

Reopening the Convergence Debate when Sharp Breaks and Smooth Shifts Wed, 1870-2010

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Abstract

This paper attempts to re-investigate the catching-up (stochastic convergence) hypothesis among the selected 16 OECD countries applying the time series approach of convergence hypothesis with annual data over one century. To reach this aim, we propose a model which specifies a trend function, incorporating both types of structural breaks – that is, sharp breaks and smooth shifts using dummy variables and Fourier function respectively. In order to detect the sharp breaks, we apply the multiple structural break models (Bai & Perron, 1998) and the Fourier function proposed in Becker et al. (2004) to capture the smooth shifts. Our results show that most divergence process occurred over World War I (WWI) and World War II (WWII). Among the 69 estimated break points occurred over the period 1870-2010, 75 % of those break points result in catching-up and the remainder results in divergence.

Keywords: Convergence, Trend Function, Smooth Shifts, Sharp Breaks, Catching-up.

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1. Introduction

The income convergence hypothesis is one of the controversial predictions of the neoclassical growth theory. The hypothesis predicts that countries with similar initial conditions will move toward a perpetual common balanced growth path, and the difference of per

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capita income among countries will disappear. But this prediction has rejected by endogenous growth theory is also due to inconclusive empirical results therein. Its empirical validity remains to be controversial. In terms of the empirical analyses to the convergence hypothesis, researchers have examined convergence with several ways-such as absolute convergence, conditional convergence, and catching-up (stochastic convergence) hypothesis and they examine employing various methodologies like cross-sectional approach, distribution approach, and time series approach.

The cross-sectional and the time series approaches are used to test the absolute and conditional notions of the convergence hypothesis. In the cross-sectional approach, the growth rate of per capita income is based on initial per capita income, and a negative (partial) relation or reverse correlation between two variables is interpreted as evidence of the absolute (conditional) convergence.

In the time series framework, the convergence hypothesis is examined by employing the unit roots or stationarity tests. Hence, its empirical validity depends on the unit root or stationarity tests (Lee et al., 2013). In this approach, the deterministic terms (intercept and/or linear trend) do not allow to be included in the unit root or stationarity test with investigating the absolute convergence hypothesis. The unit root or stationarity tests are used to test the conditional convergence hypothesis instead. In addition, the unit root and/or stationarity tests contain intercept and linear trend while testing the catching-up hypothesis. As noted by Cunado & Gracia (2006: 156), the catching-up hypothesis states “it might be appropriate in a context in which convergence is an on-going process”. Sigma convergence is based on the cross-sectional distribution of per capita income among countries. Researchers examine sigma convergence using dispersion indexes as such standard deviation and Gini coefficients and also using distribution dynamics approach. If standard deviation of per capita income across countries decreases over time, it represents that there exists the sigma convergence.

In this paper we attempt to re-examine the catching-up (stochastic convergence) hypothesis among the selected 16 Organization for Economic Co-operation and Development (OECD) countries using the time series approach of convergence hypothesis with annual data

over one century. As noted by Cunado & Gracia (2006), examining the catching-up hypothesis needs two steps. The first step (necessary condition) involves testing the unit root hypothesis in the relative per capita GDP, and the second step (sufficient condition) is related to estimate trend function for relative per capita GDP series. In other words, in the second step, the relative per capita real GDP dynamics is modeled as a trend function.

The main target and motivation of our study are that we try to develop the second step of testing catching-up process by introducing a trend function that composes both the sharp breaks and smooth shifts using annual data over one century in length. The reason for us to develop the second step of testing is that according to the demonstration of previous studies to the GDP or per capita GDP behaviors, they are well characterized by sharp breaks (e.g. Ben-David & Papell, 1998; Carrion-i-Silvestre et al., 2005) and by smooth shifts (e.g. Su & Chang, 2011; Chang et al., 2012). But to the best of our knowledge, none of them incorporates the sharp breaks and smooth shifts together in one investigated model, which is ascribed to that all mentioned above papers examine based on unit root or stationarity tests, but there is no unit root or stationarity test taking both types of structural breaks consideration in a model yet.

In this paper, followed by Carrion-i-Silvestre et al. (2005, CBL hereafter) and Becker et al. (2006, BEL hereafter) stationarity tests we show that the relative per capita real GDP series are well characterized with sharp breaks (using CBL, 2005) and with smooth shifts (using BEL, 2006). Whereas our stationarity tests identify that the relative per capita real GDP series experience both types of structural breaks, and we thereby, as the second step, specify a trend function to capture the effects of both types of breaks – that is, sharp breaks and smooth shifts. The sharp breaks are modeled by dummy variables and used to identify the break locations with the procedure of Bai & Perron (1998).¹ The smooth shifts are modeled using Fourier terms as proposed by Becker et al. (2004). We also employ this methodology to estimate the trend function, by which the coefficients of intercept

1. Whereas the procedure of Bai & Perron (1998) needs all regressors in the model be stationary, hence we estimate our trend function for series that the null of stationary is not rejected for them.

and the slope of the trend function can be time-varying for any sub-period (for example between two break dates), and we can be better indicating convergence or divergence in sub-periods.

Several previous studies, such as Greasley & Oxley (1997), Li & Papell (1999), Freeman & Yerger (2001), Strazicich et al. (2004), Datta (2003), Dawson & Sen (2007), Christopoulos & Leon-Ledesma (2008), Chong et al. (2008), and Costantini & Sen (2012) have tested the convergence hypothesis for the OECD countries using the time series framework and found that the convergence hypothesis over two decades 1990s and 2000s is successive advances in the econometric treatment of unit root tests. For example, Greasley & Oxley (1997) tested the bivariate convergence between eight OECD countries using the ADF and Perron's (1989) unit root tests. Li & Papell (1999) used the Perron (1997) unit root test for 16 OECD countries. In order to relax the structural breaks and capture dynamic behavior, Datta (2003) used the Kalman filtering to test the convergence hypothesis aiming at 15 OECD countries. Strazicich et al. (2004) used the Lee & Strazicich (2003) LM unit root test with two structural breaks. Chong et al. (2008) tested the convergence hypothesis toward the USA for 15 OECD countries using Kapetanios et al. (2003) nonlinear unit root test and found that 12 out of 15 OECD income gaps present nonlinear dynamics. Christopoulos & Leon-Ledesma (2008) developed a simple neoclassical growth theory and showed that some determinants of convergence rate may vary with time. They thereby tested the convergence hypothesis for 14 OECD countries using stationarity covariates and found strong evidence for the convergence in 12 countries. Freeman & Yerger (2001), Fleissig & Strauss (2001), and Cheung & Pascual (2004) tested the convergence hypothesis using panel unit root tests.

The remainder of paper is organized as follows: section 2 presents our methodology. In section 3 we present our data and empirical results. Conclusions are presented in the final section.

2. Methodology

2.1 Time Series Framework of Catching-up Hypothesis

The time series approach of the convergence hypotheses is introduced by Carlino & Mills (1993) and it is extended by Bernard & Durlauf

(1995), Evans & Karras (1996), and Li & Papell (1999). By this approach, country i will be converged toward the country j (as a leader or a benchmark country) if and only if:

$$\lim_{n \rightarrow \infty} (y_{i,t+n} - ay_{j,t+n} | \xi_t) = 0 \quad (1)$$

Where y is the per capita real GDP in log, a is relative per capita real GDP, and ξ_t is the information set at time t . The indices i and j denote country i and country j , respectively. We can define three versions of the convergence hypothesis using equation (1). If $a=1$ then it shows absolute convergence. In order to test this definition, researchers use unit root or stationarity test without any intercept and linear trend. If $a \neq 0$ and the series $(y_{i,t} - y_{j,t})$ performs level stationarity, then it can be named as conditional convergence or deterministic convergence. If $a \neq 0$ and the series $(y_{i,t} - y_{j,t})$ performs trend stationarity, then it can be named as stochastic convergence or catching-up process.

Testing the catching-up process proceeds with the following two steps. The first step or the necessary condition relates to testing existing unit root/stationarity to the relative per capita real GDP series. The second step or the sufficient step involves the estimation of trend functions for relative per capita real GDP series that the unit root hypothesis is rejected for it (Ranjbar et al., 2013).

2.2 Necessary Condition: Stationarity Tests

In order to test the first step or the necessary condition of catching-up hypothesis, this paper uses the univariate and panel data versions of the CBL (2005) stationarity test and also the BEL (2006) Fourier stationarity test, in which the former allows for sharp breaks and the later allows for an unknown form and a number of smooth drifts.

2.2.1 Carrion-i-Silvestre et al. (2005) Stationarity Test

The CBL (2005) stationarity test is adopted in this study due to several advantages. First, the reversal of the null and alternative hypotheses is the most appealing for the CBL test, because most of the panel unit root tests are equipped with the null hypothesis, in which the rejection of the unit root null implies only parts (but not all) of countries are stationary. By contrast, the null hypothesis of the CBL stationarity test is based on

the stationarity throughout all countries. Therefore, if the null hypothesis of the CBL test is rejected, then we say that all of the series in the panel are non-stationary. Second, the CBL method enables us to consider multiple structural breaks positioned at different unknown dates in addition to a different number of breaks for each individual. Allowing the existence of structural breaks can potentially strengthen our results more correctly in respect of specifying the model. Third and finally, we can allow for more general forms of cross-sectional correlation than previous studies through the conventional cross-sectional demeaning of the data, which assumes that a common factor affects all units with the same intensity. Carrion-i-Silvestre and German-Soto (2009) also indicate that the lack of consideration of the cross-sectional dependence might bias the analysis to conclude in favor of the stationarity of the panel data even in the case that it is non-stationary. It is important to note that the panel stationarity test controls non-parametrically for serial correlation in the error through the estimation of the long-run variance via kernels. In our study, we employ the bootstrap distribution, tailored to the error structure of panel data, in order to accommodate general forms of cross-sectional dependence.

CBL (2005) extended the approach of Hadri (2000) by further allowing for multiple structural breaks through incorporating dummy variables into the deterministic specification of a model.¹ In this case, the data generation process under the null of stationarity is based on following model:

$$y_{it} = \alpha_i + \beta t + \sum_{l=1}^m \theta_{il} DU_{l,t} + \sum_{l=1}^m \rho_{il} DT_{l,t} + \varepsilon_{it} \quad (2)$$

In equation (2), y_{it} is relative per capita real GDP of country i in year t , as well as α , t , and m are intercept, linear trend, and the optimal number of breaks, respectively. The other regressors, $DU_{l,t}$ and $DT_{l,t}$ are l^{th} break in intercept and slope of linear trend in year t respectively and are defined as the following:

1. The null hypothesis of CBL (2005) implies regime-wise stationarity for all countries, versus the alternative of non-stationarity for some countries.

$$DU_{1,t} = \begin{cases} 1 & \text{if } t > TB_1 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

$$DT_{1,t} = \begin{cases} t - TB_1 & \text{if } t > TB_1 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

The univariate test statistic ($LM(\lambda_i)$) is computed as Kwiatkowski et al. (1992, KPSS hereafter) test with multiple breaks:¹

$$LM(\lambda_i) = \hat{\omega}_i^{-2} T^{-2} \sum_{t=1}^T \hat{S}_{it}^2 \quad (5)$$

where \hat{S}_{it} is the partial sum of the estimated OLS residuals from equation (2), $\hat{\omega}_i^2$ denotes a heteroscedasticity and autocorrelation consistent estimate of the long-run variance of $\hat{\varepsilon}_{it}$. We estimate the consistent long-run variance using the new boundary condition rule proposed by Sul et al. (2005). For this end, we estimate an AR(p) autoregressive process for each unit by the OLS method:

$$\hat{\varepsilon}_{it} = \sum_{p=1}^1 \hat{\zeta}_{i,1} \hat{\varepsilon}_{it-1} + v_{it} \quad (6)$$

where $\hat{\varepsilon}_{it}$ are the estimated OLS residuals from equation (2). Then the estimator of long-run variance ($\hat{\omega}_i^2$) is constructed using Sul et al. (2005) boundary condition rule as follows:

$$\hat{\omega}_i^2 = \min \left\{ T \hat{\sigma}_{vi}^2, \frac{\hat{\sigma}_{vi}^2}{\left(1 - \sum_{p=1}^1 \hat{\zeta}_{i,1}\right)^2} \right\} \quad (7)$$

$$\text{and } \hat{\sigma}_{vi}^2 = \frac{1}{T} \sum_{t=1}^T \hat{v}_{it}^2.$$

where $\hat{\zeta}$ is the autoregressive coefficient estimates from equation (6),

1. Hadri (2000) proposed an LM panel data stationarity test without breaks. However, CBL (2005) extended the analysis to account for the presence of multiple breaks in a panel framework.

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and the optimum lag length (p) in equation (6) is determined using the BIC information criterion. λ_i is the location of the breaks related to the entire time period (T). The test statistic is dependent on the λ_i , which is important in identifying the location and the number of breaks correctly. For this end, the CBL recommend for Bai & Perron (1998) procedure, which is based upon the global minimization of the sum of squared residuals (SSR) expressed as follows:

$$(T\hat{B}_1, \dots, T\hat{B}_m) = \arg \min_{(T\hat{B}_1, \dots, T\hat{B}_m)} SSR(T\hat{B}_1, \dots, T\hat{B}_m) \quad (8)$$

where $T\hat{B}_m$ is m^{th} break date. The optimal number of breaks is selected by CBL criterion of Liu et al. (1997). CBL calculated the test statistic for the null of a stationary panel with multiple breaks as follows:

$$Z(\lambda) = \frac{\sqrt{N} \left(N^{-1} \sum_{i=1}^N LM(\lambda_i) - \bar{\mu}_{LM} \right)}{\sigma_{LM}} \xrightarrow{d} N(0,1) \quad (9)$$

where $\bar{\mu}_{LM}$ and σ_{LM} are the mean and standard deviation of $LM(\lambda_i)$. We computed the empirical distribution of $Z(\lambda)$ using Bootstrap techniques following as Maddala & Wu (1999). In step one, we run a regression with equation (2), imposing the null hypothesis of stationarity and then save the resulting residuals ($\hat{\varepsilon}_{it}$) and fitted \hat{y}_{it} . In step two, we generate bootstrap residuals e_{it} following the sampling strategy suggested by Maddala & Wu (1999), with replacement samples of $t+100$ values (and then discard the first 100 values) from the residual matrix. In step three, we calculate the bootstrap samples of observations \tilde{y}_{it} as $\tilde{y}_{it} = \hat{y}_{it} + e_{it}$. In step four, we construct the pseudo individual and panel statistics based on equations (5) and (9), respectively. And in step five, we repeat steps 1-4 for 20,000 times to derive the empirical distribution of $LM(\lambda_i)$ under the null hypothesis of regime-wise stationary.

2.2.2 Becker et al. (2006) Stationarity Test

BEL (2006) developed the standard KPSS stationarity test with a

Fourier function that allows the deterministic term in regression to be a time-dependent function. Hence the test does not need to pre-specified number and form of structural breaks. It can control for unknown number and form of structural breaks using a selected frequency component of a Fourier function. Hence this test is suitable for various series with various types of smooth structural breaks with unknown number and form. Following the BEL (2006), we consider the following data generating process (DGP):

$$y_t = \alpha_0 + \beta t + \gamma_1 \sin(2\pi kt/T) + \gamma_2 \cos(2\pi kt/T) + r_t + \varepsilon_t \quad (10)$$

$$r_t = r_{t-1} + u_t$$

where ε_t are stationarity errors and u_t are independent and identically distributed (*i.i.d*) with variance σ_u^2 . Under the null hypothesis that $\sigma_u^2 = 0$, the process described by equation (10) is stationarity. The rational for selecting $[\sin(2\pi kt/T), \cos(2\pi kt/T)]$ is based on the fact that a Fourier expression is capable of approximating functions which are not graded, to any desired degree of accuracy, where k represents the frequency selected for the approximation, and $\gamma = [\gamma_1, \gamma_2]'$ measures the amplitude and displacement of the frequency component.¹ A desire feature of equation (10) is that the standard linear specification is regarded as a special case while setting $\gamma_1 = \gamma_2 = 0$. It also follows that at least one frequency component must be present if there is a structural break. Here, if it is possible to reject the null hypothesis $\gamma_1 = \gamma_2 = 0$, the series must have a nonlinear component.² Becker et al. (2004) use this property of equation (10) to develop a test which is more powerful to detect breaks of an unknown form than the standard Bai & Perron

1. As see in equation (10), the conventional KPSS test is a one variety of BEL (2006) when trigonometric component is ignored. As noted by BEL (2006, p: 391) "the usual KPSS-type stationary tests will diverge when nonlinear trends are ignored. This leads to over-rejections of the true stationary null hypothesis in favor of the false unit-root hypothesis."

2. In order to test for presence of nonlinear terms, BEL offered a $F(k)$ test. As noted by-BEL, the presence of the nuisance parameter causes that the distribution of $F(k)$ does not have be non-standard. Hence, we calculate the critical values for any series herein. To this end, we first generate 20,000 random series using the Gauss (version 10.0.0) RNDN procedure under the null of linearity. Then using optimum frequency to any actual series, we calculate the F-statistic to any of 20,000 pseudo series. In final step we obtain the critical values from the sorted vector of pseudo F-statistic.

(1998) test. As the DGP in equation (10) nests used to generate the common KPSS (1992) test, the BEL's stationarity test with a Fourier function needs only a slight modification of the KPSS statistic. First, one needs to obtain the residuals from the following equation:

$$y_t = \alpha_0 + \beta t + \gamma_1 \sin(2\pi kt/T) + \gamma_2 \cos(2\pi kt/T) + v_t \quad (11)$$

Equation (11) tests the null of trend stationarity. The test statistic is given by:

$$\tau_{KPSS} = \frac{1}{T^2} \frac{\sum_{t=1}^T \tilde{S}_t(k)^2}{\tilde{\sigma}^2} \quad (12)$$

where $\tilde{S}_t(k) = \sum_{j=1}^t \hat{v}_j$ and \hat{v}_j are the OLS residuals from regression

(11) and $\tilde{\sigma}^2$ the long run variance. In this paper we follow Carrion-i-Silvestre & Sansó (2006) and use the Sul et al. (2005) method to compute the long run variance. BEL (2006) suggests that the frequencies in equation(11) should be obtained via the minimization of the sum of squared residuals. However, their Monte Carlo experiments suggest that no more than one or two frequencies should be used because of the loss of power associated with a larger number of frequencies.¹

As seen in equation (11), the conventional KPSS test is one variety of BEL (2006), in which trigonometric component has been ignored. In order to test the presence of nonlinear terms, BEL offered an F-test expressed as follows:

$$F(k) = \frac{SSR_{KPSS} - SSR_{BEL}(k)}{SSR_{BEL}(k) / (T - q)} \quad (13)$$

where SSR_{BEL} denotes the SSR from equation (11), T is time period, q is the number of regressors, and SSR_{KPSS} denotes the SSR from the regression without the nonlinear terms. As noted by BEL (2006), the

1. In order to determine the optimum frequency, we follow BEL (2006) and first determine the maximum frequency equal to 5 and then calculate the sum of squared residuals (SSR hereafter) for any frequency. The optimum frequency is that minimize the SSR.

presence of the nuisance parameters causes the distribution of $F(k)$ not to be non-standard. Therefore, in this paper we calculate the critical values for any series. To this end, we first generate 20,000 random series under the null of linearity. Then by using optimum frequency to any actual series, we calculate the F-statistic to any of 20,000 pseudo series and finally, we obtain the critical values from the sorted vector of pseudo F-statistic.

2.3 Sufficient Condition: Estimation of Trend Function

As noted by Tomljanovich & Vogelsang (2002), and Cunado & Gracia (2006), for the catching-up hypothesis, the trend stationarity is a necessary condition. In order to test the sufficient condition, we introduce a new methodology for estimating the trend function. Suppose series y_t is trend stationarity and we can specify its trend function as following:

$$y_t = \sum_{l=1}^{m+1} \theta_l DU_{l,t} + \sum_{l=1}^{m+1} \rho_l DT_{l,t} + \sum_{k=1}^n \gamma_{1,k} \sin\left(\frac{2\pi kt}{T}\right) + \sum_{k=1}^n \gamma_{2,k} \cos\left(\frac{2\pi kt}{T}\right) + \varepsilon_t \quad (14)$$

where m is the optimal number of breaks. The other regressors, $DU_{l,t}$ and $DT_{l,t}$ are l^{th} break points in intercept and slope of linear trend in year t respectively and are defined as the following:

$$DU_{l,t} = \begin{cases} 1 & \text{if } TB_{l-1} < t < TB_l \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

$$DT_{l,t} = \begin{cases} t - TB_{l-1} & \text{if } TB_{l-1} < t < TB_l \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

Variables DU and DT are incorporated into the model for capturing the sharp drifts. Following Gallant (1981) in respect of obtaining a global approximation from the smooth shifts, we use the Fourier approximation and incorporate terms $\sum_{k=1}^n \gamma_{1,k} \sin\left(\frac{2\pi kt}{T}\right)$ and

$\sum_{k=1}^n \gamma_{2,k} \cos\left(\frac{2\pi kt}{T}\right)$ into the model. n and k indicate the number of

frequencies, which are contained in the approximation and equal to $n \leq \frac{T}{2}$ and particular frequency, respectively.

The estimation of equation (14) involves with three questions, the choice of m , the choice of n , and the choice of k . As noted by Becker et al. (2004) it is reasonable to restrict $n=1$ because if $\gamma_{1,k} = \gamma_{2,k} = 0$ is rejected for one frequency, then the null hypothesis of time invariance should be also rejected. Also Enders & Lee (2012) noted that imposing the restriction $n=1$ is useful to save the degrees of freedom and prevent from over-fitting problem. Hence we re-specify the equation (14) as follows:

$$y_t = \sum_{l=1}^{m+1} \theta_l DU_{l,t} + \sum_{l=1}^{m+1} \rho_l DT_{l,t} + \gamma_1 \sin\left(\frac{2\pi kt}{T}\right) + \gamma_2 \cos\left(\frac{2\pi kt}{T}\right) + \varepsilon_t \quad (17)$$

In order to estimate the equation (17), we propose a two-step procedure. In the first step, we determine the optimum break points, m , and optimum frequency, k . For this propose, the allowed maximum k is set to be 5, and we select an integer frequency until allowing smooth shifts temporarily.¹ Then for any $K=k$, we estimate the equation (17) using the procedure proposed in Bai & Perron (1998) and save the sum of squared residuals (SSR). We select frequency k^* as an optimum frequency minimizing the SSR, and we further re-estimate the equation (17) with $K= k^*$ and select the obtained number and location of break points as optimum number and location of break points. In the second step, we test the absence of the nonlinear component by the equation (17). To this end, following the Becker et al. (2004; 2006), we use the usual F-test statistic as follows:

$$F(k^*) = \frac{(SSR_{restricted} - SSR_{unrestricted}(k^*)) / 2}{SSR_{unrestricted}(k^*) / T - q} \quad (18)$$

$SSR_{unrestricted}$ and $SSR_{restricted}$ denote the SSR from equation (17) with and without nonlinear component, respectively, and q is the number of regressors. As noted by Becker et al. (2006), due to the presence of

1. See Christopoulos & Leon-Ledesma (2011) for more details.

nuisance parameter, the F-test has not standard distribution and we then calculate its critical values employing Monte Carlo simulation.

For testing the catching-up (stochastic convergence) hypothesis, we follow the Carrion-i-Silvestre and German-Soto (2009) procedure. We can say that there exists evidence of catching-up process or stochastic convergence when the coefficients of the parameters of each regime are significant at least at the 10% level of significance, and $\hat{\theta}_l$ and average slope of trend function (ρ_l) have opposite sign, i.e., when $\hat{\theta}_l < 0$, average slope of trend function is positive; or when $\hat{\theta}_l > 0$, average slope of trend function is negative. If both $\hat{\theta}_l$ and average slope of trend function of each regime have the same sign, we conclude that the divergence has occurred. If both parameters (θ_l and ρ_l) are insignificant, it suggests that catching-up process has occurred.

3. Data and Empirical Results

3.1 Data and Variable

We collect annual per capita real GDP in 1990 Geary-Khamis PPP-adjusted dollars for 16 selected OECD countries including; Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Sweden, Switzerland, and the United Kingdom, and the U.S., over the period 1870-2010. The source of the data is from the New Maddison Project Database. To test the catching-up (stochastic convergence) process, we calculate the ratio of real GDP per capita of each OECD country to real GDP per capita of

OECD group exhibited as $\ln\left(\frac{y_{i,t}}{y_{OECD,t}}\right)$, where $y_{i,t}$ is per capita real

GDP for the i -th country, and $y_{OECD,t}$ is the yearly average value in the sample span; i and t indicate country i in year t , respectively. The advantage of using relative per capita real GDP is that we can utilize this approach to discuss the income convergence effect in OECD country.

3.2 Empirical Results

As mentioned in the previous section, to test the necessary condition for income convergence, we need to first run the univariate and panel CBL (2005) stationarity test. The results are presented in Table 1. Hence it is possible that relative per capita real GDP time series in our panel are dependent – that is, there is possibility that our panel data suffer from cross-country dependence, and as we know that the presence of cross-sectional dependence might bias our analyses and result in favor of the stationarity of panel data (Lee, 2013; Lee et al., 2013). Hence, before we test the panel stationarity hypothesis using CBL test, we use the Pesaran (2004) cross-section dependence (PCD, hereafter) test so that we can test the cross-sectional dependence. Pesaran (2004) developed a simple test for error cross sectional dependence, which has correct size, sufficient power, and is applied to both stationarity and non-stationarity panels. Pesaran's cross section dependence test proceeds with following three steps. First, the residuals are obtained from the ADF regression for any member of panel. Second, the average of pair-wise correlation coefficients of residuals is calculated as follows:

$$\hat{\rho}_{ij} = \frac{\sum_{t=1}^T \varepsilon_{it} \varepsilon_{jt}}{\sqrt{\sum_{t=1}^T \varepsilon_{it}^2} \sqrt{\sum_{t=1}^T \varepsilon_{jt}^2}} \quad (19)$$

where ε are residuals from standard ADF regression. The *PCD* test statistics is computed as follows:

$$PCD = \left(\frac{2T}{N(N-1)} \right)^{0.5} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^N \hat{\rho}_{ij} \right) \quad (20)$$

The PCD statistic tests the null hypothesis of cross independence, and they are distributed in standard normality.¹ The PCD statistic test results are shown in the first row of panel A of Table 1. As seen that Pesaran (2004) statistic takes the value of -6.158 with a p-value 0.000, and it means that we are able to reject the null hypothesis of cross-

1. Pesaran (2004) indicates that the PCD test has exactly mean zero for fixed T and N and is robust to heterogeneous dynamic models including multiple breaks in slope coefficients and/or error variances.

sectional independence at the 1% significance level; whereas the panel statistic of CBL (2005) stationarity test requires the individual statistics to be cross-sectional independence. Hence in order to overcome this shortcoming, we compute the empirical distribution of panel statistic of CBL test using the bootstrap techniques as suggested by Maddala & Wu (1999). The critical values for panel statistic are computed by 20,000 replications. As we can see that both versions of panel statistics (homogenous and heterogeneous long run variances) are less than the critical values at the 10% significance level. The findings show that the stationarity of all countries of panel data is not rejected.

Table 1: Carrion-i-Silvestre et al. (2005) Stationarity Test Results.

Panel A: Pesaran (2004) and Panel Carrion-i- Silvestre et al. (2005) Test					
Pesaran (2004) cross sectional dependence test	<u>Test</u>			<u>P-value</u>	
	-6.158			0.000	
Carrion -i- Silvestre et al (2005) stationarity test	<u>Bartlett</u>		<u>Critical values</u>		
		90	95	97.5	99
Homogenous long run variance	0.861	4.125	4.841	5.669	6.474
Heterogeneous long run variance	1.641	4.184	4.769	5.655	6.327
Panel B: Univariate Carrion-i- Silvestre et al. (2005) Stationarity Test with Breaks in Intercept & Trend					
Country	Bartlett	90%	95%	97.5%	99%
Australia	0.020	0.025	0.028	0.031	0.034
Austria	0.011	0.048	0.060	0.072	0.088
Belgium	0.035	0.106	0.134	0.165	0.200
Canada	0.028	0.032	0.037	0.042	0.049
Denmark	0.026	0.091	0.112	0.136	0.164
Finland	0.044	0.064	0.077	0.090	0.107
France	0.047	0.073	0.089	0.106	0.130
Germany	0.018	0.049	0.060	0.072	0.089
Italy	0.038	0.102	0.132	0.164	0.207
Japan	0.020	0.048	0.061	0.074	0.090
Netherlands	0.029	0.073	0.091	0.110	0.134
Norway	0.008	0.046	0.057	0.068	0.083
Sweden	0.010	0.034	0.038	0.043	0.049
Switzerland	0.037	0.046	0.054	0.061	0.070
United Kingdom	0.015	0.056	0.071	0.086	0.106
United States	0.021	0.054	0.068	0.082	0.100

Notes: Critical values for univariate version computed using Monte Carlo simulation and critical values for panel version computed using Bootstrap techniques. Maximum number of breaks fixed at 5.

The results of univariate version of CBL (2005) stationarity test are presented in panel B of Table 1. The critical values for univariate version are computed using Monte Carlo simulation and 20,000 replications. As we can see that the null hypothesis of stationarity is not rejected at the 10% significance level for any country.¹

The results for BEL (2006) stationarity test are provided in Table 2. In order to run the BEL test, we set maximum frequencies at 5, and we use the Sul et al. (2005) method to choose the kernel and the estimation of long run variance. The significant F-statistic showed in the third column indicates that both sine and cosine terms should be included in the estimated model for all countries. The numbers in the second column show the optimum frequency for each country. The results show that the optimum frequency of K=5 are fitted for France and Germany, K=4 for Netherlands, K=2 for Australia, Norway, and United Kingdom, and K=1 for the other countries. From comparing the BEL test statistics (Bartlett) with their critical values, we know that the null hypothesis of stationarity is not rejected for each country. In addition, both CBL and BEL stationarity tests do not reject the necessary condition in any country, and we therefore further test the sufficient condition for all the OECD countries.

Table 2: Becker et al. (2006) Stationarity Test Results.

Country	Optimum frequency	F statistic	Truncation lag	Bartlett	Critical Values			
					90%	95%	97.50%	99%
Australia	2	38.255	3	0.034	0.102	0.129	0.154	0.187
Austria	1	86.1484	1	0.031	0.046	0.051	0.053	0.059
Belgium	1	83.310	2	0.029	0.114	0.141	0.184	0.223
Canada	1	50.133	3	0.029	0.109	0.139	0.152	0.165
Denmark	1	108.734	2	0.022	0.092	0.108	0.123	0.155
Finland	1	40.148	2	0.041	0.092	0.111	0.136	0.161
France	5	10.630	5	0.083	0.114	0.137	0.144	0.15
Germany	5	10.175	2	0.038	0.121	0.148	0.161	0.19
Italy	1	132.992	3	0.032	0.129	0.169	0.180	0.203
Japan	1	35.617	1	0.018	0.049	0.056	0.064	0.071
Netherlands	4	15.695	2	0.092	0.098	0.116	0.131	0.158
Norway	2	66.393	1	0.038	0.051	0.054	0.058	0.066
Sweden	1	291.370	2	0.028	0.104	0.123	0.136	0.149

1. In order to save the space, we do not report the estimated break dates.

Switzerland	1	243.067	5	0.055	0.090	0.102	0.107	0.124
United Kingdom	2	29.443	2	0.057	0.096	0.13	0.188	0.211
USA	1	39.639	2	0.025	0.094	0.117	0.149	0.175

Notes: The finite sample critical values for flexible Fourier KPSS test (Bartlett) statistic were calculated with 20000 replications.

In order to run the second step or test the sufficient condition for the catching-up hypothesis, we estimate the equation (17) for 16 countries and report the results in Table 3. To this end, we set a maximum break point at 8 and a maximum frequency at 5. The results of a grid-search for finding the best frequency, presented in the second column of panel A in Table 3, indicate that the case $K=1$ (frequency) is fitted for Belgium and Italy, $K=2$ for Finland, France, Norway, and Sweden, $K=3$ for Japan and Netherland, $K=4$ for Denmark and the United Kingdom, and $K=5$ for Australia, Austria, Canada, Germany, Switzerland, and the U.S.

Table 3: Estimation Results for Trend Function in Equation (17), 1870-2010.

Panel A: The Results for Optimum Frequency, F Statistic and its Critical Values						
Country	Optimum frequency	F stat	90%	95%	97.50%	99%
Australia	5	20.863**	2.125	2.457	2.727	4.022
Austria	5	6.278**	2.297	3.478	4.369	4.691
Belgium	1	97.278**	2.365	3.596	3.872	4.212
Canada	5	5.412**	2.495	2.883	3.164	4.599
Denmark	4	3.817**	2.124	2.662	3.635	5.606
Finland	2	28.986**	2.923	3.577	4.246	6.956
France	2	0.168	2.310	2.670	3.381	3.434
Germany	5	8.124**	2.763	2.923	3.447	5.041
Italy	1	139.049**	2.244	2.617	3.201	4.183
Japan	3	19.984**	2.642	3.434	4.372	4.762
Netherlands	3	3.255**	2.080	2.898	3.179	3.735
Norway	2	108.428**	2.235	3.082	3.574	5.249
Sweden	2	5.755**	2.727	3.336	4.085	5.243
Switzerland	5	16.130**	2.335	3.091	3.334	4.452
United Kingdom	4	22.721**	2.464	2.858	3.312	4.265
United States	5	4.461**	2.402	3.120	3.909	5.094

Panel B: The Results for Sharp Drift Dates in Equation (17)

Country	Break dates					
	First	Second	Third	Fourth	Fifth	Sixth
Australia	1891 [1889,1892]	1927 [1924,1928]	1941 [1939,1942]	1966 [1964,1967]	1990 [1987,1991]	
Austria	1913 [1911,1914]	1930 [1929,1931]	1944 [1942,1945]	1958 [1956,1959]		
Belgium	1900	1919	1944	1977	1993	

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	[1899,1901]	[1917,1920]	[1942,1945]	[1975,1978]	[1991,1994]		
Canada	1900	1918	1932	1946	1961		
	[1898,1906]	[1916,1921]	[1930,1933]	[1944,1947]	[1959,1963]		
Denmark	1929	1943	1982				
	[1928,1930]	[1941,1948]	[1980,1989]				
Finland	1894	1916	1944	1991			
	[1890,1895]	[1914,1917]	[1942,1945]	[1989,1992]			
France	1916	1931	1945	1977			
	[1915,1917]	[1928,1932]	[1944,1946]	[1975,1978]			
Germany	1913	1931	1945	1959			
	[1910,1914]	[1928,1932]	[1943,1946]	[1957,1960]			
Italy	1886	1942	1995				
	[1884,1887]	[1940,1943]	[1994,1996]				
Japan	1889	1916	1930	1944	1973	1990	
	[1887,1890]	[1915,1917]	[1928,1931]	[1942,1945]	[1971,1974]	[1988,1991]	
Netherlands	1906	1931	1945	1981			
	[1903,1907]	[1930,1932]	[1944,1946]	[1979,1982]			
Norway	1891	1913	1929	1944	1987		
	[1890,1892]	[1912,1914]	[1927,1931]	[1942,1946]	[1985,1988]		
Sweden	1890	1913	1929	1944	1960	1991	
	[1888,1891]	[1912,1914]	[1926,1930]	[1942,1945]	[1958,1962]	[1988,1992]	
Switzerland	1884	1930	1944	1995			
	[1881,1885]	[1929,1931]	[1942,1946]	[1993,1996]			
United Kingdom	1914	1928	1946	1971			
	[1913,1915]	[1926,1929]	[1945,1947]	[1969,1972]			
United States	1931	1945	1976				
	[1930,1932]	[1943,1946]	[1974,1977]				

Panel C: Catching-up Phase after any Breaks

Country	Before the first break	1th	2th	3th	4th	5th	6th
Australia	C	C	D	C	C	C	
Austria	C	C	C	C	C		
Belgium	C	D	C	C	D	C	
Canada	C	D	C	C	C	C	
Denmark	C	C	C	C			
Finland	D	D	C	D	C		
France	C	C	C	C	C		
Germany	C	D	C	C	D		
Italy	C	C	C	D			
Japan	D	C	D	C	C	C	C
Netherlands	C	D	C	C	C		
Norway	C	D	D	D	C	C	
Sweden	D	C	D	C	C	C	C
Switzerland	D	C	C	C	D		
United Kingdom	D	C	C	C	D		
United States	C	C	D	C	C		

Notes: Critical values for F statistic were calculated with 20000 replications. Maximum breaks were fixed at 8 and maximum frequencies were fixed at 5. C and D denote the catching-up and divergence process after any break. The figures in the bracket in panel B are 95% confidence interval.

Next, the results for F-statistics and its critical values (computed from 20,000 replications) are presented in the columns 3-7 of panel A of Table 3. The results indicate that both sine and cosine terms should be included in the estimated model for all countries except for France.

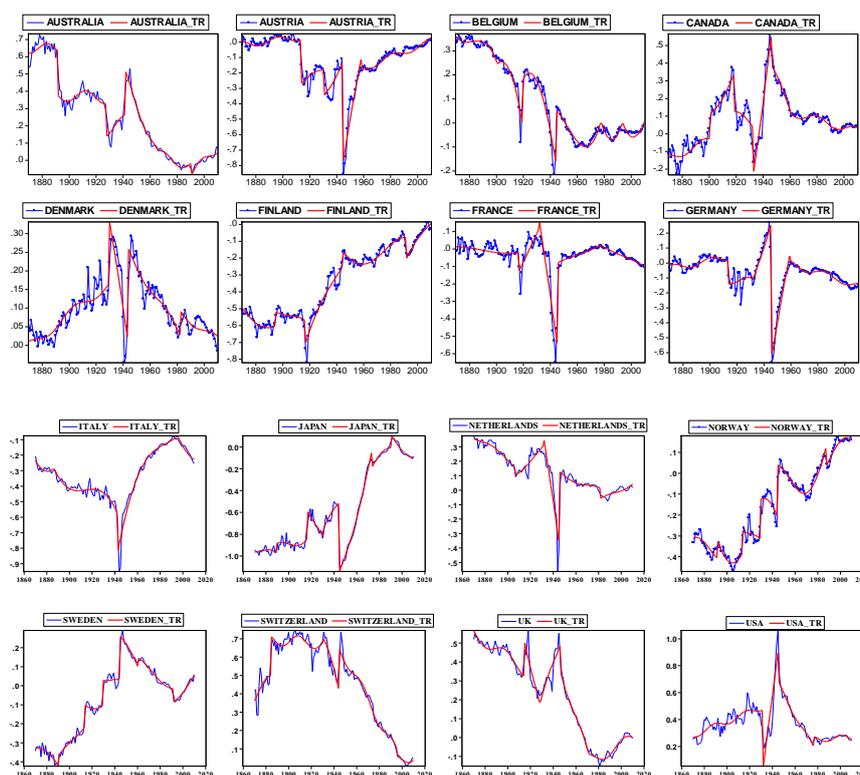
Third, we show the estimated break point locations and corresponding 95% confidence intervals which provide information about the degree of uncertainty in the estimation of the break dates in panel B of Table 3. The 95% confidence intervals appear to be very tight for all the break dates in all countries, which make us very confident that the break locations are properly estimated. The dispersion of break point dates is shown in panel B of Table 3, and it shows that all countries experience at least three sharp breaks. Japan and Sweden experience 6 sharp breaks, Australia, Belgium, Canada, and Norway experience 5 breaks, Austria, Finland, France, Germany, Netherland, Switzerland, the United Kingdom experience 4 breaks, and Denmark, Italy, and the U.S. experience 3 breaks in their catching-up processes to the real GDP per capita in OECD countries.

From 69 estimated break points, 10 out of them (i.e., Austria [1913], Belgium [1919], Canada [1918], Finland [1916], France [1916], Germany [1913], Japan [1916], Norway [1913], Sweden [1913], and United Kingdom [1914]) occurred in 1920s that are coincided with years World War I (WWI). 16 out of 69 break points (i.e., Australia [1941], Austria [1944], Belgium [1944], Canada [1946], Denmark [1943], Finland [1944], France[1945], Germany [1945], Italy [1942], Japan [1944], Netherlands [1945], Norway [1944], Sweden [1944], Switzerland [1944], United Kingdom [1946], and USA[1945]) occurred in 1940s that are coincided with years World War II (WWII). 13 out of 69 break points (i.e., Australia [1927], Austria [1930], Canada [1932], Denmark [1929], France [1931], Germany [1931], Japan [1930], Netherlands [1931], Norway [1929], Sweden [1929], Switzerland [1930], United Kingdom [1928], and USA [1931]) occurred over the period 1929-30 that are coincided with year Great depression. 12 out of 69 break points (i.e., Australia [1990], Belgium [1977], Denmark [1982], Finland [1991], France [1977], Japan [1973 and 1990], Netherlands [1981], Norway [1987], Sweden [1991], United Kingdom [1971], and USA [1976]) are coincided with oil shocks in the early and end stages of 1970s and the

early stage of 1990s.

Also our results in panel C of Table 3 show that from 69 estimated break points that occurred over the period 1870-2010, 52 cases (75%) result in catching-up, and the others result in divergence. In panel C, terms C and D denote the catching-up and divergence process after any break, respectively. Figure 1 displays the time paths of the relative per capita real GDP (blue line) and the estimated flexible trend function (red line) for each country. As we know that the actual nature of break(s) is generally unknown, and there is no specific guide as to where and how many breaks to use in testing for a unit root or stationarity. Using an incorrect specification for the form and number of breaks can be as problematic as ignoring the breaks altogether. A further examination of the figures, we can clearly observe both forms of breaks; i.e., sharp breaks and smooth shifts in the trend of the data. According to the graphs, it seems that the estimated break points using both the dummy variables and the Fourier approximations are reasonable, and these results further support our hypothesis that trend function can experience both types of breaks (i.e., sharp and smooth breaks).

Figure 1: Log Relative per capita Real GDP (Blue Line) and Estimated Trend Function with Sharp Breaks and Smooth Shifts (Red Line) for 16 Selected OECD Countries, 1870-2010.



4. Conclusion

In this paper we attempt to re-test the catching-up hypothesis among the 16 OECD countries using the time series approach of stochastic convergence hypothesis with annual data over one century. To reach this aim, we propose a model which specifies a trend function, incorporating both types of structural breaks (i.e., sharp breaks and smooth shifts) using dummy variable and Fourier function, respectively. In order to detect the sharp breaks, we use the multiple break models proposed in Bai & Perron (1998), and we apply the Fourier function proposed in Becker et al. (2004) to capture the smooth shifts. Evidences demonstrate that the null hypothesis of stationarity is not rejected by CBL (2005), and BEL (2006) tests for each countries we focus. The tests also show that all relative per capita real GDP series have experienced shapes breaks and smooth shifts. Therefore, to

investigate the sufficient condition of catching-up hypothesis, we specify a new trend function that incorporates with both types of structural breaks. The results show that most of sharp breaks are coincided with WWI, WWII, great depression, and oil shocks, and most divergence process occur over WWI and WWII.

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