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Forced vibration of piezoelectric nanowires based on nonlocal elasticity theory

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Abstract

In this paper, a numerical solution procedure is presented for the free and forced vibration of a piezoelectric nanowire under thermo-electro-mechanical loads based on the nonlocal elasticity theory within the framework of Timoshenko beam theory. The influences of surface piezoelectricity, surface elasticity and residual surface stress are taken into consideration. Using Hamilton's principle, the nonlocal governing differential equations are derived. The governing equations and the related boundary conditions are discretized by using the differential quadrature method (DQM). The numerical results are obtained for both free and forced vibration of piezoelectric nanowires. The present results are validated by available results in the literature. The effects of the nonlocal parameter together with the other parameters such as residual surface stress, temperature change and external electric voltage on the size-dependent forced vibration of the piezoelectric nanowires are studied. It is shown that the nonlocal effect (small scale effect) plays a prominent role in the forced vibration of piezoelectric nanowires and this effect cannot be neglected for small external characteristic lengths. The resonant frequency increases with increasing the residual surface stress. In addition, as the surface elastic constant increases, the resonant frequency of PNWs increases, while the surface piezoelectric constant has a decreasing effect on the resonant frequency.

Keywords: *Piezoelectric nanowire, Forced vibration, Small scale effect, Surface effects*

1. Introduction

Since laboratoryexaminations in nanoscale are difficult to perform and also molecular dynamics simulationsare costly and time consuming particularly for systems with large number of atoms, the theoretical analysis of nanostructures is very important.The continuum modelingof nanoscale structures has attracted tremendous attention in recent years. The theory of classical continuum mechanics is independent of small scale effect and is unable to predict the small scale effect on the mechanical behavior of nanomaterials.Both laboratory results and the results obtained by molecular dynamics simulations show that the small scale effect would be very important and also meaningful in the mechanical properties of materials when thedimensions of these structures arein the order of several nanometers

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[1,2].The classical elasticitytheory is incapable of predictingthe size-dependent behavior of nanostructures and by which the small scale effects cannot be considered in nanoscale. There are different theories of elasticity concerning the scale effect in their analysis, including couple stress theory, strain gradient theory and modified couple stresstheory[3], butthe theory that is widely used to analyzethe structures with smaller size is the Eringen nonlocal elasticity theory[4,5]. The nature of this theory is based on the assumption that the stress at a given point is not only a function of strain at that point, but is a function of strain at all points of the body. Recently, the nonlocal elasticity theory has been used by several researchers to study the mechanical behavior of nanorods[6], nanobeams[7,8], piezoelectric nanowires [9,10], single-layered graphene sheets [11,12], piezoelectric nanoplates[13] and magneto-electro-elastic nanoplates[14].

After the publication of the article by Pan and his colleagues[15] about the ZnO piezoelectric nanostructures in a prestigious journal(Science), various piezoelectric nanostructures such as piezoelectric nanowires have attracted considerable attention from many researchers. Ke and Wang [16] investigated transverse vibrations of piezoelectric nanobeams in a thermal environment based on the nonlocal elasticity theory. They derived governing differential equations based on the Hamilton's principle and by using the differential quadrature method, they calculated the natural frequencies of the system. In other research, Samaei and his colleagues [17] were investigated the buckling behavior of piezoelectric nanowires with considering surface effects. They used the local (classical) Timoshenko beam theory with surface effects for the modeling of piezoelectric nanowires.In their presented model, nonlocal effects were not considered. Gheshlaghi and HashemiNejad[18] studied the effects of surface energy and size on the free transverse vibrations of piezoelectric nanowires. Using the nonlocal elasticity theory and with considering surface effects, they developed a new analytical model for the piezoelectric nanowires. However, they did not take into account the influence of surface piezoelectricity. In another interesting work, Yan and Jiang [19] investigatedtheelectromechanical response of a curved piezoelectric nanobeam with the consideration of surface effects. The nonlocal effects were not taken into consideration in their work. Ke et al. [20] also studied the nonlinear vibration of the piezoelectric nanobeams based on the nonlocal beam model. More recently, Ansari et al. [21] developed a nonlocal Timoshenko beam model for the nonlinear forced vibration of magneto-electro-thermo-elastic nanobeams. These two interesting works are limited to the vibration of piezoelectric and magneto-electro-thermo-elastic nanobeams with the consideration of only nonlocal effects. In nanoscale structures, the surface-to-volume ratio is relatively high and therefore the surface-to-bulk energy is quite meaningful [19]. Hence, surface effects on the vibration of nanostructures such as graphene sheets [22], carbon nanotubes [23], piezoelectric nanobeams[24] and microtubules [25] should be considered in order to correctly determine the free and forced vibration characteristics. This motivates us to investigate the free and forced vibration of piezoelectric nanowires (PNWs) considering both surface and nonlocal effects. It should be noted that in Ref. [9], only the free vibration of PNWs was studied. Furthermore, the influence of surface piezoelectricity was not taken into account in this work.

In this study the free and forced vibrations of piezoelectric nanowires are investigated using the Eringen nonlocal elasticity theory with the consideration of surface effects. The governing differential equations of motion of PNWsarederivedbased on the Timoshenko beam theory. Both surface and small scale effects are taken into account. The natural frequencies of piezoelectric nanowires as well as their forced vibration characteristics are obtained by using the differential quadrature method as an efficient numerical tool. The present results are validated by comparing the results with available solutions in the open literature. The numerical results are presented for both simply-simply (S-S) and clamped-clamped (C-C) piezoelectric nanowires. Finally, the effect of various parameters such as residual surface stress, surface elastic modulus, surface piezoelectric constant, small scalecoefficient,temperature change, external load and applied electric voltage on the vibration of PNWsare investigated.

2. Nonlocal elasticity theory for the piezoelectric materials

In the classical (local) elasticity theory, the stress tensor at a given pointofa body depends on the strain tensor at that point, while in the nonlocal elasticity, the stress tensordepends on the strain tensor at all points of the body.This assumption is in accordance with the experimental observations on phonon dispersion[4] and the results of molecular dynamics simulations [22]. The basic nonlocal constitutive relationsofHookean piezoelectricsolidwithout any body forcescan be mathematically written as[4,16,20]

$$\sigma_{ij} = \iiint_V J_{nl}(|\mathbf{x}' - \mathbf{x}|, \chi)\big[C_{ijkl}\varepsilon_{kl}(\mathbf{x}') - e_{kij}E_k(\mathbf{x}') - \alpha_{ij}^{th}\Delta T\big]d\,\mathbf{x}' \quad (1)$$

$$D_i = \iiint_V J_{nl}(|\mathbf{x}' - \mathbf{x}|, \chi)\big[e_{ikl}\varepsilon_{kl}(\mathbf{x}') + \kappa_{ik}E_k(\mathbf{x}') + \beta_{ij}^{th}\Delta T\big]d\,\mathbf{x}' \quad (2)$$

$$\sigma_{ij,j} = \rho\ddot{U}_i, \quad D_{i,i} = 0 \quad (3)$$

$$\varepsilon_{ij} = \frac{1}{2}\big(U_{i,j} + U_{j,i}\big), \quad E_i = -\Psi_{,i} \quad (4)$$

Here, the terms σ_{ij},ε_{ij}, $D_i$and $E_i$represent the nonlocal stress, strain, nonlocal electric displacement and electric field, respectively; u_i,ΔTand Ψ stand for the displacement components, the change of environment temperatureand the electric potential, respectively;C_{ijkl}, $e_{kij}$and $\kappa_{ik}$are the fourth order elasticity tensor, piezoelectric constants anddielectric constants, respectively;α_{ij}^{th}andβ_{ij}^{th}arethermal moduli and pyroelectric constants, respectively;J_{nl} indicates the nonlocal modulus that takes into account thesmall scale effects.$|x' - x|$is the distance between pointsx and x'.$\chi = e_0\ell_i/l$is small scale parameter (nonlocal parameter) where e_0 is a calibration parameterwhich can be determined experimentally or by using simulations based on molecular dynamics.$\ell_i$and$l$ are theinternal (e.g. lattice parameter or granular size)and external (e.g. length or widthof nanostructure) characteristic lengths, respectively.The use of integral equations (1) and (2) is very difficult to model nanostructures. Therefore,Eringen[4,5] suggested the following differential forms instead of equations (1) and (2):

$$\sigma_{ij} - (e_0\ell_i)^2\nabla^2\sigma_{ij} = C_{ijkl}\varepsilon_{kl} - e_{kij}E_k - \alpha_{ij}^{th}\Delta T \quad (5)$$

$$D_i - (e_0\ell_i)^2\nabla^2 D_i = e_{ikl}\varepsilon_{kl} + \kappa_{ik}E_k + \beta_{ij}^{th}\Delta T \quad (6)$$

where∇^2denotes the Laplacian operator.It should be noted that the traditional local constitutive relations ($e_0\ell_i = 0\,nm$) cannot be employed for the nanostructures such as graphenesheets, carbon nanotubes and piezoelectric nanowires because they do not contain a small scale parameter.

3. Forced vibration of piezoelectric nanowires with surface and nonlocal effects

Based on the nonlocal elasticity theory which has been described above, in this section a nonlocal beam model including surface effects is presented for the forced vibrations of piezoelectric nanowires under the thermo-electrical loading.Fig. (1) shows a piezoelectric nanowire under an applied electric voltage. The length, width and thickness of the nanowire are denoted by*L*, *b* and *h*, respectively.

According to the Timoshenko beam theory, the components of the strain tensor of an arbitrary point(*x*, *z*) of the beam can be written as follows:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + z\frac{\partial \phi}{\partial x}, \quad \gamma_{xz} = \frac{\partial w}{\partial x} + \phi \quad (7)$$

where$u(x,t)$ and $w(x,t)$are displacement components of the middle surfacealong the *x*- and *z*-directions, respectively. Also,$\phi(x,t)$denotes the rotation of nanowire cross-section. For piezoelectric material, the distribution of the electric potential should be known as well as the displacement field. Yan and Jiang [19] assumed that the distribution of the electric potential is linear across the thickness of the nanobeams. However, this electric potential function does not satisfy the Maxwell equations. In order to satisfy the Maxwell equations, Ke and Wang [16] considered a cosine and linear function for the electric potential variations. Following them, one can easily obtain the following relationships for the electric field components of the PNWs:

$$E_x = cos\left(\frac{\pi z}{h}\right)\frac{\partial \psi}{\partial x}, \qquad E_z = -\frac{\pi}{h}sin\left(\frac{\pi z}{h}\right)\psi(x,t) - \frac{2\psi_0}{h}e^{i\omega t} \quad (8)$$

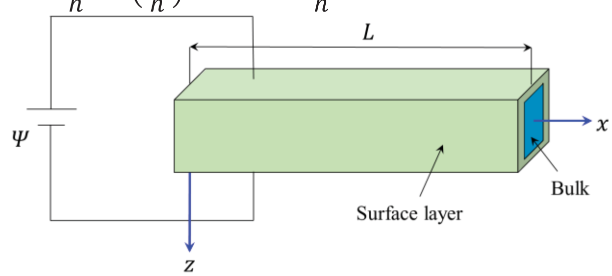

Fig.1. A nonlocal continuum model of a piezoelectric nanowire under an external voltage.

Here$\psi(x,t)$denotes the variation ofelectric potential in the middle surface; $\psi_0$and$\omega$are the appliedelectric voltage and the natural frequency of the nanowire, respectively.

Using the differential forms of the basic relations,i.e. Eqs.(5)and(6),the constitutive equationsofPNWscan be written as

$$\sigma_{xx} - (e_0\ell_i)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = C_{11}\varepsilon_{xx} - e_{31}E_z - \alpha_{11}^{th}\Delta T \quad (9)$$

$$\sigma_{xz} - (e_0\ell_i)^2 \frac{\partial^2 \sigma_{xz}}{\partial x^2} = C_{44}\gamma_{xz} - e_{15}E_x \quad (10)$$

$$D_x - (e_0\ell_i)^2 \frac{\partial^2 D_x}{\partial x^2} = e_{15}\gamma_{xz} + \kappa_{11}E_x \quad (11)$$

$$-(e_0\ell_i)^2 \frac{\partial^2 D_z}{\partial x^2} = e_{31}\varepsilon_{xx} + \kappa_{33}E_z + \beta_{11}^{th}\Delta T \quad (12)$$

In nanostructures, the ratio of surface to volume is high and thus the ratio of surface energy to bulk energy is considerable [19]. Therefore, the influences of surface energy on the bending, buckling and vibration of micro/nano structures such as single-layered graphene sheets [22],carbon nanotubes [23], piezoelectric nanowires [24]and microtubules[25]should not be neglected. Based on the surface elasticity theory [19,24], the constitutive equations of the surface layer can be written as

$$\sigma_{xx}^s = \sigma_{xx}^0 + c_{11}^s\varepsilon_{xx} - e_{31}^s E_z\,, \quad D_{xx}^s = D_{xx}^0 \quad (13)$$

where σ_{xx}^s , σ_{xx}^0, $D_{xx}^s$and$D_{xx}^0$are the surface stress, residual surface stress, surface electric displacement and residual surface electric displacement, respectively. The coefficients c_{11}^s and $e_{31}^s$are the Young's modulus and piezoelectric constant of surface layer, respectively. It should be noted that the elastic and piezoelectric properties of surface layer are different from those of the bulk material. There are two distributed tractions due to the surface stresses that act on the piezoelectric nanowires. According to the generalized Young–Laplace equations, these tractions can be expressed as[24]

$$T_x = \frac{\partial \sigma_{xx}^s}{\partial x}\,, \quad T_z = \frac{\sigma_{xx}^s}{R_c} \quad (14a,b)$$

whereR_c represents the radius of curvature of the surface.The z component of the surface traction (T_z) is zero on the left and right surface layers, namely, T_z is only non-zero at the lower and upper surface layers of the PNW. It should be noted that the electric displacement jump along the surface is assumed to be zero. Substituting Eqs. (7)and(8) into Eq. (13), we have

$$\sigma_{xx}^s = \sigma_{xx}^0 + c_{11}^s\frac{\partial u}{\partial x} + \frac{2e_{31}^s\psi_0}{h}e^{i\omega t} + c_{11}^s\frac{\partial \phi}{\partial x}z + \frac{\pi e_{31}^s\psi}{h}sin\left(\frac{\pi z}{h}\right) \quad (15)$$

Using the Hamilton's principle and considering the surface tractions which acts on the nanowire, the following differential equations of motion are obtained

$$\delta w:\quad \frac{\partial Q_x}{\partial x} + q + (N_{th} + N_{el} + N_s)\frac{\partial^2 w}{\partial x^2} = m_0\frac{\partial^2 w}{\partial t^2} \quad (16)$$

$$\delta\phi:\quad \frac{\partial M_x}{\partial x} - Q_x + c_{11}^s I_1^*\frac{\partial^2 \phi}{\partial x^2} + \frac{\pi e_{31}^s I_2^*}{h}\frac{\partial \psi}{\partial x} = m_2\frac{\partial^2 \phi}{\partial t^2} \quad (17)$$

$$\delta\Psi:\quad \int_{-h/2}^{h/2}\left[cos\left(\frac{\pi z}{h}\right)\frac{\partial D_x}{\partial x} + \frac{\pi}{h}sin\left(\frac{\pi z}{h}\right)D_z\right]dz = 0 \quad (18)$$

in which$m_0 = \rho h$and $m_2 = \rho h^3/12$; Q_x and M_x are the shear force and bending moment, respectively; q is the transverse distributed load;N_{th},$N_{el}$and$N_s$arerespectively the axial loads caused by temperature change, external electric voltage and surface traction and are defined by

$$N_{th} = -\alpha_{11}^{th}h\Delta T,\ \ N_{el} = 2e_{31}\psi_0,\ \ N_s = 2\left(\sigma_{xx}^0 + \frac{2e_{31}^s\psi_0}{h}\right) \quad (19)$$

The surface moments of inertia are defined by

$$I_1^* = \frac{h^2}{2} + \frac{h^4}{6A},\ \ I_2^* = h + \frac{4h^3}{\pi^2 A} \quad (20)$$

where$A$is the cross-sectional area of the PNW. The stress resultants are defined by using the following relations:

$$Q_x = \int_{-h/2}^{h/2}\sigma_{xz}dz\,, \qquad M_x = \int_{-h/2}^{h/2}\sigma_{xx}zdz \quad (21)$$

UsingEqs. (9)and(10) and in view of relation(21), the stress resultants can be expressed as

$$M_x - (e_0\ell_i)^2\frac{\partial^2 M_x}{\partial x^2} = D_{11}\frac{\partial \phi}{\partial x} + F_{31}\psi, \quad 22\,(a,b)$$

$$Q_x - (e_0\ell_i)^2\frac{\partial^2 Q_x}{\partial x^2} = k_s A_{44}\left(\phi + \frac{\partial w}{\partial x}\right) - k_s F_{15}\frac{\partial \psi}{\partial x}$$

Similarly, using Eqs. (11)and(12),the following equations can be obtained for the electric displacements

$\int_{-h/2}^{h/2}\cos\left(\frac{\pi z}{h}\right)\left[1-(e_0\ell_i)^2\frac{\partial^2}{\partial x^2}\right]\frac{\partial D_x}{\partial x}dz = F_{15}\left(\frac{\partial\phi}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)+H_{11}\frac{\partial^2\psi}{\partial x^2},$ $\int_{-h/2}^{h/2}\frac{\pi}{h}\sin\left(\frac{\pi z}{h}\right)\left[1-(e_0\ell_i)^2\frac{\partial^2}{\partial x^2}\right]D_z dz = F_{31}\frac{\partial\phi}{\partial x}-H_{33}\psi$	23 (a,b)

where

$$D_{11}=\int_{-h/2}^{h/2}c_{11}z^2dz=\frac{c_{11}h^3}{12},$$
$$A_{44}=\int_{-h/2}^{h/2}c_{44}dz=c_{44}h,$$
$$F_{31}=\int_{-\pi/2}^{\pi/2}\frac{e_{31}h}{\pi}z^*\sin z^*dz^*=\frac{2e_{31}h}{\pi},$$
$$F_{15}=\int_{-\pi/2}^{\pi/2}\frac{e_{15}h}{\pi}\cos z^*dz^*=\frac{2e_{15}h}{\pi},$$
$$H_{11}=\int_{-\pi/2}^{\pi/2}\frac{\kappa_{11}h}{\pi}\cos^2 z^*dz^*=\frac{\kappa_{11}h}{2},$$
$$H_{33}=\int_{-\pi/2}^{\pi/2}\frac{\kappa_{33}\pi}{h}\sin^2 z^*dz^*=\frac{\kappa_{33}\pi^2}{2h} \quad (24)$$

Here $z^*=\pi z/h$ and k_s is the shear correction factor. By applying Eqs. (16), (17) and (22), one can obtain the following relations for the bending moment and shear force of the PNW in terms of transverse displacement and cross-sectional rotation

$$M_x=D_{11}\frac{\partial\phi}{\partial x}+F_{31}\psi+(e_0\ell_i)^2\left[m_0\frac{\partial^2 w}{\partial t^2}+m_2\frac{\partial^3\phi}{\partial x\partial t^2}-q-(N_{th}+N_{el}+N_s)\frac{\partial^2 w}{\partial x^2}-c_{11}^sI_1^*\frac{\partial^3\phi}{\partial x^3}-\frac{\pi e_{31}^sI_2^*}{h}\frac{\partial^2\psi}{\partial x^2}\right],$$

$$Q_x=(e_0\ell_i)^2\left[m_0\frac{\partial^3 w}{\partial x\partial t^2}-\frac{\partial q}{\partial x}-(N_{th}+N_{el}+N_s)\frac{\partial^3 w}{\partial x^3}\right]+k_sA_{44}\left(\phi+\frac{\partial w}{\partial x}\right)-k_sF_{15}\frac{\partial\psi}{\partial x}$$

$(25a,b)$

Substituting Eqs.(23) and (25) into Eqs. (16), (17) and (18), one can obtain the governing differential equations of motion for the forced vibration of PNWs in thermal environment considering both surface and nonlocal effects

$$(e_0\ell_i)^2\left[m_0\frac{\partial^4 w}{\partial x^2\partial t^2}-\frac{\partial^2 q}{\partial x^2}-(N_{th}+N_{el}+N_s)\frac{\partial^4 w}{\partial x^4}\right]+k_sA_{44}\left(\frac{\partial\phi}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)-k_sF_{15}\frac{\partial^2\psi}{\partial x^2}+q+(N_{th}+N_{el}+N_s)\frac{\partial^2 w}{\partial x^2}=m_0\frac{\partial^2 w}{\partial t^2} \quad (26)$$

$$(e_0\ell_i)^2\left(m_2\frac{\partial^4\phi}{\partial x^2\partial t^2}-c_{11}^sI_1^*\frac{\partial^4\phi}{\partial x^4}-\frac{\pi e_{31}^sI_2^*}{h}\frac{\partial^3\psi}{\partial x^3}\right)+(D_{11}+c_{11}^sI_1^*)\frac{\partial^2\phi}{\partial x^2}+\left(F_{31}+\frac{\pi e_{31}^sI_2^*}{h}\right)\frac{\partial\psi}{\partial x}-k_sA_{44}\left(\phi+\frac{\partial w}{\partial x}\right)+k_sF_{15}\frac{\partial\psi}{\partial x}=m_2\frac{\partial^2\phi}{\partial t^2} \quad (27)$$

$$F_{15}\left(\frac{\partial\phi}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)+F_{31}\frac{\partial\phi}{\partial x}+H_{11}\frac{\partial^2\psi}{\partial x^2}-H_{33}\psi=0 \quad (28)$$

It should be noted that when the surface parameters are set to zero (i.e., $N_s=c_{11}^s=e_{31}^s=0$), the above differential equations of motion and the relations of bending moment and shear force reduce to those of the nonlocal theory of elasticity. In this work, the boundary conditions of the piezoelectric nanowires are assumed to be simply supported or clamped. In addition, the value of electric potential is equal to zero at both two ends of the nanowire. Thus, the non-classical boundary conditions can be expressed as

$$w|_{x=0,L}=\phi|_{x=0,L}=\psi|_{x=0,L}=0 \quad (29)$$

for a clamped-clamped piezoelectric nanowire and

$$\left\{D_{11}\frac{\partial\phi}{\partial x}+F_{31}\psi+(e_0\ell_i)^2\left[m_0\frac{\partial^2 w}{\partial t^2}+m_2\frac{\partial^3\phi}{\partial x\partial t^2}-q-(N_{th}+N_{el}\right.\right.$$

$$\left.\left.-c_{11}^{s}I_{1}^{*}\frac{\partial^{3}\phi}{\partial x^{3}}-\frac{\pi e_{31}^{s}I_{2}^{*}}{h}\frac{\partial^{2}\psi}{\partial x^{2}}\right]\right\}_{x=0,L}=0, \tag{30}$$

$$w\big|_{x=0,L}=\psi\big|_{x=0,L}=0$$

for a simply-simply piezoelectric nanowire.

4. Solution of the governing equations using the differential quadrature method

Assuming that the external transverse load as well as the vibration are harmonic, therefore the transverse displacements, cross-sectional rotations, electric potential and distributed load can be written in the following form

$$w(x,t)=\hat{w}(x)e^{i\omega t},\quad \phi(x,t)=\hat{\phi}(x)e^{i\omega t},\quad \psi(x,t)=\hat{\psi}(x)e^{i\omega t},\quad q(x,t)=\hat{q}(x)e^{i\omega t} \tag{31}$$

Substituting the above equations into Eqs. (26), (27) and (28), one can derive the non-dimensional equations of motion of PNWs as follows

$$-\chi^{2}\left(\hat{N}_{th}+\hat{N}_{el}+\hat{N}_{s}\right)\frac{\partial^{4}\bar{w}}{\partial\xi^{4}}+\left(\hat{N}_{th}+\hat{N}_{el}+\hat{N}_{s}+k_{s}A_{44}^{*}\right)\frac{\partial^{2}\bar{w}}{\partial\xi^{2}}+\bar{q}-\chi^{2}\frac{\partial^{2}\bar{q}}{\partial\xi^{2}}+k_{s}A_{44}^{*}r\frac{\partial\hat{\phi}}{\partial\xi}-k_{s}F_{15}^{*}\frac{\partial^{2}\bar{\psi}}{\partial\xi^{2}}+\Omega^{2}\left(\bar{w}-\chi^{2}\frac{\partial^{2}\bar{w}}{\partial\xi^{2}}\right)=0 \tag{32}$$

$$-k_{s}A_{44}^{*}r\frac{\partial\bar{w}}{\partial\xi}-\chi^{2}\bar{c}_{11}^{s}\frac{\partial^{4}\hat{\phi}}{\partial\xi^{4}}+\left(D_{11}^{*}+\bar{c}_{11}^{s}\right)\frac{\partial^{2}\hat{\phi}}{\partial\xi^{2}}-k_{s}A_{44}^{*}r^{2}\hat{\phi}-\chi^{2}\bar{e}_{31}^{s}r\frac{\partial^{3}\bar{\psi}}{\partial\xi^{3}}+r\left(F_{31}^{*}+k_{s}F_{15}^{*}+\bar{e}_{31}^{s}\right)\frac{\partial\bar{\psi}}{\partial\xi}+\bar{I}\,\Omega^{2}\left(\hat{\phi}-\chi^{2}\frac{\partial^{2}\hat{\phi}}{\partial\xi^{2}}\right)=0 \tag{33}$$

$$F_{15}^{*}\frac{\partial^{2}\bar{w}}{\partial\xi^{2}}+r\left(F_{31}^{*}+F_{15}^{*}\right)\frac{\partial\hat{\phi}}{\partial\xi}+H_{11}^{*}\frac{\partial^{2}\bar{\psi}}{\partial\xi^{2}}-r^{2}H_{33}^{*}\bar{\psi}=0 \tag{34}$$

In the above equations, the dimensionless parameters are defined as

$$\xi=\frac{x}{L},\quad \bar{w}=\frac{\hat{w}}{h},\quad r=\frac{L}{h},\quad \chi=\frac{e_{0}\ell_{i}}{L},\quad \bar{\psi}=\frac{\hat{\psi}}{\psi^{*}},\quad \psi^{*}=\sqrt{\frac{c_{11}h}{H_{33}}},\quad A_{44}^{*}=\frac{A_{44}}{A_{11}},$$
$$D_{11}^{*}=\frac{D_{11}}{A_{11}h^{2}},\quad F_{31}^{*}=\frac{F_{31}\psi^{*}}{A_{11}h},\quad F_{15}^{*}=\frac{F_{15}\psi^{*}}{A_{11}h},\quad H_{11}^{*}=\frac{H_{11}\left(\psi^{*}\right)^{2}}{A_{11}h^{2}},\quad H_{33}^{*}=\frac{H_{33}\left(\psi^{*}\right)^{2}}{A_{11}},$$
$$\hat{N}_{th}=-\frac{\alpha_{11}^{th}h\Delta T}{A_{11}},\quad \hat{N}_{el}=\frac{2e_{31}\psi_{0}}{A_{11}},\quad \Omega^{2}=\frac{m_{0}L^{2}\omega^{2}}{A_{11}},\quad \bar{q}=\frac{\hat{q}L^{2}}{A_{11}h},\quad \bar{I}=\frac{m_{2}}{m_{0}h^{2}},$$
$$\hat{N}_{s}=\frac{N_{s}}{A_{11}}=\frac{2}{A_{11}}\left(\sigma_{xx}^{0}+\frac{2e_{31}^{s}\psi_{0}}{h}\right),\quad \bar{c}_{11}^{s}=\frac{c_{11}^{s}I_{1}^{*}}{A_{11}h^{2}},\quad \bar{e}_{31}^{s}=\frac{\pi e_{31}^{s}I_{2}^{*}\psi^{*}}{A_{11}h^{2}} \tag{35}$$

To determine the free and forced vibration characteristics of PNWs, thedifferential quadrature method [26,27]is employed. DQM is an effective numerical toolto obtain the solution ofordinary and partial differential equations in engineering applications[27]. According to this numerical method, the derivatives in the differential equations arereplaced by a set of linear weighted sum of the functional values at all of the mesh points (grid points) in the domain[27]. Using DQM, the governing equations (32), (33) and (34) can be written as

$$-\chi^{2}\left(\hat{N}_{th}+\hat{N}_{el}+\hat{N}_{s}\right)\sum_{j=1}^{n}C_{ij}^{(4)}\bar{w}_{j}+\left(\hat{N}_{th}+\hat{N}_{el}+\hat{N}_{s}+k_{s}A_{44}^{*}\right)\sum_{j=1}^{n}C_{ij}^{(2)}\bar{w}_{j}+k_{s}A_{44}^{*}r\sum_{j=1}^{n}C_{ij}^{(1)}\hat{\phi}_{j}-k_{s}F_{15}^{*}\sum_{j=1}^{n}C_{ij}^{(2)}\bar{\psi}_{j}-\Omega^{2}\left(\chi^{2}\sum_{j=1}^{n}C_{ij}^{(2)}\bar{w}_{j}-\bar{w}_{j}\right)=\left(\chi^{2}\frac{\partial^{2}\bar{q}}{\partial\xi^{2}}-\bar{q}\right)_{\xi=\xi_{i}} \tag{36}$$

$$-k_{s}A_{44}^{*}r\sum_{j=1}^{n}C_{ij}^{(1)}\bar{w}_{j}-\chi^{2}\bar{c}_{11}^{s}\sum_{j=1}^{n}C_{ij}^{(4)}\hat{\phi}_{j}+\left(D_{11}^{*}+\bar{c}_{11}^{s}\right)\sum_{j=1}^{n}C_{ij}^{(2)}\hat{\phi}_{j}-k_{s}A_{44}^{*}r^{2}\hat{\phi}_{j}$$

$$-\chi^2\bar{e}_{31}^{s} r\sum_{j=1}^{n}C_{ij}^{(3)}\bar{\psi}_j + r\left(F_{31}^{*}+k_s F_{15}^{*}+\bar{e}_{31}^{s}\right)\sum_{j=1}^{n}C_{ij}^{(1)}\bar{\psi}_j - \bar{I}\Omega^2\left(\chi^2\sum_{j=1}^{n}C_{ij}^{(2)}\hat{\phi}_j-\hat{\phi}_j\right)=0 \tag{37}$$

$$F_{15}^{*}\sum_{j=1}^{n}C_{ij}^{(2)}\bar{w}_j + r\left(F_{31}^{*}+F_{15}^{*}\right)\sum_{j=1}^{n}C_{ij}^{(1)}\hat{\phi}_j + H_{11}^{*}\sum_{j=1}^{n}C_{ij}^{(2)}\bar{\psi}_j - r^2H_{33}^{*}\bar{\psi}_j=0 \tag{38}$$

where n represents the number of grid points in the ξ direction. $C_{ij}^{(r)}$ is the weighting coefficient associated with the rth-order differentiation. For more detail about the weighting coefficients and DQM, refer to the paper of Bert et al.[26].The boundary conditions (29) and (30) are also discretized using this procedure. Then, the discrete forms of boundary conditions are directly substituted into the discretized governing equations (36)-(38).Now, to determine the free and forcedvibration characteristics of the PNWs, a MATLAB program is used to obtain the matrix form of the above system of algebraic equations.

$$([K^{nl}]-\Omega^2[M^{nl}])\{\tilde{w}\}=\{\bar{q}\} \tag{39}$$

where $[K^{nl}]$ and $[M^{nl}]$ are two square matrices with the same dimension $3(n-2)\times 3(n-2)$ which are calculated using the computer program (written in MATLAB). The vibration amplitude vector $\{\tilde{w}\}$ can beexpressed as follows:

$$\{\tilde{w}\}=\begin{Bmatrix}\bar{w}_2\\ \bar{w}_3\\ \vdots\\ \bar{w}_{n-1}\\ \hat{\phi}_2\\ \hat{\phi}_3\\ \vdots\\ \hat{\phi}_{n-1}\\ \bar{\psi}_2\\ \bar{\psi}_3\\ \vdots\\ \bar{\psi}_{n-1}\end{Bmatrix} \tag{40}$$

As can be seen, the vector $\{\tilde{w}\}$ consists of $3(n-2)$ components that one-third of them are nanowire displacements and the other one-third are its rotations and the final one-third are potential energies. For the forced vibration of piezoelectric nanowires, we have

$$\{\tilde{w}\}=([K^{nl}]-\Omega^2[M^{nl}])^{-1}\{\bar{q}\} \tag{41}$$

On the other hand, the non-dimensional natural frequencies of PNWs can be obtained from the following eigenvalue problem:

$$([K^{nl}]-\Omega^2[M^{nl}])\{\tilde{w}\}=\{0\} \tag{42}$$

5. Results analysis

5.1. *Validation and convergence of the present results*

To validate the present DQsolution, the free vibration of piezoelectric nanobeams without taking into account the surface effects is considered here. The fundamental natural frequencies of piezoelectric nanowiresdetermined from Eq. (42)with *n=20*are compared with those obtained by Ke and Wang [16]. In order to make a reasonable comparison, the material and geometric properties of piezoelectric nanobeam are the same as those reported in their work.

Elastic and electrical properties:

$c_{11}=132$ GPa, $c_{44}=26$ GPa,
$\rho=7500\ Kg/m^3$, $e_{31}=-4.1\ C/m^2$,
$e_{15}=10.5\ C/m^2$, $\kappa_{11}=5.841\times10^{-9}\ C/Vm$,
$\kappa_{33}=7.124\times10^{-9}\ C/Vm$,

Thermal properties:

$\alpha_{11}^{th}=4.738\times10^{5}\ N/m^2K$, $\beta_{11}^{th}=0.25\times10^{-4}\ C/m^2K$

Geometric properties:

$L=80\ nm,\ h=10\ nm,\ k_s=5/6$

The above elastic, electrical and thermal properties are used in the following subsection except noted otherwise. The present Timoshenko beam model of PNWs is reduced to that of Ke and Wang [16], when the external transverse force and surface properties are set to zero. Table 1shows the natural frequencies (GHz) of piezoelectric nanowires for three different values of temperature change (i.e. $\Delta T=0,\ 20,\ 40$ K). Since all these values are not very high, the material properties of PNWs are assumed to be constants. In Refs. [16]and[20], the material properties of piezoelectric nanobeams are assumed to be constant within the range of 0-100 K. The numerical results are presented for various beam theories including the classical (local) beam theory (CBT), nonlocal beam theory (NBT) with and without surface effects. The surface properties of piezoelectric nanowires are as follows [24]

$\sigma_{xx}^{0}=1\ N/m,\ c_{11}^{s}=7.56\ N/m,\ e_{31}^{s}=-3\times10^{-8}\ C/m.$

No electric voltage is applied to the nanowire. Also, the results are presented for both clamped-clamped and simply-simply boundary conditions. From Table 1, it can be found that the present DQ results are in very good agreementwith those obtained byKe and Wang [16].In addition, it can be concluded that the natural frequency slightly reduces with increasing temperature change from 0 to 40 K. Another interesting result is that the natural frequency increases by considering the surface effects while the nonlocal parameter has a decreasing effect on the fundamental frequencies of PNWs.It should be noted that with increasing the number of grid points, no significant change is found in the natural frequencies. This means that the present numerical solution is converged (see Table 2).

5.2. *Forced vibration analysis of PNWs considering surface and nonlocal effects*

To investigate the influence of surface properties on the forced vibration of PNWs, non-dimensional amplitude versus the frequency parameter ($\Omega = \sqrt{m_0 / A_{11}} L\omega$) are plotted in Figs. (2), (3) and (4) for various values of residual surface stress, surface elastic and piezoelectric constants, respectively. The bulk material properties

Table 1. Comparison of the present results for the free vibration of piezoelectric nanowire ($L = 80\ nm,\ h = 10\ nm,\ \psi_0 = 0$).

Boundary conditions	Beam theory	$\Delta T = 0$ K		$\Delta T = 20$ K		$\Delta T = 40$ K	
		Present	Ref. [16]	Present	Ref. [16]	Present	Ref. [16]
SS	Classical beam theory	2.8786	2.8786	2.8701	2.8701	2.8616	2.8616
	NBT without surface effects	2.4374	2.4374	2.4274	2.4274	2.4173	2.4173
	NBT with surface effects	2.8504	---	2.8418	---	2.8332	---
CC	Classical beam theory	5.7267	5.7266	5.7219	5.7219	5.7171	5.7172
	NBT without surface effects	4.7395	4.7395	4.7298	4.7298	4.7201	4.7201
	NBT with surface effects	5.9300	---	5.9266	---	5.9233	---

Table 2. Convergence study of the natural frequencies of piezoelectric nanowires ($\Delta T = \psi_0 = 0,\ \sigma_{xx}^0 = 1$ *N/m*, $c_{11}^s = 7.56$ *N/m*, $e_{31}^s = -3\times10^{-8}$ *C/m*).

Number of grid points	S-S			C-C		
	Small scale coefficient			Small scale coefficient		
	0.0	0.25	0.5	0.0	0.25	0.5
5	3.1871	2.6067	2.0074	6.0212	5.4508	5.3373
7	3.2362	2.7108	2.1367	6.0661	5.7958	4.9800
10	3.2352	2.7040	2.1433	6.0680	5.8240	5.0266
12	3.2352	2.7037	2.1424	6.0678	5.8218	5.0205
14	3.2352	2.7038	2.1424	6.0677	5.8214	5.0203
16	3.2352	2.7038	2.1424	6.0677	5.8213	5.0203
18	3.2352	2.7038	2.1424	6.0677	5.8213	5.0203

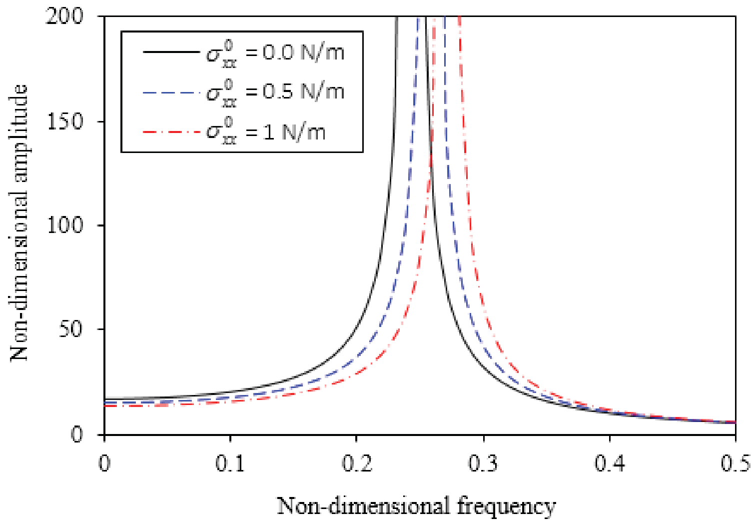

Fig. 2. Variation of dimensionless vibration amplitude against frequency for different values of residual surface stress ($\chi = 0.25,\ c_{11}^s = 7.56$ N/m, $e_{31}^s = -3\times10^{-8}$ C/m).

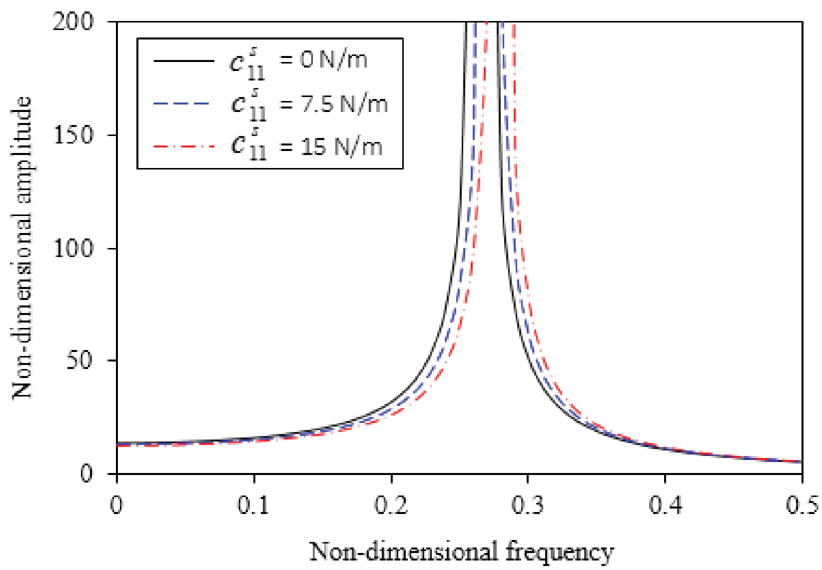

Fig. 3. Variation of dimensionless vibration amplitude against frequency for different values of surface elastic constant ($\chi = 0.25,\ \sigma_{xx}^0 = 1$ N/m, $e_{31}^s = -3\times10^{-8}$ C/m).

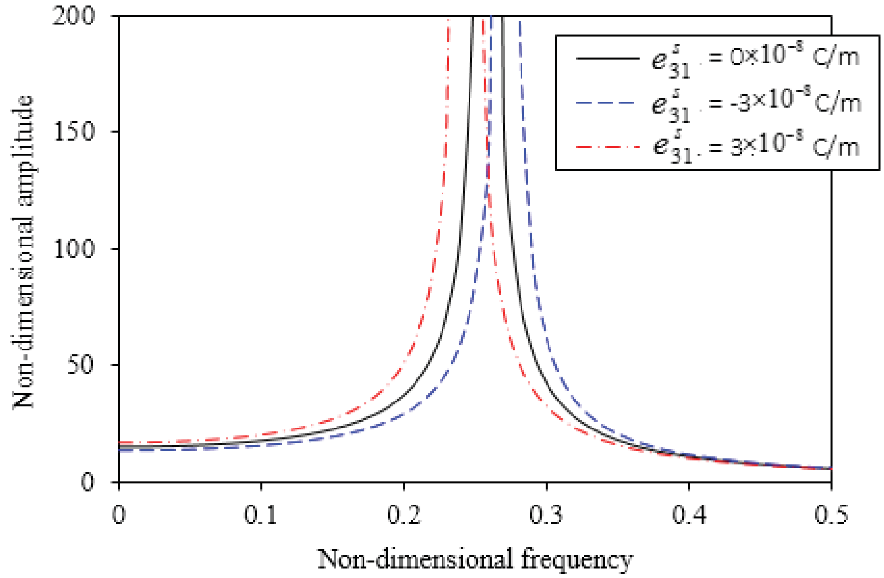

Fig. 4. Variation of dimensionless vibration amplitude against frequency for different values of surface piezoelectric constant ($\chi = 0.25,\ c_{11}^{s} = 7.56$ N/m, $\sigma_{xx}^{0} = 1$ N/m).

are taken as those given in the previous subsection. The elastic and piezoelectric properties of surface layers are different from their bulk counterparts. The length of the piezoelectric nanowire is taken as *L=100 nm*. The cross section is a square of size10×10 nm (*h=b=10 nm*).A value of 5/6 is chosen for the shear correction factor. The small scale coefficient (χ), external voltage and temperature change are set as 0.25, 0 V and 0 K, respectively. The PNW is subjected to a sinusoidal transvers load of maximum intensity $\bar{q}_m = 1$.In order to solve the governing equations numerically (with the use of the DQ method), 20grid points is considered along the nanowire.The typical range of residual surface stress is from 0.1 to 1 N/m [24]. Unless noted otherwise, the surface properties of piezoelectric nanowires are assumed to be $\sigma_{xx}^{0} = 1$ *N/m*, $c_{11}^{s} = 7.56$ *N/m*, $e_{31}^{s} = -3\times10^{-8}$ *C/m* in the present work.According toFig. (2), it is found that the resonant frequency increases with increasing the residual surface stress.Further, the surface elastic constant has an increasing effect on the resonant frequency of PNWs (see Fig. (3)), while the resonant frequency decreases with the increase of surface piezoelectric constant (refer toFig. (4)).

Now,the influence of nonlocal parameter (small scale effect) on the forced vibrations of the nanowires made of piezoelectric materials is studied. It is assumed that the nanowire is subjected toasinusoidal harmonic force ($\bar{q}_m = 1$).In order to clarify more the influence of small scale coefficient, the temperature change and external electric voltage are set to zero. Figs. (5a) and (5b) show the small scale effect on the amplitude-frequency curve of PNWs for the simply-simply and clamped-clamped boundary conditions, respectively. According to the reported values of the nonlocal parameter for piezoelectric nanobeams[16], three different values between 0 and0.5are considered. From this figure, it is observed that the nonlocal parameter has a decreasing effect on the resonant frequencies of PNWs.In other words,it can be said that increasing the small scale effect leads to the reduction in thestiffness of piezoelectric nanowires. A similar result is also reported by Ke and Wang [16] in the case of free vibration without taking into account the influences of surface residual stress, surface elasticity and surface piezoelectricity.

The effect of the external applied voltage on the vibrations of piezoelectric nanowires under harmonic external forces is shown in Fig. (6). The nonlocal parameter (small scale coefficient) is assumed to be 0.25. The length, thickness and width of the nanowire are *L=100 nm*, *h=10 nm* and *b=10 nm*, respectively. Thermal effects are not taken into account ($\Delta T = 0$ K). The numerical results are presented for various values of applied electric voltage. Both S-S and C-C boundary conditions are considered. From Fig.(6), it can be seen that the non-dimensional amplitude of oscillationsis suddenly increases at a certain non-dimensional frequency ($\Omega = \sqrt{m_0/A_{11}}L\omega$) between 0.2 and 0.3 for S-S boundary conditions and

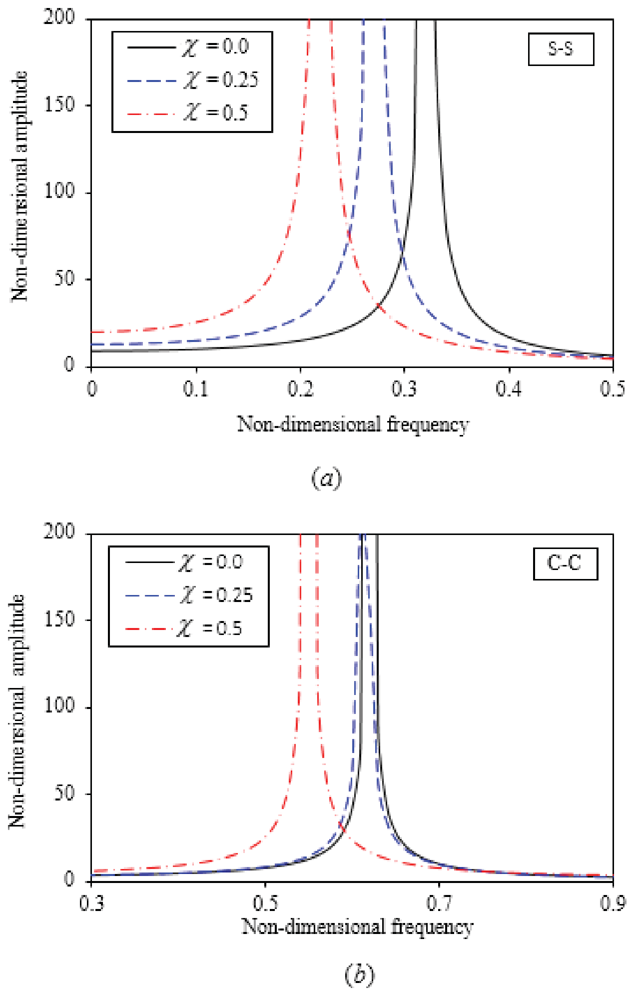

Fig. 5. Variation of dimensionless vibration amplitude against frequency for different values of small scale coefficient for (*a*) SS and (*b*) CC boundary conditions $(\Delta T = 0,\ \psi_0 = 0)$.

between 0.6 and 0.7 for C-C boundary conditions. As the external electric voltage increasesfrom 0 to 0.1 V, the curve shifts to the left (resonance occurs at lower excitation frequency).This means that the resonant frequency decreases with increasing the external electric voltage for piezoelectric nanowires with two ends simply supported. This observation can be used in the electric frequency tuning of piezoelectric nanobeams. However, the effect of applied electric voltage on the dimensionless resonant frequency of PNWs with clamped-clamped boundary conditions is negligible compared with PNWs with two ends clamped.

Fig. (7a) and (7b)depicts the effect of the magnitude of external harmonic load applied along the lateral direction on the vibrations of the piezoelectric nanowires for simply-simply and clamped-clamped boundary conditions, respectively. The small scale coefficient is taken as $\chi = 0.25$. The geometric properties of the PNW are *L=100 nm*, *h=10 nm* and *b=10 nm*. In order to clarify more the influence of external transverse load, the temperature change and external electric voltage are set to zero. The applied harmonic force is of the form $\bar{q}(x) = \sum \bar{q}_m \sin(m\pi x/L)$. Different values of the magnitude of external harmonic load are considered in Fig. (7). From this figure, it is found that the resonant frequency of PNWs is independent of $\bar{q}_m$.

6. Conclusions

The effectsof surface elasticity, surface piezoelectricity and residual surface stress as well as thesmall scale effecton the forced vibration of piezoelectric nanowiresare investigated based on the nonlocal elasticity theory. For a more comprehensive study of the piezoelectric nanowires, the thermal effects arealso considered. It is observed that the vibration behavior of the piezoelectric nanowires is described using three differential equations. To solve

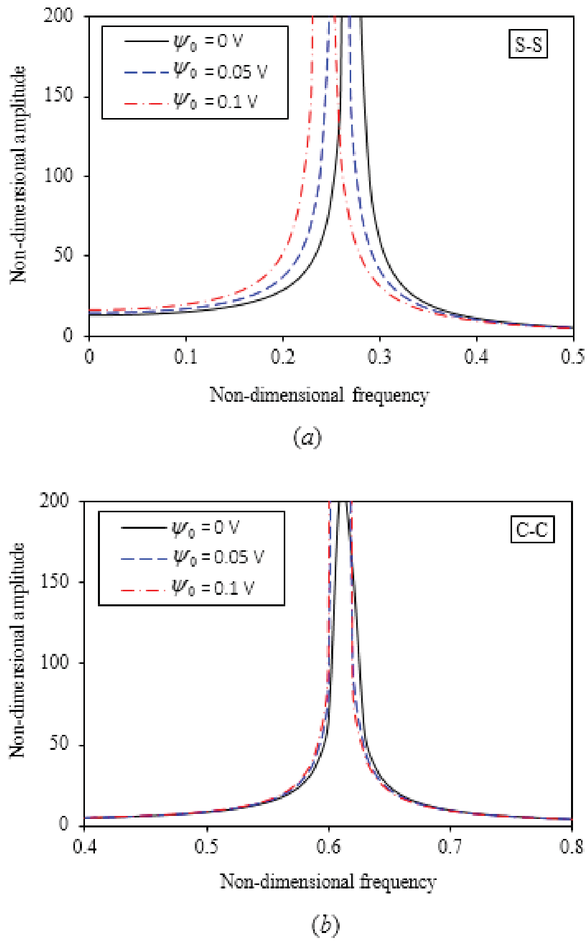

Fig. 6. Variation of dimensionless vibration amplitude against frequency for different values of external electric voltage for (*a*) SS and (*b*) CC boundary conditions $(\Delta T = 0,\ \chi = 0.25)$.

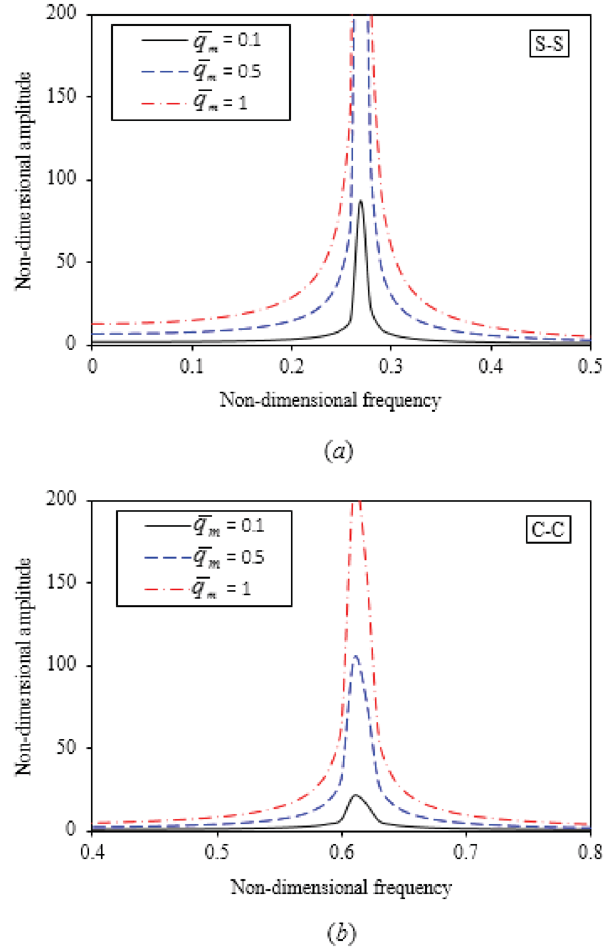

Fig. 7. Variation of dimensionless vibration amplitude against frequency for different values of external load for (*a*) SS and (*b*) CC boundary conditions $(\Delta T = 0,\ \psi_0 = 0,\ \chi = 0.25)$.

this set of differential equation, the DQ method is employed. The accuracy of the obtained results are checked by comparing them with the available results in the literature. It is observed that the residual surface stress has an increasing effect on the resonant frequency of PNWs.Furthermore,the resonant frequencyincreases with the increase of surface elastic constant. On the other hand, the resonant frequency decreases with the increase of surface piezoelectric constant. The small scale coefficient has a decreasing effect on the resonant frequencies of PNWs. Also, it is found that the amplitude-frequency curve of the nanowire shifts to the left with increasing the external electric voltage for simply-simply boundary conditions. However, the rate of this reduction is relatively small for the clamped-clamped boundary conditions

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