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Structural Reliability: An Assessment Using a New and Efficient Two-Phase Method Based on Artificial Neural Network and a Harmony Search Algorithm

Kazemi Elaki, N.[1*], Shabakhty, N.[2], Abbasi Kia, M.[3] and Sanayee Moghaddam, S.[4]

[1] M.Sc. in Structural Engineering, University of Sistan and Baluchestan, Zahedan, Iran.
[2] Invited Lecturer, Ph.D. in Offshore Engineering, Faculty of Civil Engineering, Iran University of Science and Technology, Tehran, Iran.
[3] M.Sc. in Computer Science, Department of Mathematics, University of Lorestan, Khorramabad, Iran.
[4] M.Sc. in Hydraulic Structures, University of Sistan and Baluchestan, Zahedan, Iran.

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ABSTRACT: In this research, a two-phase algorithm based on the artificial neural network (ANN) and a harmony search (HS) algorithm has been developed with the aim of assessing the reliability of structures with implicit limit state functions. The proposed method involves the generation of datasets to be used specifically for training by Finite Element analysis, to establish an ANN model using a proven ANN model in the reliability assessment process as an analyzer for structures, and finally estimate the reliability index and failure probability by using the HS algorithm, without any requirements for the explicit form of limit state function. The proposed algorithm is investigated here, and its accuracy and efficiency are demonstrated by using several numerical examples. The results obtained show that the proposed algorithm gives an appropriate estimate for the assessment of reliability of structures.

Keywords: Artificial Neural Network, Failure Probability, Harmony Search Algorithm, Implicit Limit State Function, Reliability Index.

INTRODUCTION

Generally, in the analysis of structural reliability, we need to define the functional relationship between strength (S) and load (L) parameters as follows:

$$M = S - L = g(X_1, X_2, ..., X_n) \quad (1)$$

where M: is the limit state function (LSF), and sometimes refers to the safety margin or the performance function. $X = (X_1, ..., X_i, ..., X_n),\ i = (1, 2, ..., n)$, denote n basic random variables, and $g(.)$: is the functional relation between them. In general, the function $g(X)$ takes a specific form, so that the failure of the structure is observed when $M \le 0$, and to the contrary, the survival of the structure occurs when $M > 0$.

* Corresponding author E-mail: n.kazemi87@yahoo.com

Therefore, the failure probability can be estimated by performing the following integration over the failure region:

$$P_f = \int_{g(X\leq 0)} \int \ldots \int f_x(x_1, x_2, \ldots, x_n)\, dx_1 . dx_2 \ldots dx_n \tag{2}$$

where f_x: denotes the joint probability density function (PDF) of the basic random variables $X_1, X_2, \ldots, X_n$. For most practical reliability problems, the structural responses have to be calculated by a numerical procedure such as Finite Element analysis. This takes the reliability analysis to another level of complexity, because the LSF $g(X)$ is not available in the explicit closed form. In other words, the functional relationship between the basic design, variable X, and the LSF $g(X)$ is not explicitly available. This situation arises, for instance, when a large-scale engineering structure is analyzed using Finite Element software. Similarly, in vibration problems, when the governing field equations are nonlinear, and/or contain parametric excitation terms, the definition $g(X)$ can only be given implicitly. Several computational approaches could be pursued for the reliability analysis of structures with implicit LSF. These can be broadly divided into three categories based on their main approaches: (1) Monte Carlo simulation (MCS) including efficient sampling methods and variance reduction techniques; (2) response surface method; and (3) sensitivity-based analysis.

As long as the specific algorithm is available to compute the structural response, MCS can be used for problems with implicit LSF. The inherent disadvantage of MCS is the tremendous computational effort required for solving problems involving a low probability of failure, or for problems that require a considerable amount of computation in each sampling cycle (Stapelberg and Rudolph, 2009). To reduce the computational cost, different techniques for reduction of variance, such as importance sampling, by Harbiz (1986) and adaptive sampling by Bucher (1998) are presented.

Within the response surface method, a first or second-order polynomial approximation of $g(X)$ can be determined through Eq. (1) and a few selected simulations in the neighborhood of the most likely failure point; and Eq. (2) applies the regression analysis of these results or solve a set of linear equations (Allaix and Carbone, 2011). Then, the obtained closed-form polynomial expression of the LSF is used to calculate the failure probability of structure. This approach will be called ‘polynomial-based response surface method’ in order to distinguish it from the proposed method, which will be called later as ‘ANN-HSA’. The main limitation of the polynomial-based response surface is that when the number of random variables increases, the number of deterministic analyses increases substantially, thus making the method more time consuming and expensive (Cheng and Li, 2008).

In the sensitivity-based approach, the sensitivity of the structural response to the input variables is computed and incorporated in the FORM/SORM algorithm (Zang et al., 2015). Thus, the value of the performance function is calculated from deterministic analysis, and the gradient is computed using sensitivity analysis. However, the sensitivity-based reliability analysis approach is more elegant and more efficient (Castillo et al., 2008) than the simulation or response surface methods. Complexity, discontinuity and nonlinear behavior of the LSF may cause severe problems while using these approaches, however, a recent study by Lopez et al, deals with some of these drawbacks (Lopez et al., 2015).

As is clear, the reliability assessment of structures by means of the above mentioned methods needs to approximate the explicit form of LSF. Therefore, developing a new technique for assessing reliability without any need to approximate the explicit form of LSF is very efficient, and this will be indispensable. For this purpose, a new two-phase technique based on meta-heuristic optimization algorithms and artificial neural network (ANN) is presented in this paper.

One of the methods used in the reliability assessment is to convert the reliability problem to the constrained optimization problem, and solve it by optimization techniques (Liu and Armen, 1991). Among optimization techniques, stochastic optimization algorithms based on swarm intelligence may efficiently help to solve global optimization problems (Marti, 2008). Actually, the stochastic optimization methods provide a means of coping with inherent system noise, models or systems that are highly nonlinear and have, high dimensions, or otherwise inappropriate for classical deterministic methods of optimization. Meta-heuristic algorithms implement some form of stochastic optimization (Mucherino and Seref, 2009). Their application in the field of structural reliability not only presents the advantage of its feasibility of implementation, but also the possibility of being able to obtain the failure probability with good accuracy, without the need for evaluation of the derivatives of LSF (Elegbede, 2005). Many different optimization algorithms are also presented to solve reliability problems, such as genetic algorithm (GA) used by Cheng (2010), Huang (2015), Coelho (2009), evolutionary algorithm given by de Castro Rodrigues et al. (2016) and Ramirez (2008).

Harmony Search Algorithm (HSA) is based on the swarm intelligence optimization algorithm, and has been recently developed, which is inspired by the phenomenon of a musician tuning his instrument (Geem, 2001). HSA does not require differential gradients, thus, it can consider discontinuous functions as well as continuous functions. It does not require an initial value setting for the variables and is free from divergence. Also the main advantage in using HSA is that it may overcome the drawback of GA’s building block theory which works well only if the relationship among variables in a chromosome is carefully considered. If neighbor variables in a chromosome have a weaker relationship than remote variables, then the building block theory may not work well because of crossover operation. However, HSA explicitly considers the relationship using an ensemble operation (Mun and Cho, 2012).

The ANNs can also be used to derive a good approximation for the LSF or structural responses. The motivation for applying ANN in this research is to develop a methodology to improve the efficiency and/or accuracy of estimating reliability in comparison to the aforementioned methods.

Shao and Murotsu (1997) developed a technique to use ANN when analyzing reliability. They used ANN to approximate an explicit form for LSF. Similarly, ANN is applied to approximate the LSF, and failure probability is then estimated by a general reliability method such as, FORM, SORM, and MCS (Deng et al., 2005). Cheng (2007) developed a method for analyzing reliability based on ANN and GA. The ANN model is used to approximate an explicit form for LSF, and GA is also used to determine probability of failure. All of the aforementioned research works also need to define the explicit form of LSF. The method proposed by the authors of this paper try to solve this problem and conquer it.

In this study, a new two-phase algorithm based on the ANN and HSA (ANN-HSA) has been proposed to assess the reliability of structures with the implicit LSF. Since the proposed method doesn't need the explicit form of LSF, there is not any problem for explicitly estimation of LSF, and only care should be given to the cost of vector solution in search process. Therefore, we offer to utilize this new technique based on (ANN) to reduce the estimation cost of a structure in response to the input solution vectors in the process of optimization.

FAILURE PROBABILITY ASSESSMENT

In this section, the problem of reliability assessment is presented, and the reliability index is introduced for assessment of reliability of structures.

One method in the reliability analysis is to convert the reliability assessment to the constrained optimization problem, and solve it by optimization techniques. In this approach, we are looking at the closest distance from LSF to the origin when all the variables are transformed from their physical space to an independent standard normal space. This close distance is known as the Hasofer-Lind reliability index (β_{HL}) which was defined by Hasofer and Lind (1974) for the first time. Hasofer-Lind reliability index is actually proposed for an independent normal that we shown here as the Π space in the Figure 1. Let U be any vector in this space, Λ the *n*-dimensional surface defined by the LSF $g(X)$ in the physical variables space **Ω**, and Y=T(Λ) its image in the standard Gaussian Π space. The Hasofer-Lind reliability index (β_{HL}) is, therefore, estimated as $\beta_{HL} = Min - (d(O,P))$, where O is the center or origin of Π space, and *P* is the closest point on the surface Y to this origin (Figure 1). This index enables us to have a first-order approximation (FORM) of the reliability by the relation $P_f \approx \Phi(-\beta)$, and it can be exact when the LSF is linear or nearly linear in Π space: $P_f = \Phi(-\beta)$.

There are three main transformation techniques which enable us to change the random vector X of variables from their physical or original space Ω to the random vector U of variables in the standard independent Gaussian or normal space Π. The first technique is the Rosemblatt transformation method. This technique is used when the PDF of all random variables X is known, and have some correlation with each other. The second transformation method is Nataf transformation. We can use this method when all or some PDFs of the random variables are unknown, but the correlation matrix of random variables is specified. The last transformation is plain linear transformation (Lebrun and Dutfoy, 2009). This method is used when the random vector X is Gaussian without any correlation between its components, and the transformation $T = U(X)$ may be simply related by $(U_i = (X_i - \mu_i) / \sigma_i)$, where μ_i and σ_i are the mean and standard deviation of the i^{th} random variable.

To estimate the reliability index *β*, one has to solve an optimization problem that is:

$$\begin{cases} Minimize \ \ \sum_{i=1}^{n} u_i^2 \\ Subject\ to\ g(T^{-1}(u)) = 0 \end{cases} \tag{3}$$

where u_i, $i = (1, 2, ..., n)$ are the design variables in the standard normal space. The problem in Eq. (3) has a constrained nonlinear optimization shape. Solving Eq. (3) is equivalent to solving the relaxed form obtained by the penalty method as:

$$Minimize \sum_{i=1}^{n} u_i^2 + \lambda\xi(g(T^{-1}(u))) \tag{4}$$

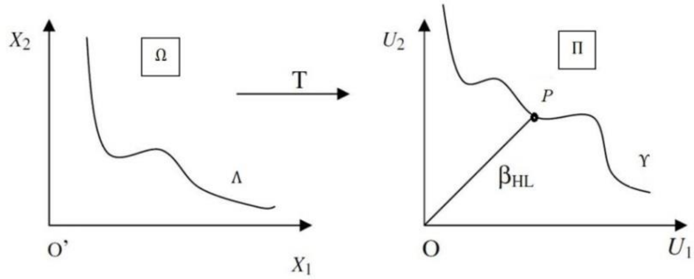


Fig. 1. Hasofer-lind reliability index

where ξ : is the penalty function, and λ: is the penalty coefficient (strictly positive). The solution u^* of Eq. (3) or (4) is called the design point, and enables us to calculate the reliability index as the distance of the origin to the design point with, $\beta = \| u^* \|$.

Selection of the penalty coefficient δ in Eq. (4) is crucial for the convergence of the search towards the solution of Eq. (3). In case of equality constraints as has been addressed in this research, the penalty coefficient will be located by an iterative process from a low value because the search space is a hyper-surface (Fiacco and McCormick, 1968). According to our investigation, an appropriate sequence for λ is λ_i, in such a way that $\lambda_{i+1} = 4\lambda_i$ and $\lambda_0 = 0.05$. The value of λ will be considered suitable when the expression $\xi(g(T^{-1}(u)))$ in Eq. (4) is small enough, namely, less than 10^{-5}. Generally, for the penalty function ξ, we can use one of the two following functions: (1) a quadratic function $x \to x^2$; (2) an exact penalty function $x \to |x|$. The first penalty function is the one used in this research. In case of any constraints in inequality, the penalty function is equal to 0 when the constraint is satisfied and equal to ξ, otherwise.

HARMONY SEARCH ALGORITHM

Originally, the HSA which was first presented by Geem et al. (2001) drew inspiration from the process of a natural musical performance, when a musician searches for a better rendition of harmony, such as during a jazz improvisation. Jazz improvisation endeavors to find the optimum musically pleasing harmony (a perfect state) as determined by an aesthetic standard, just as the optimization process seeks to find a solution that is globally perfect, through an objective function. The pitch of each musical instrument determines the aesthetic quality, just as the objective function value is determined by the set of values assigned to each decision variable. The main steps of the HS algorithm are given in the following sections.

Initializing Algorithm Parameters and Problems

The optimization problem is defined as a minimized function $f(x)$ subjected to $X_{iL} \le X \le X_{iU}$ $(i-1,...,N)$, where X_{iL} and X_{iU} are the lower and upper bounds for design variables. The algorithm parameters are also specified in this step. They are the Harmony Memory Size (HMS), Harmony Memory Considering Rate (HMCR); Bandwidth (bw); Pitch Adjusting Rate (PAR); and the number of improvisations, or stopping criterion.

Initializing the Harmony Memory

In this step, the Harmony Memory (HM) matrix is filled with as many randomly generated solution vectors as the HMS.

These random variables are generated from normal distribution in the ranges $\left[X_{iL}, X_{iU}\right], (i = 1, 2, ..., N)$ as in the following Eq. (5):

$$HM = \begin{bmatrix} x_1^1 & x_2^1 & \dots & x_{N-1}^1 & x_N^1 \\ x_1^2 & x_2^2 & \dots & x_{N-1}^2 & x_N^2 \\ \dots & \dots & \dots & \dots & \dots \\ x_1^{HMS-1} & x_2^{HMS-1} & \dots & x_{N-1}^{HMS-1} & x_N^{HMS-1} \\ x_1^{HMS} & x_2^{HMS} & \dots & x_{N-1}^{HMS} & x_N^{HMS} \end{bmatrix} \cdots \tag{5}$$

Improving a New Harmony

Generating a new harmony is called 'improvisation' (Lee and Geem, 2001). A new harmony vector, $x' = (x'_1, x'_2, ..., x'_N),$ is usually generated based on three rules: (1) memory consideration, (2) pitch adjustment and (3) random selection. In the memory consideration, the value of the first decision variable (x'_1) for the new vector is chosen from any of the values in the specified HM range $(x'_1, x_1^{'HMS})$. Values of the other decision variables $(x'_2, ..., x'_N)$ are chosen in the same manner. The HMCR, which varies between 0 and 1, is the rate of choosing one value from the historical values stored in the HM, while (1-HMCR) is the rate of randomly selecting one value from the possible range of values as shown in Eq. (6).

$$x'_i \leftarrow \begin{cases} x'_i \in \left\{x_i^1, x_i^2, ..., x_i^{HMS}\right\} \\ \text{with probability } HMCR \\ x'_i \in X_i \\ \text{with probability } (1-HMCR) \end{cases} \tag{6}$$

Every component obtained by the memory consideration is examined to determine whether the pitch should be adjusted. This operation uses the PAR parameter, which is the rate of pitch adjustment, as shown in Eq. (7).

Pitch adjusting decision for

$$x'_i \leftarrow \begin{cases} Yes \\ \text{with probability } PAR \\ No \\ \text{with probability } (1-PAR) \end{cases} \tag{7}$$

The value of (1-PAR) sets the rate of doing nothing. If the decision of pitch adjustment for x'_i is yes, x'_i is replaced as shown in Eq. (8)

$$x'_i \leftarrow x'_i + rand() \times bw \tag{8}$$

where *bw*: is an arbitrary distance bandwidth, *rand()*: is a random number between 0 and 1. In step 3, HM consideration, pitch adjustment, or random selection is applied to each variable of the new harmony vector in turn.

To improve the performance of the HS algorithm and eliminate the drawbacks that lie with fixed values of HMCR and PAR, Mahdavi et al. (2007) proposed an Improved Harmony Search (IHS) algorithm that uses the variables *PAR* and *bw* in an improvisation step. In fact, the IHS dynamically updates *PAR* and *bw* according to the following two Eqs. (9) and (10):

$$PAR(n) = PAR_{min} + \frac{PAR_{max} - PAR_{min}}{NI} \times n \tag{9}$$

$$bw(k) = bw_{max} \exp\left(\frac{\ln\left(\frac{bw_{min}}{bw_{max}}\right)}{NI} k\right) \tag{10}$$

where *NI*: is the maximum number of iterations, and *k*: is the current number of iterations; PAR_{min} and PAR_{max}: are the minimum adjusting rate and the maximum adjusting rate, respectively; bw_{min} and bw_{max}: are the minimum *bw* and the maximum *bw*, respectively. Results and studies reveal that the IHS based on improved *PAR* and *bw* has better optimization performance than HS in most cases (Mahdavi et al., 2007).

Updating HM

If the new harmony vector, $x' = (x'_1, x'_2, ..., x'_N)$, is better than the worst harmony in the HM, judged in terms of the objective function value, the new harmony is included in the HM, and the existing worst harmony is excluded from the HM.

Check the Stopping Criterion

If the stopping criterion is satisfied, computation is terminated, otherwise, steps 3 and 4 are repeated. In this research, the following stopping criteria have been used:

- The average reliability index of the current harmony solution vectors does not show significant improvement over the former harmony solution vectors: $\beta^{k+1} > \lambda\beta^k$ and λ can set to 0.95; and
- The first three different minimum reliability indexes of the current harmony solution vectors remain the same as those of the previous solution vectors of the HM matrix.

ARTIFICIAL NEURAL NETWORK

ANN is the numerical algorithm inspired in the functioning of biological neurons. This concept was introduced by McCulloch and Pitts (1943), who proposed a mathematical model to simulate neuron behavior. Use of ANN has become widespread in several fields of engineering, such as structural mechanics and structural reliability (Cheng, 2010). Deng et al. (2005) presented an approach in which an ANN was used in the structural reliability analysis of problems with implicit LSFs. There are a number of ANN paradigms, a multilayer, feed-forward, back-propagation network, which is one of the well-known and most widely used among ANN techniques. Some basic concept of the proposed ANN method is briefly presented in the following sections.

ANN Architecture and Training Algorithm

The proposed ANN structure consists of three layers; an input layer, one hidden layer, and an output layer. Each layer has its corresponding neurons or nodes and weight connections. The number of neurons or nodes in the input and output layers is determined by the number of input and output parameters, respectively. However, the selection of an optimal number of neurons or nodes in the hidden layer is a difficult task and there is no general rule for selecting the number of neurons or nodes in a hidden layer. It depends on the complexity of the structures being modeled. In this paper, the optimal number of neurons or nodes in the hidden layer is determined by a trial-and-error process. Figure 2 shows a typical architecture of an ANN model, in which the left column is the input layer, the column on the right is the output layer, and the middle column is the hidden layer. Each neuron in the network operates by taking the sum of its weighted inputs and passing the result through a nonlinear activation function (transfer function). In this study, unless stated, a logistic transfer function, $f(z) = tanh(z)$ is used to transfer the values of the input layer nodes to the hidden layer nodes, whereas the linear transfer function $f(z) = z$ is adopted to transfer the values from the hidden layer to the output layer. The training phase of the proposed model is based on the back-propagation training algorithm.

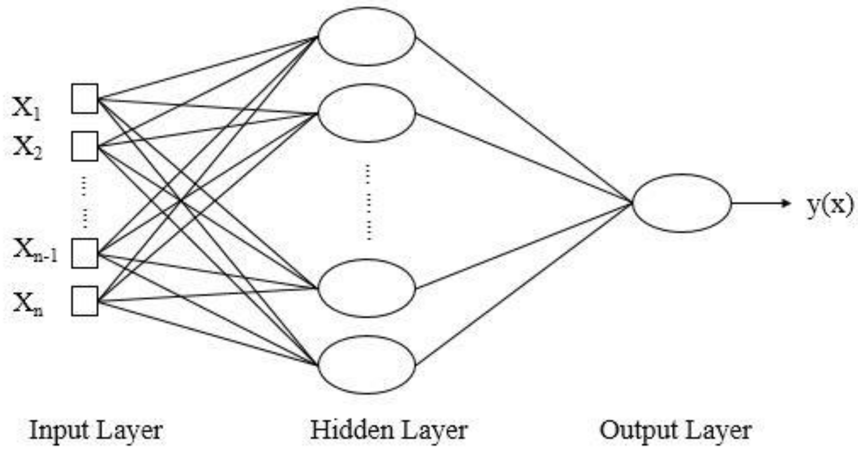


Fig. 2. Typical architecture of an ANN model

Data Preparation and Processing

In this study, a set of input and output data is prepared for developing the ANN model. A subset of data is used for training, while the other one is used for testing the model. For simplicity purposes, sufficient numbers of data are randomly generated from the distributions of the variables to be used in developing a Finite Element analysis code to obtain the response of the structure. Thereafter, the trained ANN model is used in the process of reliability assessment to estimate the structural response.

Evaluation of ANN Performance

Once the ANN model is trained, the relationship between the LSF and the various designvariables is readily retrieved. The next step is to validate and evaluate the trained model. This can be done by using common error parameters such as the Mean Absolute Error (MAE) or Root-Mean-Squared Error (RMSE). The two error functions can be expressed as shown in Eqs. (11) and (12):

$$MAE = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left|(P_{ij} - T_{ij})\right|}{n.m} \quad (11)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}(P_{ij} - T_{ij})^2}{n.m}} \quad (12)$$

where n: is the number of patterns in the validation data (i.e., the test data); m: is the number of components in the output vector; P: is the output vector from the ANN model (predicted structural response); and T: is the desired output vector from the deterministic Finite Element analysis (obtained structural response by FEM).

THE PROPOSED ALGORITHM

In the reliability assessment of complex structures, the LSF $g(X)$ may not be expressible explicitly in the basic design variables. When the above-mentioned HSA is applied to analyze the reliability of complex structures, the LSF needs to be evaluated implicitly through the sophisticated numerical methods, such as the Finite Element method. This process could be so computationally time consuming that makes it hard to use. To deal with this drawback in the reliability analysis, a new two-phase algorithm based on a combination of ANN and HSA is developed. In the proposed method, a trained ANN model is applied to approximate the structure response and incorporate in optimization process to define the reliability index with HSA. Once the ANN model is appropriately trained, we can directly apply the ANN model instead of a deterministic Finite Element analysis, (Figure 3). It requires hours of computation time to perform a large number of Finite Element analysis. By contrast, evaluation of a quadratic function requires only a fraction of a second, and the computing time is substantially reduced. The proposed algorithm in this research is called the ANN-HSA method. For some further clarifications on this, the implemented code of the proposed algorithm has been stated as follows.

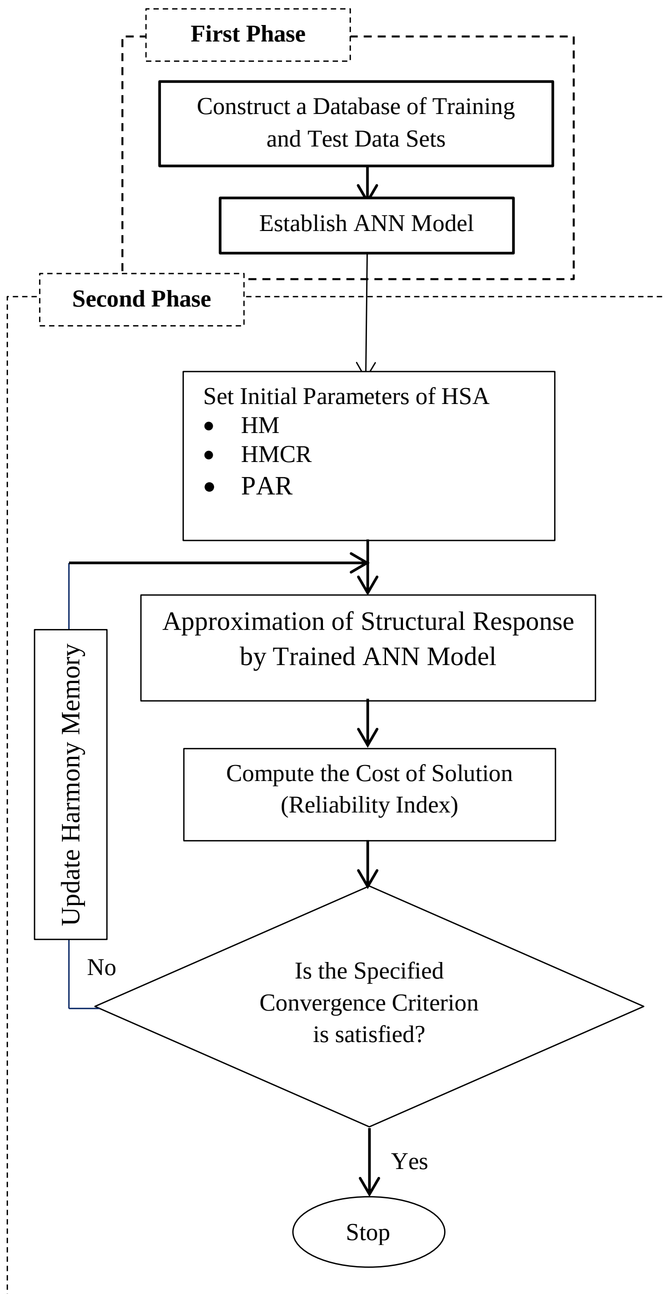


Fig. 3. Flow chart of the proposed method

Step 1. Construction of database for ANN

This step involves producing the needed datasets for training and testing datasets of ANN models. Each of these datasets includes input values (random variables) and output values (structural response). The input values are generated by uniform distribution when random variables X_i are mapped from physical space Ω to standard normal space Π by means of mentioned transformations. Output values can be obtained by use of a developed analytical model of structures in a Finite Element software, or utilizing infield measurement equipment such as strain or stress gauges (Moore et al., 2012). A vector's initial number of ANN datasets can be considered from 10 to 20 times the number of input variables of the problem.

Step 2. Establishment of an ANN model

In this stage, an ANN model is trained with the database obtained from step 1. The efficiency and precision of a trained ANN will be measured based on the common error values like MAE or RMSE. The most efficient ANN model in terms of structure (number of layer and neuron) and minimum error values will be chosen for predicting structural responses.

Step 3. Initializing algorithm parameters

The HS algorithm parameters are specified in this step. They are the HMS or the number of solution vectors in the HM, HMCR and PAR.

Step 4. Determination of structural response by a trained ANN model

In this step, structural responses are computed for each vector of the HM matrix. For this purpose, a trained ANN model is used, and the structural responses are computed to correspond with each input vector of HM.

Step 5. Computing reliability index

To compute the reliability index, Eq. (3) is applied to all the vectors of the HM matrix (solution vectors), and the fitness of these solutions is calculated. Thereafter, solutions will be sorted based on their objective fitness (minimization). The best solution is the one that has the minimum value among all the available solutions.

Step 6. Checking stopping criterions

In this step, two stopping criterions are checked. In the first criterion, if the average of the reliability index of the current Harmony solution vectors does not show significant improvement in comparison with the former Harmony solution vectors ($\beta^{k+1} > \lambda\beta^k$, λ sets to 0.95), the process will be terminated. In the second criterion, if the first three different minimum reliability indexes of the current Harmony solution vectors remain the same as those of the previous solution vector of the HM matrix, the algorithm will be terminated.

Step 7. Improving new harmony, updating HM and repeating steps 4 to 6

The new harmony vector $x' = (x'_1, x'_2, \ldots, x'_3)$ is determined as follows:

for $each\ i \in [1, N]$ **do**
if $rand() \leq HMCR$ **then**
$x'_i = x_i^j (j = 1,2, \ldots, HMS)$% memory consideration
if $rand \leq PARi$ **then**
$x'_i = x'_i \pm r \times bwi$ % pitch adjustment
if $x'_i > x_{iU}$
$x'_i = x_{iU}$
elseif $x'_i < x_{iL}$
$x'_i = x_{iL}$
end
end
else
$x'_i = x_{iL} + rand() \times (x_{iU} - x_{iL})$% random selection
end

end

$x_i'(i = 1,2,\dots,n)$ is the ith component of x', and $x_i^j(j = 1,2,\dots,HMS)$ is the ith component of the jth candidate solution vector in HM. Both r and $rand()$ are uniformly generated random numbers in the region of [0,1], and bw is an arbitrary distance bandwidth. If the fitness of the improvised harmony vector $x' = (x_1', x_2', \dots, x_3')$ is better than that of the worst harmony, then replace the worst harmony in the HM with x'. Thereafter, steps 4 to 6 are repeated until stopping criteria are satisfied.

The main advantage of this method can be compared to the reliability procedures of others such as MSC, response surface, or sensitivity-based analysis, and there is no need for an explicit form of the LSF and its derivation. In fact, the HSA just needs to evaluate the value of LSF per input random design variables (solution vector or Harmony vector) in an optimization process that can be obtained by a trained ANN model very quickly and easily.

NUMERICAL EXAMPLES

In order to demonstrate and validate the proposed algorithm, three examples are considered herein to check accuracy and efficiency. The results are also compared with other reliability methods. The parameters of the proposed algorithms are listed in Table 1. Note that some of these parameters are kept constant throughout the whole process of reliability analysis.

Example 1: 2D frame structure

This example is taken from Sondipon (2010). The structure is shown with element numbering, node numbering, and coordinates of the nodes in meters, as shown in Figure 4. It is assumed that the axial stiffness (EA) and the bending stiffness (EI) of each member have Gaussian random variables, so that there are, in total, six random variables, $x \in R^6$. Further, it is also assumed that the EA and EI of the different members are uncorrelated.

$$\begin{aligned} &\left\langle EA_i , EA_j \right\rangle = 0 , \forall i \neq j ; \\ &\rho\left\langle EI_i , EI_j \right\rangle = 0 , \forall i \neq j ; \\ &\rho\left\langle EA_i , EI_j \right\rangle = 0 , \forall i , j . \end{aligned} \tag{13}$$

Table 2 shows the numerical values of the mechanical properties for different members. The vertical force applied in node 3 is 100 KN and is deterministic.

Table 1. Parameters of the proposed algorithm

Parameters	Name	Example 1	Example 2	Example 3
ANN Parameters	**Number of Training Dataset**	**750**	**1000**	**900**
	Number of Nodes in Hidden Layer	**6**	**10**	**12**
HSA Parameters	HMS	4	8	6
	HMCR	0.9	0.9	0.85
	PAR_{min}	0.35	0.35	0.35
	PAR_{max}	0.99	0.99	0.99
	bw_{min}	10^{-4}	10^{-4}	10^{-4}
	bw_{max}	0.01	0.01	0.01

Table 2. Element properties of the random 2D frame

Member Id	EA(KN)		EI(KNm^2)	
	Mean	**Standard Deviation (%)**	**Mean**	**Standard Deviation (%)**
1	1.0×10^9	3.0	2.0×10^4	10.0
2	5.0×10^9	7.0	6.0×10^4	5.0
3	3.0×10^9	10.0	4.0×10^4	9.0

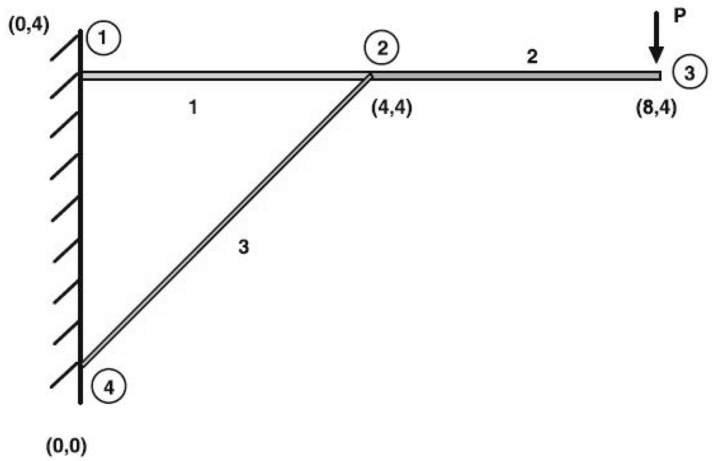


Fig. 4. 2D frame geometry properties

The failure condition is given by specifying a maximum allowable vertical displacement at node 3, say d_{max} . The LSF is $g(x) = d_{max} - |\delta v_3(x)|$, where the random variable δv_3 shows the vertical displacement at node 3. The structure is unsafe when $g(x) < 0$ that is, when $\delta v_3 > d_{max}$. For numerical calculations, in this example, $d_{max} = 0.095m$. Figure 5 shows the convergence process of reliability index. As is clear from the proposed algorithm, convergence with the solution occurred after nearly 700 iterations in comparison with the MCS method which has required about 10^6 computations for convergence. The required time for convergence with the solution for this example is about 15.6 seconds.

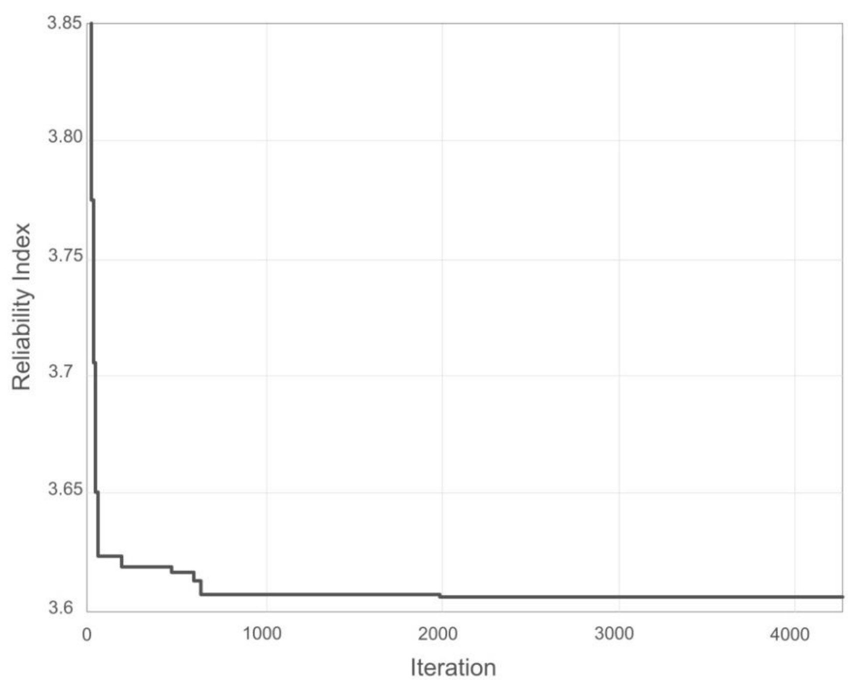


Fig. 5. Convergence process to reliability index for a 2D frame

Numerical results obtained according to the MC and FORM, and using the proposed method, are shown in Table 3. It is clear that the proposed algorithm produces satisfactory agreement with the usual FORM and the MCS, which is considered as the benchmark method.

Table 4 summarizes the performance of the ANN model based on two parameters MAE and RMSE, both for the training and test data. It is clear that the ANN model gives a good precision for both MAE and RSME.

Example 2: Multistory portal frame

This example includes a linear portal frame with 12 stories and three bays as shown in Figure 6 based on Cheng (2007). Different cross-sectional areas A_i and horizontal load P are considered here as independent random variables. Table 5 shows the statistical characteristics of these random variables. The sectional moments of inertia are expressed with the formulation $I_i = \alpha_i A_i^2$ where $\alpha_1 = \alpha_2 = \alpha_3 = 0.08333, \alpha_4 = 0.26670, \alpha_5 = 0.200$.

The Young's modulus, E is deterministic with the value of $2.0 \times 10^7 KN / m^2$.

The failure condition is stated by a maximum allowable horizontal displacement at node A, say d_{max}. Therefore, the LSF is given with $g(x) = d_{max} - |\delta h_A(x)|$, where the random variable δh_A is the horizontal displacement at node A. For numerical calculations, in this example, the maximum horizontal displacement at node A is taken as 0.096 m. Thus, the final LSF is expressed as in Eq. (14).

$$g(A_1, A_2, A_3, A_4, A_5, P) = 0.096 - u_A(A_1, A_2, A_3, A_4, A_5, P) \quad (14)$$

Figure 7 shows how the results converge to the reliability index. As is clearly shown, the proposed algorithm converges to the solution after just about 3600 iterations in comparison with the MCS method, which has required about 10^4 computations for convergence. The time for convergence with the solution for this example is about 27.3 seconds.

Table 3. Comparison of reliability index and failure probability

Method	Reliability Index	Failure Probability
MC Simulation	3.598	0.16×10^{-3}
FORM (Adhikari, 2010)	3.590	0.165×10^{-3}
Proposed Method	3.610	0.153×10^{-3}

Table 4. Performance of ANN model

Error Function	Horizontal Displacement at Node 11	
	Training Data	Test Data
RMSE	0.27×10^{-2}	0.44×10^{-2}
MAE	0.036	0.087

Table 5. Statistics of the random variables for Example 3

Variable	Mean	Standard Deviation	Dimension	Distribution
A_1	0.25	0.025	m^2	Lognormal
A_2	0.16	0.016	m^2	Lognormal
A_3	0.36	0.036	m^2	Lognormal
A_4	0.20	0.020	m^2	Lognormal
A_5	0.15	0.015	m^2	Lognormal
P	30.0	7.5	KN	Type I Largest

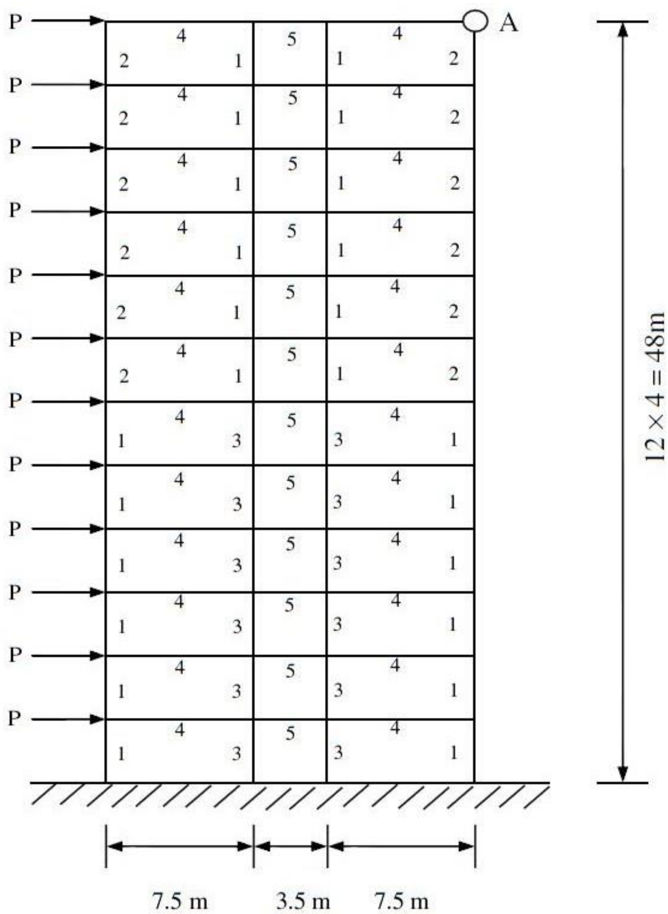


Fig. 6. Multistory liner portal frame

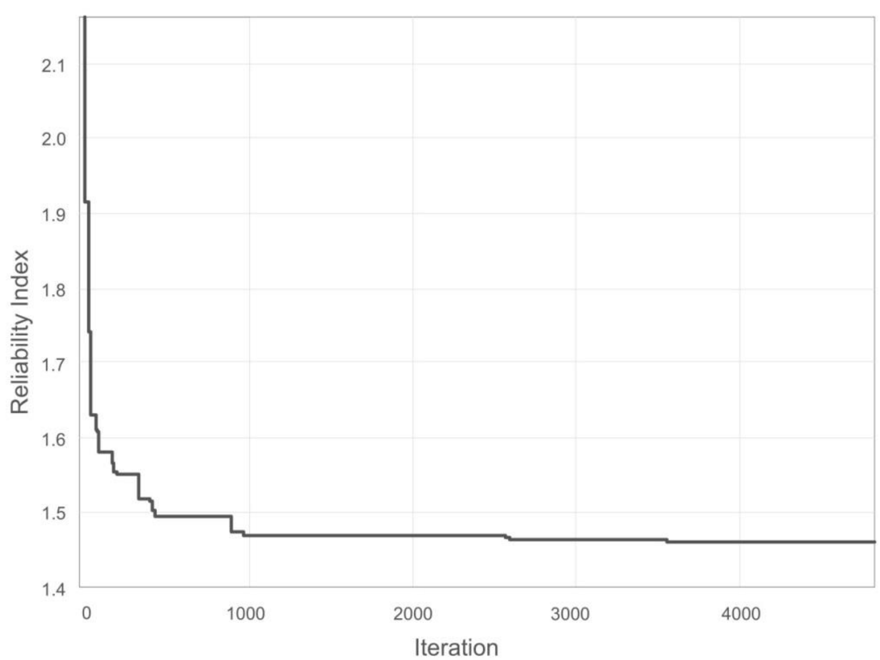


Fig. 7. Convergence process to reliability index for liner portal frame

The numerical results obtained for this example are shown in Table 6. As is clear from this table, the proposed method gives good accuracy in comparison with the other methods mentioned. Table 7 summarizes the performance of the ANN model for MAE and RMSE, both for the training and test data. It is clear that the ANN model gives good approximation to the reliability result based on RMSE and MAE.

Example 3: Seismic reliability assessment of steel frame structure

This example has been taken from Achintya Haldar (2006) to check the performance of a proposed method for assessment of seismic reliability. This involves a two-story steel frame structure that consists of W27×84 for all beams and W14×426 for all columns. A36 steel is used. The frame is excited for 15 seconds by the actual acceleration time, and the history was recorded at Canoga Park during the Northridge earthquake of 1994 (Figure 8).

The serviceability limit state is considered in this example. The statistical propriety of a random variable is listed in Table 8.

Table 6. Comparison of reliability index and failure probability

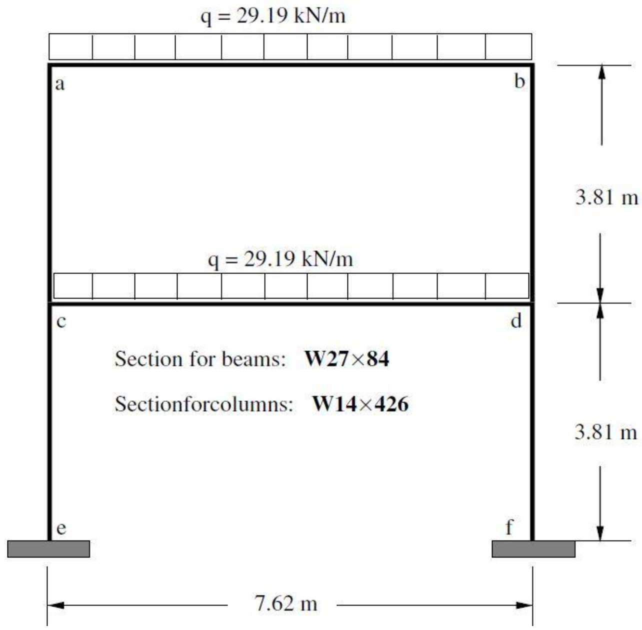
Method	Reliability Index	Failure Probability
ANN-FORM (Cheng, 2007)	1.438	0.0751
Proposed Method	1.440	0.07493

Table 7. Performance of ANN model

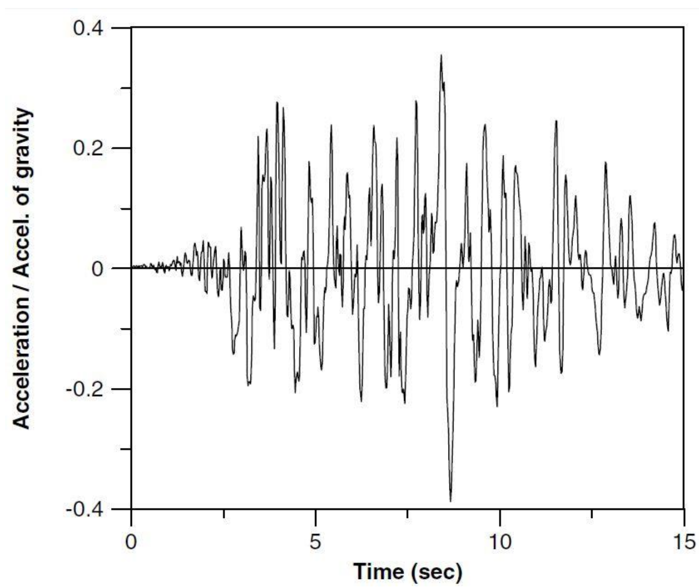
Error Function	Horizontal Displacement at Node 11	
	Training Data	Test Data
RMSE	0.15×10^{-2}	0.56×10^{-2}
MAE	0.0046	0.0096

Table 8. Statistical description of random variable (b: beam, c: column)

Random Variable	Mean Value	Serviceability Limit State	
		COV	Distribution
$E(KN/m^2)$	1.9994×10^{8}	0.06	Lognormal
$A^b(m^2)$	1.600×10^{-2}	0.05	Lognormal
$I_x^b(m^4)$	1.186×10^{-3}	0.05	Lognormal
Z_x^b	3.998×10^{-3}	Deterministic	Deterministic
$A^c(m^2)$	8.065×10^{-2}	0.05	Lognormal
$I_x^c(m^4)$	2.747×10^{-3}	0.05	Lognormal
$F_y(KN/m^2)$	2.4822×10^{5}	Deterministic	Deterministic
ξ	0.05	0.15	Lognormal
g_e	1.00	0.20	Lognormal



(a) Two-story steel frame



(b) Northridge earthquake (N-S) time history

Fig. 8. Two-story steel frame excited by the Northridge earthquake

For the serviceability limit state, the permissible lateral displacement at the top of the frame is assumed not to exceed *h*/400, where *h* is the height of the frame. Thus, $\delta_{allowable}$ is 1.905 cm for this example, and the corresponding limit state is $g(x) = 1.905 - \delta h_b$, where δh_b, is the horizontal displacement of node b that will be approximated by ANN. Figure 9 shows the convergence process to the reliability index. As can be clearly seen, the proposed algorithm converges to the solution after nearly 2100 iterations, in comparison with the MCS method which has required about 0.5×10^4 computations for convergence. The time for convergence with the solution for this example is about 18.2 seconds.

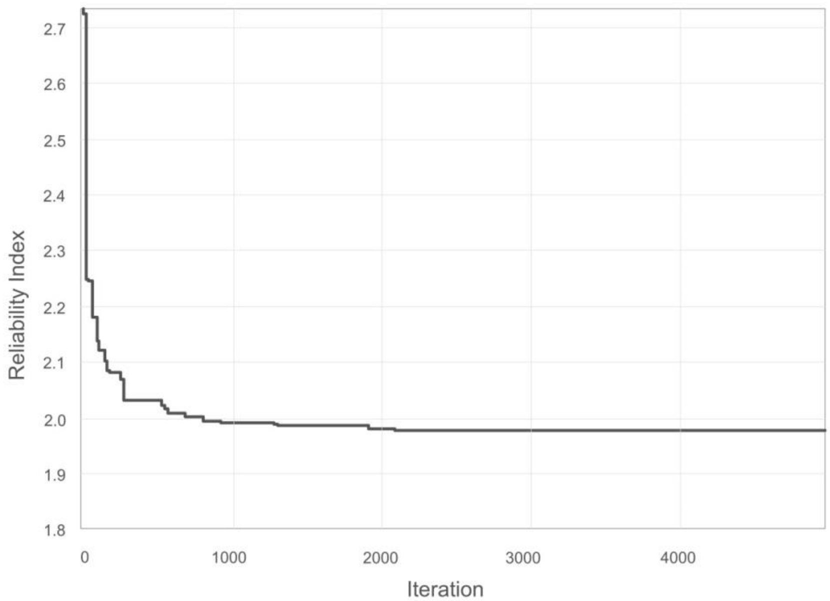


Fig. 9. Convergence process to reliability index for liner portal frame

The numerical results obtained using the proposed method for reliability index and failure probability are shown in Table 9.

Table 10 summarizes the performance of the ANN model for MAE and RMSE, both for the training and test data. Results show that the proposed method shows a good estimation of the failure probability of structures.

SENSITIVITY ANALYSIS OF PROPOSED ALGORITHM

In this section, the effect of changing parameters in the proposed algorithm has been studied. This investigation is mainly based on parameters such as the number of training datasets; the number of nodes in the hidden layer; HMS; and the HM consideration rate. Different values are chosen for these parameters in order to determine their effect on the final results separately. The aim is to obtain some general guidelines in the use of the proposed algorithm. For simplicity, only Example 1 in the previous section is considered here.

Sensitivity on the Number of Training Data Sets

All the parameters of the proposed algorithm are given in Table 1 and kept constant here, except the number of training datasets on which a sensitivity analysis is performed. The results of failure probability are given in Table 11.

Table 9. Comparison of reliability index and failure probability

Method	Reliability Index	Failure Probability
MCS	1.898	0.02884
Response Surface (Haldar, 2006)	1.914	0.02779
Proposed Method	1.912	0.02793

Table 10. Performance of ANN model

Error Function	Horizontal Displacement at Node 11	
	Training Data	Test Data
RMSE	0.23×10^{-2}	0.67×10^{-2}
MAE	0.0052	0.0198

Table 11. Estimated value of P_f for different number of training datasets

Number of Training Datasets	P_f	Error (%)
250	0.000214	28.91
500	0.000157	9.78
750	0.000159	5.42
1000	0.000159	3.89

As is clearly seen from Table 11, changing the number of training datasets has a significant impact on the estimated failure probability. Using less than 500 training datasets results in a poor estimation value of P_f for the ANN-HSA method. Using a small number of training datasets causes numerous errors, but using a large number of training datasets results in little improvement in accuracy and efficiency of the ANN-HSA.

Sensitivity on the Number of Nodes in a Hidden Layer

In this study, four different numbers of nodes in the hidden layer ranging from three to 12 are investigated, and the results are listed in Table 12.

From the results, it can be seen that changing the number of nodes in the hidden layer has an influential impact on the accuracy of the calculated values of P_f. For the ANN-HSA estimation of P_f, the best accuracy is associated with six nodes in the hidden layer in Example 1.

Sensitivity on the HM Size

To study the effect of HM size on the estimation of failure probability by the proposed method, different values for this parameter are taken ranging from four to 10. The results are shown in Table 13.

According to Table 13, change of HMS has a minor effect on the failure probability, but CPU time is increased by increasing the number. Thus, for a problem on which computing time must be minimized, the population size may be chosen in the lower value. Therefore, the optimum value for this parameter at Example 1 is selected as 4.

Sensitivity on the HM Consideration Rate

In this section, the effect of the HM consideration rate value on the calculated probability of failure by the proposed method has been studied. For this purpose, the convergence of the proposed method as per different values ranging from 0.75 to 0.95 with steps of 0.05 are estimated and presented in Table 14.

The results show that changing of this parameter has some impact on the final failure probability; however, it could change it insignificantly. The optimum value of 0.9 is selected for this parameter in Example 1.

Table 12. Effect of the number of nodes on predicted

Number of Nodes in Hidden Layer	P_f	Error (%)
3	0.000183	10.24
6	0.000157	5.42
9	0.000152	8.43
12	0.000147	11.44

Table 13. Effect of HMS on predicted P_f

Harmony Memory Size	P_f	Error (%)	Time
4	0.000157	5.42	15.8
6	0.000157	5.42	16.6
8	0.000156	6.02	17.4
10	0.000157	5.42	24.2

Table 14. Effect of HMS on predicted P_f

Harmony Memory Consideration Rate	P_f	Error (%)
0.75	0.000154	7.22
0.80	0.000154	7.22
0.85	0.000155	6.62
0.90	0.000157	5.42
0.95	0.000156	6.02

CONCLUSIONS

In this paper, a new two-phase algorithm based on the ANN and HSA is proposed for estimation of reliability. The proposed method shows an efficient, accurate, and robust algorithm to solve the reliability problem with implicit response functions, and that does not have any requirement to approximation of explicit form for the LSF. The application of ANN to structural problems, while leading to satisfactory precision, enables to significantly speed up the computation of structural response. This feature is particularly relevant in reliability analysis, where a very substantial computing effort is normally required in order to accurately evaluate the probability of failure. With integration, the concepts of the ANN method, and the HSA, the number of deterministic response analyses is dramatically reduced, and there is no need for any explicit form of LSF for reliability assessment. On the other hand, sensitivity analysis on the parameters of the proposed method shows that the number of training datasets and the number of nodes in a hidden layer have significant impact on the convergence of the estimated failure probability. In all the examples considered in this research, the results are close to those obtained by conventional reliability assessment methods. From the research presented herein, it can be concluded that the application of ANN in conjunction with HSA seems promising, and appears to offer great potential for structural reliability problems.

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REFERENCES

Adhikari, S. (2010). “Sensitivity based reduced approaches for structural reliability analysis”, *Sadhana Academy Proceeding in Engineering Science*, 35(3), 319-339.

Allaix, D.L. and Carbone, V.I. (2011). “An improvement of the response surface method”, *Structural Safety*, 33(2), 165-172.

Bucher, C.G. (1998). “Adaptive sampling– an iterative fast Monte Carlo procedure”, *Structural Safety*, 5(2), 119-126.

Castillo, E., Mínguez, R. and Castillo, C. (2008). “Sensitivity analysis in optimization and reliability problems”, *Reliability Engineering & System Safety*, 93(12), 1788-1800.

Cheng, J. (2010). "An artificial neural network based genetic algorithm for estimating the reliability of long span suspension bridges", *Finite Elements in Analysis and Design*, 46(8), 657-668.

Cheng, J. and Li, Q.S. (2008). "Reliability analysis of structures using artificial neural network based genetic algorithms", *Computer Methods in Applied Mechanics and Engineering*, 197(45), 3742-3750.

Cheng, J. (2007). "Hybrid genetic algorithms for structural reliability analysis", *Computers and Structures*, 85(19), 1524-1533.

Cheng, J., Zhang, J., Cai, C.S. and Xiao, R.C. (2007). "A new approach for solving inverse reliability problems with implicit response functions", *Engineering Structures*, 29(1), 71-79.

Coelho, L.S. (2009). "An efficient particle swarm approach for mixed-integer programming in reliability–redundancy optimization applications", *Reliability Engineering and System Safety*, 94(4), 830-837.

Deng, J., Gu, D., Li, X. and Yue, Z. (2005). "Structural reliability analysis for implicit performance functions using artificial neural network", *Structural Safety*, 27(1), 25-48.

Elegbede, C. (2005). "Structural reliability assessment based on particles swarm optimization", *Structural Safety*, 27(2), 171-186.

Fiacco, A.V. and McCormick, G.P. (1968). *Nonlinear programming: sequential unconstrained minimization techniques*, Wiley, New York.

Geem, Z.W., Kim, J.H. and Loganathan, G.V. (2001). "A new heuristic optimization algorithm: harmony search", *Simulations*, 76(2), 60-68.

Haldar, A. (2006). *Recent developments in reliability-based civil engineering*, World Scientific Publishing, Arizona, USA.

Harbiz, A. (1986). "An efficient sampling method for probability of failure calculation", *Structural Safety*, 3(2), 109-116.

Hasofer, A.M. and Lind, N. (1974). "An exact and invariant First-Order Reliability Format", *Engineering Mechanics, ASCE*, 100, 111-121.

Huang, C.L. (2015). "A particle-based simplified swarm optimization algorithm for reliability redundancy allocation problems", *Reliability Engineering & System Safety*, 142, 221-230.

Lebrun, R. and Dutfoy, A. (2009). "Do Rosenblatt and Natafiso probabilistic transformations really differ?", *Probabilistic Engineering Mechanics*, 24(4), 577-584.

Lee, K.S. and Geem, Z.W. (2004). "A new meta-heuristic algorithm for continues engineering optimization: harmony search theory and practice", *Computer Method Applied Mechanic and Engineering*, 194(36), 3902-3933.

Liu, P.L. and Der Kiureghian, A. (1991). "Optimization algorithms for structural reliability", *Structural Safety*, 9(3), 161-177.

Lopez, R.H., Torii, A.J., Miguel, L.F.F. and Souza Cursi, J.E. (2015). "Overcoming the drawbacks of the FORM using a full characterization method", *Structural Safety*, 54, 57-63.

Mahdavi, M., Fesanghary, M. and Damangir, E. (2007). "An improved harmony search algorithm for solving optimization problems", *Applied Mathematics and Computation*, 188(2), 1567-1579.

Marti, K. (2008). *Stochastic optimization methods*. SpringerVerlag, Berlin Heidelberg.

McCulloch, W.S. and Pitts, W. (1943). "A logical calculus of ideas immanent in nervous activity", *Bull Math Biophysics*, 5(4), 115-33.

Moore, J.C., Glenncross-Grant, R., Mahini, S.S. and Patterson, R. (2012). "Regional timber bridge girder reliability: Structural health monitoring and reliability strategies", *Advances in Structural Engineering*, 15(5), 793-806.

Mucherino, A. and Seref, O. (2009). "Modeling and solving real life global optimization problems with meta-heuristic methods", *Advances in Modeling Agricultural Systems*, 25, 1-17.

Mun, S. and Cho, Y.H. (2012). "Modified harmony search optimization for constrained design problems", *Expert Systems with Applications*, 39(1), 419-423.

Ramirez-Marquez, J.E. (2008). "Port-of-entry safety via the reliability optimization of container inspection strategy through an evolutionary approach", *Reliability Engineering and System Safety*, 93(11), 1698-1709.

Rodrigues, M., de Lima, B.S.L.P. and Guimarães, S. (2016). "Balanced ranking method for constrained optimization problems using evolutionary algorithms", *Information Sciences*, 327(C), 71-90.

Shao, S. and Murotsu, Y. (1997). "Structural reliability using a neural network", *JSME International*, 40, 242-246.

Stapelberg and Rudolph, F. (2009). *Handbook of reliability, availability, maintainability and safety in engineering design*, Springer-Verlag, London.

Zhang, Z., Jiang, C., Wang, G.G. and Han, X. (2015). "First and second order approximate reliability analysis methods using evidence theory", *Reliability Engineering & System Safety*, 137, 40-49.