., & Lewis, R. (2000). Coupling Temperature to a Double Porosity Model of Deformable Porous Media. *International Journal for Numerical Methods in Engineering* , 49, pp.421-438.
  - 51- Minkoff, S., Stone, C., Arguello, J., Bryant, S., Eaton, J., Peszynska, M. (1999). Staggered in Time Coupling of Reservoir Flow Simulation and Geomechanical Deformation: Step 1 - One-Way Coupling. *SPE Reservoir Simulation Symposium*. Houston, Texas. SPE 51920.
  - 52- Minkoff, S., Stone, C., Bryant, S., Peszynska, M., & Weeler, M. (2003). Coupled Fluid Flow and Geomechanical Deformation Modeling. *Journal of Petroleum Science and Engineering* , 38, pp.37-56.
  - 53- Narr, W., Schechter, D.W. & Thompson, L.B. (2006). *Naturally Fractured Reservoir Characterization*. Textbook Series, SPE, Richardson, Texas.
  - 54- Nelson, R.A. (2001). *Geologic Analysis of Naturally Fractured Reservoirs*. 2nd ed. Gulf Professional Publishing.
  - 55- Nur, A. & Byerlee, J. (1971). An Exact Effective Stress Law for Elastic Deformation of Rock with Fluid. *Journal of Geophysical Research* , 76, pp.6414-6418.
  - 56- Ohkami, T., Mitsui, Y., & Kusama, T. (1985). Coupled Boundary Element/Finite Element Analysis in Geomechanics Including Body Forces. *Computers and Geotechnics*, 1, pp.263–278.
  - 57- Osorio, J., Chen, H., Tuefel, J., & Schaffer, S. (1999). A Two-Domain, Fully Coupled Fluid Flow/Geomechanical Simulation Model for Reservoirs with Stress-Sensitive Mechanical and Fluid-Flow Properties. *SPE/ISRM Rock Mechanics in Petroleum Engineering*, (pp. 455-464). Trondheim, Norway. SPE 47397.
  - 58- Pao, W., Lewis, R., & Masters, I. (2001). A Fully Coupled Hydro-Thermo-Poromechanical Model for Black Oil Reservoir Simulation. *International Journal for Numerical and Analytical Methods in Geomechanics* , 25, pp.1229-1256.
  - 59- Pao, W., & Lewis, R. (2002). Three-Dimensional Finite Element Simulation of Three-Phase Flow in a Deforming Fissured Reservoir. *Computer Methods in Applied Mechanics and Engineering* , 191, pp.2631-2659.
  - 60- Perkins, T.K. & Gonzalez, J.A. (1985). The Effect of Thermoelastic Stresses on Injection Well Fracturing. *Society of Petroleum Engineers Journal*, 25(1), pp.78-88.
-

- 
- 61- Rice, J., & Cleary, M. (1976). Some Basic Stress-Diffusion Solutions for Fluid Saturated Elastic Porous Media with Compressible Constituents. *Reviews of Geophysics and Space Physics*, 14, pp.227-241.
  - 62- Rutqvist, J., Wu, Y., Tsang, C., & Bodvarsson, G. (2002). A Modeling Approach for Analysis of Coupled Multiphase Fluid Flow, Heat Transfer, and Deformation in Fractured Porous Rock. *International Journal of Rock Mechanics & Mining Sciences*, 39, pp.429-442.
  - 63- Salamon, M.D.G. (1963). Elastic Analysis of Displacements and Stresses Induced by the Mining of Seam or Reef Deposits. *Journal of the South African Institute of Mining and Metallurgy*, 64, pp.128-149.
  - 64- Salamon, M.D.G. (1968). Two-dimensional treatment of problems arising from mining tabular deposits in isotropic or transversely isotropic ground. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts* 5(2), pp.159-185
  - 65- Schiffman, R. (1971). A Thermoelastic Theory of Consolidation. *Environmental and Geophysical Heat Transfer, ASCE*, pp.78-84.
  - 66- Schrefler, B., Matteazzi, R., Gawin, D., & Wang, X. (2000). Two Parallel Computing Methods for Coupled Thermo-Hydro-Mechanical Problems. *Computer-Aided Civil and Infrastructure Engineering*, 15, pp.176-188.
  - 67- Settari, A., & Mourits, F. (1994). Coupling of Geomechanics and Reservoir Simulation Models. (Siriwardane, & Zaman, Eds.) *Computer Methods and Advances in Geomechanics*, pp. 2151-2158.
  - 68- Settari, A., & Mourits, F. (1998). A Coupled Reservoir and Geomechanical Modeling System. *SPE Journal*, SPE 50939, pp.219-226.
  - 69- Settari, A., & Walters, D. (1999). Advances in Coupled Geomechanical and Reservoir Modeling with Applications to Reservoir Compaction. *Reservoir Simulation Symposium*. Houston, Texas. SPE 51927.
  - 70- Settari, A., Walters, D., & Behie, G. (2001). Use of Coupled Reservoir and Geomechanical Modelling for Integrated Reservoir Analysis and Management. *Journal of Canadian Petroleum Technology*, 40 (12), pp.55-61.
  - 71- Shi, G.H. (1989). Discontinuous Deformation Analysis - A New Numerical Model for the Statics and Dynamics of Block Systems. Lawrence Berkeley Laboratory, Report to DOE OWTD, Contract AC03-76SF0098.
  - 72- Skempton, A. (1954). The Pore Pressure Coefficients A and B. *Geotechnique*, 4, pp.143-147.
  - 73- Spanos, T., Udey, N., & Dusseault, M.B. (2002). Completing Biot Theory, Proc. 2<sup>nd</sup> Biot Conf. on Poromechanics. Grenoble France. pp. 819-826.
  - 74- Stone, T., Xian, C., Fang, Z., Manalac, E., Marsden, R., & Fuller, J. (2003). Coupled Geomechanical Simulation of Stress Dependent Reservoirs. *SPE Reservoir Simulation Symposium*. Houston, Texas. SPE 79697.
  - 75- Sulak, R., Thomas, L., & Boade, R. (1991). 3D Reservoir Simulation of Ekofisk Compaction Drive. *Journal of Petroleum Technology*, 43, pp.1272-1278.
  - 76- Tortike, W., & Farouq Ali, S. (1993). Reservoir Simulation Integrated with Geomechanics. *Journal of Canadian Petroleum Technology*, 32 (5), pp.28-37.
  - 77- Tran, D., Nghiem, L., & Buchanan, L. (2005). An Overview of Iterative Coupling between Geomechanical Deformation and Reservoir Flow. *International Thermal Operations and Heavy Oil Symposium*. Calgary. SPE 97879.
  - 78- Universal Distinct Element Code (UDEC) Manual, ITASCA Consulting Group, Inc. 2006, [www.itascacg.com](http://www.itascacg.com).
  - 79- Van der Knaap, W. (1959). Nonlinear Behavior of Elastic Porous Media. *Trans. AIME*, 216, pp.179-187.
-

- 
- 80- Van Lingen, P., Daniel, J.M., Cosentino, L., & Sengul, M. (2001). Single Medium Simulation of reservoirs with Conductive Faults and Fractures. SPE Middle East Oil Show. Bahrain. SPE 68165.
  - 81- Varadarajan, A., Sharma, K.G., & Singh, R.B. (1985). Some Aspects of Coupled FEBEM Analysis of Underground Openings. *International Journal for Numerical and Analytical Methods in Geomechanics*, 9, pp.557–571.
  - 82- Von Estorff, O., & Firuziaan, M. (2000). Coupled BEM/FEM Approach for Non-Linear Soil/Structure Interaction. *Engineering Analysis with Boundary Elements*, 24, pp.715–725.
  - 83- Walton, O.R. (1980). Particle Dynamic Modeling of Geological Materials. Lawrence Livermore National Laboratory, Report UCRL-52915.
  - 84- Wang, Y., & Dusseault, M.B. (2003). A Coupled Conductive-Convective Thermo-poroelastic Solution and Implications for Wellbore Stability. *Journal of Petroleum Science and Engineering*, 38, pp.187-198.
  - 85- Warburton, P.M. (1981). Vector Stability Analysis of an Arbitrary Polyhedral Rock Block with any Number of Free Faces. *International Journal of Rock Mechanics and Mining Science & Geomechanics Abstracts*, 18, pp.415-427.
  - 86- Warren, J., & Root, P. (1963). The Behavior of Naturally Fractured Reservoirs. *SPE Journal*, SPE 426 , pp.245-255.
  - 87- Yin, S., Dusseault, M., & Rothenburg, L. (2007a). Analytical and Numerical Analysis of Pressure Drawdown in a Poroelastic Reservoir with Complete Overburden Effect Considered. *Advances in Water Resource*, 30, pp.1160-1167.
  - 88- Yin, S., Dusseault, M., & Rothenburg, L. (2007b). Coupled Multiphase Poroelastic Analysis of Reservoir Depletion Including Surrounding Strata. *International Journal of Rock Mechanics and Mining Sciences*, 44, pp.758-466.
  - 89- Zienkiewicz, O.C., Best, B., Dullage, C., Stagg, K. (1970). Analysis of Nonlinear Problems in Rock Mechanics with Particular Reference to Jointed Rock Systems. *Proceedings of the 2<sup>nd</sup> International Congress on Rock Mechanics*. Belgrade.
  - 90- Zienkiewicz, O.C., Kelly, D.W., & Bettess, P. (1977). The Coupling of the Finite Element Method and Boundary Solution Procedures. *International Journal for Numerical Methods in Engineering*, 11, pp.355–375.
-