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Numerical Modelling of the Segmental Lining of Underground Structures

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Abstract

There are several methods for analysing the behaviour of underground structures under different loading conditions. Most of these methods have many simplifications; therefore, in some cases, the results are too conservative and a very high safety factor, usually of more than 2 is needed. On the other hand, for stability analysis and the designing of support systems, these methods consider segmental lining and its joints as a uniform lining or a lining with pin connections. In this study, numerical modelling of the segmental lining of a tunnel was analysed using a sensitivity analysis of the static modelling. The numerical results were obtained by using a finite difference method (FLAC2D). Using this form of analysis, a new simple methodology was introduced so that more reliable results can be obtained. By comparing the frame analysis results obtained by the SAP2000 software with those obtained by the proposed method, it was concluded that the suggested method can be used as a simple and reasonable approach for the segmental lining of underground structures such as tunnels.

Keywords: *frame analysis, underground structures, numerical modelling, segmental tunnel lining, sensitivity analysis.*

1. Introduction

Tunnelling projects are often deemed as some of the most challenging and costly underground projects. This has prompted many to research ways for improving the constructability and stability of tunnels and to reduce the high costs involved in such underground structures. In recent years, by using segmental lining to improve the speed of construction and eventually reduce the costs of tunnelling, more attention has been focused on the design of this type of lining.

The first concrete segmental support on record appears to date back to 1903 and was developed by the British contractor McAlpine in Glasgow, Scotland. Since the use of pre-cast concrete segmental lining for the eastward extension of the London Transport Central Line in 1937, a large number of different concrete linings have been developed [1].

The effect of joints between segmental linings is one of the most influential factors affecting the stresses induced in tunnel lining, which is often ignored in tunnel design. If the joints in the lining are not considered, a conservative simplification will result in the overestimation of the bending moments, whilst the normal forces will remain unaffected by this assumption [2].

The most important problem of segmental lining design theory is whether the design model can reflect the actual stresses of the segments. Commonly used methods for design calculations are classified by joint evaluations as follows: i) the usual calculation; ii) the modified usual calculation; iii) the ring with multiple hinged joints calculation; iv) the beam-spring model calculation.

i) The usual calculation method assumes that the segmental ring is a ring with uniform bending rigidity and ignores the decrease of rigidity at segment joints. In this method, neither the segment (longitudinal) nor the ring (circumferential) joints are considered [3].

ii) In the modified usual calculation method, a coefficient of the effective ratio of bending rigidity ($\eta \leq 1$) for evaluating the rigidity of joints, a bending rigidity as ηEI, a transfer ratio of bending moment ξ, the moment of the main section as $(1+\xi)M$ and the moments of the joints as $(1-\xi)M$ are calculated. Then, the value of η and ξ are primarily determined by experiences that consider the joint performance test results and records of other projects, which are random and uncertain [4].

iii) The ring with multiple hinged joints calculation method is utilized under good ground conditions. The longitudinal joints are treated as hinges and their influence is exaggerated; however, the influence of circumferential joints is not considered [5].

iv) The beam-spring model calculation method assumes the segmental ring as a ring with rotational and shear springs. In this method, the reduction of bending rigidity and the splice effects of staggered arrangement are evaluated by using a special model that considers the segment as a curved or straight beam. Furthermore, the longitudinal and circumferential joints are assumed as a rotational and shear spring, respectively [6].

For the usual cases where loads and the structure do not change in the longitudinal direction, the three-dimensional behaviour of the segments has no significant influence on the system. This means that for these types of load configurations, two-dimensional analyses are sufficient. For special cases such as openings in the lining, various loads on the rings (e.g., swelling in partial areas), varying bending conditions for the rings (e.g., if the grouting of the tail gap had not been done properly at one ring), etc., internal forces and deformations of the lining can be only predicted by 3D analysis [6]. In 3D calculations, the stiffness coefficients of the longitudinal and circumferential joints were determined by laboratory tests using the actual joints or theoretical calculations. The lateral earth pressure coefficient and the coefficient of subgrade reaction were determined from previous case studies using similar ground conditions [7].

In this study, the induced bending moments were calculated from simple static analyses using $FLAC^{2D}$, on account of the progressive reduction of some parts of the lining corresponding to the segments' connections. Finally, by conducting the sensitivity analysis, a model was chosen so that the bending moment of lining in all joint locations were close to zero. As a case study, the results of the segmental lining modelling of the Tabriz Urban Railway (TUR) were compared with the frame analysis results obtained by the *SAP2000* software. This comparison showed that the suggested method can be used as a simple and rational method for the lining of segmented tunnels.

2. Analytical methods for designing of segmental lining with equal segments

From a mechanical point of view, the ground pressure on the longitudinal joints can be assumed as the compressive axial forces and bending moment (REF). The most important point is that the amounts of bending moment in the vicinity of joints will be reduced due to the reduction of the moment of inertia. Thus, as shown in Figure 1, these regions can be modelled in the form of the reduced thickness of lining corresponding to the contact area of the lining segments [3].

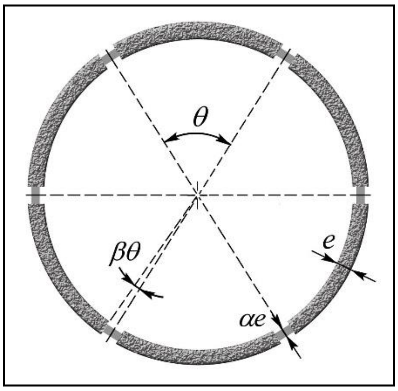


Fig. 1. A typical model of a segmental ring with uniform and regular segments (modified from [3])

In design practice, there are many known structural systems for calculating internal force within the tunnel lining. In the simplest of these, a rigid bedded ring is used. This model does not take the behaviour of the joints into account. For an uncoupled system of hinged rings, the estimated bending moments are too high and should give conservative results.

Muir Wood [3] developed a very easy to use empirical formula for the estimation of the effects of the longitudinal joints of uncoupled rings in a calculation with a homogenous rigid ring by reducing the bending stiffness of the lining. The maximum bending moments calculated with this approach are close to those of uncoupled hinged rings. In the relations proposed by Muir Wood, the theoretical axial stiffness modulus for a cylindrical shell is defined by the following equation [3].

$$E = \frac{\alpha}{\alpha(1-\beta)+\beta} E_m \qquad (1)$$

where, according to Figure 1, e is the thickness of the ring, E is equilibrium modulus, α is reduced thickness corresponding to the longitudinal joints, θ is the central angle related to each segment, ß is the average circular width of each longitudinal joint, R is the radius of the ring and E_m is elasticity modulus of ring material.

The dimensionless coefficient ß is very small (several times of 10^{-3}); thus, the joints do not change the axial stiffness modulus of the lining. Under these conditions, the bending stiffness modulus can be calculated by Equation 1 and the corresponding moment of inertia of the lining can be defined as Equation [2].

$$I = I_j + \left(\frac{4}{n}\right)^2 \frac{e^3}{12} \qquad (2)$$

where I_j is the moment of inertia in joints and n is the number of existing joints in a ring. This formula was accepted for $n>4$. It is also assumed that the joints of ring are not opened in the inner or outer face of the contacts. Thus, if the moment of the inertia of joints is considered as $I = 0.57\frac{e^3}{12}$, then the equal moment of inertia can be expressed as:

$$I = \left[\alpha^3 + \left(\frac{4}{n}\right)^2\right]\frac{e^3}{12} \qquad (3)$$

where, for α = 0.5 and n = 6, it can be shown that $I = 0.57\frac{e^3}{12}$. This moment of inertia can be defined as $I = \frac{e^3}{12}$ for a cylindrical shell. Therefore, the bending stiffness of the segmental lining must be lower than that of the continual lining with the same properties; also, the bending moment will be decreased at the joints [3].

On the other hand, for four or fewer segments, Muir Wood (1975) suggested that the existence of joints will not affect the rigidity of the lining. However, for a lining with multiple segments, the stiffness at the joint may be appreciably less than that of the lining; thus, reducing the bending moment in the lining is an approach for considering the joints.

In the equations suggested by Muir Wood [3], it is assumed that the joints at all rings are regular (i.e., the longitudinal joints are in a line), whereas this is not truly the case and a joint on the nth ring increases the bending moment on the same location of the joints, i.e., on the adjacent rings ((n-1) and (n+1) rings). Thus, non-consecutive joints decrease the lining deformation. Generally, the bending stiffness of the lining is exceeded at levels high above its actual value and the Muir Wood formula cannot predict this. However, this approach is nonetheless useful for gaining a first idea of the forces in the lining; α and *ß* coefficients can be determined by experiences of past similar projects or laboratory models.

However, Muir Wood did not take into consideration certain structural effects such as the joint arrangement. Lee and Ge [8] did, however, consider these structural effects. They found that the joint stiffness ratio and the radius of the tunnel had a large influence on the effective rigidity ratio, whereby either a reduction in the joint stiffness ratio or the radius of tunnel will result in a substantial drop of effective rigidity ratio. This result implies that the bending moment of the lining also decreases. If the number of joints were to increase, the effective rigidity ratio and the bending moment of the lining will decrease substantially, due to the increasing flexibility of the lining. Other factors such as joint distribution do not affect the bending moment in the lining very much. Though Lee and Ge took into consideration some of the effects ignored by Muir Wood, their design method often requires rather long iterations in addition to approximations in the assumed loadings. Therefore, in continuation of these studies, Hefny et al. [9] investigated the factors affecting the stresses induced in the tunnel lining with an emphasis on the effects of the number and orientation of longitudinal joints. Additionally, they proposed a simple design methodology for determining the stresses induced in jointed linings, which can be carried out without incorporating the joints in the analysis. In Hefny et al.'s model, an equivalent tunnel is defined as an unjointed tunnel that has a lining thickness with the same maximum stresses as those found in a jointed tunnel lining. The thickness of unjointed tunnel lining is varied to achieve the maximum stresses were induced in the jointed tunnel lining. These maximum stresses are matched with the maximum stresses obtained in the both conditions of critical and most favourable orientation of the joints. [9]. However, this design method requires more iteration to compare the jointed and unjointed linings.

3. Geological and geotechnical characteristics of the Tabriz Urban Railway (TUR) site

The case study of this research is the Tabriz Urban Railway (TUR) twin tunnels, which is located in Tabriz city, in the centre of East-Azerbaijan province (Fig. 2). This province is located in the northwest of Iran, i.e., N 38°, E 46° [10].

Fig. 2. The location of Tabriz city [11]

Line 1 of the TUR tunnels for a total length of 7.2 km was excavated by two TBM-EPB machines. The depth of overburden in the cross section of the tunnels was about 7 m and consists of three layers: gravelly sand, silty/clayey sand and clay (respectively, starting from the surface). The average values of the geotechnical parameters for these three soil layers are listed in Table 1 [12].

The groundwater level in the tunnel section under consideration in this study was at a 21.5 m depth, i.e., about 7 m below the tunnel bottom [14]. Therefore, in the all models and analyses of this research, the groundwater and its effects on the tunnels were neglected. The geometry of the tunnels' cross-section is described in Figure 3.

Table 1. Geotechnical parameters for three layers of the studied section [13]

Thickness of layer (*m*)	Soil Class.	γ (Kn/m^3)	*C* (*kPa*)	φ (*deg.*)	*E* (*MPa*)
6	Gravelly Sand	20.0	8	23	23
24	Silty/Clayey Sand	18.0	5	35	23
10	Clay	15.0	34	19	23

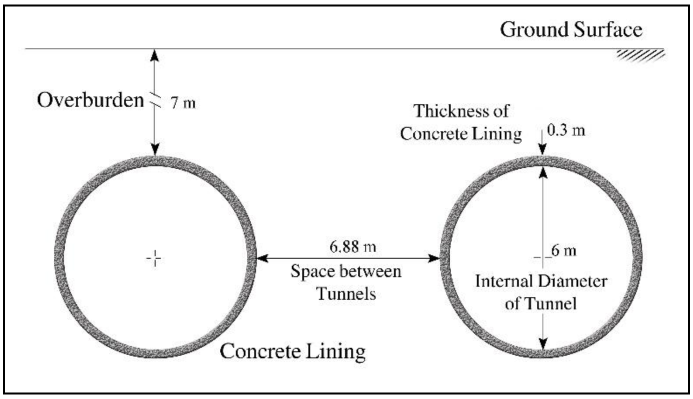


Fig. 3. Geometry of the tunnels

4. Numerical modelling of segmental lining with non-uniform segments

According to numerical modelling principles, the modelling steps taken by *FLAC*^{2D} include [15]:

1. the boundary limitation around the tunnels by assuming the plane strain condition and selecting the model dimensions as 40×150 m^2;
2. defining the constitutive model and its properties, where the Mohr-Coulomb material model and elastic beam element have been used to consider the plastic behaviour of alluvial ground material and the behaviour of tunnel lining, respectively;
3. drawing of the tunnels' geometry;
4. model solving and its establishment prior to starting tunnelling;
5. beginning tunnels' excavation and exerting the appropriate stress relaxation;
6. installing the tunnel lining, so that as shown in Figure 4, the lining may consist of 5+1 segments (1 corresponds to the key segment) with a 30 cm thickness [16].

The original idea of using the numerical modelling of segmental tunnel lining was that the induced bending moment in a jointed lining is reduced in the joints location. Thus, the segments could be simulated by a reduced thickness of the lining at these locations. This is clearly shown in Figure 4 below.

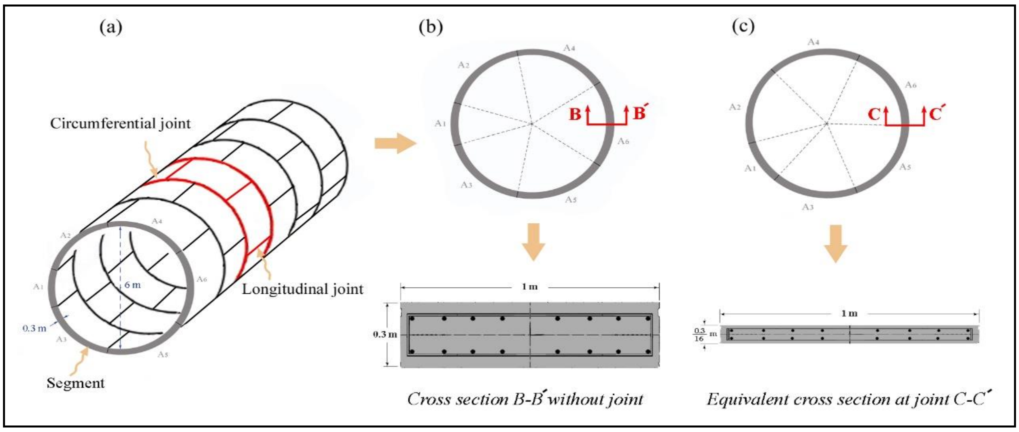


Fig. 4. The original idea of using segmental lining modelling with non-uniform segments

In the numerical modelling of TUR tunnels lining, the lining consisted of six non-uniform segments (5+1), as shown in Figure 4. Two series models with 15 and 10 cm of mesh dimensions were created by *FLAC*^{2D}. The reason for making a fine mesh model (10 cm meshing) was to control the deflection effect, i.e., the reduction of beam elements bending moment in the joints.

Finally, one may observe that deflection had no effect on the bending moment reduction of these beam elements. In each series of modelling, after finding the joint locations on the beam elements, the thickness of these elements was reduced by 1/2, 1/4, 1/8, 1/16, 1/32 and 1/64 times of the original thickness. The effect of this process on the induced bending moment in the lining could then be estimated, i.e., the best state may happen when the bending moment of the lining in all joint locations were close to zero.

The charts in Figures 5 and 6 show the induced bending moments in the mentioned status for models with 15 and 10 cm mesh dimensions, respectively. As shown in these charts, a reduced thickness of about 1/8 to 1/16 had the best situation in the non-uniform segmental lining simulation for models with 15 cm meshing. In the models with 10 cm meshing, this reduced thickness was 1/16. However, in this series of models, the induced bending moments, instead of six points (joint locations) were close to zero in an additional point. To remove this difficulty, the same problem was modelled by *SAP2000* structural analysis software.

In the modelling of tunnel lining, a 24-piece frame (as the element number applied in *FLAC*) was used as concrete beams. To appropriately consider the surrounding ground effects on the tunnel lining in the *SAP2000* model (according to Fig. 7), the engineering and design guidance of tunnels and shafts (EM 1110-2-2901) were used, the tangential and radial springs were applied and the soil-structure interactions were restricted. The stiffness of radial and tangential springs was obtained from Equations 4 and 5, respectively [17].

$$k_r = \frac{E_m b\theta}{(1+\nu_m)} \tag{4}$$

$$k_t = k_r \frac{G_m}{E_m} = \frac{k_r}{2(1+\nu_r)} \tag{5}$$

where, E_r and G_r are the elasticity and shear modulus, respectively, V_m is the Poisson ratio of the surrounding ground materials and b is the tunnel length in the longitudinal direction or ring width. In this study, b was taken as 1 m for all models. The parameter θ is an arc angle of the beam element in the radian.

After exerting the loads due to overburdening and ground convergence on the tunnel lining, the under tension springs were omitted. Finally, the induced bending moment on the segmental lining due to the in situ stresses of the ground is shown in Figure 8. Comparing Figure 6 (concerned with segment t/16 in the graph) and Figure 8 provided a good resemblance between the bending moment results obtained from the *SAP2000* and *FLAC*^{2D} models.

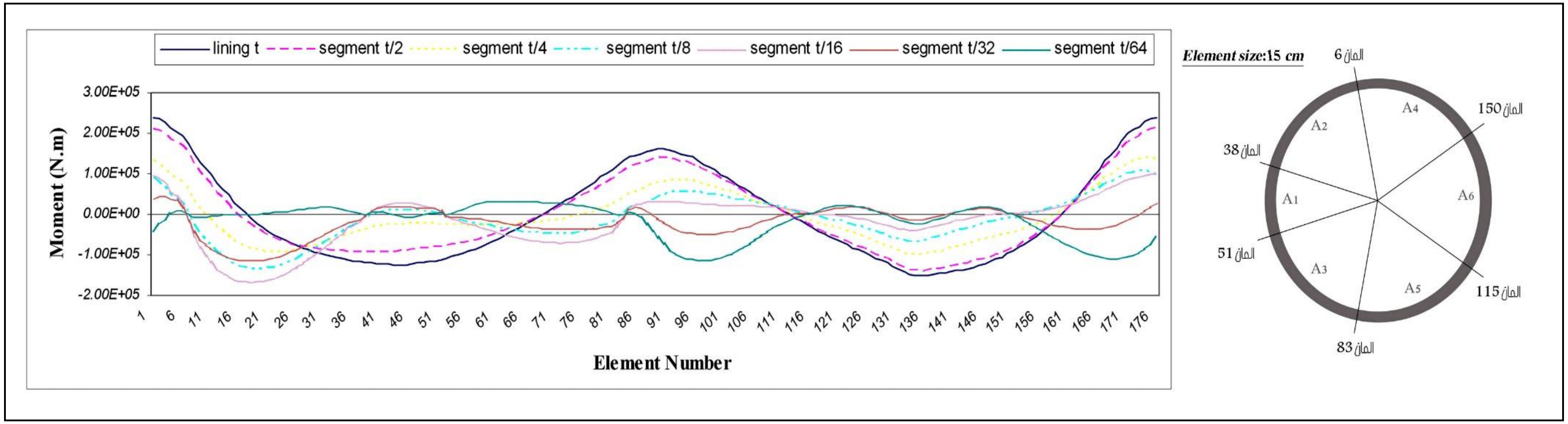


Fig. 5. The induced bending moment in different elements of segmental lining in models with 15 *cm* meshing

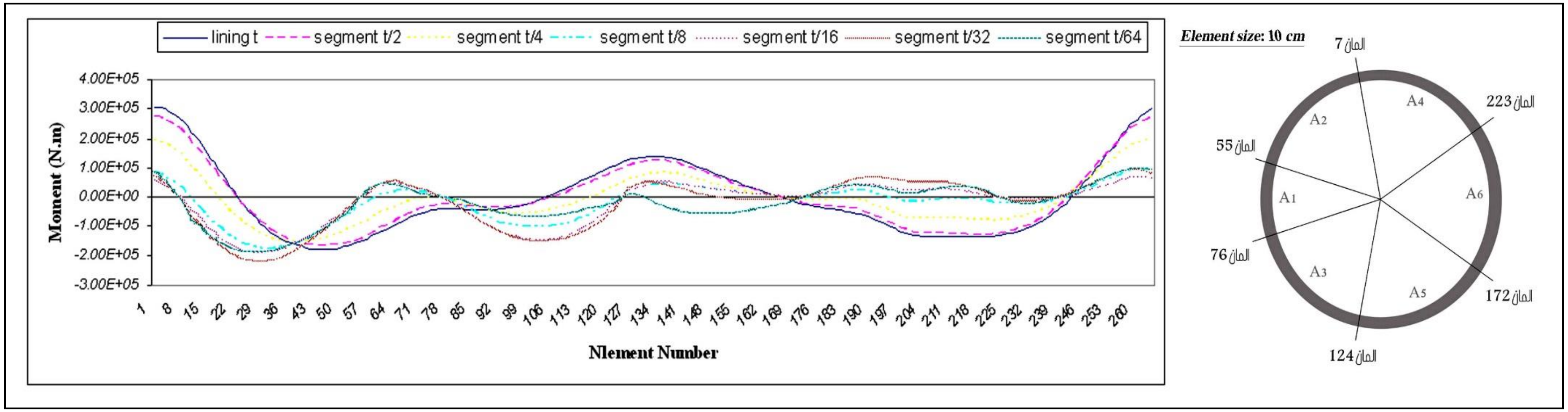


Fig. 6. The induced bending moment in different elements of segmental lining in models with 10 cm meshing

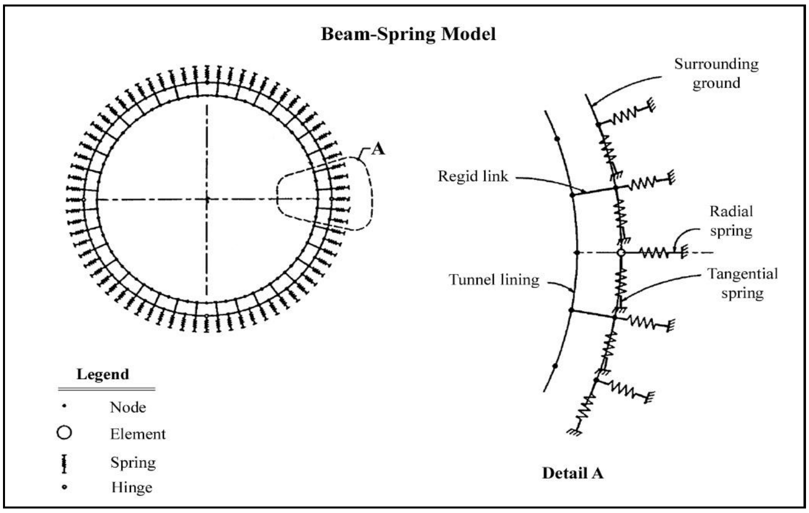


Fig. 7. The beam-spring model applied in the *SAP2000* model containing soil-structure interaction (taken from [17])

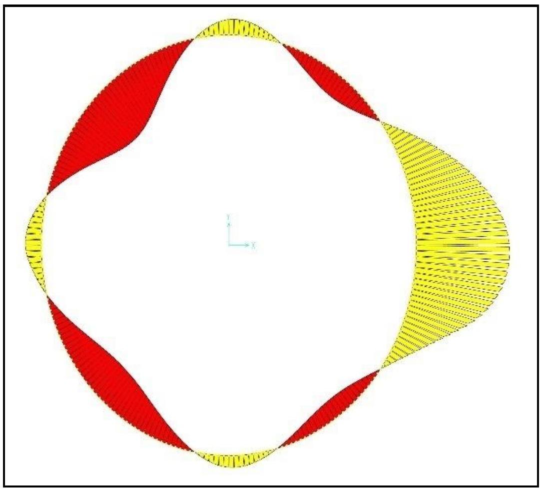


Fig. 8. The induced bending moment on the segmental lining model in SAP20005. Comparing the behaviour of continual and segmental linings

5. Comparing the behaviour of continual and segmental linings

In this part of the paper, the behaviour of both continual and segmental linings duos to the distribution of induced axial and shear forces and bending moments are compared, using the same model with 10 cm meshing. The results of the static analysis of these two models are shown in Figures 9 to 16 below.

Comparing the above results shows some differences between these two modelling results; therefore, the effect of joints must be considered in the analyses. To observe the further differences in the results of these two modelling schemes, the graphs shown in Figures 17 and 18 must be considered, as they illustrate the different induced bending moments in the two models.

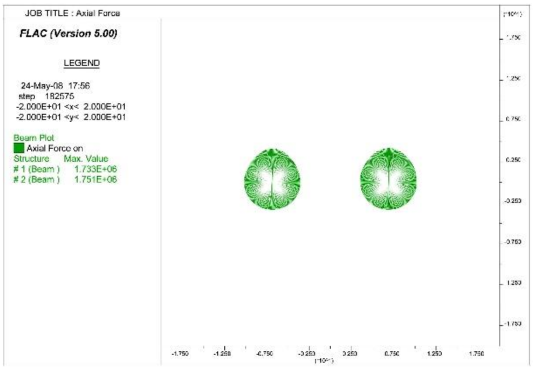


Fig. 9. The induced axial force on the continual lining

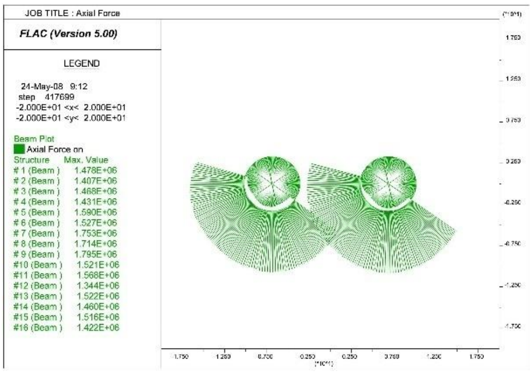
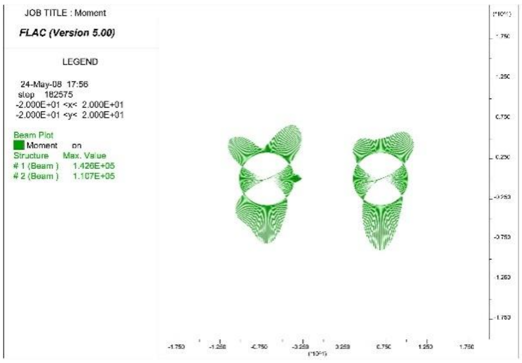
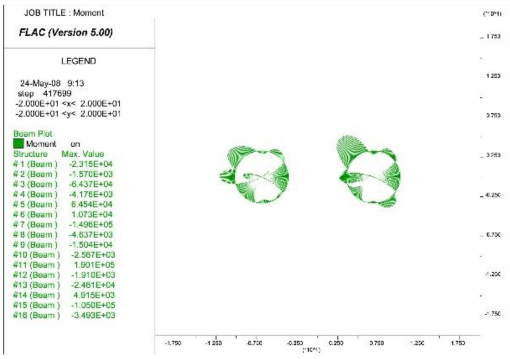


Fig. 10. The induced axial force on the segmental lining



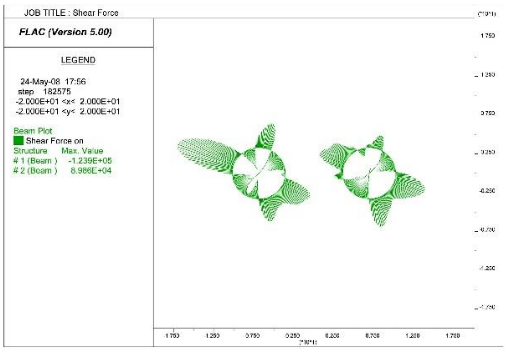
Structure	Max. Value
# 1 (Beam)	1.426E+05
# 2 (Beam)	1.107E+05

Fig. 11. The induced bending moment on the continual lining



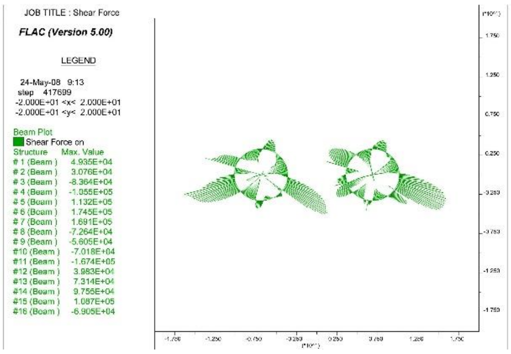
Structure	Max. Value
# 1 (Beam)	-2.315E+04
# 2 (Beam)	-1.570E+03
# 3 (Beam)	-6.437E+04
# 4 (Beam)	-4.176E+03
# 5 (Beam)	6.454E+04
# 6 (Beam)	1.073E+04
# 7 (Beam)	-1.496E+05
# 8 (Beam)	-4.637E+03
# 9 (Beam)	-1.504E+04
#10 (Beam)	-2.567E+03
#11 (Beam)	1.901E+05
#12 (Beam)	-1.910E+03
#13 (Beam)	-2.461E+04
#14 (Beam)	4.915E+03
#15 (Beam)	-1.050E+05
#16 (Beam)	-3.493E+03

Fig. 12. The induced bending moment on the segmental lining



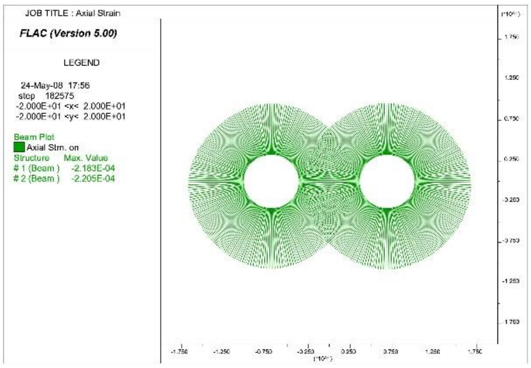
Structure	Max. Value
# 1 (Beam)	-1.239E+05
# 2 (Beam)	8.986E+04

Fig. 13. The induced shear force on the continual lining



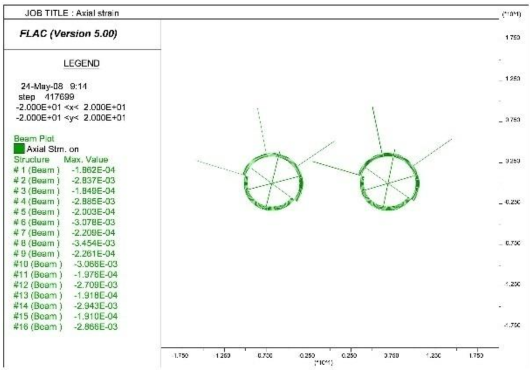
Structure	Max. Value
# 1 (Beam)	4.935E+04
# 2 (Beam)	3.076E+04
# 3 (Beam)	-8.364E+04
# 4 (Beam)	-1.055E+05
# 5 (Beam)	1.132E+05
# 6 (Beam)	1.745E+05
# 7 (Beam)	1.691E+05
# 8 (Beam)	-7.264E+04
# 9 (Beam)	-5.605E+04
#10 (Beam)	-7.018E+04
#11 (Beam)	-1.674E+05
#12 (Beam)	3.983E+04
#13 (Beam)	7.314E+04
#14 (Beam)	9.755E+04
#15 (Beam)	1.087E+05
#16 (Beam)	-6.905E+04

Fig. 14. The induced shear force on the segmental lining



Structure	Max. Value
# 1 (Beam)	-2.183E-04
# 2 (Beam)	-2.205E-04

Fig. 15. The induced axial strain on the continual lining



Structure	Max. Value
# 1 (Beam)	-1.862E-04
# 2 (Beam)	-2.837E-03
# 3 (Beam)	-1.849E-04
# 4 (Beam)	-2.885E-03
# 5 (Beam)	-2.003E-04
# 6 (Beam)	-3.078E-03
# 7 (Beam)	-2.209E-04
# 8 (Beam)	-3.454E-03
# 9 (Beam)	-2.261E-04
#10 (Beam)	-3.068E-03
#11 (Beam)	-1.976E-04
#12 (Beam)	-2.709E-03
#13 (Beam)	-1.918E-04
#14 (Beam)	-2.943E-03
#15 (Beam)	-1.910E-04
#16 (Beam)	-2.866E-03

Fig. 16. The induced axial strain on the segmental lining

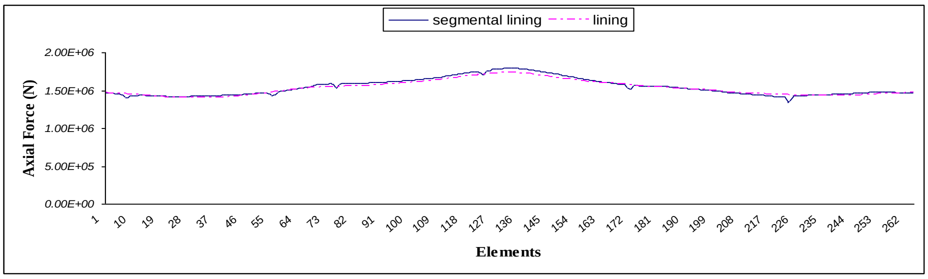


Fig. 17. The induced axial force variation graphs in the beam elements of continual and segmental linings

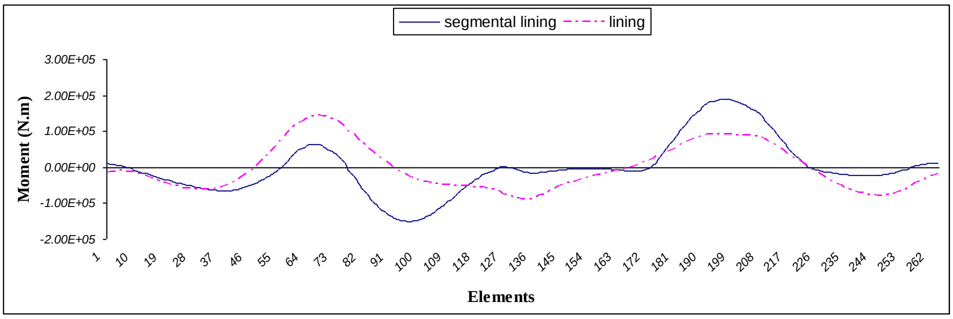


Fig. 18. The induced bending moment variation graphs in the beam elements of continual and segmental linings

According to these graphs (Figures 17 and 18), the axial force values in different parts of the linings did not change more, but the corresponding bending moment values changed considerably.

6. Conclusion

The results obtained in this paper show that the structural analyses of segmental lining require the consideration of joints' behaviour. Some of the suggested methods and models for considering joint behaviour ignore some of the joint effects, while others require laboratory results or experiments gained from past similar projects for their completion. Therefore, they may not provide trustworthy results with sufficient accuracy. In addition, in nearly all of the previous analytical methods, the segments have been assumed in a similar way. However, 3D modelling may be the best method for estimating the longitudinal and circumferential joints' effect on segmental lining. This method is very complex, however, as well as time consuming and requires some real properties such as the stiffness of joints, which require expensive laboratory tests or results gained from ambiguous experiments conducted for past similar projects. Therefore, in this paper, two dimensional numerical modelling of the segmental lining with non-uniform segments was proposed by employing a sensitivity analysis of the induced bending moments in the tunnel lining. Comparing the *FLAC* modelling results with the structural analysis, accomplished by using *SAP2000* software, clearly confirmed the validity and accuracy of the proposed method. The primary advantages of this method may be its simplicity, good efficiency and flexibility for tunnels with various cross sections, its ability to model the different segmental arrangements, the lack of need for expensive laboratory tests or other experimental data, the capability of using different loading, as well as other facilities of modelling. For example, by comparing the results of continual and segmental linings, it can clearly be shown that the bending moment values in different parts of the tunnel lining have changed considerably, while the axial force values have not changed. Finally, it must be noted that all of the models in this research were applied to TUR tunnels site conditions; in other cases, a wider scope may need to be applied to research such as ignoring the circumferential joint effects and considering the effects of consecutive rings on each other, disregarding the special nature of segment junctions, as well as other modelling simplifications, some of which may considerably affect modelling results.

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