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Evaluation Approaches of Value at Risk for Tehran Stock Exchange

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Abstract

The purpose of this study is estimation of daily Value at Risk (VaR) for total index of Tehran Stock Exchange using parametric, nonparametric and semi-parametric approaches. Conditional and unconditional coverage backtesting are used for evaluating the accuracy of calculated VaR and also to compare the performance of mentioned approaches. In most cases, based on backtesting statistics Results, accuracy of calculated VaR is approved for historical, Monte Carlo and Volatility-Weighted historical simulation methods. It is also approved for GARCH type of volatility models under normal distribution and Riskmetrics model under student-t distribution. On the other hand, it is observed that parametric approach measures VaR value more than non-parametric and semi-parametric approaches. This result indicates that GARCH type of volatility models under student-t distribution overestimate magnitude of value at risk. Finally, four volatility models of parametric approach including NARCH, NAGARCH and APGARCH under normal distribution and Riskmetrics under student-t distribution are selected best methods to measure accurate value of VaR.

Keywords: Nonparametric Approach, Parametric Approach, Semi-Parametric Approach, Value at Risk.

1. Introduction

Risk is considered as one of important issue in financial markets. The crisis of financial markets in 2007-2008 confirms this fact. In addition, historical volatility in financial markets such as switching from fixed exchange rate regime to flexible one, U.S. stock market crash in black Monday 1987, Bursting the bubble of Japanese stock price in 1989, Asian southeast financial crisis in 1997, closure of financial markets and fall of U.S. stock price indices in 11th September of 2001 represents the necessity of risk management.

In general, there are several types of risk in financial markets. Market risk, liquidity risk, credit risk and operational risk are the main types of financial

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risks. Value at Risk (VaR) is mainly related to market risk and considered as an important measure of risk. From the perspective of financial institution, VaR can be defined as maximum loss of financial position during specific time period (1 day, 1 week, 1 month) at a given confidence level. VaR is considered as a key instrument for risk management of financial institutions. Panning (1999) expressed it as an approach for risk evaluation, risk management and making decision about risk.

After introduction of VaR by JPMorgan in 1994, it is considered as an important measure for calculating risk in financial institutions. The application of VaR expanded from securities houses to commercial banks, insurance companies and etc. Since VaR responds to complications of financial instruments and summarizes different types of risk in one number, therefore it can be used for risk regulation and also determination of adequate capital requirement for financial institutions.

Although VaR is a simple concept to understand, but it is difficult to calculate and estimate its value. In fact finding probability distribution of returns which is not constant over time creates problems for estimating critical values at desired probability level and therefore leads to difficulties in calculation of VaR. In general there are several techniques to measure VaR. The purpose of this study is to evaluate different methods VaR for Tehran stock exchange and select the more accurate approach by comparing them through backtesting.

In the next section a brief overview of last studies relating to VaR is presented .Literature review and different approaches of VaR evaluation is expressed in section III. In section IV conditional and unconditional coverage tests of back testing are reviewed. Empirical results are presented in section V. general conclusion of this paper is presented in the last section.

2. An Overview of Last Studies

Even if the concept of VaR was first introduced by Bamoul (1963) in a model as "expected-gain confidence limit criterion", the word VaR was not officially entered the finance literature until the early 1990s. Before 1990 and mainly due to fall of stock market in October 1988, many countries decided to have better control over the risk of financial institutions in order to guarantee themselves against unexpected great losses. In 1988, countries of group G-10 formed the first Basel capital accord. In 1990, due to other financial crisis many financial institutions proposed new approach to evaluate risk that was the VaR. In October 1994, VaR was used in Riskmetrics model by JPMorgan. In 1996 Basel committee on bank supervision offered commercial banks to calculate minimum regulatory capital using internal model and based on the VaR threshold. Much attention is paid to calculation of VaR after current financial crisis (2007-2008). In this section some previous studies related to VaR are reviewed.

Samir Mabrouk and Samir Saadi (2012) used GARCH type of volatility models under normal, student-t and skewed student-t distributions in order to estimate the one-day-ahead VaR for 7 U.S. stock indices. Their result shows that the skewed Student-t FIAPARCH model included more realistic assumptions of financial markets such as fat tails, asymmetry, volatility clustering and long memory for all stock market indices.

Chen, Gerlach, Hwang and McAleer et al. (2012) used conditional autoregressive value at risk (CAViaR) models to predict VaR and employ Bayesian method to estimate them. Results show that in comparison with other models threshold CAViaR is more accurate and efficient.

Chaker and Mabrouk (2011) estimated VaR by ARCH and GARCH type models such as FIGARCH, FIAPARCH, and HYGARCH. These models were estimated based on normal, student-t and skewed t-student distributions. Results show that by considering features of financial time series data such as long memory, fat tail and asymmetrical performance, daily VaR predictions would be more accurate. Also they indicate FIGARCH has better performance compared to other models. P.T. Wu and Shieh (2007), and T.L. Tang (2006) are also investigated Value-at-risk analysis for long-term interest rate futures and long memory in stock index future markets.

Enocksson and Skoog (2011) studied different volatility models (ARCH, GJR-GARCH, GARCH) in order to identify proper model to estimate VaR for some exchange rates (including dollar, euro, pound, and yen). Their research findings show that GARCH (1,1) and GJR-GARCH (1,1) under normal distribution are more appropriate model to estimate conditional variance and VaR.

Qi Chen and Rongda Chen (2013) used equally weighted moving average, exponentially weighted moving average, historical simulation and Monte Carlo simulation methods to calculate VaR for Shanghai stock market. Their paper shows that Monte Carlo simulation is a best method of VaR calculation.

3. Literature Review

Based on value at risk definition, the decrement of portfolio value will not be more than calculated VaR, at a given confidence level in the future. Therefore, this downside risk criteria measures the worst expected loss at determined confidence level under normal conditions of market over a specified period of time. According to the definition, VaR has two main parameters. One is time horizon shown in form of days and the other is confidence level.

In general by assuming N days as time horizon and C=1-α as confidence level, VaR (which is expressed in terms of currency) is the loss equivalent with (1-α) of probability distribution curve of portfolio value change during future N days. In other words, we have C percent sure that our loss will not be more than V in N future days. Mathematically speaking, it can be written as:

$$VaR(C) = \inf\{v \in R : P(V > v) \leq (1-\alpha)\} = \inf\{v \in R : F_V(v) \geq \alpha\} \qquad (1)$$

where F_V is loss distribution function. In general in order to calculate quantity, VaR is defined as negative (1-α) quantile of return distribution as below:

$$VaR_t^c = -Q_{1-\alpha}(r_t|\Omega_{t-1}) = -\inf\{r|R : P(r_t \leq VaR|\Omega_{t-1}) \geq 1-\alpha\} \qquad (2)$$

where Ω_{t-1} is available information set at time t-1.

For more explanation, a time series of financial returns is considered which follows a stochastic process as below (Abad, Benito & Lopez, 2013):

$$r_t = \mu_t + \varepsilon_t = \mu_t + \sigma_t z_t \qquad (3)$$
$$z_t \sim iid(0,1), \quad E(\varepsilon_t|\Omega_{t-1}) = 0, \ \sigma_t^2 = E(e_t^2|\Omega_{t-1})$$

where μ_t is expected mean of returns at time t according to information of time t-1, ε_t is innovation of returns, σ^2_t is conditional variance and Z_t is sequence of N(0,1) i.i.d random variable. So, at confidence level of C on information of last period VaR will be equal to:

$$VaR_t^c = -Q_{1-\alpha}(R_t|\Omega_{t-1}) = -(\mu_t + \sigma_t Q_{1-\alpha}(z)) \qquad (4)$$

where $Q_{1-a}(z)$ is (1-α) quantile of z distribution. As it can be seen from equation (4), VaR is defined based on both return distribution (r_t) quantile and z distribution quintile. By assuming f and F as density function and cumulative distribution function (CDF) of r, g and G as density function and cumulative distribution function of z, then:

$$Q_{1-\alpha}(r) = -F_r^{-1}(1-\alpha), \qquad Q_{1-\alpha}(z) = -G_z^{-1}(1-\alpha) \qquad (5)$$

Therefore, a VaR model will be achieved as:

$$VaR_t^c = F_r^{-1}(1-\alpha) = \sigma_t G_z^{-1}(1-\alpha) - \mu_t \qquad (6)$$

Thus, calculation of VaR contains inverse specification of return’s CDF (F^{-1}_r) or estimation of conditional variance and determining the type of z distribution.

In a general classification, there are three main approaches including nonparametric, parametric and semi-parametric to calculate VaR. nonparametric approach is related to estimation of returns quantile and does not impose any restrictions on the distribution of returns. Historical simulation and Monte Carlo simulation methods belong to this approach. For parametric approach VaR is calculated based on determined assumptions about selection of conditional or unconditional return distribution and also dynamics of model. GARCH-type volatility models and Riskmetrics models are examples of this approach. Finally, the third approach is called semi-parametric approach which defines default for dynamics of model but not for type of innovation distribution. Volatility-Weighted historical simulation and filtered historical simulations are examples of this approach.

3.1. Parametric Approach

For the first time, parametric approach is described in detail by J.P. Morgan in Riskmetrics programming. Assumptions of parametric approach are as follows: firstly, returns and risk factors follow the certain distribution such as normal or student-t distribution. Secondly, asset return is time independent and also there is a linear relationship between market risk factors an asset value.

According to equation (6), for calculation of VaR through parametric approach the main focus is on second part of equation: $VaR^{c}_{t} = \sigma_t G^{-1}_{z}(1-\alpha) - \mu_t$. Based on this equation calculation of VaR includes estimates of μ_t, σ_t and G^{-1}_{z}. Since mean of return (μ) can be simply achieved from mean equation of ARMA (p,q), therefore the main focus in this method is to determine the type of z distribution and estimate conditional variance. In this study standard normal and student-t distributions will be considered for z distribution, thus VaR^{c}_{t} will be calculated as:

$$VaR_{t}^{c} = \sigma_t \phi^{-1}\left(1-\alpha\right) - \mu_t \tag{7}$$

$$VaR_{t}^{c} = \sigma_t \sqrt{(d-2)d^{-1}}\, T_{d}^{-1}\left(1-\alpha\right) - \mu_t \tag{8}$$

where $\phi^{-1}(1-\alpha)$ is inverse CDF of standard normal distribution and $T_d^{-1}(1-\alpha)$ is inverse PDF of student-t distribution with d degree of freedom.

According to above discussion, calculation of σ_t that is one of the main indicators for measuring market volatility plays an important role in parametric approach. Thus, in different methods of VaR calculation the main focus is on prediction of σ_t. In this section different models of volatility equations are presented.

3.1.1. Riskmetrics Model (RM)

In this method, exponentially weighted moving average model (EWMA) is used for conditional variance prediction. Since more weight is given to new innovations, variance of return responses faster related to innovations that occur in market. Also, after innovation occurrence, volatility decreases exponentially. In this case by defining λ which is known as decay parameter between 0 and 1, risk metrics model is expressed as:

$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + \left(1-\lambda\right)\varepsilon_{t-1}^2 \tag{9}$$

As it can be observed from above equation, new innovations have more impact on variance when λ has smaller value. Selection of optimized is an empirical issue. Many researchers select λ=0.94 and λ=0.97 for daily and monthly volatility, respectively.

3.1.2. GARCH Type Models

In this paper, in order to examine the performance of GARCH type models in explaining the behavior of mean, variance and VaR for returns of total price index of Tehran stock exchange, several types of these models are estimated. General form of such model is as GARCH (p,q) and specified as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{10}$$

In many studies p and q is considered equal to 1 and research findings show that these models provide acceptable results in financial time series discussion (So & Yu, 2006). Nine types of GARCH models are given in Table 1 by assuming p=q=1. (See Bollerslev, 1986; Nelson, 1991; Glosten & Jagannathan Runkle, 1993; Engle, 1982 & 1990; Engle & Bollerslev, 1986; Engle & V.K. Ng, 1993; Higgins & Bera, 1992; Granger & Engle, 1993).

Table 1. GARCH models

Model	Formulation
GARCH(1,1)	$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$
IGARCH(1,1)	$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$
EGARCH(1,1)	$\ln(\sigma_t^2) = \alpha_0 + \alpha_1 \lvert\ \varepsilon_{t-1}\ \rvert - \gamma \varepsilon_{t-1} + \beta_1 \ln(\sigma_{t-1}^2)$
GJR(1,1)	$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2\ \lvert \varepsilon_{t-1} < 0 + \beta_1 \sigma_{t-1}^2,$
AGARCH(1,1	$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1} + \beta_1 \sigma_{t-1}^2$
NAGARCH(1,1)	$\sigma_t^2 = \alpha_0 + \alpha_1 [\varepsilon_{t-1} + \gamma \sigma_{t-1}]^2 + \beta_1 \sigma_{t-1}^2$
NARCH(1,1)	$\sigma_t^2 = \alpha_0 + \alpha_1 \lvert\ \varepsilon_{t-1}\ \rvert^{\delta} + \beta_1 \sigma_{t-1}^2$
NGARCH(1,1)	$\sigma_t^{\delta} = \alpha_0 + \alpha_1 \lvert\ \varepsilon_{t-1}\ \rvert^{\delta} + \beta_1 \sigma_{t-1}^{\delta}$
APGARCH(1,1)	$\sigma_t^{\delta} = \alpha_0 + \alpha_1 [\lvert\ \varepsilon_{t-1}\ \rvert + \gamma \varepsilon_{t-1}]^{\delta} + \beta_1 \sigma_{t-1}^{\delta}$

3.2. Nonparametric Approach

Nonparametric approach is an alternative approach for VaR calculation which includes simulation methods. These methods do not impose any assumption on distribution of financial returns and use empirical distribution of returns to evaluate risk. Historical simulation and Monte Carlo simulation are two main methods of nonparametric approach which will be discussed bellow.

3.2.1. Historical Simulation (HS)

This method is the simplest way for calculation of VaR and discards some problems related to parametric method such as main assumptions of normal distribution returns and constant correlation between risk factors. In fact, the main advantage of this method is that there are no distributional assumptions about the data. The only assumption is that distribution of returns in the past is similar to distribution of returns in the future. In fact, the main assumption of historical simulation is that past can be the good measure to predict future. This method uses historical distribution of portfolio returns to simulate VaR by assuming that combination of portfolio does not change during the period of historical returns Collection (Gupta, 2008).

Historical simulation method for VaR estimate introduced during investigations of Boudoukh (1998) and Barone-Adesi (1999). In this method first, series of asset return of one portfolio is collected, and then current weight of each asset is multiplied by its historical returns in order to achieve historical return of portfolio as: $r_{pt} = \sum_{i=1}^{n} w_i r_{it}$ Where n is the number of assets, r_{it} is the return of asset i at time t, w_i is share of each asset in portfolio and r_{pt} is simulated historical returns of portfolio. Therefore, VaR is calculated as quantile of time series simulated historical returns.

In general, for series of stock index data which is the purpose of this study, VaR is equal to (1-α) quantile of past returns distribution extracted from this index as shown:

$$\mathrm{VaR}_{t+1}^{c} = Q^{1-\alpha}\left(\left\{r_t\right\}_{t=1}^{n}\right) \tag{11}$$

3.2.2. Monte Carlo Simulation (MCS)

Monte Carlo simulation method is one of the powerful tools in risk analysis which is similar to the historical simulation method in some aspects. In this method, by using of stochastic processes and simulations, future returns data are predicted. VaR is calculated through quantile of new returns distribution similar to historical simulations.

Steps of VaR calculation in Monte Carlo simulation method are as follows:

1. Determination of stochastic process and its parameters for financial variables.
2. Virtual simulation of price for all used variables.
3. Determination of financial assets price at time t, determination of assets return from simulated prices and calculation of portfolio value at time t.
4. Repeating step 2 and 3 for many times in order to create distribution of portfolio value.
5. Measurement of VaR from simulated distribution returns at confidence level of 1-α.

3.3. Semi-Parametric Approach

Semi-parametric approach combines both parametric and nonparametric approaches in a way that there are pre assumptions for dynamics of volatility model but no pre assumptions regarding innovation distribution. Volatility-Weighted historical simulation and filtered historical simulation are two important methods of this approach.

3.3.1. Volatility-Weighted Historical Simulation Method (WHS)

As it is expressed, traditional historical simulation method only use historical data and does not consider recent changes in volatility. In 1998 Hull and White presented a new approach that combines benefits of traditional historical simulation method with volatility models. The main purpose of this approach is to update return information in order to consider recent changes in volatility.

Assume that $r_{t,i}$ is return on asset i at time t, $\sigma_{t,i}$ be the prediction of volatility at time t in the end of t-1 and $\sigma_{T,i}$ is the most recent of forecast volatility. However, it should be noted that $\sigma_{t,i}$ and $\sigma_{T,i}$ are achieved using GARCH models. In this case, the adjusted return ($r^{*}_{t,i}$) based on volatility models are obtained as:

$$r^{*}_{t,i} = \frac{\sigma_{T,i} r_{t,i}}{\sigma_{t,i}} \tag{12}$$

In this method VaR at confidence level (C) is achieved through (1-α) quintile of adjusted return distribution. In fact, in this new approach, volatility changes are considered directly and calculated VaR based on this approach is properly sensitive to recent volatility changes.

3.3.2. Filtered Historical Simulation Methods (FHS)

This method is proposed by Barone-Adesi et al. which combines advantages of historical simulation methods with conditional volatility models. In this method mean and variance of returns are predicted using parametric approach (GARCH type models) and standardized returns quantile is used for VaR calculation.

In general, the filtered historical simulations are done in three stages: at first step, an appropriate conditional volatility model is selected (usually GARCH or EGARCH is selected) and then the standardized residuals of the model are predicted as $\hat{e}_t = \frac{\hat{z}_t}{\hat{\sigma}_t}$ in which $\hat{z}_t$ is residuals of GARCH or EGARCH model and $\hat{\sigma}_t$ is conditional variance. The second step includes bootstrapping and simulation of standardize residual so that new series of returns achieved as:

$$\{\hat{r}_i\}_{i=1}^{n} = \{\hat{e}_i\}_{i=1}^{n}.\hat{\sigma}_{t+1} \tag{13}$$

where n is the number of simulations. At third step, VaR is obtained through quantile of simulated returns as bellow:

$$VaR^{c}_{t+1} = Q^{1-\alpha}\left(\left\{\hat{r}_t\right\}_{t=1}^{n}\right) \tag{14}$$

4. Backtesting

The essence of VaR measure is such that its forecasting is different with other prediction variables in many aspects. The most obvious difference is that the actual value of VaR cannot be observed and the only measure for comparison is actual observations. So, the concept of error prediction in VaR approaches also differs. While the main concern in usual prediction models is that to what extent predictions are close to actual data, in VaR models the main concern is that how many times is the actual loss greater than the predicted loss.

Therefore, many common criteria to measure accuracy of prediction models such as mean square error (MSE) and Mean absolute deviation error (MADE) are not applicable in VaR predictions. For this reason, backtesting is used to evaluate the accuracy of VaR models. In this study, the accuracy of calculated VaR and performance of different VaR approaches are examined through conditional and unconditional coverage tests.

4.1. Kupiec's Proportion of Failure Test

This test is an unconditional coverage test and has a null hypothesis of($H_0: \alpha = \hat{\alpha} = \frac{X}{N}$), where α is probability level or predicted failure proportion and ($\hat{\alpha}$) is actual failure proportion. N is the number of observations and X is the number of failures or the number of times that actual loss are greater than VaR estimated loss. Kupiec (1995) shows that assuming the probability of failure is constant, then the number of failures (x) follows binomial distribution B(N,α).The likelihood ratio statistic of this test is represented as:

$$LR_{POF} = 2Ln\left[\frac{\hat{\alpha}^{x}\left(1-\hat{\alpha}\right)^{T-x}}{\alpha^{x}\left(1-\alpha\right)^{T-x}}\right] \tag{15}$$

4.2. Kupiec's Time Until First Failure (TUFF) Test

Main assumption of this test Like Kupiec's POF test is that the number of failures follows binomial distribution but null hypothesis of Kupiec's TUFF test will be:

$$H_0:\ \alpha = \hat{\alpha} = \frac{1}{\mathrm{v}}$$

where V is the first time that failure occurred. In this situation statistic of likelihood ratio for this unconditional test as:

$$LR_{TUFF} = 2Ln\left[\frac{\frac{1}{\mathrm{v}}\left(1-\frac{1}{\mathrm{v}}\right)^{\mathrm{v}-1}}{\alpha\left(1-\alpha\right)^{\mathrm{v}-1}}\right] \tag{16}$$

It's should be noticed that both of LR_{TUFF} and LR_{POF} is asymptotically distributed $\chi^2(1)$.

4.3. Christoffersen Interval Forecast Test

Christoffersen (1998) proposed conditional coverage test. For This test, equality hypothesis of actual and expectation failure rate is not considered. It examines serial independence against first-order Markov dependence. In fact based on independent null hypothesis, failure of today should not depend on previous or next day. The likelihood ratio statistic of this test which is asymptotically distributed $\chi^2(1)$ as below:

$$LR_{IND} = -2Ln\left(\frac{(1-\pi)^{n_{00}+n_{10}}\pi^{n_{01}+n_{11}}}{(1-\pi_0)^{n_{00}}\pi^{n_{01}}(1-\pi)^{n_{10}}\pi^{n_{11}}}\right) \tag{17}$$

$$\pi_0 = \frac{n_{01}}{n_{00}+n_{01}}, \pi_1 = \frac{n_{11}}{n_{10}+n_{11}}, \pi = \frac{n_{01}+n_{11}}{n_{00}+n_{01}+n_{10}+n_{11}}$$

where $n_{i,j}$ indicate the number of observation that situation j after situation i occurred and π_i is a probability of observing a failure conditional on condition i on the previous day.

4.4. Joint Test

This test is the combination of independent and POF tests, so the hypothesis test not only examines the equality of observed and execrated failures but also considers independent of failures. The statistic likelihood ratio of this conditional coverage test is specified as:

$$LR_{MIX} = LR_{POF} + LR_{IND} \tag{18}$$

LR_{MIX} Statistic is Chi-square distributed with 2 degree of freedom.

5. Empirical Research

5.1. Statistical Description of Data

In this study daily price index of Tehran stock exchange from 14/09/2004 to 14/09 /2014 is used. This time series data includes 2350 observations which are divided to 1880 in sample observations and 470 out of sample section in order to estimate volatility models and predictions, respectively. The series of returns are achieved from equation $r_t = \ln(\frac{P_t}{P_{t-1}})$ where P is a price index. The index and return time series diagram of Tehran stock exchange in mentioned period is given in Figure 1.

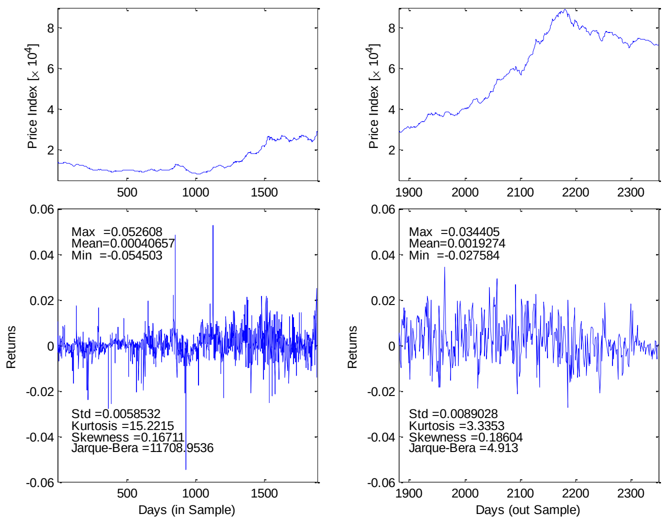

Figure 1. Time series plot of index and return

Table 2 shows some statistical description of Tehran exchange return. The mean of Daily returns is equal to 0.0004 with standard error about 0.0058. Skewness statistic is equal to 0.16 and close to zero which shows that return distribution of Tehran stock exchange is close to symmetrical distribution. Kurtosis index has great value which represents conditional distribution of Tehran exchange return is fat tail. Big value for Jarque–Bera statistic also shows that null hypothesis of normality for return distribution of Tehran stock exchange is rejected at probability level of 1%.

Table 2. Descriptive statistics

mean	Minimum	Maximum	Std.Dev.	Skewness	Kurtosis	Jarque–Bera
0.0004	-0.0545	0.0525	0.0058	0.1671 (0.00)	15.22 (0.00)	11708.95 (0.00)

5.2. Estimation Results

5.2.1. First Step: Parameters Estimates of Volatility Models

The purpose of this study is to calculate and evaluate VaR for Tehran stock exchange through parametric, nonparametric and semi-parametric approaches. As mentioned in above sections, in nonparametric approach, VaR is directly calculated based on distribution of historical returns or predicted returns. In parametric method, VaR is achieved through mean and volatility equations and also distribution of returns innovations. In semi-parametric approach VaR is estimated through combination of volatility equations and historical returns. Thus at first step volatility models are estimated and then VaR is estimated through three mentioned approaches.

Maximum likelihood is used to estimate volatility models. In estimates of Riskmetrics and GARCH (1,1) type models, by assuming that the conditional mean of returns follows an AR(m) process, the best model for total price index is selected based on Akaike criteria. Results show that mean of return index follows an AR (1) process as: $r_t= a_0+a_1r_{t-1}+\varepsilon_t$. In this section, proposed GARCH type models and Riskmetrics model are estimated using data related to total price index of Tehran stock exchange. All models are estimated by assuming normal and student-t distributions. Estimation results of these models are given in Table 3.

According to the results of Tables 3.1 to 3.3, some points can be explained. First, based on the results of t-statistics, all estimated parameters of mean and variance equations for nine GARCH type models and Riskmetrics model are significant at 95 percent confidence level. Also, Akaike criteria results show that these volatility models have goodness of fit in sample. Moreover, for volatility models with student-t distribution, degree of freedom is greater than 3 which ensure existence of first, second and third order conditional moment. Finally, negative sign of γ for some asymmetric volatility models such as GJR, AGARCH and APGARCH confirm the leverage effect in Tehran Stock Exchange. In short, by substituting of forecasted conditional μ and σ from these volatility models into equation 7 and 8, parametric VaR are calculated. Note that predicted VaR via this method is presented in the next step.

Table 3.1. Estimated Parameters of GARCH, IGARCH and RM Models

		GARCH-N	GARCH-T	IGARCH-N	IGARCH-T	RM-N	RM-T
Mean equation	α_0	-0.0002	-0.0000	-0.0002	-0.0000	-0.0000	-0.0001
	T-Statistic	-1.4976	-0.0000	-2.1878	-0.1488	-0.2286	-0.7327
	α_1	0.4755	0.4936	0.4656	0.4936	0.4592	0.4644
	T-Statistic	69.103	0.0005	16.728	21.826	18.774	21.041
Volatility equation	α_0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	T-Statistic	706.29	0.0000	8.2232	3.4193	0.0000	0.0000
	α_1	0.3283	0.4257	0.5353	0.4250	0.0342	0.00001
	T-Statistic	47.271	0.0001	11.611	6.6045	8.3949	0.0000
	β_1	0.4948	0.5743	0.4647	0.5750	0.9658	0.99999
	T-Statistic	306.53	0.0001	8.7612	6.1045	3. 8535	0.0000
	df		3.2179		3.2149		3.5920
	T-Statistic		0.0001		21.953		21.665
	Log likelihood	7375	7653	7360	7653	7207	7444
	AIC	-14739	-15293	-14712	-15295	-14405	-14850

Table 3.2. Estimated Parameters GJR, AGARCH, EGARCH and NAGARCH Models

		GJR-N	GJR-T	AGARCH-N	AGARCH-T	EGARCH-N	EGARCH-T	NAGARCH-N	NAGARCH-T
Mean equation	α_0	-0.0002	-0.0000	-0.0001	0.0000	-0.0002	0.0000	0.0108	0.0086
	T-Statistic	-2.8781	-0.0000	-0.6932	0.0001	-2.3270	0.1348	2.8513	50990
	α_1	0.4565	0.4858	0.4675	0.4906	0.4648	0.4945	-0.5804	-0.4700
	T-Statistic	11.2563	0.0027	19.010	0.1389	18.098	22.004	13.2431	-7491142
Volatility equation	α_0	0.0000	0.0000	0.0000	0.0000	-2.7736	-0.9638	0.0003	0.0001
	T-Statistic	10.828	0.0000	6.7054	0.0004	-9.890	-3.8419	4.2987	1606.5
	α_1	0.5132	0.4765	0.3029	0.3989	0.5009	0.6256	0.0413	0.0265
	T-Statistic	67.677	0.0001	7.5071	0.0007	12.622	7.2436	23.3428	432428
	β_1	0.4072	0.6042	0.5290	0.6011	0.7343	0.8939	0.2286	0.2523
	T-Statistic	4.0734	0.0001	10.893	0.0220	27.779	38.351	6.1284	1815148
	γ	-0.098	-0.1615	-0.0011	-0.0006	0.0370	0.0942	0.3646	0.4014
	T-Statistic	-0.4180	-0.0000	-3.8830	-0.0053	1.3935	2.7937	1.7884	2334751
	df		3.2397		3.2258		3.6609		2.8746
	T-Statistic		0.0002		0.0022		12.931		828828
	Loglikelihood	7241	7655	7382	7656	7368	7667	4028	4806
	AIC	-14472	-15296	-14752	-15297	-14723	-15321	-8043	-9598

Table 3.3. Estimated Parameters of NARCH, NGARCRCH and APGARCH Models

		NARCH-N	NARCH-N	NGARCH-N	NGARCH-T	APGARCH-N	APGARCH-T
Mean equation	α_0	-0.0002	0.0000	-0.0003	0.0000	-0.0001	0.0001
	T-Statistic	-1.8141	0.1570	-6.2240	0.6192	-0.1280	0.0002
	α_1	0.4740	0.4858	0.4477	0.4816	0.4538	0.4726
	T-Statistic	17.924	20.229	18.170	0.5355	10.843	588.36
Volatility equation	α_0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	T-Statistic	6.4707	0.6499	0.7919	0.0001	0.3359	0.0000
	α_1	0.1679	0.0263	0.0037	0.0066	0.0002	0.0029
	T-Statistic	1.3835	1.7423	1.8758	0.1586	0.9375	0.0001
	β_1	0.5026	0.5856	0.5252	0.5151	0.4585	0.4996
	T-Statistic	10.325	10.901	13.252	0.1299	3.1726	0.0001
	δ	1.8575	1.3113	0.9473	0.9790	1.5381	1.1515
	T-Statistic	12.399	14.170	10.419	3.7319	10.557	78.852
	γ					-0.1540	-0.1365
	T-Statistic					-3.6939	-0.0002
	df		3.6498		3.2179		2.6703
	T-Statistic		18.349		0.2662		0.0006
	Loglikelihood	7375	7677	7373	7670	7367	7672
	AIC	-14738	-15340	-14694	-15326	-14720	-15328

5.2.2. Second Step: Calculation of VaR and Statistics of Backtesting

In this section, calculated VaR through different approaches and backtesting statistics are presented. Daily VaR of total price index based on parametric approach (including Riskmetrics model and nine GARCH type volatility models under normal and student-t distributions) and two other simulation methods along with expected and real failures at two confidence levels of 95% and 99% are reported in Table 4. It should be noted that V (the first day that failure occurs) is required to calculate Kupiec's TUFF Test statistics. Also, decision criteria are needed to achieve independence statistic of Christoffersen. If failure occurs the index value would be equal to 1, otherwise index value would be equal to 0, thus contingency matrix is formed as matrix 2x2 with 4 members. The first value (n_{00}) is equal to the number of days that no failure occurs for two consecutive days. The second value ((n_{10})) shows the number of days that the failure on the first day accompanied with no failure on the next day. The third (n_{01}) is the number of days that the absence of failure on the first day accompanied with no failure on the next day. Finally, the fourth value ((n_{11})) is the number of days that failure occurs in two consecutive days. Results related to V, n_{ij} and probability ratios (π_i) for different approaches at two confidence levels of 95% and 99% are given in Tables 5 and 6. Results related to statistics of four backtesting tests including Kupiec's POF Test, Kupiec's TUFF Test, Christoffersen's interval forecast test and joint test at two confidence levels of 95% and 99% are given in Tables 7 and 8.

Table 4. VaR Statistics at 95% and 99% Confidence Levels

		C=0.95				C=0.99			
		VaR	**Expectation Failures**	**No. Failures**	**Failure Rate**	**VaR**	**Expectation Failures**	**No. Failures**	**Failure Rate**
Non parametric	MCS	-0.01389	23.5	17	0.036	-0.0203	4.7	3	0.006
	HS	-0.00752	23.5	64	0.14	-0.01779	4.7	7	0.015
Semi parametric	WHS	-0.0078	23.5	60	0.13	-0.01882	4.7	3	0.006
	FHS	-0.0079	23.5	60	0.13	-0.02867	4.7	0	0
parametric	GARCH-N	-0.01355	23.5	18	0.038	-0.02205	4.7	1	0.002
	GARCH-t	-0.02716	23.5	1	0.002	-0.06	4.7	0	0
	IGARCH-N	-0.0182	23.5	7	0.015	-0.0256	4.7	1	0.002
	IGARCH-t	-0.0272	23.5	1	0.002	-0.0576	4.7	0	0
	RM-N	-0.0092	23.5	41	0.087	-0.0138	4.7	18	0.038
	RM -t	-0.0128	23.5	23	0.049	-0.0165	4.7	7	0.015
	GJR-N	-0.016	23.5	7	0.015	-0.0235	4.7	1	0.002
	GJR-t	-0.02606	23.5	1	0.002	-0.05522	4.7	0	0
	AGARCH-N	-0.01298	23.5	21	0.045	-0.02122	4.7	2	0.004
	AGARCH-t	-0.02668	23.5	1	0.002	-0.05655	4.7	0	0
	EGARCH-N	-0.0146	23.5	16	0.034	-0.0203	4.7	3	0.006
	EGARCH-t	-0.02991	23.5	0	0	-0.06	4.7	0	0
	NAGARCH-N	-0.0133	23.5	20	0.042	-0.0188	4.7	3	0.006
	NAGARCH-t	-0.18776	23.5	0	0	-0.28	4.7	0	0
	NARCH-N	-0.01289	23.5	22	0.047	-0.02109	4.7	2	0.004
	NARCH-t	-0.03100	23.5	0	0	-0.18	4.7	0	0
	NGARCH-N	-0.0154	23.5	11	0.023	-0.0214	4.7	1	0.002
	NGARCH -t	-0.02569	23.5	1	0.002	-0.06008	4.7	0	0
	APGARCH-N	-0.0128	23.5	22	0.046	-0.0179	4.7	7	0.015
	APGARCH -t	-0.01979	23.5	3	0.006	-0.04746	4.7	0	0

Table 5. Input Data for Kupiec TUFF and Independence Back-Test at 0.95 Confidence Level

		V	N_{00}	N_{01}	N_{10}	N_{11}	Π_0	Π_1	π
Non parametric	MCS	21	435	17	17	0	0.038	0	0.036
	HS	5	360	45	45	19	0.11111	0.29688	0.13646
Semi parametric	WHS	5	367	42	42	18	0.10269	0.3	0.12793
	FHS	5	367	42	42	18	0.10269	0.3	0.12793
Parametric	GARCH-N	21	433	18	18	0	0.039911	0	0.03838
	GARCH-t	306	467	1	1	0	0.0021368	0	0.0021322
	IGARCH-N	73	455	7	7	0	0.0151	0	0.0149
	IGARCH-t	306	467	1	1	0	0.0021368	0	0.0021322
	RM-N	5	397	31	31	10	0.072	0.24	0.087
	RM -t	21	423	23	23	0	0.051	0	0.049
	GJR-N	73	455	7	7	0	0.0151	0	0.0149
	GJR-t	306	467	1	1	0	0.0021368	0	0.0021322
	AGARCH-N	21	427	21	21	0	0.046875	0	0.044776
	AGARCH-t	306	467	1	1	0	0.0021368	0	0.0021322
	EGARCH-N	21	441	14	14	0	0.0307	0	0.0298
	EGARCH-t	na	na	na	na	na	na	na	na
	NAGARCH-N	21	429	20	20	0	0.0445	0	0.0426
	NAGARCH-t	na	na	na	na	na	na	na	na
	NARCH-N	21	425	22	22	0	0.049217	0	0.046908
	NARCH-t	na	na	na	na	na	na	na	na
	NGARCH-N	62	447	11	11	0	0.0240	0	.0234
	NGARCH -t	306	467	1	1	0	0.0021368	0	0.0021322
	APGARCH-N	21	425	22	22	0	0.0492	0	0.0469
	APGARCH -t	215	463	3	3	0	0.0064378	0	0.0063966

Table 6. Input data for Kupiec TUFF and Independence Back-Test at 0.99 Confidence Level

		V	N_{00}	N_{01}	N_{10}	N_{11}	Π_0	Π_1	π
Non parametric	MCS	215	463	3	3	0	0.0064	0	0.0063
	HS	73	455	7	7	0	0.015152	0	0.014925
Semi parametric	WHS	215	463	3	3	0	0.0064378	0	0.0063966
	FHS	na	na	na	na	na	na	na	na
Parametric	GARCH-N	306	467	1	1	0	0.0021368	0	0.0021322
	GARCH-t	na	na	na	na	na	na	na	na
	IGARCH-N	306	467	1	1	0	0.0021368	0	0.0021322
	IGARCH-t	na	na	na	na	na	na	na	na
	RM-N	21	433	18	18	0	0.039911	0	0.03838
	RM -t	73	455	7	7	0	0.015152	0	0.014925
	GJR-N	306	467	1	1	0	0.0021368	0	0.0021322
	GJR-t	na	na	na	na	na	na	na	na
	AGARCH-N	215	465	2	2	0	0.0042827	0	0.00426
	AGARCH-t	na	na	na	na	na	na	na	na
	EGARCH-N	215	463	3	3	0	0.0064	0	0.0063
	EGARCH-t	na	na	na	na	na	na	na	na
	NAGARCH-N	215	463	3	3	0	0.0064	0	0.0063
	NAGARCH-t	na	na	na	na	na	na	na	na
	NARCH-N	215	465	2	2	0	0.0042827	0	0.00426
	NARCH-t	na	na	na	na	na	na	na	na
	NGARCH-N	306	467	1	1	0	0.0021	0	0.0021
	NGARCH -t	na	na	na	na	na	na	na	na
	APGARCH-N	73	455	7	7	0	0.0151	0	0.0149
	APGARCH -t	na	na	na	na	na	na	na	na

5.2.3. Third step: results analysis

In this section, the accuracy of estimated VaR is analyzed and evaluated based on different backtesting criteria. At first glance it can be seen from Table 4 that in most cases, parametric approach estimate the value of VaR more than two other methods which this value under student-t distribution is greater than normal distribution. In order to provide more accurate evaluation of results, confidence level, and type of backtesting and number of out of sample observations should be taken in to account.

Table 7. Backtesting result at confidence level 0.95

		LR_{POF}	H_0	LR_{TUFF}	H_0	LR_{ind}	H_0	LR_{uc}	H_0
Non parametric	MCS	2.08	accept	0.0025	accept	1.28	accept	3.36	accept
	HS	51.03	reject	1.3978	accept	13.3771	reject	64.4081	reject
Semi parametric	WHS	42.54	reject	1.3978	accept	14.7022	reject	57.252	reject
	FHS	42.54	reject	1.3978	accept	14.7022	reject	57.252	reject
parametric	GARCH-N	1.47	accept	0.0025	accept	1.4372	accept	2.906	accept
	GARCH-t	39.80	reject	23.84	reject	0.0042735	accept	39.8055	reject
	IGARCH-N	16.65	reject	2.81	accept	0.2121	accept	16.8592	reject
	IGARCH-t	39.80	reject	23.84	reject	0.0042735	accept	39.8055	reject
	RM-N	11.33	reject	1.40	accept	10.13	reject	21.44	reject
	RM -t	0.011	accept	0.002	accept	2.37	accept	2.38	accept
	GJR-N	16.65	reject	2.81	accept	0.2121	accept	16.8592	reject
	GJR-t	39.80	reject	23.84	reject	0.0042735	accept	39.8055	reject
	AGARCH-N	0.29	accept	0.0025	accept	1.9695	accept	2.2594	accept
	AGARCH-t	39.80	reject	23.8365	reject	0.0042735	accept	39.8055	reject
	EGARCH-N	3.69	accept	0.0025	accept	0.86	accept	4.55	accept
	EGARCH-t	na	reject	na	reject	na	reject	na	reject
	NAGARCH-N	0.57	accept	0.0025	accept	1.78	accept	2.35	accept
	NAGARCH-t	na	reject	na	reject	na	reject	na	reject
	NARCH-N	0.11	accept	0.0025449	accept	2.1664	accept	2.2693	accept
	NARCH-t	na	reject	na	reject	na	reject	na	reject
	NGARCH-N	8.64	reject	2.01	accept	0.52	accept	9.16	reject
	NGARCH -t	39.80	reject	23.8365	reject	0.0042735	accept	39.8055	reject
	APGARCH-N	0.102	accept	0.002	accept	2.16	accept	2.26	accept
	APGARCH -t	29.58	reject	15.2084	reject	0.038627	accept	29.6154	reject

Table 8. Backtesting result at confidence level 0.99

		LR_{POF}	H_0	LR_{TUFF}	H_0	LR_{ind}	H_0	LR_{uc}	H_0
Non parametric	MCS	0.71	accept	0.77	accept	0.038	accept	0.75	accept
	HS	0.98825	accept	0.090431	accept	0.21213	accept	1.20	accept
Semi parametric	WHS	0.7125	accept	0.77527	accept	0.038627	accept	0.75	accept
	FHS	na	reject	na	reject	na	reject	na	reject
parametric	GARCH-N	4.3342	accept	1.8971	accept	0.0042735	accept	4.34	accept
	GARCH-t	na	reject	na	reject	na	reject	na	reject
	IGARCH-N	4.3342	accept	1.8971	accept	0.0042735	accept	4.34	accept
	IGARCH-t	na	reject	na	reject	na	reject	na	reject
	RM-N	22.125	reject	1.5717	accept	1.4372	accept	23.56	reject
	RM -t	0.98825	accept	0.090431	accept	0.21213	accept	1.20	accept
	GJR-N	4.3342	accept	1.8971	accept	0.0042735	accept	4.34	accept
	GJR-t	na	reject	na	reject	na	reject	na	reject
	AGARCH-N	1.998	accept	0.77527	accept	0.017131	accept	2.01	accept
	AGARCH-t	na	reject	na	reject	na	reject	na	reject
	EGARCH-N	0.71	accept	0.77	accept	0.038	accept	0.75	accept
	EGARCH-t	na	reject	na	reject	na	reject	na	reject
	NAGARCH-N	0.71	accept	0.77	accept	0.038	accept	0.75	accept
	NAGARCH-t	na	reject	na	reject	na	reject	na	reject
	NARCH-N	1.998	accept	0.77527	accept	0.017131	accept	2.01	accept
	NARCH-t	na	reject	na	reject	na	reject	na	reject
	NGARCH-N	4.33	accept	1.89	accept	0.004	accept	4.33	accept
	NGARCH -t	na	reject	na	reject	na	reject	na	reject
	APGARCH-N	0.99	accept	0.09	accept	0.21	accept	1.2	accept
	APGARCH -t	na	reject	na	reject	na	reject	na	reject

For example, in POF test, number of real failures should be close to expected failures (α.T) at confidence level of (1-α) %. Results related to number of failures and failure rates at two confidence level of 95% and 99% are given in Table 4. Likelihood ratio statistics for POF test is presented in second column of Tables 7 and 8 for confidence level of 95% and 99%, respectively. Reject or accept results of null hypothesis (α=$\hat{\alpha}$) are given in third columns of Tables 7 and 8.

It can be observed that based on unconditional coverage test of POF, for confidence level of 95%, null hypothesis is only accepted for some volatility models (including GARCH, AGARCH, NARCH, EGARCH,NAGARCH APGARCH under normal distribution and Riskmetrics under student-t distribution) and Monte-Carlo simulation method. But for confidence level of 99%, H_0 is accepted for all nine GARCH type models under normal distribution and Riskmetrics model under student-t distribution. 1t is also accepted for historical, Monte Carlo and volatility-weighted historical simulation.

TUFF statistics and reject or accept result of H_0 for all mentioned approaches are given in fourth and fifth columns of Tables 7 and 8, respectively. Results show that based on TUFF test the accuracy of estimated VaR at confidence level of 95% is approved for semi-parametric approach, nonparametric approach and all nine GARCH type models under normal distribution. Also null hypothesis according to Riskmetrics model under student-t distribution is accepted. These results at 99% confidence level are the same except for filtered historical simulation.

Results of independence statistics about calculated VaR are given in fifth column of Tables 7 and 8 and the result of acceptation or rejection are given in 6th column of these tables. It can be found that at confidence level of 95%, independence hypothesis is accepted for Monte Carlo simulation and also for all models of parametric approach except than EGARCH, NAGARCH and NARCH models under normal distribution and Riskmetrics under student-t distribution. At 99% confidence level, independence hypothesis is accepted for Monte-Carlo simulation, historical simulation and volatility-weighted historical simulation. In addition to, it is also accepted for all ten volatility models under normal distribution and Riskmetrics under student-t distribution.

Finally, 8th and 9th column of Tables 7 and 8 are related to conditional coverage test which is the combination of Kupiec failure test and Christoffersen independence test. Results show that at 95% confidence level, Monte Carlo simulation method, Riskmetrics volatility model under student-t distribution and some of GARCH type models under normal distribution such as GARCH, AGARCH, NARCH, NAGARCH, EGARCH and APGARCH have the both characteristics of optimum failures and independent failures. But, at 99% confidence level accuracy of calculated VaR is accepted based on this test via all GARCH family models under normal distribution and Riskmetrics under student-t distribution. Also validity of Monte Carlo simulation, historical simulation and volatility-weighted historical simulation are approved.

As mentioned before, proposing VaR as a risk measure created an important evolution in risk management. In fact, major application of this measure for financial institution is determination of capital requirement in order to loss coverage. Therefore, accurate forecasting of VaR is essential. According the results of this paper, correct amount of VaR for total index is obtained close to 0.013 and .020 at 95 and 99 percent confidence levels respectively. This means that for an individual who hold one million Rial stock, he will exposure up to 13000 Rial loss for next day with 95 percent probability. In other words, capital adequacy ratio for risk coverage is 0.013. Since accuracy of these results is tested by backtesting statistics, findings of this study can be proposed to financial investors and participants for their investing and analyzing.

6. Conclusion

In this study, daily VaR is estimated for total index of Tehran stock exchange through parametric, nonparametric and semi-parametric approaches between 2004 and 2014. Also, the accuracy of calculated VaR is evaluated using conditional and unconditional coverage backtesting tests.

The results show that, null hypothesizes based on equality of actual and expectation failures and serial independence of failures are accepted for Monte-Carlo simulation, historical simulation and volatility-weighted historical simulation at confidence level of 99%. They are also accepted for all GARCH type models of parametric approach under normal distribution and Riskmetrics model under student-t distribution. But, at confidence level of 95%, accuracy of calculated VaR is approved only for Monte-Carlo simulation and some of volatility models including GARCH, AGARCH, EGARCH, NARCH, NAGARCH and APGARCH models under normal distribution and Riskmetrics model under student-t distribution.

We also found that volatility models measure VaR value more than non-parametric and semi-parametric approaches. Moreover, this value under student-t distribution is more than normal distribution. In fact by comparing of obtained backtesting statistics and results of occurred failures, volatility models of parametric approach under student-t distribution overestimate the magnitude of VaR. Finally, it can be concluded that four volatility models of parametric approach (including NARCH, NAGARCH, and APGARCH under normal distribution and Riskmetrics under student-t distribution) provide more accurate VaR estimates.

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