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Morphological analysis of glaciated valleys in the Zardkuh Mountains, Iran

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Abstract
The morphology of glacial valley can be described in terms of power law or quadratic equations fitted to valley cross-profiles. These two models are used to study the cross-profiles of 86 valley cross-profiles in the Zardkuh Mountain in order to understand the evolutional patterns of valleys. Assessment of using the power law function indicates that b values for both valley sides range from 1.0 to 2.5 with values showing an increase within this range as valley floor altitude increases. Analysis of b and FR of the valleys in the Zardkuh Mountains does not fit in with the Rocky Mountain model of Hirano and Aniya, but has a similar trend to the Patagonia-Antarctica model. The analyses also show a more efficient widening process in higher altitudes and a more efficient over deepening of valleys in lower altitudes. The results of the analysis show a relatively efficient glacial process in the elevated region of the Zardkuh. Application of quadratic function show similar conclusions and also indicates that most of the valleys are roughly symmetrical in the cross-profile. Apparently, greatest degree of "U-ness" showing glacial modifications was observed in altitudes above ELA during Last Glacial Maximum.

Keywords
glacial valley, power law function, quadratic function, Zardkuh.

1. Introduction
Several studies have used geomorphological indicators as evidence of climatic change during past glacial periods in Iran. Based on these studies, one of the main centers of former glaciations has been in the Zardkuh (Desio, 1934; Wright, 1962; Mc Quillan, 1969; Grunert et al., 1978; Pedrami, 1982; Preu, 1984; Ferrigno, 1988; Yamani, 2007; Moussavi et al., 2009). As in other glacial regions, glacial processes had an important role in shaping the landscapes, and they were responsible for carving out geomorphic features such as the valleys in the Zardkuh Mountains.

The quantitative description of valley cross profiles is of great significance to morphological research on glacial regions. Empirical studies have established the general concept that many glaciated valleys have approximately parabolic (U-shaped) cross profiles in contrast to the V-shaped valley cross profiles typically produced in areas dominated by fluvial erosion. Different univariate mathematical functions have been used to describe the cross profiles morphology of glacial valleys. Among these, power law functions have been widely used in the analysis of glacial valley morphology and its evolution (Svensson, 1959; Graf, 1970; Doornkamp and King, 1971; Jiao, 1981; Liu, 1989; Li et al., 2001; Brook and Brock, 2005). The quadratic or polynomial model is another empirical function widely used in the analysis of glacial valleys (Wheeler, 1984; Augustinus, 1992; James, 1996; Li et al., 2001; Brook and Brock, 2005). Although both models have advantages and limitations, the power law function is especially

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useful in comparing different cross-profiles and examining form development. The quadratic equation is valuable as it provides a steady description of glacial valley cross profiles (Li et al., 2001).

The main purpose of this study is to use the functions for studying morphology of the valleys in the Zardkuh Mountain (located in the Zagros Mountain Range, Iran), in order to determine possible glacial influences on the development of the valleys and to examine their evolution. For these purposes, morphometric parameters of the main valleys have been analyzed and the results have been compared with the other similar research (Graf, 1970; Li et al., 2001; Brook and Brock, 2005; Kassab and Harbor, 2013).

The main hypothesis of this research may be summarized as follows: (i) The morphometric characteristics of the valleys in Zardkuh Mountain were affected by the combined effects of glacial and fluvial processes; (ii) The power law function is a suitable means of assessing the role of glacial processes in the development of the valleys in Zardkuh Mountain; (iii) The morphometric characteristics of valleys have a close relationship with equilibrium line altitude (ELA) during the Last Glacial Maximum (LGM).

2. Study area

The study area is Zardkuh Mountain in the Zagros Mountain Range (32°14' to 32°38'N; 49°50' to 50°15'E) in Chaharmahal and Bakhtiari province, Iran (Fig. 1). The highest summits of this mountain chain are Kolonchin with 4220 m altitude and Shahe Shahidan with 4163 m altitude (National Cartographic Center of Iran). The climate of the Zagros Mountain is typically Mediterranean with dry summers and precipitation during fall, winter, and spring. Spring and summer precipitation is a result of cyclonic storms moving as cold fronts and is characterized by snow at higher elevations. Winter precipitation is as a result of gentler anti-cyclonic fronts. In winter, several meters of snow cover are normal in the mountainous areas. Based on the recorded data in Chelgerd station (2400 m asl), the northeast of Zardkuh receives 1468 mm/y precipitation in its piedmont area. The maximum and the minimum annual precipitations measured at Chelgerd station are 2555 mm and 925 mm, respectively.

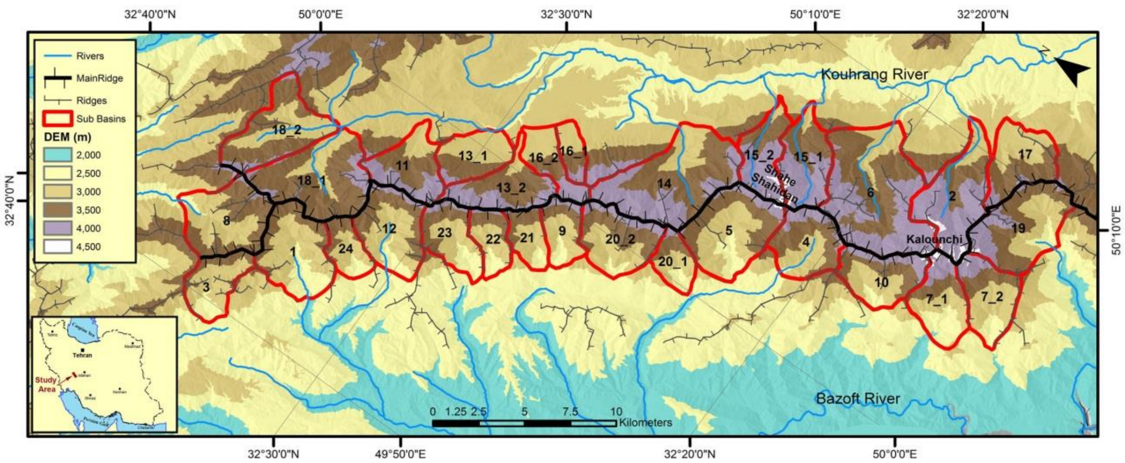

Fig. 1. Location map of Zardkuh Mountain with main hydrological sub basins on Digital Elevation Model of Zardkuh Mountain (Seif and Ebrahimi, 2014).

From a geological point of view, the Zagros Mountain range is a branch of the Alpine-Himalayan orogenic belt formed by collision of two tectonic plates: the Eurasian and Arabian (Falcon, 1974). The occurrence of numerous main faults such as Bazoft, Haftanan, Panbe kal and Chamal indicates strong tectonic activity in the area. Recent GPS measurements in Iran have shown that this collision is still active (Nilforoushan et al., 2003). They have also shown a high rate of deformation within the Zagros. The GPS results show that the current rate of shortening in the SE of Zagros is about 10 mm/yr, and it is 5mm/yr in the NW of Zagros (Nilforoushan et al., 2003). Regular series of Precambrian, Paleozoic, Mesozoic and Cenozoic sedimentary rocks are exposed in Zardkuh region (Fig. 2). The most exposed geological formations are Cretaceous (Sarvak formation) and Oligomiocene (Asmari-Jahrom formation) limestone, marlstone and marly limestone whose general trends are NW-SE. Salt domes are common features of the Zagros Mountains, partly outcropping on the SW flanks of Zardkuh. The geomorphology of the region is directly related to its geological and structural features. It is characterized by high mountains whose southern slopes are steeper than the northern slopes. The high relief region is composed of Sarvak Limestone which is the most resistant formation in the area. The high relief, active tectonics and abundant precipitation result in various karstic features such as sinkholes, vertical shafts, poljes, karstic springs and karrens. Indeed, the karstic limestone of Sarvak and Asmari-Jahrum formations are the underlying bedrock of the Zardkuh glaciers (Moussavi et al., 2009).

The first published observations of small glaciers (mostly cirque glaciers) on the northern slopes of Zardkuh have been provided by Desio (1934) who described four small glaciers with a total area of 150 ha. Each glacier had an altitudinal range of 200 m, with the minimum occurring at about 3600 and the maximum at 4200 m asl. Desio (1934) noted that the small glaciers of Zardkuh are relicts of the bigger ones during to the LIA (The Little Ice Age about 1850 AD). In 1963, McQuillan (1969) photographed the "Ghiacciaietto and estimated its width about 400 m. By comparison of McQuillan's photograph (1969) with Desio's sketch (1934), Ferrigno (1988) has shown that the glaciers of Zardkuh have thinned considerably and the toes have lost at least 20 m of the total 100 m thickness. Wright (1962) has pointed out cirques on Zardkuh which have altitudes about 3000 m asl. Wright (1962) has mentioned moraines at an altitude of ~2600 m asl in the small valleys on the northern faces of the Zardkuh, and a large outwash fan at the northern face of the mountain. Grunert et al. (1978) described and sketched the location of five modern glaciers. The largest was described as 500 m wide and spanned an altitudinal range of 150 m, from 3900 to 4050 m asl. Preu (1984) has reported two small glaciers in cirques on the lee side of Zardkuh, surrounded by moraine deposits, and has mentioned small recent glaciers in some valleys on the northern faces of Zardkuh.

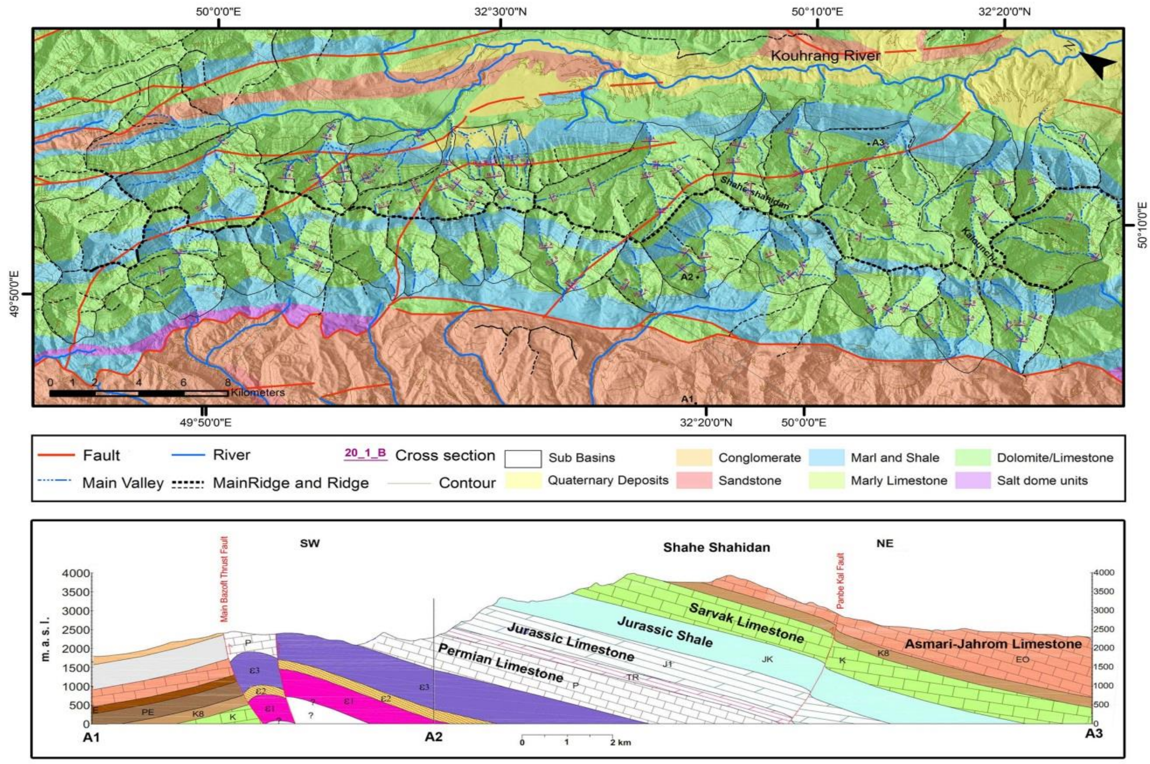

Fig. 2. General geologic map of the study are with a geologic cross-section perpendicular to Zardkuh axis and pass through Shahe Shahidan summit (A1, A2-A3).

Yamani (2007) outlined 15 glacial cirques at 3400 and 4000 m asl on the NE flanks of Zardkuh, and pointed out moraines at ~2500 m asl. Based on this study, the height of the modern snowline is considered to lie above 4800 m asl. Yamani (2007) also mentioned some old alluvial fans located at the outlet of some glacier basins. Moussavi et al. (2009) prepared a new glacier inventory for the glaciers of Iran (according to the GLIMS guidelines and remote sensing supported by fieldwork). According to their research, the greatest glacier concentration in Zardkuh is observed around Joft-zarde and Shahe Shahidan (Zardkuh) summits, around Sirdan summit, around Haft-tanan (Iluk) region with an approximate total area of 20 km^2.

According to Seif and Ebrahimi (2014), at least nine typical cirques with the classical characteristics of cirques have developed in the lee side of the Zardkuh Mountain at altitudes above 3650 m asl. In addition, 19 cirques were classified as "Well-defined" and "Definite cirques". Their study has indicated that the vertical development of the cirques in Zardkuh Mountain increases slower than their length and width (that is, they develop allometrically). Their study has also shown that the elevation of the glacial cirque floor has not had a close relationship with ELA during Last Glacial Maximum (ELA_{LGM} = 3100 m asl; Ebrahimi, 2015; Ebrahimi & Seif, 2016). In other words, former glaciers in Zardkuh Mountains, as in other glaciated tropical mountain ranges, expanded beyond cirques to form valley glaciers, with ELAs lays below the altitudes of cirques floors (~550 m in Zardkuh).

Some photographs of the study area are presented in Figure 3 which are showing general view of the valleys in northern (a and c) and southern flanks (b) of Zardkuh mountain. Figure 3a was taken from Khersan basin (Basin 15) and a panorama of the Joftzardeh and Kouhrang basins (basin No. 14 and 16) is presented in Figure 3c. Various types of glacial landforms and deposits, such as cirques, glacial outwash fans and moraines can be observed in these figures.

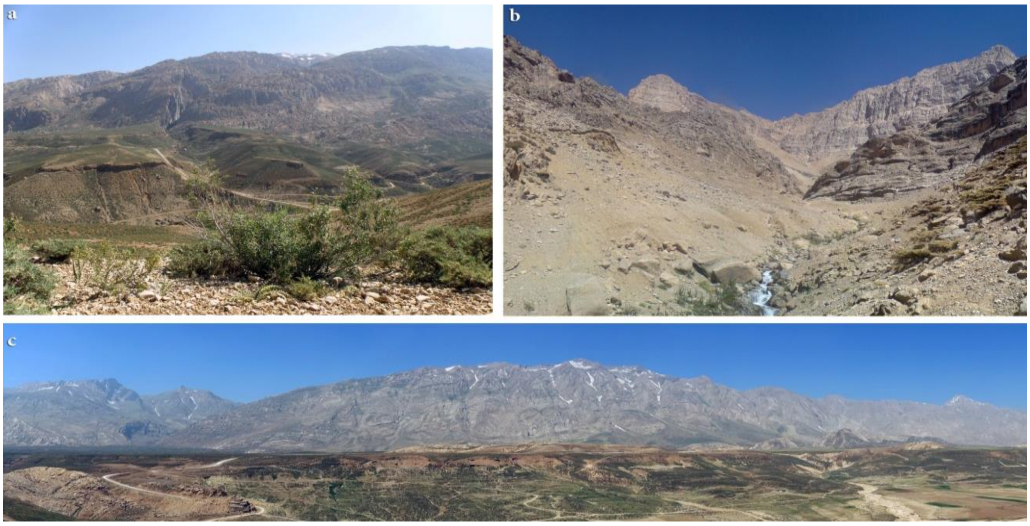

Fig. 3. General views of the valleys in northern (a) and southern flanks (b) of Zardkuh with a panorama of the Chalmishan, Joftzarde and Kouhrang basins in lee side of Zardkuh Mountains (c).

3. Materials and Methods

The power law or parabolic function ($y=ax^b$) was first introduced by Svensson (1959) and has been used in the analysis of glacial valley by Graf (1970), Doornkamp and King (1971), Jiao (1981), Liu (1989), Li et al. (2001), Brook and Brock (2005) and Kassab and Harbor (2013). Parameters x and y in the power function are horizontal and vertical distances from the lowest point on a cross-profile, and a, b are constants. As the results of the power law method are sensitive to location of the coordinate system origin, Kassab and Harbor (2013) have systematically assessed four alternative approaches for selecting the coordinate system origin. Kassab and Harbor (2013) recommended using the lowest point on a valley cross-profile as the coordinate system origin. Various studies have shown that b values range from 1.3 to 2.0 indicating a parabolic form. As intensity of erosion increases in the glacial valley system, the b value also increases indicating relatively deeper and narrower valley cross profiles (Graf, 1970; Li et al., 2001; Brook and Brock, 2005). Li et al. (2001) showed that the range of a and b values for fluvial channel cross profiles is evidently larger than that for glacial valleys. The b values in the range of 1.5-2.5 are most common for glacial valley cross-profiles; however, they range from far less than 1 to several hundred in fluvial channels (Li et al., 2001). Li et al. (2001) also indicated that the plot of |A| (logarithmic value of the *a* constant) against its corresponding *b* value has a clear linear trend. They have developed strong linear regression functions, between |A| and b for glacial and fluvial valleys (|A|=6.582b - 6.133 for glacial valleys and |A|=6.691b - 3.435 for fluvial valleys). Indeed, the slopes of A-b relationships of the two valley types are nearly the same, but the intercept for fluvial channels is larger than that for glacial valleys. Finally, Li et al., (2001) concluded that the |*A*|-*b* relationship and the ranges of *b* values (commonly from 1.5 to 2.5) may be helpful in differentiating valleys formed by different processes.

Hirano and Aniya (1988, 1990) have introduced two models illustrating how valley cross-profiles develop on the basis of the relationship between the power law exponent b and a form ratio (ratio of depth to width of a valley (FR)) defined by Graf (1970). These models are referred to as the Rocky Mountain and Patagonia-Antarctica models. The Rocky Mountain model, in which b increases with an increase in FR, is appropriate for alpine glacial valleys and depicts an over-deepening development of glacial valleys. The Patagonia-Antarctica model is appropriate for glacial valleys formed by continental ice masses with larger b and smaller FR and reflects a widening process for glacial valley. However, based on work in valleys in the Tian Shan Mountains, Li et al. (2001) have indicated that the Rocky Mountain model cannot be applied to all alpine glacial areas.

Brook and Brock (2005) applied both models to valleys in the Tararua Range (southern North Island, New Zealand) and found a clear relationship between variation of the exponent b and distance down-valley (b decreases as distance increases). Indeed, they have shown that the b values are higher in up-valleys with greater a degree of "U-ness".

The quadratic or polynomial model ($y=a+bx+cx^2$) is another empirical function used in the analysis of glacial valleys (James, 1996; Harbor and Wheeler, 1992; Li et al., 2001). The x and y in the quadratic function are horizontal and vertical distances from a datum. The, b and c are constants. Polynomial functions are valuable for entire valley modeling and can provide valid and statistically significant expressions of glacial valley forms (James, 1996). In this model, the *a* and *b* coefficients control position of valley cross-profiles in the coordinate system and they have not direct link to valley form. The valley form is mainly controlled by the *c* value, and the larger *c* is due to the narrower valley floor. The quadratic equation provides a concise description of the entire valley form; however, this approach relies upon the priori assumption that cross-profile form is parabolic and symmetrical (Harbor and Wheeler, 1992; James, 1996). As a result, the model may lose some ability in describing asymmetrical valleys and therefore contributes less to the understanding of valley form evolution (Li et al., 2001).

In the present study, 30 basins around Zardkuh Mountain were extracted from a DEM with a 10m horizontal resolution (National Cartographic Center of Iran) using ArcGIS 10 (Spatial Analyst Tools - Hydrology). The main valleys were extracted and overlapped on the geological map. In next step, cross-profiles of the selected valleys were extracted using 3D Analyst Tools. Morphometric parameters of valley cross-profiles, including the Form Ratio (FR) valley width (W) were also calculated. The power function parameters were calculated by plotting regression functions through the x-y data. As the power law represents only one valley side, two power-law equations for cross-profiles of the studied valleys were used, with a "pair" of power law b exponents for each cross-profile (Brook and Brock, 2005). To reduce errors, four major steps were used to analyze the morphology of the glacial valley cross profiles: (1) to reduce the effect of lithology, all cross-profiles selected were in area of limestone bedrock; (2) all cross-profiles were selected at locations where bedrock was exposes; (3) following Kassab and Harbor (2013), the lowest point on the valley cross-profiles were selected as the coordinate system origin; and 4) the trim line of each profile was determined by direct observation of the profiles convexities. The flow chart of the methodology is shown in Figure 4.

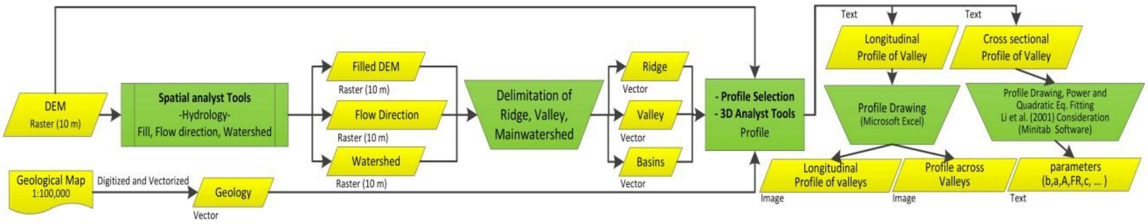

Fig. 4. Flow chart of methodology for delimitation of valleys profiles in GIS

4. Results and Discussion

A total of 86 cross profiles of the inner valleys including 31 valleys on the SW flanks and 55 valleys on the NE flanks of Zardkuh Mountains have been extracted (Figs. 2 and 5). Some representative cross profiles of the nearly symmetrical valleys are shown in Figure 6. Statistical information relating to the Power Law coefficients, Quadratic model and FR and W parameters are summarized in Table 1.

Power-law functions were fitted to both side of each cross profile. An example of the power law model fitting on cross profiles no. 13_2_C is shown in Figures 7a and b. As noted, two power-law equations have been used for 86 cross-profiles of the studied valleys. The means of b1 and b2 values (parameters of power-law function of both valley flanks) are 1.530 (from 1.458 to 1.601 using the 95% confidence limit) and 1.593 (from 1.511 to 1.675 using the 95% confidence limit), respectively. The mean b value (the average for the two valley sides (b1 and b2)) is 1.561 (from 1.494 to 1.629 using the 95% confidence limit). Figure 8a shows the probability plot of b1 and b2 values. The probability plot of b values (the average for the two valley sides (b1 and b2)) is also presented in Figure 8b. These plots allow us to compare the observed distributions with each other and with the normal distribution model. Using a 2-Sample t-test, the means of b1 and b2 were not found to be different at the 0.05 level of significance. This result is clearly deduced from Figures 8a, c. The frequency histograms of b1, b2 and b values are shown in Figures 8c, d. These plots allow comparison of the observed distributions and the normal distribution model and clearly show a bimodal distribution for the b values.

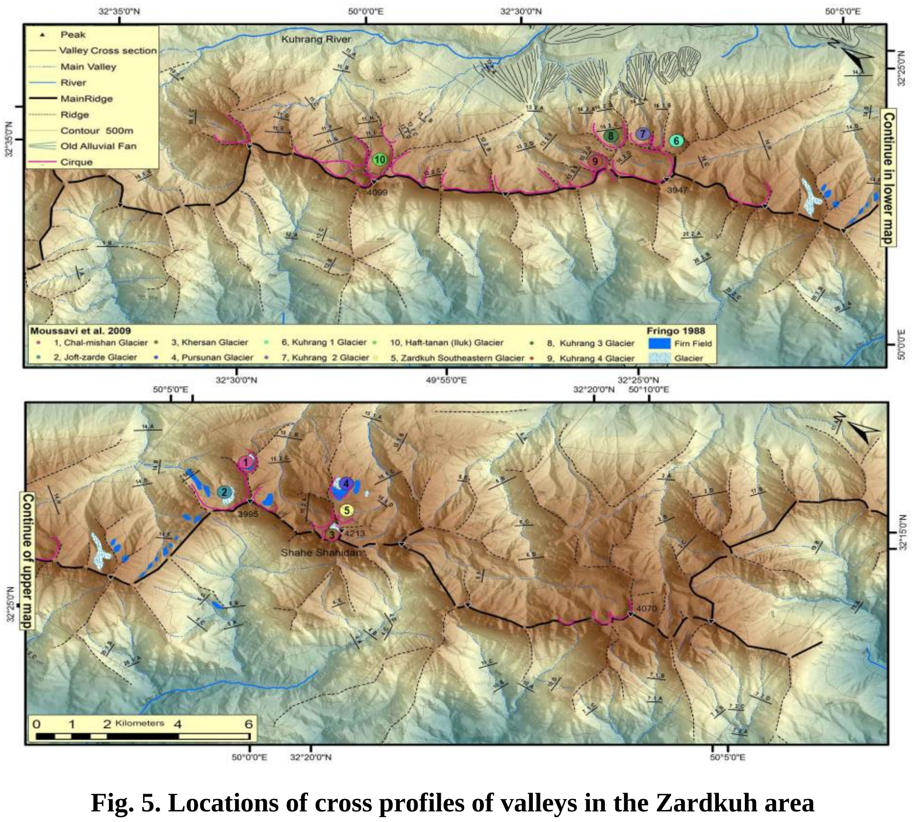

Fig. 5. Locations of cross profiles of valleys in the Zardkuh area

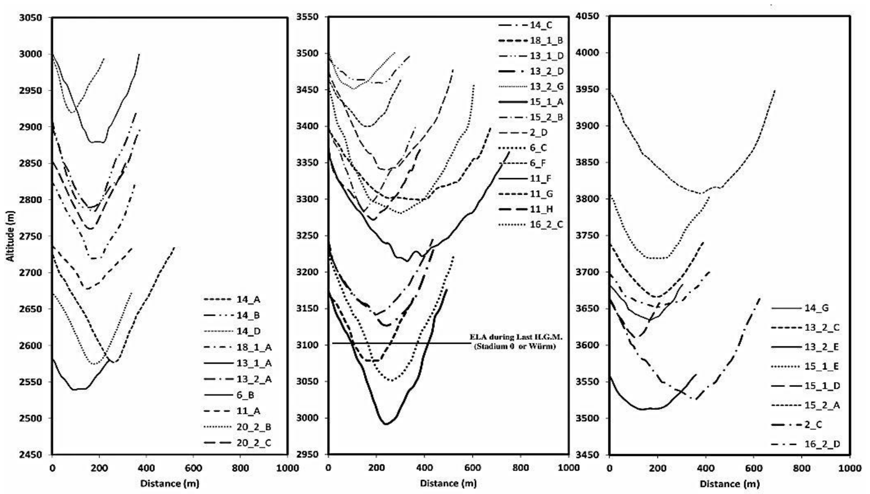

Fig. 6. Some valley cross-profiles from Zardkuh area with various floor altitudes

Table 1. Statistical parameters relating to power law and quadratic coefficients of valley cross profiles in Zardkuh (the b and |A| values used here are the average values for the two valley sides)

Statistical Parameters	VFA	C (x1000)	r^2 Quadratic	b1	b2	r_1^2 Power law	r_2^2 Power law	b	|A|	FR	W (m)
				For 86 cross profiles in both flanks of Zardkuh Mountain							
Min	2074	0.800	0.881	1.009	0.921	0.836	0.809	1.091	0.957	0.116	135
Max	3808	14.10	1.000	2.020	2.480	1.000	1.000	2.190	7.210	0.640	686
Mean	2977	3.472	0.960	1.530	1.593	0.975	0.972	1.561	3.389	0.295	350
SD	403	2.154	0.023	0.258	0.296	0.028	0.031	0.243	1.342	0.093	130
L.B.	2865	2.874	0.953	1.458	1.511	0.968	0.963	1.494	3.016	0.270	314
U.B.	3088	4.070	0.966	1.601	1.675	0.983	0.980	1.629	3.762	0.321	386
Skewness	-0.08	2.568	-0.789	0.055	0.442	-3.117	-2.628	0.194	0.314	0.696	0.67
				For 55 cross profiles in NE flank of Zardkuh Mountain							
Min	2539	0.800	0.881	1.103	1.130	0.906	0.908	1.153	1.324	0.116	135
Max	3808	6.100	0.992	2.021	2.476	0.999	0.999	2.190	7.207	0.420	686
Mean	3197	2.942	0.961	1.575	1.619	0.978	0.974	1.597	3.647	0.266	362
SD	297	1.484	0.025	0.251	0.305	0.020	0.022	0.254	1.433	0.077	143
L.B.	3093	2.426	0.952	1.488	1.513	0.971	0.967	1.509	3.149	0.239	312
U.B.	3300	3.457	0.969	1.663	1.725	0.985	0.982	1.685	4.145	0.293	411
Skewness	-0.36	0.593	-0.873	-0.001	0.663	-1.903	-1.319	0.162	0.167	0.088	0.51
				For 31 cross profiles in SW flank of Zardkuh Mountain							
Min	2074	1.700	0.914	1.009	0.921	0.836	0.809	1.091	0.957	0.175	138
Max	3109	14.10	0.986	1.965	2.130	0.999	0.999	1.876	5.099	0.638	651
Mean	2586	4.413	0.958	1.450	1.547	0.970	0.967	1.498	2.931	0.348	329
SD	232	2.784	0.019	0.253	0.277	0.037	0.042	0.212	1.033	0.098	100
L.B.	2479	3.125	0.949	1.333	1.418	0.953	0.948	1.400	2.453	0.302	283
U.B.	2693	5.701	0.967	1.567	1.675	0.988	0.987	1.596	3.409	0.393	375
Skewness	0.13	2.550	-0.619	0.194	-0.171	-2.892	-2.421	-0.069	-0.078	0.955	1

SD: Standard Deviation; U.B.: Upper Bound of Mean with 95% Confidence Level; L.B.: Lower Bound of Mean with 95% Confidence Level; VFA: Valley floor altitude (m asl).

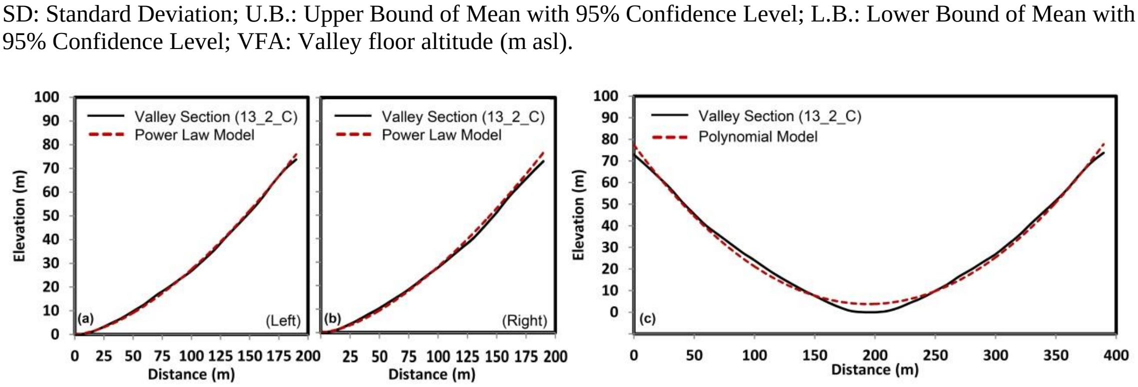

Fig. 7. A pair of Power law (a, b) and quadratic function (c) fitting on cross profiles of valley no. 13_2C

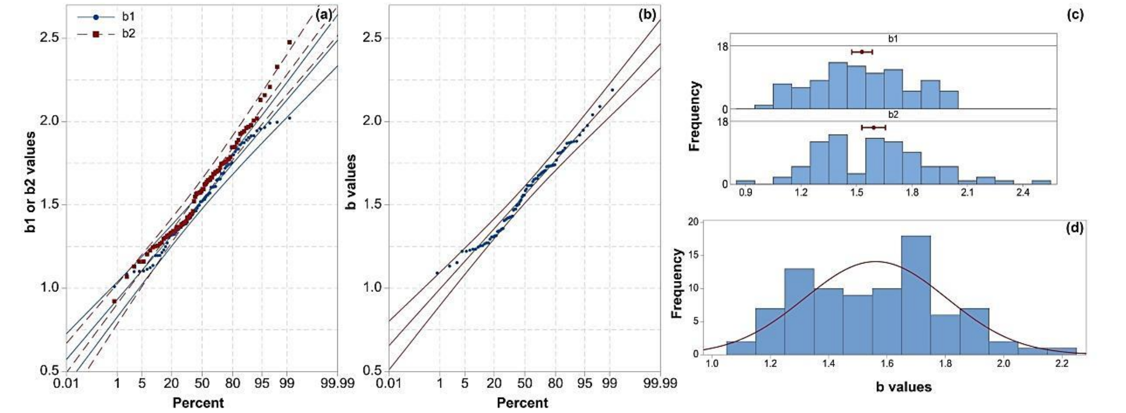

Fig. 8. Probability plot and Histograms of b values in the left (b1) and right (b2) of the valley sides (a and c); Probability plot and Histograms of the average of b value for the two valley sides (b and d)

Some Statistical parameters of the Power Law coefficients (and also the Quadratic models) with statistical parameters of FR and W for all 86 valley cross profiles in Zardkuh are shown in Table 1. Probability plots of the FR and W parameters are also presented in Figure 9. Linearity of characteristics of the parameters in the probability plots confirms that most data relating to the valley cross profiles parameters (W, FR and |A|) are approximated by the normal distribution model. The average of W for all 86 valley cross profiles is 350 m and its value varies between 314 m and 386 m (95% confidence limit). The FR varies between 0.270 and 0.321 (95% confidence limit) and the values of |A| range from 3.016 to 3.762 (95% confidence limit).

Although 86 cross profiles have been outlined in the study area, our attention in what follows will be mainly on the 55 valley crosses profiles on the northern slopes of Zardkuh. This is because of the relatively steep slopes, less developed valleys and the inability to accurately estimate the former ELAs on southern flanks. Statistical parameters for the Power Law and the Quadratic models coefficients for 55 valley cross profiles on northern flanks and 31 valley cross profiles on southern flanks of Zardkuh Mountain are presented in Table 1.

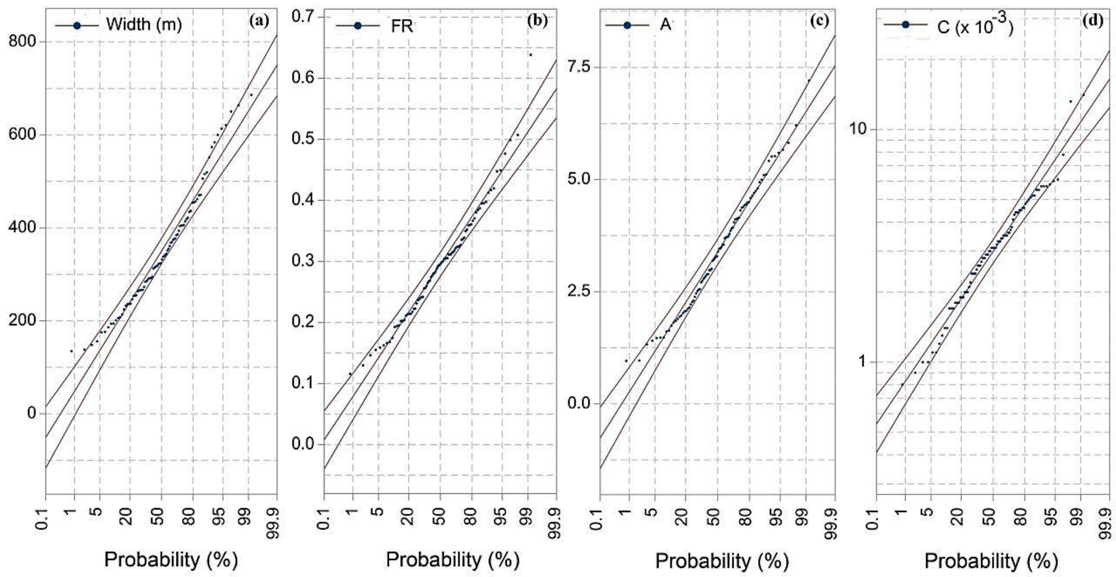

Fig. 9. Probability plot (quantile plots) of valley cross profiles parameters in the Zardkuh area. a) Width of valley cross section; b) Form ratio FR; c) The average value of |A| (logarithm value of a constant in parabolas function) for two valley sides; d) c-value of quadratic (polynomial) function. (Medians can be read off from intersection with the 50% line)

Figure 10c shows the frequency histograms of b values for 55 valley cross profiles on the northern slopes of Zardkuh. Similar to the frequency histograms of b values for all valley cross profiles (Fig. 8), the frequency histogram of b values for 55 valley cross profiles on northern slopes demonstrate a bimodal distribution. In the present research, among the different classification algorithms of clustering techniques, the k-means algorithm has been applied to classify cross profiles on the northern slopes of Zardkuh based on their b values. As expected and observed in Figure 10d, it is possible to classify cross profiles on the NE flanks into two main clusters with centers at 1.79 (for cluster I) and 1.36 (for cluster II). The number of cross profiles in each cluster is 30 and 25 for the clusters I and II, respectively. The 2-Sample t Test has been conducted on the means of b for both clusters. Based on the analysis, it could be concluded that the mean of b in the first cluster (No. I) is greater than in the 2nd cluster (No. II) at the 0.05 level of significance (Fig. 10e). This result can be clearly deduced from Figure 10a, which shows the probability plot of b values for both clusters. Figures 10b and 10f show the probability plot and Valley Floor Altitude (VFA) for both clusters of valley cross profiles in the northern slopes. Based on the probability plot and the 2-Sample t Test for the VFA of both clusters, it is possible to conclude that the mean of VFA in the first cluster is greater than the 2nd cluster at the 0.05 level of significance.

The statistical parameters for Power Law coefficients and some morphometric factors (For example, FR and W) for valley cross profiles of both clusters on the northern slopes of Zardkuh are shown in Table 2. The means of b values are 1.79 (from 1.72 to 1.86 within 95% confidence limit) and 1.36 (from 1.30 to 1.42 within 95% confidence limit) for clusters I and II, respectively. The mean W of the first cluster is 375 m, varying between 303 m and 446 m (95% confidence limit). The mean W of the second cluster is 346 m (from 278 m to 414 m within 95% confidence limit). The mean of form ratios are 0.24 (from 0.21 to 0.27 within 95% confidence limit) and 0.30 (from 0.26 to 0.34 within 95% confidence limit) for the clusters I and II, respectively (Table 2).

The relationship between the VFA and b was checked for both flanks of Zardkuh. Figure 11a shows the b and VFA regression relationship for valley cross profiles on the SW flanks of Zardkuh Mountain. Based on this analysis, the relationship between b and valley floor altitude is very weak (r=0.3) and it is not statistically significant within 95% confidence limit (it is significant at 90% confidence limit). Figure 11b shows a similar relationship for valley cross profiles on the NE flanks of the study area, too. Based on the regression analysis, the relationship between b and floor altitude is statistically significant within 95% confidence limit (P<0.01) and its positive significant correlation (r=0.6) indicates that b tends to increase as valley floor altitude increases.

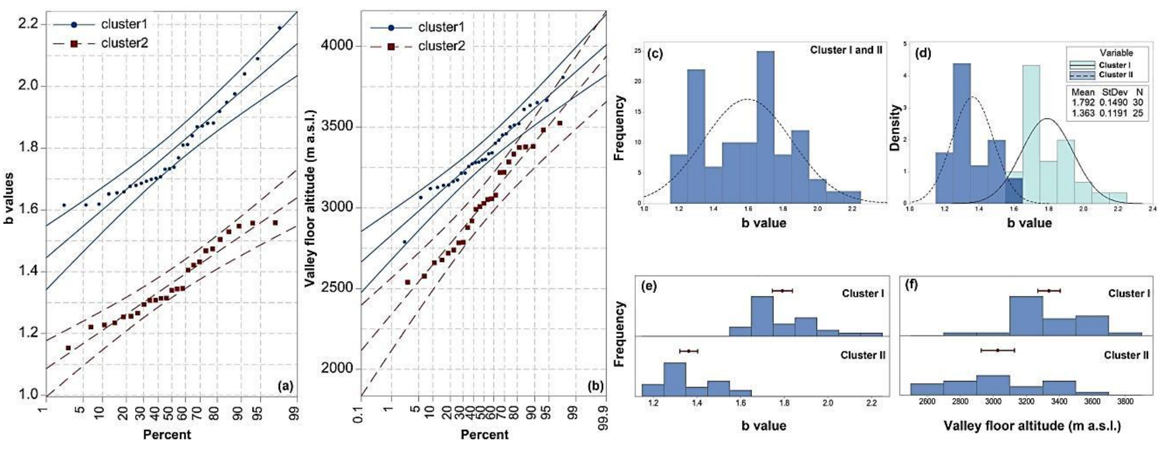

Fig. 10. Probability plots of b values (a) and valley floor altitude (b) for clusters I and II; Histogram of b values of all 55 valley cross profiles in northern slopes (c); Histogram of b values of both cluster of valley cross profiles in northern slopes (d); comparison of b value distribution and b value means for both cluster (e) and comparison of VFA distribution and their means for both cluster (f)

Table 2. Statistical parameters of valley floor altitude (VFA), form ratio (FR), width of valley (W) and power law and quadratic model coefficients (b, A and C) of a cluster of valley cross profiles on the northern slope of Zardkuh Mountain

Statistical Parameters	VFA (m asl)	C (x1000)	W (m)	b	|A|	FR
For 30 cross profiles in cluster I of northern flanks of Zardkuh Mountain						
Min	2789	0.80	148	1.62	3.19	0.12
Max	3808	6.00	686	2.19	7.21	0.39
Mean	3338	2.61	375	1.79	4.73	0.24
SD	218	1.51	153	0.15	0.88	0.07
L.B.	3235	1.90	303	1.72	4.32	0.21
U.B.	3440	3.32	446	1.86	5.14	0.27
Skewness	0.00	0.91	0.49	0.98	0.68	0.42
Lower Quartile	3170	1.38	260	1.68	4.13	0.19
Upper Quartile	3504	3.58	482	1.88	5.44	0.27
For 25 cross profiles in cluster II of northern flanks of Zardkuh Mountain						
Min	2539	1.00	135	1.15	1.32	0.13
Max	3525	6.10	621	1.56	3.55	0.42
Mean	3027	3.34	346	1.36	2.35	0.30
SD	295	1.38	132	0.12	0.69	0.07
L.B.	2875	2.63	278	1.30	1.99	0.26
U.B.	3179	4.05	414	1.42	2.71	0.34
Skewness	0.03	0.46	0.46	0.28	0.26	-0.50
Lower Quartile	2761	2.35	235	1.26	1.80	0.24
Upper Quartile	3309	4.45	458	1.47	2.92	0.36

SD: Standard Deviation; U.B.: Upper Bound of Mean with 95% Confidence Level; L.B.: Lower Bound of Mean with 95% Confidence Level; VFA: Valley floor altitude.

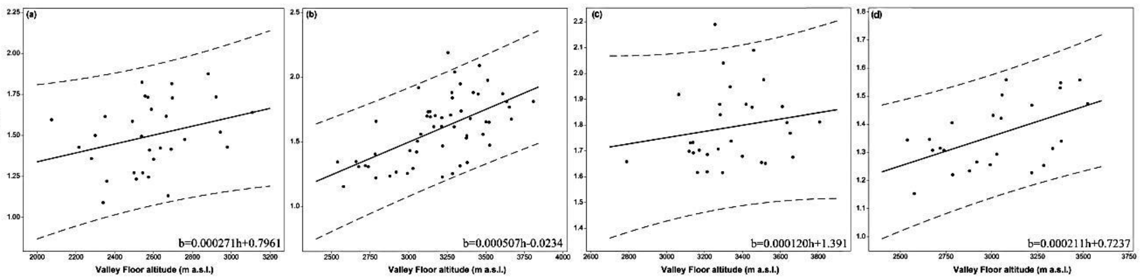

Fig. 11. The b-VFA diagrams for all valley cross profiles in the SW flanks (a) and NE flanks (b) and similar relationship for valley cross profiles in the cluster I (c) and Cluster II (d) of Zardkuh Mountain. The fitted line shows the predicted b for any VFA and the dashed lines show the 95% prediction interval.

The relationship between the VFA and b was also checked for both clusters of valley cross profiles on the NE flanks of Zardkuh. Figure 11c shows the b and VFA regression relationship for valley cross profiles in cluster I. Based on this analysis, the relationship between b and floor altitude is very weak (r=0.25) and is statistically significant at a 65% confidence limit. Figure 11d shows a similar relationship for valley cross profiles in cluster II. Cosidering regression analysis, the relationship between b and VFA is statistically significant within a 95% confidence limit (P=0.007) and its positive significant correlation (r=0.55) indicates that b tends to increase as valley floor altitude increases.

Figure 12a shows the relationship between b and FR for all valley cross profiles of Zardkuh. The analysis indicates a negative relationship between b and FR (r=-0.26) that is statistically significant within 95% confidence limits (P=0.018). The relationship between the b and FR has been checked for cross-profiles on the NE flanks of Zardkuh, too (Fig. 12b). Based on the analysis, a negative correlation between b and FR is observed (r=-0.4) which is statistically significant at P=0.003.

The relationship between the VFA and FR has been checked. Figure 12c shows the regression relationship between the VFA and FR for all valley cross profiles in Zardkuh. Regarding the analysis, a moderate negative correlation (r=-0.56) exists between the VFA and FR statistically significant at P=0.001. A similar conclusion is obtained from the analysis of the VFA and FR relationship on the NE flanks of Zardkuh (Fig. 12d). Based on the analysis, the relationship between the valley floor altitude and form ratio is statistically significant within 95% confidence limits (P=0.001) and the negative significant correlation (r=-0.43) indicates that FR tends to increase as valley floor altitude decreases.

Following Li et al. (2001), the relationship between the A and b has been checked for Zardkuh valleys. The data values of b and |A| shown in Figure 15 come from both flanks of the Zardkuh valleys (the b value and |A| used here are the average values for the two valley sides). In Figure 13 the regression functions of the glacial and fluvial valleys (based on Li et al., 2001) have been drawn for comparison.

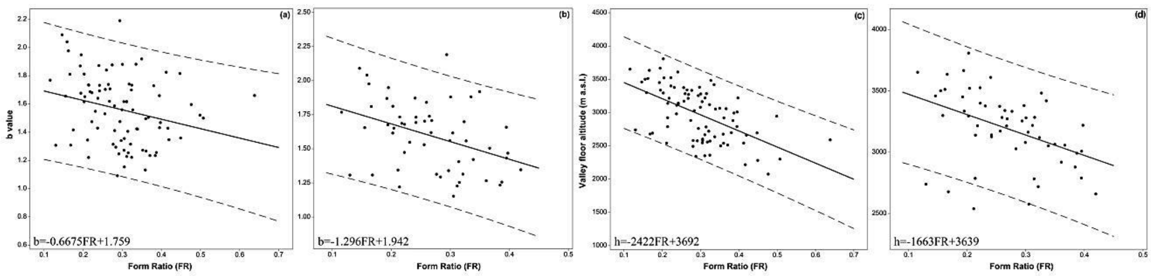

Fig. 12. The b-FR diagrams for all valleys cross profiles (a) and valley cross profiles in the north flank (b) of Zardkuh Mountain. The VFA-FR diagrams for all valleys cross profiles (c) and valley cross profiles in the north flank (d) of Zardkuh Mountain. The fitted line shows the predicted b or VFA for any form ratio and the dashed lines show the 95% prediction interval.

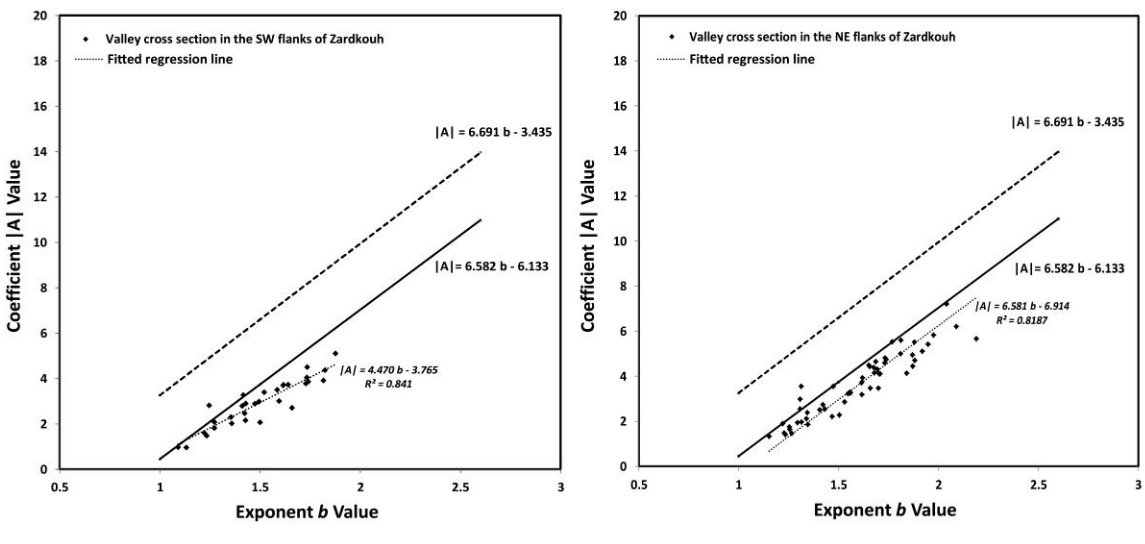

Fig. 13. The b-|A| diagrams for valley cross-profile in the both flanks of Zardkuh Mountain. Solid and dashed lines are regression functions of glacier and fluvial valleys based on Li et al. (2001)

In this study, the quadratic equation has been used to analyze all 86 valley cross-profiles. An example of the quadratic equation fitted to cross profiles no. 13_2_C is illustrated in Figure 7c. The statistical descriptions of c coefficients for all valleys cross-profiles and cross profiles on the NE and the SE flanks of Zardkuh are presented in Tables 1 and 2. Figure 9d shows the probability plots of c values in more detail. Linearity of characteristics of the c values in the probability plots confirms that the c values are approximated by the log-normal distribution model. The c values for all 86 valley cross-profiles range from 0.00080 to 0.01410 with an average of 0.00347. The means of c values (x1000) for valley cross profiles (the NE flanks of Zardkuh) with floor altitudes above (ca) and below (cb) the altitude of the ELALGM (3100 m asl) have also been calculated (Table 3). The means of cb and ca values (on the north flank of Zardkuh) are 3.2 (between 2.44 and 3.97 within 95% confidence limit) and 2.8 (between 2.14 and 3.48 within 95% confidence limit), respectively. Result shows that the mean of c values for valleys with floor altitudes below the ELALGM is higher than the c values for valleys with floor altitudes above the ELALGM significant within 86% confidence limits.

Following Li et al. (2001), the relationship between FR and c was checked for Zardkuh valleys. Figure 14 shows a significant nonlinear relationship for the FR and c exponent in the study area. The diagram shows that the exponent c significantly increases as FR increases.

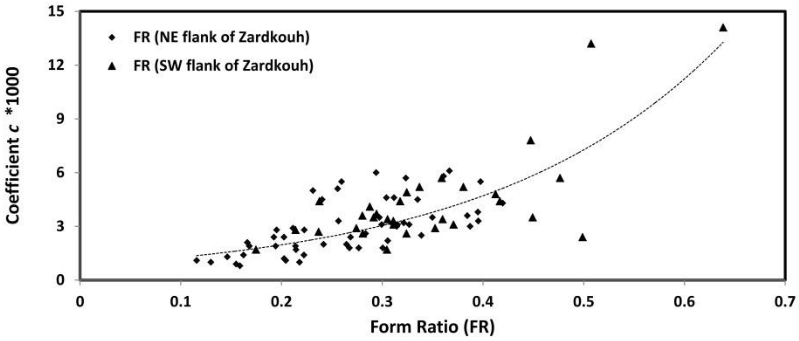

Fig. 14. Relationship between c and FR of glacial valley cross profiles in the both flanks of Zardkuh Mountain

Table 3. Statistical parameters of C (x1000) and Valley Floor altitude for valley cross profiles on the NE flanks of Zardkuh, with floor altitude above and below the altitude of the ELALGM

Parameters	Min	Max	Mean	SD	L.B.	U.B.	Skewness	Lower Quartile	Upper Quartile
VFA (m asl) (below the ELA):	253 9	307 9	2853	179	2744	2961	-0.225	2709	3034
c_b (1000*C):	1	6.1	3.21	1.27	2.44	3.97	0.864	2.43	3.65
VFA (m asl) (above the ELA):	311 9	380 8	3364	172	3291	3437	0.62	3219	3492
c_a (1000*C):	0.8	6	2.81	1.58	2.14	3.48	0.649	1.55	4.5

SD: Standard Deviation; U.B.: Upper Bound of Mean with 95% Confidence Level; L.B.: Lower Bound of Mean with 95% Confidence Level; VFA: Valley floor altitude; ca and cb value for cross profile of valley above and below the ELA, respectively.

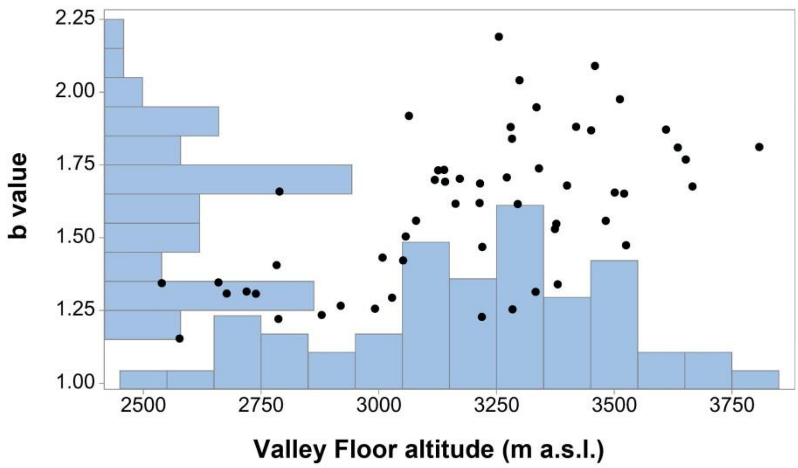

Fig. 15. A scatterplot diagram with histograms (marginal plot) of the b value and VFA in NE flanks of Zardkuh

The statistical analyses have shown that a relatively strong significant correlation exists between b and VFA on the NE flanks of the Zardkuh. In other words, b values for valleys on the NE flanks of the Zardkuh tend to increase as valley floor altitudes increase. These analyses have not shown any significant relationship between b values and floor altitudes on the SW flanks. This may indicate a more efficient fluvial process in valleys on the SW flanks, and relatively efficient glacial process at higher altitudes on the NE flanks of Zardkuh. Similar analyses have not shown any significant relationship between b and floor altitudes in the first cluster of valley cross profiles on the NE flanks. In other words, the b value at a mean altitude of 3338 m asl is about 1.8 without any significant relationship with altitude (the standard deviation of b is 0.15). On the other hand, the statistical analyses have shown a relatively strong significant positive relationship between b and VFA in the second cluster of valley cross-profiles on the NE flanks of the Zardkuh. In other words, the b value at a mean altitude of 3000 m asl is about 1.36 and its value increases at the rate of 0.012 per 100 meters. Figure 15 shows the marginal plot with related histograms of b value and VFA on the NE flanks of Zardkuh. Indeed, the marginal plot is a scatterplot diagram with histograms of the data, used to assess relationship between the two variables and examine their distributions. Regarding the pattern of b values in Figure 15, it shows that the b value at altitudes lower than 3000 m asl is low and its value increases as altitude increases. In other words, that fluvial erosion dominates at altitudes lower than the altitude of ELALGM (3100 m asl). The highest b values occur at a mean altitude of 3338 m asl (ranging from about 3100 to 3500 m asl), suggesting that glacial modification is most effective in this area. Above 3500 m asl, the b values are lower than those in the 3100-3500 m asl elevation range, perhaps suggesting less glacial modification in the highest parts of the valleys. Although the highest b is found at about the 3100 to 3500 m asl, it should be noted that b values have maximum variance in this range. This could be due to the fact that rock structure can influence the shape of the valley cross profiles (Harbor, 1995).

Analysis of the b-FR diagram indicates that the valleys in the Zardkuh Mountains (especially on the northern flanks) do not fit the Rocky Mountain model of Hirano and Aniya (1988, 1990) and show a statistically significant negative trend similar to the Patagonia Antarctica model. This result is similar to the result of Li et al. (2001) in the Tian Shan Mountains. The FR-b and FR-VFA diagrams have shown that FR increases as the floor altitude of the valleys decreases. These variations suggest a more efficient widening process in the higher altitudes and a more efficient over-deepening in the lower altitudes.

The regression analyses of b-|A| values of valleys on the NE flanks of Zardkuh have shown similar trend and slope to the regression line of glacial valley; however, the intercept for Zardkuh (NE) is a little smaller than that of the regression function of glacial valleys. Similar analysis has been conducted for b-|A| values of valleys on the SW flanks of Zardkuh. The analysis has shown that b-|A| values of the valleys are close to the regression function of glacial valleys but their slopes and intercepts are different.

The quadratic functions have been tested on all the 86 Zardkuh valleys and their parameters (a, b and c) have been calculated for all valley cross profiles. The calculated values of b and c have shown similar order of magnitudes to the previous studies around the world (e.g., Graf, 1970; James, 1996; Li et al., 2001; Brook and Brock, 2005). Among the quadratic function parameters, only the c values reflect the shape of the valley cross section. The relationship between c and FR is also important in revealing the morphology of the valley cross profiles similar to the b-FR diagram of the power law (Li et al., 2001). The relationship of c-FR has shown that exponent c significantly increases as FR increases (an increase in c represents steepening and narrowing of the fitted curve). Since the average of FR values of the cross-profiles at higher altitudes is commonly smaller than FR values at lower altitudes, valley floors of the cross-profiles at higher altitudes are generally wider than the cross-profiles at lower altitudes. In addition, the significant correlation of c and FR has shown that most of the valleys are roughly symmetrical in cross-profile. The reason is that the quadratic equation (from which c is derived) has the priori assumption that a cross-profile is symmetrical (Li et al., 2001). Hence, if the valleys were indeed asymmetrical, the correlation between c and FR would be poor (Brook and Brock, 2005).

5. Conclusions

In this study, morphological characteristics of the valleys in Zardkuh Mountain have been analyzed and discussed. Morphometric study of the valleys has been carried out by application of the power and quadratic functions and analysis of their parameters (such as A, c, b). To reduce the effect of lithology, all cross profiles have been selected on the limestone valleys. The main conclusions of the study are summarized as follows:

a. Values of c coefficient in the quadratic function, range from 0.00080 to 0.01410 (with the mean equal to 0.00347). The means of the c coefficient for valley cross profiles with altitudes above and below ELALGM (on NE flanks) are 0.00028 and 0.00032, respectively. There is a difference in the means of the c coefficient above and below ELALGM, significant at the 86% confidence level. Application of this model has shown that most of the valleys are roughly symmetrical in their cross-profile.
b. The values obtained for b, c and FR are generally consistent with those reported from other study areas, known to have been glaciated during the Last Glacial Maximum.
c. The values of b in the study area for both valley sides range from 1.0 to 2.5 with the averages of 1.5 and 1.6 for left and right valley sides (looking down valley), respectively. The mean b value (the average for the two valley sides) for all 86 valley cross profiles is 1.56, varying from 1.5 to 1.6 within 95% confidence limits. The regression analysis of VFA and b has not shown a significant relationship between b and floor altitude on the SW flanks of the study area. Similar analysis for cross profiles on the NE flanks of Zardkuh demonstrates a moderate positive correlation indicating that b tends to increase as valley floor altitude increases. This may indicate a relatively efficient glacial process in highlands on the NE flanks of Zardkuh.
d. The statistical analysis has shown a bimodal distribution for b values. This behavior has also been observed in the distribution of b values in 55 valley cross profiles on the northern slopes of Zardkuh. Based on application of the k-means algorithm, cross profiles on the northern slopes of Zardkuh have been classified into two main clusters with mean b values of 1.79 (mean VFA = 3338 m asl) and 1.36 (mean VFA = 3338 m asl) for cluster no. I and II, respectively. The regression analysis of VFA and b has not shown any significant relationship between b and floor altitude in cluster No. I. Similar analysis for cross profiles of cluster No. II has shown a relatively strong significant positive relationship. The statistical analysis has shown that the b value at altitudes above ELALGM (3100 m asl) is relatively high (b=1.79), but its value is independent of the valley floor altitude. This analysis has shown that the b value at altitudes below ELALGM is relatively low (b=1.36) and their values significantly increase as valley floor altitude increase. In other words, fluvial erosion dominates at altitudes lower than the ELALGM, and glacial modification is most effective in altitudes above the ELALGM. Above 3500 m asl, b values are lower than those occurring in the 3100-3500 m asl elevation range, perhaps suggesting less glacial modification in the highest parts of valleys.
e. The values of |A| range from 0.957 to 7.210 (3.016 to 3.762 within 95% confidence limits). The linear relationship of |A|-b for the Zardkuh valleys especially on the NE flanks coincides with the relationship for glacial valleys introduced by li et al. (2001). The general trends of b variations (against altitude) on both flanks of Zardkuh are similar but their slopes in the leeside of Zardkuh are significantly higher than the SW flank. This may indicate a more efficient fluvial process in valleys of the SW flanks and relatively efficient glacial process in highlands of the NE flanks of Zardkuh.
f. The FR value ranges from 0.116 to 0.640 (0.270 to 0.321 within 95% confidence limit) with an average of 0.295. The mean of W for all valley cross profiles is 350 m and it varies from 135 m to 686 m (314 to 386 m within 95% confidence limit). Analysis of b and FR of the valleys in the Zardkuh Mountains does not confirm the Rocky Mountain model of Hirano and Aniya (1988, 1990); however, carries a similar trend to the Patagonia-Antarctica model (similar to the results of Li et al., 2001). Based on the analysis, the FR increases with a decrease in floor valley altitude, showing a more efficient widening process at higher altitudes and a more efficient over deepening of the

valleys in lower altitudes. A similar conclusion has been obtained by applying the quadratic equation. The significant nonlinear relationships between FR and the c exponent, and the significant correlation between c and FR have shown that the valley floors at higher altitudes are generally wider than the valleys in lower altitudes. In addition to the several mechanism of transversal valley and deep gorge formation (such as the effect of tectonic uplift) in Zagros Mountain (Oberlander, 1965), glacial melt waters could be a possible factor influencing development of deep gorges on the NE flanks of Zardkuh. In other words, down cutting (or vertical erosion) of valleys below the altitude of ELA to the local base level may have resulted from the corrosive action of melt water on the limestone bedrocks controlled by structural weaknesses such as transversal faults or master joints.

g. It seems that glaciers in the studied area would have developed initially at altitudes above 3600 m asl, as cirque glaciers and they spread down-valley during the coldest periods following pre-existing valleys. At present, greatest modifications of the valleys (highest b values) are observed at altitudes above the ELALGM (~ 3100 m asl) and the valleys have fluvial characteristics at lower altitudes, showing shorter duration of the glacial process.

h. This study is concerned with the effect of glacial processes on the shape of valley cross-profile. Although to reduce the effect of the geological factor, all cross profiles have been selected on the limestone valleys, the shape of valley cross-profile could be affected by local structural factors that could affect the results of the research and should be considered in future research.

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