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Studying Transition Behavior of Neutron Point Kinetics Equations Using the Lyapunov Exponent Method

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Abstract

The neutron density is one of the most important dynamical parameters in a reactor. It is directly related to the control and stability of the reactor power. Any change in applied reactivity and some of dynamical parameters in the reactor causes a change in the neutron density. Lyapunov exponent method is a powerful tool for investigating the range of stability and the transient behavior of the reactor power. In contrast to the other linear stability methods, this method can be used for large perturbations and is not needed to construct Lyapunov function. In this work, the range of stability using Lyapunov exponent method is evaluated for neutron point kinetics equations with six-groups delayed neutrons. Here, effects of four set of applications, namely, step, ramp, sinusoidal, and temperature feedback reactivities on power reactor were investigated with Lyapunov exponent method. The results of qualitative analysis were compared with traditional methods and were in good agreement with other works.

Keywords: Temperature feedback; Neutron density; Delayed neutron; Lyapunov exponent.

Introduction

The dynamic behavior of the reactor can be described via a set of ordinary differential equations known as the neutron point reactor kinetics (NPK) equations [1, 2]. NPK equations describe the variation of in neutron population due to the change of reactivity [2]. The neutron density and the delayed neutron precursor concentration are the most important parameters to be studied, for the purpose of safety and the transient behavior of the reactor power [3]. These parameters are directly affected by reactivity [4,5].

There are several methods for stability analysis of a nuclear reactor. In literature [6-8], these methods are divided into linear and nonlinear; Routh Hurwitz criterion [9], Nyquist stability [8], Bode diagram [10] are some of linear stability analysis methods. The linear stability analysis methods are valid for nonlinear systems with small perturbations [8]. Nonlinear methods: such as Lyapunov second method and

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Lyapunov exponent method [4, 11-13] are the most important methods for nonlinear stability analysis. Lyapunov second method is based on the construct of Lyapunov function for which there are several methods such as Aizerman, Szego, Rosen, variable gradient methods, etc, but there is no systematic method for the construction of this function [8, 11, 13]. Some of the researchers- Fu, Chen, Ergen, etc have used this method [4, 12, 13]. While, for Lyapunov exponents method there is no such problem, so another important method for analyzing and diagnosing instability of nuclear reactors is the spectrum of Lyapunov exponents method, that is based on eigenvalues and eigenvectors of the Jacobian matrix [14-16]. Estimating Lyapunov exponents, which is one of the most important problems of the control theory, has been investigated in the following papers: [17] for discrete time varying linear systems, [18] for continuous time linear systems and [19] for discrete stochastic linear systems. Lyapunov exponents describe necessary and sufficient conditions for asymptotic stability in the linear systems [19]. So, Lyapunov exponent method in the nonchaotic systems can be used for the analysis of stability.

The purpose of the present study is to introduce the mean Lyapunov exponent approach on stability analysis of NPK equations in nuclear reactors with multi-group delayed neutron in the presence of step, ramp, sinusoidal, and temperature feedback reactivities.

This paper is organized as follows. In Sec. 2, a brief description of NPK equations is presented. In Sec. 3, analysis tools are reviewed and finally, the authors are discussed and interpreted the results.

Materials and Methods

In this section we introduce neutron point kinetic equations and Lyapunov exponent method for analysis reactor dynamics.

II-1. Neutron point kinetic equations

NPK equations by considering the temperature feedback reactivities are the stiff non-linear ordinary differential equations [20]. The general form of the equations with m-group delayed neutron precursors is [21-23]:

(1)

$$\frac{dn(t)}{dt} = \left(\frac{\rho(t)-\beta}{l}\right) n(t) + \sum_{i=1}^{m} \lambda_i c_i(t) + q(t)$$

$$\frac{dc_i(t)}{dt} = \frac{\beta_i}{l} n(t) - \lambda_i c_i(t)$$

Where $\rho(t)$ is the net reactivity which is the sum of external reactivity ($\rho_{ext}(t)$) and feedback reactivity $\rho_f(t)$ [24]. $n(t)$ is the neutron density, $c_i(t)$ is the concentration of precursor delayed neutrons, β_i is the relative fraction of $i-th$ group delayed neutron, β is the total effective fraction of delayed neutrons, l is the prompt neutron generation time, λ_i is the $i-th$ group constant decay, and $q(t)$ is the external neutron source. The effects of arising from the temperature feedback reactivities can be written as [1, 25, 26]:

$$\rho(t) = \rho_0 - \alpha\left[T(t) - T_0\right] \tag{2}$$

Here, α is the temperature coefficient of reactivity, ρ_0 is initial reactivity, $T(t)$ is the temperature of the reactor, and T_0 is the initial temperature of the reactor. After the reactivity ρ_0 is inserted into the reactor, the power responds quickly and the adiabatic model can be used for the calculation of reactor temperature as follows [8]:

$$\frac{dT(t)}{dt} = k_c n(t) \tag{3}$$

Where, k_c is the reciprocal of the thermal capacity of the reactor core.

II-2. Analysis Tools

In this study we were focused attention on dynamical systems described by a set of ordinary differential equations particularly NPK equations with six group delayed neutron.

Lyapunov exponents and entropy measures on the other hand can be considered "dynamic" measures of attractors complexity which are called "time average" [27]. The Lyapunov exponent¸ is useful for distinguishing various orbits. Lyapunov exponents quantifies sensitivity of the system to initial conditions and gives a measure of predictability. The Lyapunov exponent is a measure of the rate at which the trajectories separate one from another [28, 29] A negative exponent implies that the orbits approach to a common fixed point. A zero exponent means that the orbits maintain their relative positions; they are on a stable attractor. Finally, a positive exponent implies that the orbits are on a chaotic attractor, so the presence of a positive Lyapunov exponent indicates chaos. Even though an m dimensional system has m Lyapunov exponents, in most applications it is sufficient to compute only the Lyapunov exponents.

The Lyapunov exponents are defined as follows:

Consider two nearest neighboring points (usually the nearest) in phase space at time 0 and t, with distances of the points in the $i-th$ direction $\|\delta d_i(0)\|$ and $\|\delta d_i(t)\|$, respectively. The Lyapunov exponent is then defined through the average growth rate Λ_i of the initial distance,

$$\Lambda_i = \lim_{t\to\infty}\frac{1}{t}\log_2\frac{\|\delta d_i(t)\|}{\|\delta d_i(0)\|} \tag{4}$$

The Lyapunov exponents for a region of N-dimentional state space near a fixed point are the characteristic values Λ_i that of fixed point.

If $\forall\Lambda_i \in \text{State Space} \to \Lambda_i < 0$, then the trajectories approach the fixed point exponentially.

If $\forall\Lambda_i \in \text{State Space}, \exists\Lambda_i > 0$, then the trajectories repelled from the fixed point exponentially [30].

Commonly, Lyapunov exponents (Λ_i) can be extracted by observed signals in the following different method:

- Based on the opinion of following the time-evolution of nearby points in the state space.
- Based on the estimation of local Jacobi matrices.

The first method is usually called Wolf algorithm [31] and it only provides an estimation of the largest Lyapunov exponent. The second method is capable of estimating all the Lyapunov exponents. Using one of these methods, the Lyapunov exponent is calculated rather than a given control parameter. So, there is a little increase in the value of the control parameter and the Lyapunov exponent is calculated for the new control parameter. By continuing this method the Lyapunov exponent spectrum of the point reactor kinetics is

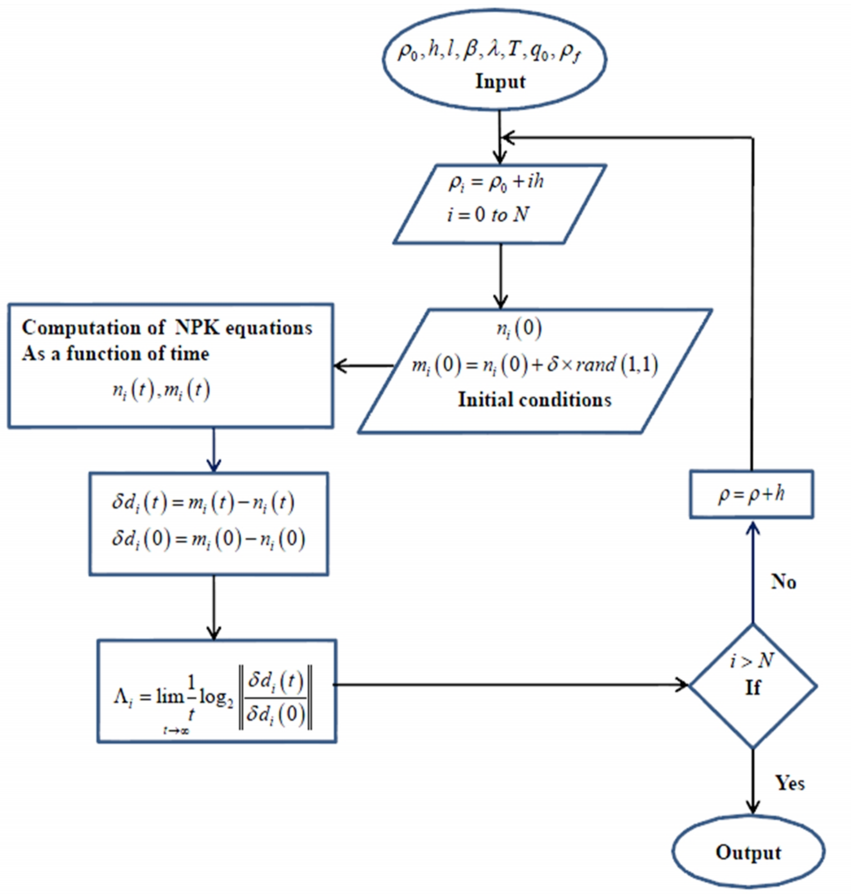


Figure 1. Calculating Lyapunov Exponents algorithm.

plotted versus the control parameter. In this study, for calculating Lyapunov exponent spectrum was applied Wolf algorithm. This algorithm is shown in Fig.1. According to the Fig.1, we have solved the NPK equations for a arbitrary value of the control parameter (ρ_i) for two initial condition

$$\left(n_i(0), m_i(0) = n_i(0) + \delta \times rand(1,1), \delta << 1, \delta \in R\right)$$

which are very close to each other. Next we have calculateed time series depends on the neutron density that is $n_i(t)$ and $m_i(t)$. Finally by using the following relation, we have calculated Lyapunov exponent.

$$\Lambda_i = \lim_{t\to\infty} \frac{1}{t} \log_2 \frac{\left\|\delta d_i(t)\right\|}{\left\|\delta d_i(0)\right\|} = \lim_{t\to\infty} \frac{1}{t} \log_2 \frac{\left\|m_i(t) - n_i(t)\right\|}{\left\|m_i(0) - n_i(0)\right\|} \tag{5}$$

Results

Here, we study the dynamical behavior of systems Eqs.1 and 3 by calculating their Lyapunov exponents. The set of all Lyapunov exponents of discrete time varying linear systems is called the spectrum of this system ($\{\Lambda_1, \Lambda_2, \ldots, \Lambda_n\}$) [17]. According to the theorem of Oseledec, the almost certain stability of the trivial solution of a system can be determined by the largest Lyapunov $\Lambda = \Lambda_{\max}$; i.e., when $\Lambda < 0$ the trivial solution is almost certainly stable and when $\Lambda > 0$ the trivial one unstable [18]. All of the coordinates in this phase space are given as follows: $n(t)$, $c_1(t)$, $c_2(t)$, $c_3(t)$, $c_4(t)$, $c_5(t)$ and $c_6(t)$. If we want to count of the effect of temperature feedback another coordinate, $T(t)$ is added too. Therefore stability analysis should be followed with the eight Lyapunov exponents $\{\Lambda_1, \Lambda_2, \ldots, \Lambda_8\}$. It is impossible to obtain these quantities analytically. Therefore numerical and estimation methods are required for their approximation [18]. Here, numerical calculations have been done with $ODE45$ method. Initial conditions are [3, 20, 21, 32]:

$$n_0 = 1, \quad c_{0i} = \frac{n_0 \beta_i}{l \lambda_i} \tag{6}$$

The data in this study are taken from references [3, 20, 32]. In the following each reactivity will be discussed in a subsection.

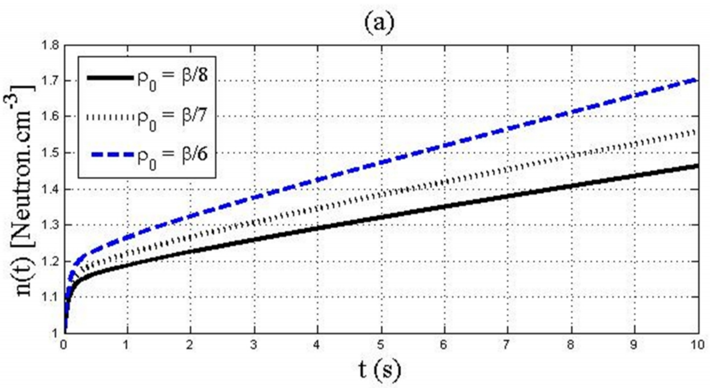
III-1. Step Reactivity

We study the dynamical behavior of NPK equations with six-groups delayed neutrons for step reactivity with steps $-\beta$, -0.5β, 0.5β and β. Lyapunov exponent method, is applied to the solving and the stability analysis of NPK equations in the thermal reactor with the following parameters [3, 20]: $\lambda_1 = 0.0127\, s^{-1}$, $\lambda_2 = 0.0317\, s^{-1}$, $\lambda_3 = 0.115\, s^{-1}$, $\lambda_4 = 0.311 s^{-1}$, $\lambda_5 = 1.4\, s^{-1}$, $\lambda_6 = 3.87\, s^{-1}$, $l = 0.0005\, s$, $\beta_1 = 0.000285$, $\beta_2 = 0.0015975$, $\beta_3 = 0.00141$, $\beta_4 = 0.0030525$, $\beta_5 = 0.00096$, $\beta_6 = 0.000196$ and $\beta = 0.0075$.

Numerical calculations have been performed with $ODE45$ method and relative errors are compared with the exact values of the neutron density (See Table 1) [3, 33]. For positive and negative step reactivities, it can be observed from Figure 2a and 2b, that the neutron density gradually increased and

Table 1. The neutron density and relative error of the thermal reactor with step reactivity [3].

ρ_0 (\$)	t_0 (s)	TSM *h=01*	AEM *h=01*	GAEM *h=01*	*RK* (*ODE* 45) *h=01*	Exact
-1	0.1	2.2×10^{-4}	1.92×10^{-7}	0.0	1.4×10^{-6}	0.5206043
-1	1	0.0	0.0	0.0	-1.85×10^{-6}	0.4333335
-1	10	0.0	0.0	0.0	-9.74×10^{-7}	0.2361107
-0.5	0.1	3.15×10^{-6}	1.43×10^{-7}	0.0	1.43×10^{-6}	0.6989252
-0.5	1	0.0	0.0	0.0	-1.6×10^{-6}	0.6070536
-0.5	10	0.0	0.0	0.0	-1.01×10^{-6}	0.3960777
+0.5	0.1	0.0	0.0	0.0	6.52×10^{-7}	1.533113
+0.5	1	3.98×10^{-7}	0.0	0.0	-1.11×10^{-6}	2.511494
+0.5	10	0.0	-7.53×10^{-7}	-7.53×10^{-7}	-1.26×10^{-6}	14.21503
+1	0.1	0.0	0.0	0.0	1.19×10^{-6}	2.515766
+1	1	0.0	0.0	0.0	0.0	10.36253
+1	10	0.0	-3.11×10^{-7}	-3.11×10^{-7}	-1.24×10^{-7}	32.18354



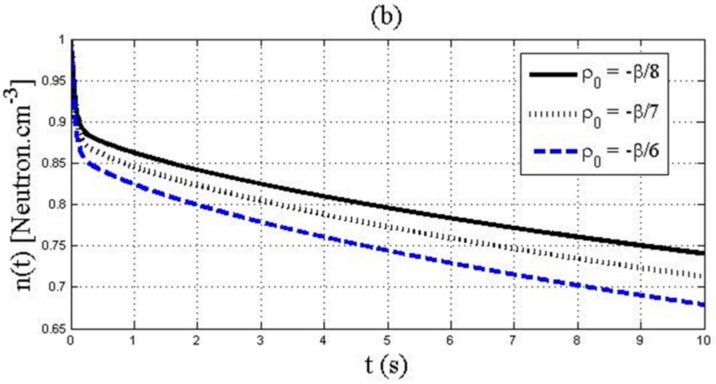


Figure 2. The neutron density as a function of time for three cases of the positive step reactivity (a), and three cases of the negative step reactivity (b).

decreased respectively. According to Table 2, in long term $(t \to \infty)$, all of the Lyapunov exponents for negative steps ($-\beta$ and -0.5β) are negative. It means that NPK equations are stable in the three dimensional space. So, the neutron density was decreased during time and reactor goes into shutdown. The step reactivity with steps β and 0.5β have a positive Lyapunov exponent, (See Table 2). In this situation, the fixed points of NPK equations with these steps are unstable so, the neutron density will be increased exponentially and reactor with these reactivities cannot remain in critical state [8].

Figure 3, shows the results of Lyapunov exponent method with respect to step reactivity in short term ($t = 10s$). Thus the transient behavior of reactor is stable for $\rho_0 < 0.003156$ in short term ($t = 10s$).

Table 2. Lyapunov exponent with respect to time for step reactivity.

$\rho_0(\$)$	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Λ_6	Λ_7
-1	-0.0124	-0.0258	-0.0980	-0.2475	-1.3115	-3.8161	-30.2281
-0.5	-0.0116	-0.0222	-0.0896	-0.2283	-1.2830	-3.7964	-22.8087
0.5	0.1725	-0.0134	-0.0283	-0.1601	-1.0939	-3.6107	-8.4932
1	2.0443	-0.0131	-0.0119	-0.1395	-0.7320	-2.4761	-4.3845

Table 3. Lyapunov exponent with respect to time for ramp rate reactivity.

$r\ (\$s^{-1})$	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Λ_6	Λ_7
-0.01	-0.0128	-0.0311	-0.1137	-0.3051	-1.31920	-3.8656	-159.0096
-0.1	-0.0116	-0.0222	-0.0896	-0.2283	-1.2830	-3.7964	-158.9926
0.1	155.2998	9.1515	109.6477	95.5988	97.0994	97.2085	61.7751
0.01	239.3204	43.6724	193.9540	184.2730	186.1184	186.8532	158.3794

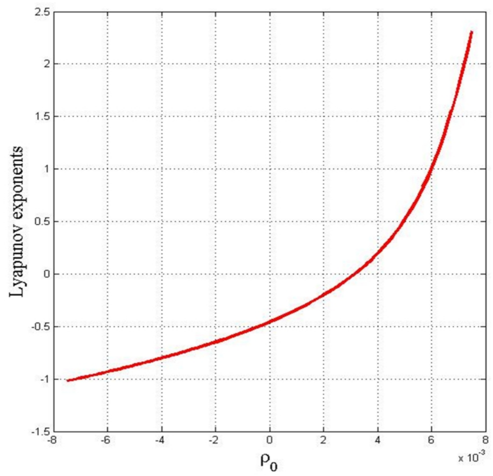


Figure 3. Variation of the Lyapunov exponents with respect to the step reactivity.

III-2. Ramp Reactivity

The analyses are presented for the thermal reactor with the following parameters [3, 20]:

$\lambda_1 = 0.0127\,s^{-1}, \lambda_2 = 0.0317\,s^{-1}$, $\lambda_3 = 0.115\,s^{-1}$, $\lambda_4 = 0.311\,s^{-1}$, $\lambda_5 = 1.4\,s^{-1}$, $\lambda_6 = 3.87\,s^{-1}$, $l = 0.00002\,s$, $\beta_1 = 0.000266$, $\beta_2 = 0.001491$, $\beta_3 = 0.001316$, $\beta_4 = 0.002849$, $\beta_5 = 0.000896$, $\beta_6 = 0.000182$ and $\beta = 0.007$.

Regarding the results listed in Table 3, the reactor for all of the negative values of ramp rate reactivities $(r \le 0)$ is stable in long term, and unstable for all of the positive values of ramp rate reactivities $(r > 0)$. Thus, for $r \le 0$, neutron density reduces gradually, and in long term, reactor goes into shutdown (see Fig. 4b). For $r > 0$, the neutron density rises rapidly and in the same vein, the period of reactor decreases. Therefore, reactor control will be problematic (see Figure 4a). Numerical results listed in Table 4 imply that.

Numerical calculations are performed and compared with TSM, GAEM and Pade methods. Regarding Table 4 the results are in good agreement [20,21]. Ramp rate reactivity influenced on the neutron density behavior is investigated in short term $(0 \le t \le 10s)$. According to Figure 5, Lyapunov exponent with respect to control parameter, (r), will be increased. For $r > 0.000798$, all of the Lyapunov exponents are positive so, system is

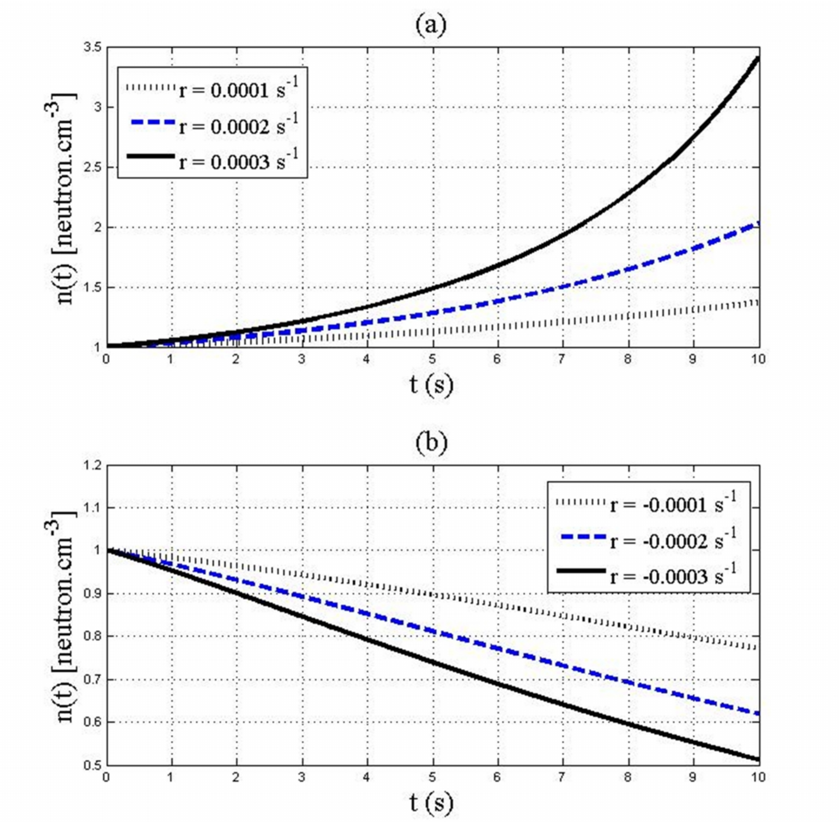


Figure 4. The neutron density as a function of time for three cases of the positive ramp rate reactivity (a), and three cases of the negative ramp rate reactivity (b).

Table 4. The neutron density of the thermal reactor with positive and negative ramp rate reactivity [3].

Ramp reactivity with $r=+0.1(\$ s^{-1})$				
$t(s)$	$TSM(h=0.0001)$	$GAEM(h=0.1)$	$Pade(h=0.001)$	$ODE45(h=0.0001)$
2	1.3382	1.3382	1.3382	1.3382
4	2.2284	2.2284	2.2284	2.2285
6	5.5822	5.5820	5.5820	5.5821
8	42.789	42.786	42.786	42.7867
10	4.5143×10^5	4.5116×10^5	4.5116×10^5	4.5117×10^5
Ramp reactivity with $r=-0.1(\$ s^{-1})$				
$t(s)$	$TSM(h=0.0001)$	$GAEM(h=0.1)$	$Pade(h=0.001)$	$ODE45(h=0.0001)$
2	0.792001	0.792007	0.792007	0.792016
4	0.613018	0.613020	0.613018	0.613021
6	0.474058	0.474065	0.474058	0.474059
8	0.369169	0.369172	0.369169	0.369168
10	0.290654	0.290653	0.290654	0.290653

Table 5. Lyapunov exponent with respect to time for different values amplitude ,(a), and period ,($\tau(s)$), of the sinusoidal reactivity.

$\tau(s)$	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Λ_6	Λ_7
1	0.0005	-0.0142	-0.0647	-0.1935	-1.0228	-2.8960	-216.1836
30	0.0008	-0.0142	-0.0646	-0.1938	-1.0229	-2.8960	-216.1598
70	0.0010	-0.0142	-0.0647	-0.1938	-1.0228	-2.8959	-216.0854
100	0.0011	-0.0143	-0.0648	-0.1938	-1.0228	-2.8959	-216.0374
$a(\$)$	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Λ_6	Λ_7
0.1	0.0003	-0.0142	-0.0648	-0.1936	-1.0230	-2.8961	-216.1830
0.2	0.0017	-0.0141	-0.0637	-0.1925	-1.0215	-2.8947	-216.1886
0.5	0.0169	-0.0136	-0.0547	-0.1835	-1.0097	-2.8832	-216.2451
1	0.9158	-0.0129	-0.0223	-0.1517	-0.9281	-2.7800	-217.3801

unstable (see Figure 5). In long term, the boundary of stability tend to become zero $(r\to 0)$.

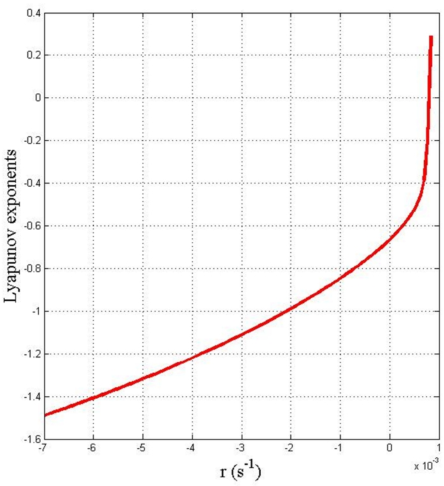


Figure 5. Variation of the Lyapunov exponents with respect to the ramp rate reactivity.

III-3. Sinusoidal Reactivity

In this subsection, sinusoidal reactivity, $\rho = a\sin\left(\frac{2\pi t}{\tau}\right)$, has been investigated, where τ and a are respectively periods and amplitude of sinusoidal reactivity. It is applied to the thermal reactor with the following parameters [32]:

$\lambda_1 = 0.0124\,s^{-1}$, $\lambda_2 = 0.0305\,s^{-1}$, $\lambda_3 = 0.111\,s^{-1}$, $\lambda_4 = 0.301\,s^{-1}$, $\lambda_5 = 1.14\,s^{-1}$, $\lambda_6 = 3.01\,s^{-1}$, $l = 0.00003\,s$, $\beta_1 = 0.000214$, $\beta_2 = 0.001423$, $\beta_3 = 0.001247$, $\beta_4 = 0.002568$, $\beta_5 = 0.000748$, $\beta_6 = 0.000273$ and $\beta = 0.006473$.

The variation of Lyapunov exponents with respect to time have been shown in Table 5. In long term the reactor is unstable for all of the values of amplitude (a) and period (τ) of reactivity. It means that the density of neutron will be increased exponentially (see Figures 6a and 6b).

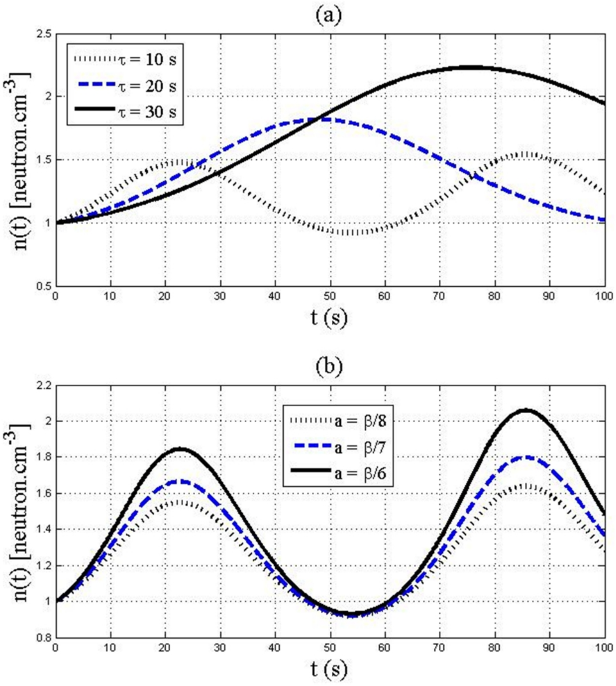


Figure 6. The neutron density as a function of time for three cases of the period of sinusoidal reactivity (a), and three cases of the amplitude of sinusoidal reactivity (b).

In short term, the reactor can be stable or unstable. For example, the neutron density in 10 s with $\tau = 1s$ and $a = 0.00073$ will be decreased (see Table 6). In this range of time reactor is stable. Lyapunov exponents with respect to control parameters are shown in Figures 7 and 8. Figure 7, shows that in the range of $1s \le \tau \le 100s$ the reactor is stable. According to Figure 8, in the range of $-\beta \le a \le \beta$ for $a < 0.0034522$ reactor is stable.

III-4.Temperature Feedback Reactivity

To complete the analysis, we now consider the NPK equations in presence of temperature feedback reactivity with the following parameters [3,20,21]: $\lambda_1 = 0.0124\,s^{-1}, \lambda_2 = 0.0305\,s^{-1}$, $\lambda_3 = 0.111\,s^{-1}$, $\lambda_4 = 0.301\,s^{-1}$, $\lambda_5 = 1.13\,s^{-1}$, $\lambda_6 = 3.0\,s^{-1}$, $l = 0.00005\,s$, $\beta_1 = 0.00021$, $\beta_2 = 0.00141$, $\beta_3 = 0.00127$, $\beta_4 = 0.00255$, $\beta_5 = 0.00074$, $\beta_6 = 0.00027$, $\beta = 0.00645$, $\alpha = 0.00005K^{-1}$ and $k_c = 0.05\,MW^{-1}s^{-1}K^{-1}$.

Table 7 shows that all of the Lyapunov exponents are negative for various control parameters. Therefore, system is stable in long term $(t \to \infty)$.

Figures 9a, 9b and 9c illustrate the change in the neutron density as a function of time for different values of initial reactivity, temperature coefficient of reactivity and the reciprocal of the thermal capacity of reactor, respectively. In these Figs, the neutron density is increased until tend to the maximum point and

Table 6. The neutron density of the thermal reactors with sinusoidal reactivity [31].

Sinusoidal reactivity with $a = 0.00073$ و $\tau = 1(s)$			
$t\,(s)$	$DM\,(h=0.01)$	$Hansen$	$ODE45(h=0.001)$
1	1.12351	1.12396	1.12394
2	1.16816	1.16880	1.16889
3	1.07429	1.07442	1.07448
4	0.95527	0.95380	0.95383
5	0.90454	0.90737	0.90735
10	0.98172	0.98464	0.98468

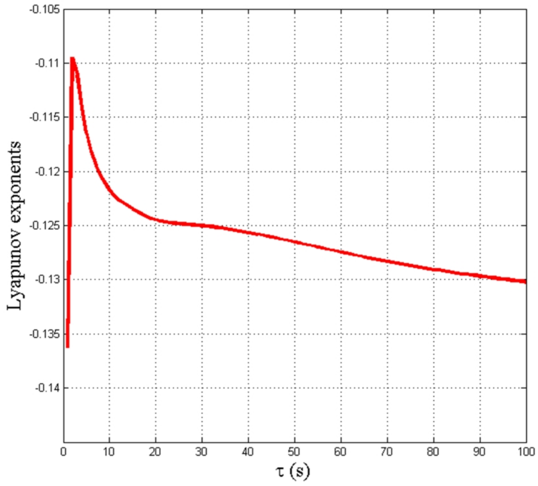


Figure 7. Variation of the Lyapunov exponents with respect to the period of sinusoidal reactivity.

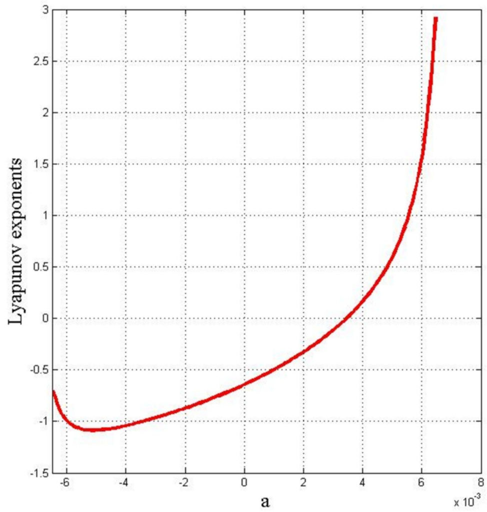


Figure 8. Variation of the Lyapunov exponents with respect to the amplitude of sinusoidal reactivity.

decreased until going into zero, due to the effects of temperature feedback reactivity. So, reactor goes into stability.

Table 8, illustrate the response of the neutron density to increase initial reactivity. Numerical calculations with ODE45 are performed and compared with TSM, GAEM and NAM methods. The results are in good agreement [20, 21]. In this work the transient behavior of reactor for range of time $(0 \le t \le 350s)$ and initial reactivity $(0 \le \rho_0 \le 2\beta)$ are considered. According to Fig. 10, all of the Lyapunov exponents with respect to initial reactivity are in the negative range. Therefore, in short term reactor is stable. By changing some dynamical parameters in the reactor such as k_c and α, reactor power can be changed, so here we considered the effect of changing these two parameters using Lyapunov exponent method. Figure 11, imply that the variation of Lyapunov exponents with respect to α parameter. They are negative in the range of $0.00001 \le \alpha \le 0.001$ and $(0 \le t \le 350s)$, therefore reactor goes into stability (see Table 7). Figure 12, shows that the variation of Lyapunov exponents with respect to k_c parameter are negative in the range of $0.001 \le k_c \le 0.1$ and $(0 \le t \le 350s)$. The results of Table 7, are in good agreement with the results of Figures 10, 11 and 12.

Discussion

Predicting the dynamic behavior of a nuclear reactor

Table 7. Lyapunov exponent with respect to time for different values control parameters, with temperature feedback reactivity.

α	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Λ_6	Λ_7	Λ_8
0.00001	-0.0007	-0.0116	-0.0246	-0.0947	-0.2415	-1.0683	-2.9400	-257.8245
0.00005	-0.0007	-0.0118	-0.0246	-0.0948	-0.2416	-1.0685	-2.9402	-258.3478
0.00010	-0.0007	-0.0118	-0.0247	-0.0949	-0.2417	-1.0686	-2.9403	-258.9940
0.00100	-0.0007	-0.0119	-0.0252	-0.0958	-0.2441	-1.0710	-2.9427	-270.0838
k_c	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Λ_6	Λ_7	Λ_8
0.001	-0.0011	-0.0114	-0.0246	-0.0947	-0.2414	-1.0683	-2.9400	-257.6987
0.010	-0.0009	-0.0116	-0.0246	-0.0947	-0.2415	-1.0683	-2.9400	-257.8243
0.050	-0.0007	-0.0118	-0.0246	-0.0948	-0.2416	-1.0685	-2.9402	-258.3438
0.100	-0.0007	-0.0118	-0.0247	-0.0949	-0.2417	-1.0686	-2.9403	-258.9941
$\rho_0(\$)$	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Λ_6	Λ_7	Λ_8
0.5	-0.0007	-0.0111	-0.0211	-0.0867	-0.2238	-1.0497	-2.9217	-194.2564
1	-0.0007	-0.0118	-0.0246	-0.0948	-0.2416	-1.0685	-2.9402	-258.3748
1.5	-0.0008	-0.0120	-0.0263	-0.0990	-0.2530	-1.0802	-2.9517	-322.6741
2	-0.0008	-0.0121	-0.0272	-0.1015	-0.2609	-1.0882	-2.9594	-387.0629

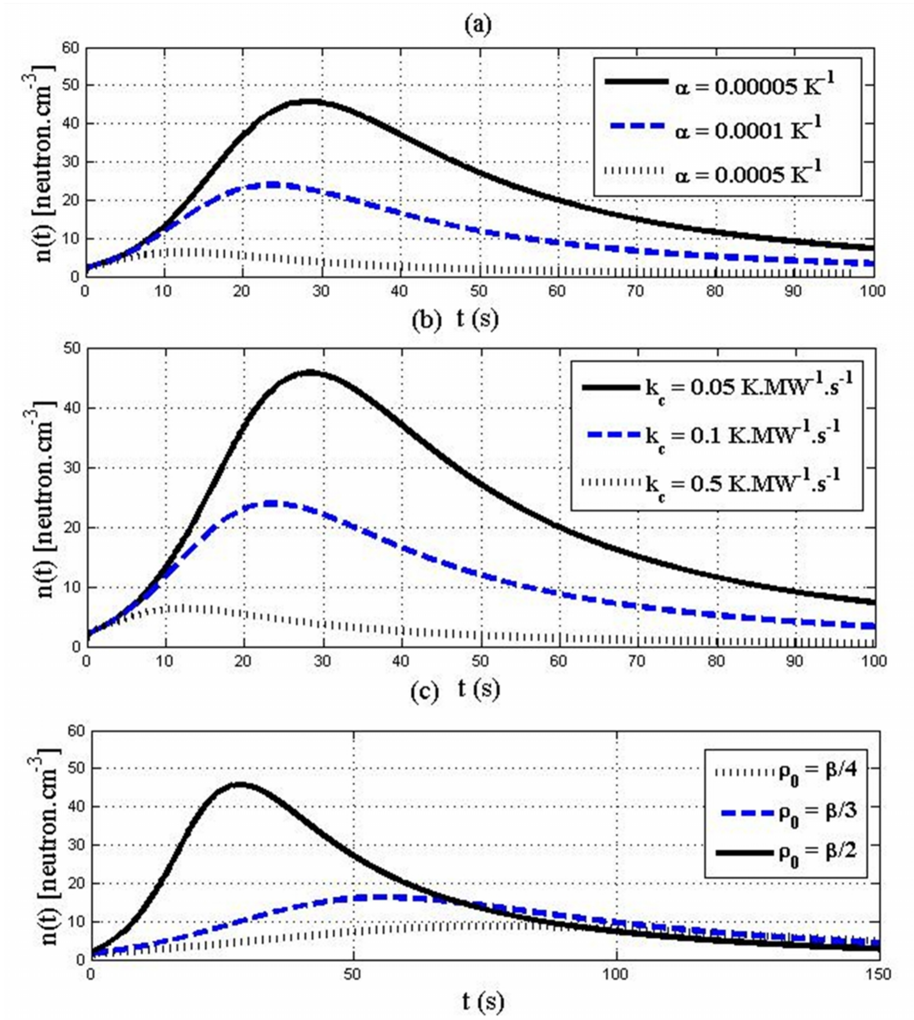


Figure 9. The neutron density as a function of time for three cases of the temperature coefficient of reactivity (a), three cases of the reciprocal of the thermal capacity of the reactor core (b), and three cases of the initial reactivity (c).

Table 8. The peak of the neutron density with temperature feedback reactivity [20].

	The peak of the neutron density				Time of the peak			
ρ_0($)	*TSM*	*GAEM*	*NAM*	*ODE*45	*TSM*	*GAEM*	*NAM*	*ODE*45
0.5	45.75429	45.75212	45.75240	45.75243	8.293	28.293	28.293	28.293
1	807.8765	807.8672	807.8666	807.8676	0.953	0.953	0.953	0.954
1.2	8020.848	8020.919	8020.916	8020.795	0.317	0.317	0.317	0.317
1.5	43021.00	43024.69	43025.93	43020.82	0.168	0.168	0.168	0.168
2	167739.3	167800.4	167856.6	167738.9	0.098	0.098	0.098	0.099

due to changes in the parameters of dynamical systems is very important. Lyapunov exponent method is a powerful tool that can help to determine the range of linear and nonlinear systems stability for the changes control parameters of the system. In this work the influence of step, ramp, sinusoidal and temperature feedback reactivities on stability and the neutron of density with Lyapunov exponent method were investigated, also the analysis of Lyapunov exponent respect to time for reactivities above in long-term scale ($t \to \infty$) was studied. In order to validate the method, the neutron density changes due to changes in the parameters of dynamical systems with *ODE*45 method was carried out and the results of compared with *Pade*,

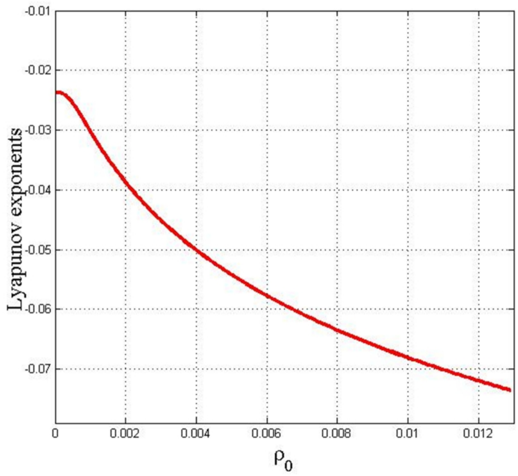


Figure 10. Variation of the Lyapunov exponents with respect to the initial reactivity.

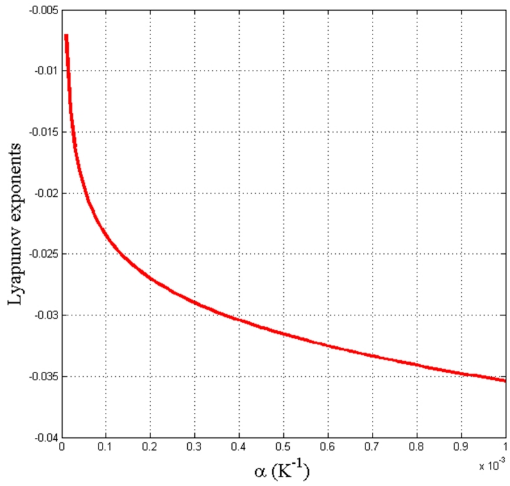


Figure 11. Variation of the Lyapunov exponents with respect to the temperature coefficient of reactivity.

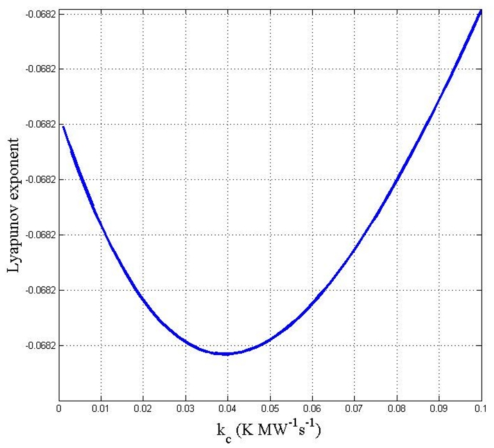


Figure 12. Variation of the Lyapunov exponents with respect to the reciprocal of the thermal capacity of the reactor core.

TSM, *GAEM*, *NAM*, *DM*, and *Hansen* methods [3,20,32]. The results are in good agreement with each other, so the *ODE*45 method can be used instead of the above methods. The quantitative results from *ODE*45 method, confirm the qualitative results obtained from Lyapunov exponent method.

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