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An Analysis the Effect of Capital Taxation on Allocation of Resources: A Dynamic Equilibrium Model Approach

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Abstract

The return of capital is fundamental to the intertemporal allocation of resources by changing the consumption behavior and capital accumulation over time. Taxation on return of capital increases the marginal product of capital, meaning that capital stock is lower than when capital is not taxed, which results decreased growth and welfare in steady state. This paper studies the impact of capital income taxation on capital stock, output and welfare in a dynamic optimization model. Theoretical and experimental results indicate that any attempt to decrease taxation on return of capital in Iran's economy, will be eventually reached to a higher capital formation, higher output and consumption per capita in the steady state. Finally, leads to higher welfare level in the steady state.

Keywords: Optimal Control Theory, Optimal Capital Taxation, Distortionary Taxation.
JEL Classification: C6, H21.

1. Introduction

An important question in tax policy analysis is whether using capital income taxation to redistribute accumulated fortunes is desirable. As in most tax policy problems, there is a classical equity vs. efficiency trade-off[3]. Progressive capital income tax can redistribute income from the wealthy to the non-wealthy but might distort savings and consumption behavior and hence reduce wealth accumulation (Saez, 2013). Also, the rate of growth can be affected by policy through the effect that taxation has upon economic

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3. This trade-off was at the center of the political debate on the introduction of progressive taxation in western countries.

decisions. An increase in taxation reduces the return to investment. Lower return means less accumulation and innovation and hence a lower rate of growth. This is the negative aspect of taxation. So, popular views on capital income taxation are mixed: on the one hand, there is the view that taxing capital income is a way to restore the balance between capital owners and workers. Advocates of capital income taxation also stress that taxing labor only would encourage firms to become more capital intensive, which in turn will translate in lower wage or higher unemployment. Another argument against capital income taxation is that it discourages entrepreneurship and therefore employment and growth. Yet some observers point out that several developed countries with relatively lower unemployment rates and higher growth rate among OECD countries have capital income tax. For example, in Sweden, capital income is taxed at a flat rate of 30%. The European Union currently raised capital taxes to about 9% of GDP and the US rose to about 8% of GDP in capital taxes[1].

The substantial gap between optimal capital tax theory and practice motivates our present work on taxation. So, the purpose of this research is to study the impacts of capital income taxation on capital stock, output and welfare in a dynamic optimization model. Finally, by reducing the tax rate on capital income and using parameters related to Iran's economy, sensitivity analysis is done.

The rest of the paper is organized as follows. Section 2 points out literature on capital income taxation. Section 3 presents the model. Section 4 discusses equilibrium effects. Section 5 presents the empirical evidence. Section 6 summarizes and concludes the results.

2. Literature on Capital Taxation

The literature on optimal taxation leads to two central conclusions[2]. First, labor taxes should be roughly constant. Second, capital taxes should be zero in the long run and very high in the short run[3]. These conclusions have very different implications for time-inconsistency. While there seems to be little time-inconsistency in labor taxation, this problem is very severe in capital taxation. The argument for a zero tax often rests on the disincentive effects on the intertemporal allocation of resources. However, the studies of second best taxation indicate that the optimal capital income tax rates are not zero, except under quite restrictive assumptions about individual preferences. A

1. See European Commission (2011), p.282 (total taxes) and p.336 (capital taxes), for GDP-weighted EU 27 averages, and OECD (2011) for the United States.
2. See Chamley (1986) & Judd (1985).
3. Conesa et al. (2012), illustrated that in life-cycle models and economies with borrowing constraints, the optimal capital tax is not zero in the long run.

number of studies on optimal dynamic taxation have suggested that capital tax might have very large efficiency costs (see e.g., Lucas, 1990; Atkeson et al., 1999).

The Ramsey tax system advocates a high tax on capital income in the initial period and a zero tax in future times[1]. The Ramsey results hinge on the assumption that the government commitment is permanent. On the other hand, many studies argue that consumption taxes usually dominate either wage tax or uniform income tax in welfare terms[2]. Chamley (1986) in his seminal paper argues that the Ramsey problem with an infinite horizon, in representative agent general equilibrium model obtained zero taxation of capital in the long run. Auerbach and Kotlikoff (1987) build an overlapping generation model with representative agents, certain lifetimes and complete markets. They find that switching from a 30% income tax to a consumption tax raises the capital stock by 61% and improves welfare by the equivalent of 2.32% of assets plus the present value of full-time lifetime earnings.

Lucas (1990) employs an infinite-horizon, representative agent, endogenous growth model and examines the impact of eliminating the tax on income from capital. His main steady-state finding is that eliminating the tax on capital income and raising the lost revenue through a higher labor income tax lead to a 32% increase in capital stock, and that the welfare benefit from this tax reform is equivalent to 6% of aggregate consumption. Chari et al. (1994) find that 80% of the welfare gains of switching from the current tax system to the Ramsey system comes from the high initial capital taxes. As the incentives to deviate from the announced zero capital taxes are paramount, some economists have suggested not taxing capital at all[3].

Imrohoroglu (1998) studies the quantitative impact of eliminating capital income tax on capital accumulation and steady-state welfare in a general equilibrium model with overlapping generations of 65-period-lived individuals who face idiosyncratic earnings risk, borrowing constraints, and life-span uncertainty. Under a wide range of parameter configurations, the capital income tax rate that maximizes steady-state welfare is positive, even though eliminating it completely would raise the steady-state capital stock toward the Golden Rule. This is because the tax burden is shifted toward the younger and liquidity constrained years, reducing the individuals' ability to self-insure.

Aiyagari (1995) & Chamley (2001) in models with credit constraints have shown that capital income taxation may be desirable, even in the long

1. See Judd (1985), Chamley (1986), and Chari, Christiano and Kehoe (1994).
2. See Summers (1981), Seidman (1984), Auerbach and Kotlikoff (1987), Pecorino (1994), Devereux and Love (1994), Turnovsky (2000), and Davies, Zeng and Zhang (2000).
3. Mankiw et al. (2009) argue that capital income ought to be untaxed.

run as capital income taxes can redistribute from the rich who are not credit constrained toward the poor who are credit constrained. Similarly, Golosov et al., (2006) have shown that dynamic labor productivity risk leads to non-zero capital income taxes. Kocherlakota (2010) in dynamic optimal taxation upon informational assumptions have shown that optimal tax structures are very complex and history dependent.

Saez (2013) analyzes optimal progressive capital income taxation in an infinite horizon model where individuals differ only through their initial wealth. He has indicated that when the intertemporal elasticity of substitution is not too large and the top tail of the initial wealth distribution is infinite and thick enough, the optimal exemption threshold converges to a finite limit. As a result, the optimal tax system drives all the large fortunes down a finite level and produces a truncated long-run wealth distribution.

3. The Model

We consider an economy populated by identical infinitely lived households. The representative household is endowed with one unit of time, which is allocated to leisure l_t and labor $1-l_t$. There is no uncertainty in the form of shocks in preferences and technology.

3.1 Household Behavior

Frank Ramsey (1928) posed the question of how much a nation should save and solved it by using a model that is now the optimal intertemporal allocation of resources. In this model the population, N_t, grows at rate n. the labor force is equal to the population, with labor supplied inelastically. Output is produced using capital, K, and labor. So, following the Ramsey, it is assumed that individuals have an infinite horizon and the preferences of the family for consumption over time are represented by the utility integral:

$$W = \int_0^\infty u(c_t)\exp(-\theta t)dt \tag{1}$$

The welfare function is the discounted sum of instantaneous utility function. The utility function $u(c_t)$ is known as the instantaneous utility function, nonnegative and a concave increasing function of the per capita consumption c_t of family members. The parameter θ is the rate of time preference, which is assumed to be strictly positive. The only choice that has to be made at each moment of time is how much the representative family should consume and how much it should add to the capital stock to provide consumption in the future. The planner has to find the solution to the following problem.

The output is either consumed or invested, that is, added to the capital stock. Both families and firms have perfect foresight. They know both current and future values of w and r and take them as given. Each family maximizes equation (1) subject to the budget constraint:

$$c_t + \frac{dk_t}{dt} + \delta k_t + nk_t = w_t + r_t k_t \tag{2}$$

Where k_t is the per capita capital stock, w_t is the real wage rate, r_t is the real interest rate, δ is rate of capital depreciation and n is population growth rate. The current-value Hamiltonian function is formulated as:

$$H = \{u(\mathrm{c}_t) + \lambda_t [w_t + r_t k_t - \delta k_t - nk_t - c_t]\} \exp(-\theta\,\mathrm{t}) \tag{3}$$

Where, λ_t is a co-state variable. The first-order conditions are:

$$H_c = 0 \tag{4}$$

$$\frac{d\,\mu_t}{dt} = -H_k \tag{5}$$

$$\lim_{t\to\infty}\ \mathrm{k}_t \mu_t = 0 \tag{6}$$

Where, $\mu_t = \lambda \exp(-\theta\,\mathrm{t})$. Using the definition of $H(.)$ and replacing μ by λ, we get:

$$u'(\mathrm{c}_t) = \lambda_t \tag{7}$$

$$\frac{d\,\lambda_t}{dt} = \lambda_t [\theta + n + \delta - r_t)] \tag{8}$$

$$\lim_{t\to\infty}\ \mathrm{k}_t u'(\mathrm{c}_t) \exp(-\theta\,\mathrm{t}) = 0 \tag{9}$$

Equations (7) and (8) can be consolidated to remove the co-state variable λ_t, yielding:

$$\frac{du'(\mathrm{c}_t)/dt}{u'(\mathrm{c}_t)} = \theta + n + \delta - r_t \tag{10}$$

So, we have:

$$\frac{c_t u''(\mathrm{c}_t)}{u'(\mathrm{c}_t)} \frac{\mathrm{dc}_t/dt}{\mathrm{c}_t} = \theta + n + \delta - r_t \tag{11}$$

The expression $c_t u''(\mathrm{c}_t)/u'(\mathrm{c}_t)$ is equal to the elasticity of marginal utility with respect to consumption at two points of time. Equation (11) can be rewritten as:

$$\frac{\mathrm{dc}_t/dt}{\mathrm{c}_t} = \frac{1}{\sigma(\mathrm{c}_t)}\left[r_t - \theta - n - \delta\right] \tag{12}$$

Where $\sigma(\mathrm{c}_t)$ is the negative inverse of the elasicity of marginal utility with respect to consumption. Equation (12) links consumption growth to the gap between the marginal product of capital (net of population growth and capital depreciation rate) over the discount rate.

3.2 Firms

There is a representative firm that chooses its use of capital and labor to maximize profits. A single final good is produced by using capital K_t and labor N_t according to the Cobb–Douglas technology:

$$Y_t = \mathrm{A}K_t^{\alpha} N_t^{1-\alpha},\ 0 < \alpha < 1 \tag{13}$$

Where Y_t is final output, A is total factor productivity, and α measures the importance of capital relative to labor in production function. By dividing both sides of (13) to N_t per capita output is derived as follows:

$$f\ (\mathrm{k}_t) = A\ \mathrm{k}_t^{\alpha} \tag{14}$$

We assume $f\ (k_t)$ to be strictly concave and to satisfy the following condition, known as Inanda condition:

$$f\ (0) = 0, \quad \mathrm{f}'(0)=\infty, \quad \mathrm{f}'(\infty)=0 \tag{15}$$

Firm in turn maximizes profit at each point in time. First-order conditions for profit maximization imply that:

$$r_t = A\alpha k^{\alpha-1} \tag{16}$$

$$w_t = f\ (\mathrm{k}_t) - \mathrm{k}_t \mathrm{f}'(\mathrm{k}_t) \tag{17}$$

Factor prices, of capital and labor services in each period are compensated according to their marginal products, where w_t is real wage and r_t is the real interest rate.

4. Equilibrium Effects

In order to avoid the complexity in tracking down transitional dynamics, we only focus on the steady state equilibrium.

4.1 Steady State Equilibrium

The optimal path is characterized by equations (9) and (12). In steady state, the capital stock, k, and the level of consumption per capita, c, are constant. Setting $dc_t/dt = 0$ in equation (12) we obtain the modified golden rule relationship:

$$f'(\mathrm{k}^{ss}) = \rho + n + \delta \tag{18}$$

The marginal product of capital in steady state is equal to the sum of the rate of time preference, the growth rate of population and the rate of capital depreciation. Substituting equation (18) in to equation (16), the steady state per capita capital stock can be determined. The golden rule itself is the condition, $f'(\mathrm{k}) = \mathrm{n}$. This is the condition on the capital stock that maximizes the steady state per capita consumption. The modification in equation (18) is that the capital stock is reduced below the golden rule level by an amount that depends on the rate of time preference and the rate of capital depreciation.

Setting $dk_t/dt = 0$ in the budget constraint in equation (2), the level of consumption in the steady state obtained by:

$$c^{ss} = f\ (\mathrm{k}^{ss}) - (n+\delta)k^{ss} \tag{20}$$

With capital stock that is reduced below the golden rule, reduced optimal consumption is in the steady state.

4.2 Distortionary Taxation of Capital

Distortionary taxation certainly affects the allocation of resources. Suppose that the government taxes on the return of capital is at rate, τ_k , and remits the proceeds in lump-sum transfer to the private sector. If r_t is the pre-tax rate of return on capital, $(1-\tau_k)\mathrm{r}_t$ is the after tax return on capital. The family's flow budget constraint is now:

$$c_t + \frac{dk_t}{dt} + \delta k_t + nk_t = w_t + r_t(1-\tau_k)k_t + x_t \tag{21}$$

Where x_t is the per capita lump-sum transfers (equal to the government's receipts from the taxation of capital) made to the family. Setting up the Hamiltonian for this problem yields a modification of equation (10):

$$\frac{du'(c_t)/dt}{u'(c_t)} = \theta + n + \delta - r_t(1-\tau_k) \tag{22}$$

Note that the taxation on return of capital affects the steady state capital stock. When, $dc_t/dt = 0$, with use the equation (22) we have:

$$r_t(1-\tau_k) = n + \delta + \theta \tag{23}$$

By substituting r_t in equation (23) to equation (16), the steady state per capita capital stock is given by:

$$k^{ss} = \left[\frac{A\alpha(1-\tau_k)}{n+\delta+\theta}\right]^{\frac{1}{1-\alpha}} \tag{24}$$

The after tax rate of return to capital is lower than pre-tax rate of return. So, the marginal product of capital in the steady state is accordingly higher, meaning that the steady state capital stock is lower than when capital is not taxed. According to equation (20), the steady state per capita consumption is lower than it was in the absence of distortionary taxation.

5. Empirical Evidence

In the first step, we determine the optimum values for capital stock, output, consumption per capita and welfare in the steady state. In the second step, conducting sensitivity analysis of the effect of a change in the tax on return of capital on optimum level of per capita capital stock, per capita output, per capita consumption and welfare. We need to estimate or use values for time preference, depreciation rate and population growth rate. Data for population growth rate and depreciation rate are collected from reliable annual statistics reports in Iran's economy.

By considering the constant relative risk aversion (CRRA) utility function[1], $u(c) = c^{1-\varepsilon}/1-\varepsilon$, where ε shows the elasticity of substitution of consumption at two points of time. The quantitative implications of the results are illustrated in Table 1 and 2. Using numerical solutions based on the parameterization, $\alpha = 0.3$, $A = 1$, $\theta = 0.10$, $\delta = .05$, $n = 0.0129$, and $\varepsilon = 0.5$ in Iran's economy, sensitivity analysis is done. In the benchmark, the tax on return of capital is 0.25%, $\tau_k = 0.25$. In the first scenario, with $\alpha = 0.30$, tax on return of capital has declined from 25% to 9% in Iran's economy . In the steady state, the resulted change in capital stock, output, consumption per capita, and welfare are showed in table 1[2].

1. The constant relative risk aversion (CRRA) utility function is a famous form of a standard utility function which is used in texts.
2. We can use Excel's software and Mathematica to solve for the equilibrium values.

Table 1: With $\alpha = 0.30$ the Resulted Change in Selected Macroeconomic Variables with Reducing Tax on Return of Capital in the Steady State in First Scenario

τ_k	k^{ss}	$f(k^{ss})$	c^{ss}	w^{ss}
0.25	1.586257	1.148450	1.048674	20.48096
0.23	1.647029	1.161476	1.057878	20.57064
0.21	1.708482	1.174311	1.066847	20.65766
0.19	1.770605	1.186961	1.075590	20.74213
0.17	1.833389	1.199434	1.084114	20.82416
0.15	1.896825	1.211736	1.092426	20.90384
0.13	1.960904	1.223875	1.100534	20.98126
0.11	2.025617	1.235854	1.108443	21.05652
0.09	2.090957	1.247681	1.116160	21.12969

This table shows the effects of reduction of tax on return of capital from 0.25% to 0.09% with $\alpha = 0.30$ (measures the importance of capital relative to labor in production function) that increases the capital stock per capita, output per capita, consumption per capita, and welfare in the first scenario.
Source: Researchers Computations.

In the steady state in first scenario, a reduction in tax on return to capital from 0.25% to 0.09%, increased capital stock per capita by 31.8%, output per capita by 8.6%, consumption per capita by 6.4%, and welfare by 3.16%.

In the second scenario, importance of capital relative to labor in production function increase from 0.30 to 0.36. Also, in this case, tax on return of capital has declined from 25% to 9%. In the steady state the resulted change in capital stock per capita, output per capita, consumption per capita and welfare in the second scenario showed in table 2.

Table2: With $\alpha = 0.36$, Resulted Change in Selected Macroeconomic Variables with Decreasing of Tax on Return of Capital in the Steady State in Second Scenario

τ_k	k^{ss}	$f(k^{ss})$	c^{ss}	w^{ss}
0.25	2.202314	1.328729	1.190204	21.81929
0.23	2.294762	1.348545	1.204205	21.94725
0.21	2.388572	1.368138	1.217896	22.07167
0.19	2.483727	1.387514	1.231288	22.19268
0.17	2.580213	1.406682	1.244387	22.31042
0.15	2.678015	1.425649	1.257202	22.42500
0.13	2.777121	1.444422	1.269741	22.53656
0.11	2.877517	1.463007	1.282011	22.64519
0.09	2.979190	1.481410	1.294019	22.75099

This table shows the effects of reduction of tax on return of capital from 0.20% to 0.09% and an increasing 20% in importance of capital relative to labor in production function ($with\ \alpha = 0.36$), increases the capital stock per capita, output per capita, consumption per capita, and welfare in the second scenario.
Source: Researchers Computations.

In the steady state in second scenario, a reduction in tax on return to capital from 0.25% to 0.09%, increased capital stock per capita by 35.27%, output per capita by 11.5%, consumption per capita by 8.7%, and welfare by 4.27% in steady state. In table 3, compared the results in two scenarios.

Table3: The Differences Between Selected Macroeconomic Variable in Two Scenarios

τ_k	Δk^{ss}	$\Delta f(k^{ss})$	Δc^{ss}	Δw^{ss}
0.25	0.616057	0.180279	0.141529	1.338332
0.23	0.647733	0.187069	0.146327	1.376612
0.21	0.680090	0.193827	0.151049	1.414008
0.19	0.713122	0.200553	0.155697	1.450548
0.17	0.746823	0.207248	0.160273	1.486259
0.15	0.781190	0.213913	0.164776	1.521166
0.13	0.816217	0.220548	0.169208	1.555294
0.11	0.851899	0.227153	0.173568	1.588665
0.09	0.888233	0.233729	0.177859	1.621301

Source: Researchers Computations.

6. Conclusion

The return to capital is fundamental to the intertemporal allocation of resources by change in the consumption behavior and capital accumulation over time. Progressive capital income taxation can redistribute from the wealthy to the non-wealthy but might distort savings and consumption behavior and hence reduce wealth accumulation. Also, taxation on return to capital increases the marginal product of capital in steady state, meaning that the steady state capital stock is lower than when capital is not taxed, which in this result decreased output and growth.

The results indicate that with reduction in tax on return to capital increased capital stock, output and consumption per capita, and welfare level in the steady state. Our ultimate goal is suggesting fiscal policy with optimal tax system in order to have a high economic growth and welfare. It seems that reduction tax on return to capital will be eventually reached to a higher capital formation, higher output and consumption per capita in the steady state. Finally, the welfare level increased in the steady state.

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