Investigation of the Influences of Track Superstructure Parameters on Ballasted Railway Track Design

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Abstract: The main design criteria of ballasted railway tracks include rail deflections, rail bending stresses, rail wheel contact stresses, sleeper bending moments and ballast sleeper contact pressures. Various criteria have been defined for the design of ballasted railway tracks due to the various mechanical properties of track components and their complex interaction. Therefore, the railway track design is a difficult and time-consuming process. These complications highlight the need to focus attention on the necessity of a thorough investigation into the effect of the track and rolling stock parameters on the design criteria. In an attempt to overcome this problem, a sensitivity analysis of the main railway track design criteria was conducted in this study. Consequently, the roles of track and rolling stock parameters (including track modulus, sleeper spacing, train speed and axle load) in the design of railway track were investigated for various track design criteria. The research findings provide new and practical suggestions in the analysis and design of ballasted railway tracks.

Keywords: Ballasted Railway Track, Design Approach, Sensitivity Analysis, Track Superstructure Components

INTRODUCTION

Railway is a means of smooth and safe transpiration of passengers and goods. Track structures are supposed to sustain the lateral, longitudinal and vertical loads as considered in the design. Steel rails, rail fasteners, timber, steel or concrete sleepers, granular ballast, sub-ballast, and subgrade materials are the main components of a ballasted railway track (Sadeghi, 2012). Due to the wide range of mechanical characteristics of the track complements as well as their complex interaction, there is a lack of a comprehensive and precise understanding of the mechanical behaviour of ballasted railway tracks, particularly when the nonlinear and dynamic properties of the track are considered (Remennikov and Kaewunruen, 2008). According to this diversity, a broad range of design criteria have been suggested by railway authorities and practitioners, based on theoretical and experimental investigations. Subsequently, the railway track design procedure usually seems to be a challenging task. Several important criteria have been defined in the conventional track design methods to ensure safe transmission of the load into the ground. These criteria include the limits on the rail and rail fastener stresses, rail deflections, sleeper stresses, contact pressure between sleepers and the ballast,

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and the pressure transferred to the supporting layers underneath the track (Esveld, 2001). Thus, a deeper understanding of the track operating conditions and a more precise analytical and mechanistic approach is needed to make more accurate and appropriate assumptions in the design process. Moreover, a thorough identification and simplification of the current procedure of ballasted railway track design seems necessary. Herein, a sensitivity analysis of design criteria was performed to achieve a desirable design process and a profound understanding of the behaviours of ballasted railway tracks components. Similarly, the main design criteria of ballasted railway tracks have been presented and their affecting factors have been identified.

**BALLASTED RAILWAY TRACKS DESIGN PROCEDURE**

Despite the minor differences in the design methods of ballasted railway tracks, one basic approach is followed in almost all available codes. Due to the availability of a great variety of structural elements of a track system, the railway standards consider each track component as a single structural unit and suggest independent design of each component. Subsequently, such an approach includes the interaction between track components by defining suitable boundary conditions and load transfer patterns (Sadeghi, 2008). Therefore, design criteria for different components of ballasted railway tracks are developed independently. Rail bending stress, rail deflection along vertical direction and the contact stress between wheel and rail are considered as the criteria for rail design; load on the rail seat, the contact pressure between the sleepers and the ballast layer, and the sleeper bending stress are considered as the criteria for the sleeper design; and the least required depth is regarded as the main criterion for the ballast and sub-ballast layers design procedure (Sadeghi, 2008; Gu and Choi, 2013). Design procedure of ballasted railway tracks is generally based on allowable stresses. Thus, internal forces in railway components are calculated and compared with the allowable amounts specified by the standards. Several important criteria that have been defined for each component of the railway track are discussed below.

Current practices in the calculation of rail bending moments and vertical deflections are mainly based on the theory of “beam on elastic foundation model” (Greisen et al., 2009). This model was proposed for the first time by Winkler (1867) and thereafter developed by Zimmerman (1888). The basic assumption in the Winkler model is that the deflection of rail at any point is proportional to the supporting pressure under the rail. Acknowledging the Winkler theory of beam on elastic foundation, bending moment (bending stress) and vertical deflection of rail were calculated and compared with the allowable values. Accordingly, the following equations were developed to calculate the rail bending moment and rail deflections:

\[
M(x) = \sum_{i=1}^{n} \frac{P_{x_i}}{2} \frac{e^{-\beta x_i}}{x_i} \left( \cos x_i - \sin x_i \right)
\]

\[
\sigma_x(x,y) = \frac{M(x,y)}{I}
\]

\[
y(x) = \sum_{i=1}^{n} \frac{P_{x_i} \beta}{2} \frac{e^{-\beta x_i}}{x_i} \left( \cos \beta x_i - \sin \beta x_i \right)
\]

where \(y(x)\) and \(M(x)\): are the vertical deflection and the bending moment of the rail at the distance “x” from the load point, respectively. Parameter \(\beta\) is defined by the Eq. (4).
\[ \beta = \left( \frac{u}{AEI} \right)^{0.25} \] (4)

where \( u \) is the track modulus in \( N/mm^2 \), \( E \) is the rail modulus of elasticity in \( N/mm^2 \) and \( I \) is the rail moment of inertia in \( m^4 \).

Recognition of rail-wheel interactions remains an interesting area of research. However, considering the complexity of this concept and its uncertainties, some authors (Van Dyk, 2014; Sadeghi, 2012) have proposed simplified solutions in railway track design. Eqs. (5) and (6) were applied to calculate the contact stress between the wheel and the rail and the maximum shear stress.

\[ \sigma_{\text{mean}} = 43.4 \sqrt{\frac{Q \times 10^7}{r}} \] (5)
\[ \tau_{\text{min}} = 0.3 \sigma_{\text{mean}} \] (6)

where \( Q \) is the static wheel load in kN and \( r \) is the wheel radius in mm.

The longitudinal stress in rail was obtained using Eq. (7), as follows:

\[
\sigma_L = \sigma_t + \sigma_d + \sigma_a + \sigma_g + \sigma_c 
\approx \frac{F_t}{A} + \frac{F_d}{A} + \frac{F_a}{A} + \frac{F_g}{A} + \frac{F_c}{A}
\] (7)

where \( \sigma_t, \sigma_d, \sigma_a, \sigma_g, \sigma_c \) are the longitudinal stresses due to temperature changes, aerodynamic resistance, acceleration, gradient, and curves, respectively. These values are obtained using the following equations (Sadeghi, 2008).

\[
F_t = E.A. \alpha. \Delta t \quad , \quad F_d = C.V^2 \\
F_a = M. \alpha. q \quad , \quad F_g = p. i \\
F_c = \frac{k}{R}
\] (8)

where \( F_t, F_d, F_a, F_g, F_c \) are the longitudinal forces due to temperature changes, aerodynamic resistance, acceleration, gradient, and curves, respectively. The detailed parameters of Eqs. (7) and (8) have been discussed in (Sadeghi, 2008).

The current practices in the analysis and design of sleepers comprise of three steps: i) estimation of vertical rail seat load, ii) assuming a stress distribution pattern under the sleeper, and iii) applying vertical static equilibrium to a structural model of the sleeper (Kerr, 2003). Vertical wheel load is transferred through the rail and distributed on certain numbers of sleepers, due to rail continuity; this process is generally referred to as vertical rail seat load. The exact magnitude of the load applied to each rail seat depends upon several parameters, including the rail weight, sleeper spacing, track modulus per rail, the amount of play between the rail and sleeper, and the amount of play between the sleeper and ballast (Powrie and Clayton, 2007). As vertical loads on railway track have significant effects on the ultimate design, results of each track components and vertical load on the rail seat have important effects on the sleeper design process (Plenge and Lammering, 2003). Various equations have been proposed for rail seat loads. Based on the theory of continuous beam on an elastic foundation, Eq. (9) is applied to calculate the vertical component of the load on the rail seat, as follows.

\[ q_r = s.u.y_{\text{max}}.F_1 \] (9)

where \( s \) is the sleeper spacing in mm, \( u \) is the track modulus in \( N/mm^2 \), \( y_{\text{max}} \) is the maximum vertical deflection in mm, and \( F_1 \) is the track support variation safety factor.

The exact contact pressure distribution between the sleeper and the ballast and its variation with time are highly important in the structural design of sleepers. Eq. (10) is applied to calculate the contact pressure between timber sleepers and ballast layer, as follows.
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\[ P_a = \left( \frac{q_r}{B . L} \right) F_2 \]  

(10)

where \( B \): is sleeper width in m, \( L \): is sleeper effective length in m and \( F_2 \): is a factor accounting for adjacent wheels interactions.

Once the sleeper loading pattern is determined, the sleeper can be analysed. Another important design criterion is the bending stresses due to the wheel load. Calculation of sleeper bending stress is required to determine the values of their established bending moments. Values of bending moments are usually calculated in three cross-sections along the sleepers’ length (two at rail seats and at the central part of the sleeper). Eqs. (11) and (12) can be used to calculate the bending moments on the rail seat point and at the centre of the sleepers, respectively.

\[ M_r = q_r \left( \frac{l - g}{8} \right) \]  

(11)

\[ M_c = q_r \left( \frac{g}{2} - \frac{1}{4} \right) = q_r \left( \frac{2g - l}{4} \right) \]  

(12)

where \( g \): is the distance between the centre of the rails in mm. Consequently, the bending stresses at the rail-seat positions and at the middle section of the sleepers can be calculated by the Eqs. (13) and (14).

\[ \sigma_r = \frac{3}{4} q_r \left( \frac{l - g}{B . t^2 \times 10^3} \right) \]  

(13)

\[ \sigma_c = \frac{3}{2} q_r \left( \frac{2g - 1}{B . t^2 \times 10^3} \right) \]  

(14)

where \( t \): is the thickness of timber sleeper in mm.

The sizes of the aggregates in the ballast and sub-ballast layers have been proposed in the majority of railway standards or codes of practices. Theoretical, semi-empirical and empirical methods are used to determine the depth of these layers. Appropriate thickness for ballast and sub-ballast layers is the most important aim in the design of these layers (Prakoso, 2012). Eq (15) is considered as the basis for the calculation of the thickness in the ballast and sub-ballast layers (Talbot, 1919).

\[ h_{\text{min}} = \sqrt{\frac{0.17P_a}{P_c}} \]  

(15)

Sensitivity Analysis

Several assumptions have been made in this paper, including that track structure is ballasted; the rail is a continuous welded rail; track gauge is 1435 mm; the distance between central axes of two rails is 1500 mm; the rail is UIC60; the range of track modulus is 10-40 MPa; type of the sleeper is timber; dimensions of the sleeper are 2400, 2500 and 2600 mm in length, 220, 230, and 240 mm in width, and 115 mm in thickness, range of the sleeper spacing is 500 - 700 mm; allowable bending stress for timber sleeper is 7.6 MPa; effective length for the sleeper is one third of the total sleeper length; and allowable contact pressure between ballast layer and the sleeper is 450 KPa. In this study, three wagons including one passenger wagon and two freight wagons were considered. A schematic view of the selected wagons is presented in Figure 1. Some of the significant characteristics of these wagons are summarized in Table 1.

Using Eqs. (1-15) and considering the above assumptions for the track and the rolling stock, sensitivity analysis of design criteria was performed. The results of the investigation are presented in the form of charts and tables. Track modulus, sleeper spacing, speed and axle load of railway vehicle are the most important track superstructure parameters that influence the criteria of ballasted railway track design. The parameters are discussed in Figure 1 and Tables 1 and 2.
The effect of different track superstructure parameters on the track responses was investigated in this research. The obtained results are presented in the following figures. Figure 2 represents the effects of track modulus variations on bending stress and vertical deflection of the rail. Figure 2(a) indicates an inverse and nonlinear relationship between the vertical deflection and the track modulus for values >10 MPa. On the other hand, a linear relationship exists between rail deflections and track axle loads. According to Figure 2(b), there is a considerable increase in the bending stress for track modulus < 10 MPa.

![Passenger wagon](image1)

![4-axles freight wagon](image2)

![6-axles freight wagon](image3)

**Fig.1.** Schematic view of selected wagons.

**Fig. 2.** Variations of a) rail deflection and b) bending stress as a function of track modulus and wagon speeds.

<table>
<thead>
<tr>
<th>Wagon Type</th>
<th>Wheel Diameter (mm)</th>
<th>Axle Load (Ton)</th>
<th>Distance Between Center (mm) of Bogies</th>
<th>Distance Between Axes (mm)</th>
<th>Axle Numbers</th>
<th>Gross Weight (Ton)</th>
<th>Tare Weight (Ton)</th>
<th>Maximum Speed (Km/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passenger</td>
<td>920</td>
<td>11.5</td>
<td>19000</td>
<td>2900</td>
<td>4</td>
<td>45.8</td>
<td>40.8</td>
<td>160</td>
</tr>
<tr>
<td>4-axles Freight</td>
<td>920</td>
<td>20</td>
<td>11140</td>
<td>1800</td>
<td>4</td>
<td>80</td>
<td>22.5</td>
<td>120</td>
</tr>
<tr>
<td>6-axles Freight</td>
<td>920</td>
<td>20</td>
<td>12800</td>
<td>1500</td>
<td>6</td>
<td>120</td>
<td>30</td>
<td>120</td>
</tr>
</tbody>
</table>
Variations in the contact stress for different wheel radiiuses are presented in Figure 3. According to Figure 3, the contact stress between wheel and rail has a linear relationship with the axle load and an inverse relationship with the wheel radius.

Figure 4 represents the shear stress variations as a function of effective load and wheel radius. According to Figure 4, the rail shear stress was less than the allowable limit, which is in good agreement with the suggestion of AREMA.

![Fig. 3. Variations of contact stress spalling with axle load for different wheel radiiuses.](image)

![Fig. 4. Shear stress variation in railhead for different wheel radiiuses.](image)
The effects of temperature, speed (wagons) and curve radius on the longitudinal stress are presented in Figures 5 and 6. According to Figure 5, there exists a direct and linear relationship between the longitudinal stress and the rail temperature. At a constant temperature, increasing the speed of rail vehicles slightly affects the rail longitudinal stress. Figure 6 presents the longitudinal stress in UIC60 rail section, indicating suitability of 1000 m for curve radius.

Figure 7 represents the variations in the vertical load applied to the rail seat as a function of sleeper spacing and the distance between sleepers for the passenger and freight trains. As depicted, some of the graphs are very similar and even overlapped which indicates that different values of speed and track stiffness impose the same amount of vertical load on the rail seat. By selecting an appropriate rail cross-section for the 4-axle freight wagons, a range of 48- 65 kN is expected to be suitable for the vertical rail seat load. These results indicate a larger amount of vertical load on the rail seat for freight wagons in comparison to those for passenger wagons.

**Fig. 5.** Longitudinal stress variation along the UIC60 rail due to temperature variation for different wagons.

**Fig. 6.** Longitudinal stress variation along UIC60 rail for different curve radiiuses and wagons.
Fig. 7. Variations of vertical load on rail seat for a) passenger train, b) 4-axles freight train, c) 6-axles freight train.

Table 2 presents a summary of the effects of track modulus variations on the magnitude of vertical load transferred onto the rail seat. An increase in the track modulus leads to an increase in the rail seat load.

The graphs depicted in Figure 8 illustrate the variations in the contact pressure between timber sleeper and ballast layer. These graphs indicate that by increasing sleeper spacing, the contact pressure between the sleeper and ballast layer also increases. A similar relationship exists between the rolling stock speed and the contact pressure between the sleeper and ballast layer. However, an inverse relationship was noted between the lower face of the sleeper area and the stress intensity between the sleeper and the ballast layer. In terms of the structural design (i.e., when the maximum stress under the sleeper does not exceed the allowable stress value), the results indicate that the passenger wagons (except 4- and 6-axle freights) satisfy the design criteria. Based on the graphs presented in Figures 9 and 10, variations in sleeper bending stress occur at two critical cross-sections (rail seat and sleeper centre), which act as a function of rolling stock speed, sleeper length and the spacing between them. Moreover, the spacing between sleepers has a direct relationship with the established bending stress. In case of a constant value for the sleeper length and the spacing, an increase in the speed leads to an increase in the bending stress in the sleeper. On the other hand, if the spacing of the sleeper and the speed of the rolling stock remain unchanged, an increase in sleeper length leads to an increase in the sleeper bending stress in the rail seat along with a decrease in the centre of the sleeper. In addition, the rate of the established bending stress in a sleeper has a direct relationship with the sleeper spacing. These results indicate that timber sleeper is not suitable for railway freight tracks (heavy haul).
Table 2. Effect of track modulus changes on vertical load on rail seat for speed of 100 km/h and sleeper spacing of 600mm ($F^6$, $F^4$ and $P$ represent the 6-axles freight train, 4-axles freight train and passenger train, respectively).

<table>
<thead>
<tr>
<th>Percent Increase in Vertical Load on Rail Seat</th>
<th>Vertical Load on Rail Seat (kN) Speed 100 km/h and Sleeper Spacing 600 mm</th>
<th>Percent Increase in Track Modulus</th>
<th>Track Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>50.28</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>14</td>
<td>53.28</td>
</tr>
<tr>
<td>10</td>
<td>14</td>
<td>25</td>
<td>55.44</td>
</tr>
<tr>
<td>14</td>
<td>18</td>
<td>35</td>
<td>57.36</td>
</tr>
</tbody>
</table>

![Graph showing contact pressure between sleeper and ballast vs. sleeper spacing](image-url)

(a) 166
Fig. 8. Variations in contact pressure between sleeper and ballast for a) passenger train, b) 4-axles freight train, c) 6-axles freight train.
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(a) Sleeper Bending Stress on Rail Seat (MPa) vs. Sleeper Spacing (mm)

- P (2400*115 - 80 km/h)
- P (2400*115 - 100 km/h)
- P (2400*115 - 120 km/h)
- P (2500*115 - 80 km/h)
- P (2500*115 - 100 km/h)
- P (2500*115 - 120 km/h)
- P (2600*115 - 80 km/h)
- P (2600*115 - 100 km/h)

(b) Sleeper Bending Stress on Rail Seat (MPa) vs. Sleeper Spacing (mm)

- F4 (2400*115 - 80 km/h)
- F4 (2400*115 - 100 km/h)
- F4 (2400*115 - 120 km/h)
- F4 (2500*115 - 80 km/h)
- F4 (2500*115 - 100 km/h)
- F4 (2500*115 - 120 km/h)
Fig. 9. Variations in the sleeper bending stress on the rail seat under loading for a) passenger train, b) 4-axles freight train, c) 6-axles freight train.
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Fig. 10. Variations in the sleeper bending stress at the center of the sleeper under loading for a) passenger train, b) 4-axles freight train, c) 6-axles freight train.
Figure 11 indicates the variations in the thickness of ballast and sub-ballast layers against the sleeper spacing, speed of the rolling stock and the allowable limit of subgrade layer stress. According to these graphs, a linear relationship exists between the required thickness of ballast and sub-ballast layers and the sleeper spacing. In other words, with similar rolling stock characteristics, the greater the sleeper spacing, the higher thickness is required for the ballast and sub-ballast layers. This observation indicates a direct relationship between the thickness of ballast and sub-ballast layers and the sleeper spacing. If the sleeper spacing and rolling stock speed remain constant, increased allowable stress of subgrade layer reduces the need for thicker ballast and sub-ballast layers; and if the sleeper spacing and allowable stress of subgrade layer remain constant, a larger rolling stock speed leads to an increase in the required thickness of the ballast and sub-ballast layers. In fact, increasing the allowable stress of the subgrade layer reduces the required thickness of the ballast and sub-ballast layers. Notably, these effects have a descending rate. According to Table 3, increasing the subgrade allowable stress leads to a decrease in the required thickness of the ballast and sub-ballast layers. In Table 3, some rows are blank, which indicate that the contact stresses between the sleeper and ballast layer exceed the allowable limits. These limits are in accordance with the considered operating conditions, including axle load, speed, sleeper spacing, track modulus and rail type.
Fig. 11. Changes in the required thickness of the ballast and sub-ballast layers for a) passenger train, b) 4-axles freight train, c) 6-axles freight train.
**Table 3.** Effect of sub grade allowable stress variation on the ballast and sub-ballast layers thickness (sleeper spacing 600mm, UIC 60 rail section).

<table>
<thead>
<tr>
<th>Decreased Percent in The Thickness of The Ballast And Sub-Ballast Layers</th>
<th>The Thickness of Ballast and Sub-Ballast Layers (m) (for Track Modulus of 10 MPa and Speed of 100 Kn/h)</th>
<th>Increase Percent in Subgrade Allowable Stress</th>
<th>Subgrade Allowable Stress (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>f⁹</td>
<td>f⁹</td>
<td>p</td>
<td>f⁹</td>
</tr>
<tr>
<td>***</td>
<td>0</td>
<td>0</td>
<td>***</td>
</tr>
<tr>
<td>***</td>
<td>43</td>
<td>41</td>
<td>***</td>
</tr>
<tr>
<td>***</td>
<td>58</td>
<td>59</td>
<td>***</td>
</tr>
</tbody>
</table>

**CONCLUSIONS**

The main design criteria of railway track superstructure were discussed in this paper, including those of rail (i.e., bending stress, vertical deflection and contact stress between the wheel and the rail), sleepers (i.e., vertical load on rail seat, sleeper bending stress and contact pressure between the sleeper and ballast layer), and the ballast and sub-ballast layers (i.e., thickness of the ballast and sub-ballast layers). The effects of railway track superstructure parameters on the design criteria were investigated by conducting sensitivity analyses of the track systems. The analyses were performed for various track parameters characteristics and track operation conditions. The results of sensitivity analysis have been presented in the form of graphs and tables.

Based on the results obtained, a linear relationship exists between the magnitude of bending stress and the train speed. In addition, a non-linear and inverse relationship exists between the vertical deformation and track modulus. By increasing the track modulus, the vertical load applied on the rail also increases. Similarly, increasing the sleeper spacing leads to an increase in the contact pressure between the sleeper and the ballast layer. Timber sleepers are not suitable for heavy haul tracks (freight railway tracks) as they do not meet the design requirements. A linear relationship was observed between the sleeper spacing and the required thickness of the ballast and sub-ballast layers. The more the sleeper spacing, the greater was the required thickness of the ballast and sub-ballast layers.

**REFERENCES**


