Study on Fundamental Frequencies of Cylindrical Storage Tanks Obtained from Codes and Finite Element Method

Yazdanian, M. * and Ghasemi, S.

Young Researchers and Elite Club, Ahvaz Branch, Islamic Azad University, Ahvaz, Iran.

ABSTRACT: In the case of study on the seismic behavior of tanks, one of the most important subjects is selection of convective and impulsive frequencies for storage tanks. These two frequencies are defined by Housner. The major utility of these frequencies is for Rayleigh damping which is a must for time history analysis. API 650, EUROCODE 8 have suggested some analytical solutions for finding convective and impulsive frequencies and other parameters. In addition, these frequencies can be obtained from modal analysis by finite element software. In current study, these frequencies are obtained using Finite Element (FE) software and performing modal analysis. In the next stage, these modes are compared with analytical methods from standards. To compare tanks with different (H/R) ratios, all these tanks are modeled with the same volume and weight. This study showed that there is a good agreement between convective frequencies extracted from Finite Element method and those from standards. Furthermore, an increase in liquid level led to a rise and dip in convective and impulsive frequencies, respectively. In addition, results show that a 1.47 times increase in the viscosity leads to a 17 percent decrease in the values of impulsive frequencies.

Keywords: Convective, Finite Element, Frequency, Impulsive, Tanks.

INTRODUCTION

Liquid storage tanks are used widely in petroleum industries, urban water storage facilities, and nuclear water places. Many parameters are associated with the seismic behavior of liquid storage tanks and these include many issues (Ghateh et al., 2010; Chen and Kianoush, 2005). Experience of past earthquakes in different regions showed that these types of structures are vulnerable along the severe earthquakes. The performance of these structures is different from the other structures like bridges and buildings and these different is due to the effect of hydrodynamic pressure on the tank wall. Hence, the stability of the liquid storage tanks under earthquake conditions must be investigated cautiously (González, 2013). Many damages of tanks have been observed around the world. For example, During the Niigata earthquake severe tanks have been damaged. This earthquake led to a widespread eruption in tanks. These tanks burned for 15 days and it caused to break out to other area (Korkmaz et al., 2011). A further explanation is Kocaeli earthquake, along this earthquake that happened in northwest Turkey many of tanks were destroyed and most damages of tanks were observed at 19 km from the earthquake epicenter (Sezen et al., 2008). The first study on the dynamic

* Corresponding author E-mail: m.yazdanian@iauahvaz.ac.ir

behavior of tanks regarding liquid-structure interaction was conducted by Housner (1963). He proposed a new method for rigid tanks by using analytical technique. In Housner method, hydrodynamic pressure is divided into convective and impulsive part. The Housner's method has been selected for seismic analysis in most of the past codes and regulations and this method has been considered for basis of many standards. An expanded application of Housner's method, in terms of an advanced design method, was presented by Epstein (1976). However, attention to interaction between the solid and fluid is not a matter just related to storage tanks. Different subjects can be found regarding this on the literature review (Ardebili, 2013).

Many conducted research on storage tanks has been done with an emphasis on Finite Element method. Haroun and Tayel (1985) proposed a FE method for investigating the seismic response of tanks using Housner model. Further research on the behavior of tanks under dynamic loads using FEM was investigated by Ruiz and Gutiérrez (2015), Kazem and Mehrpouya (2012), Ghateh et al. (2015), Hosseinzadeh et al. (2013), Kianoush and Ghaemmaghami (2011) and Moslemi and Kianoush (2012). Livaoglu et al. (2011) investigated the dynamic behavior of the backfill on rectangular tanks through the FE method and by considering both the liquid–structure and soil–structure interaction effects as well as non-linear interaction effect of the soil and contact nodes between the soil and the wall. Ranjbar et al. (2013) used FE to model an elevated water tank system. The elevated tank was placed on a framed structure. They showed that the structural responses are highly influenced by the earthquake characteristics. Chen and Kianoush (2005) used a sequential method for estimating the hydrodynamic pressure along the walls of concrete rectangular tanks in a 2D space. Also, the parameter of wall

flexibility was considered in the suggested method. Kianoush and Chen (2006) investigated the impact of horizontal and vertical ground motion combination in rectangular tanks in a 2D space. They separated the impulsive and convective parts of the tanks and they compared different structural responses according to these separated parts.

Kianoush and Ghaemmaghami (2011) investigated the effect of different frequency contents of earthquakes on the dynamic response of two rectangular storage tanks using the FE model regarding soil-liquid-tank interaction. They focused on the both convective and impulsive parts of the tanks and for this purpose they obtained structural responses of these two separate parts. Ormeño et al. (2015) provided methods for seismic ground motion scaling in dynamic analysis of liquid storage tanks. They used three different methods for doing several analyses of earthquake responses. Ruiz et al. (2015) proposed an efficient computational method for seismic analysis of liquid storage tanks. The proposed model was based on the theory of potential flow, in which the continuity equation is solved through the Finite Element method. Yazdanian et al. (2016a) used FE method to evaluate the seismic behavior of two concrete rectangular tanks regarding fluid-structure interaction. They concluded that the structural responses obtained from time history analysis are more accurate than those obtained from response spectrum analysis. Yazdanian et al. (2016b) used FE method to study on the dynamic behavior of cylindrical storage tanks. They investigated different analysis to show how a tank behaves under different analysis. They concluded that the peak ground acceleration of an earthquake is highly important in the dynamic analyses of tanks. In addition, their method investigated different parameters in detail. Eskandari-Ghadi et al. (2013) proposed an analytical solution for 2D

coupled in a cylinder. They used numerical inversion of Laplace transform to obtain the stresses, displacements and temperature in terms of time.

There is also some researches on the fundamental frequencies of tanks. However the majority of them just regarded vibration analysis. Virella et al. (2006) studied on the fundamental modes of tank-liquid systems under horizontal motions. They found similar fundamental periods and mode shapes from the free vibration (modal) analyses and from the harmonic response analyses, using the added mass formulation and the model with liquid acoustic finite elements. Nicolici and Bilegan (2013) used both computational fluid dynamics analyses and the FE to obtain both convective and impulsive frequencies. They concluded that convective frequency depends on liquid height and other geometry parameters, while impulsive frequency changes with the elastic properties of the tank material. Jaiswal (2008) carried out an experimental research on different scaled tank models, then, tanks excited by a shake table. To compare with numerical study, the FE model of solid-fluid system was developed. They observed a good agreement between the results obtained from FE and experimental method. Furthermore, it can be found some studies on the analytical approach to assess the response of storage tanks. Shahmardani et al. (2014) evaluated the impacts of different fluid field models and the mass of equipment at the top of offshore structure which was simulated as lumped mass on the responses of offshore structures. They showed that different approximations regarding the fluid field could largely change the value of maximum displacement evaluated by the models. Amiri and Sabbagh-Yazdi (2012) investigated the effect of roof on dynamic characteristics of dome roof tanks partially filled with liquid. They concluded that at low liquid levels, equal to 1.80 and 4.80 m, the tank roof does affect the

axial modes of the tallest and medium height tanks. Maekawa et al. (2010) described the results of vibration tests using a 1/10 reduced scale model of large-scale cylindrical water storage tanks to clarify their dynamic behavior under seismic excitation. Their examination showed that the oval-type vibration, which is a high-order vibration mode, occurring on the sidewall of the tanks affects the distribution shape and magnitude of dynamic fluid pressure. Gnitko (2011) used both Boundary Element and FE methods to solve coupled problem. In their model differential equations of transient problem were solved numerically by Runge-Kutta method of 4th and 5th order. Tedesco (1982) presented an in-depth investigation regarding the vibrational characteristics and dynamic analysis of cylindrical tanks subject to a horizontal component of earthquake ground motion. They developed simple analytical expressions in the form of cubic polynomials which accurately predicted frequencies and radial mode shapes corresponding to the fundamental mode of vibration.

The seismic behavior of the storage tanks is highly dependent on their dynamic characteristics such as natural frequencies, mode shapes and damping ratios. In investigation of seismic behavior of tanks, one of the most crucial issues is how to find the convective and impulsive frequencies in storage tanks (Bayraktar et al., 2010). For finding convective and impulsive frequencies and other parameters, API 650 (2008) and EURO 8 (2003) codes for steel tanks have suggested some analytical solutions. Regarding concrete tanks, ACI (2006) is a key code which present some formulation for these type of tanks accordingly. Alternatively, these frequencies can obtain through modal analysis using FE software. In current study, these frequencies are extracted using FE software (2014) and performing modal analysis. Also these modes are

compared with analytical methods from API and EURO 8 standards for cylindrical tanks.

Considering the previous studies carried out on this topic, it can be observed many studies on dynamic behavior of tanks but most of them have just considered the seismic behavior of the tanks and there is a limited research on these significant frequencies. In addition, none of the previous studies has investigated the impact of different parameters on vibration analysis of tanks. As a result, there is a shortcoming in the recent references related to storage tanks. It is worthwhile that in this paper a comprehensive study has been carried out on these two frequencies, as well as the other different parameters. For this purpose, a parametric study has been conducted and different parameters are investigated. To account for the effect of liquid density and thickness, different liquid and thickness are taken into account. In addition, tanks with different liquid height are modeled to consider the impact of this parameter. Overall, in this paper, different parameters are vastly considered for cylindrical tanks with a focus on frequencies analysis of these types of structures.

MATERIALS AND METHODS

Governing Equation of Dynamic Forces

Time history analysis can take all the nonlinear factors into the analysis, so it is used to estimate the exact amount of structural response. The discretized formulation for a multi degree of freedom system subjected to external dynamic forces can be defined as Eq. (1) (Chopra, 2000).

$$[\mathbf{M}]\{\ddot{\mathbf{u}}\} + [\mathbf{C}]\{\dot{\mathbf{u}}\} + [\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{f}\} \qquad (1)$$

in which $[\mathbf{M}]$ is the mass matrix of the system; $[\mathbf{C}]$ is the damping matrix of the system; $[\mathbf{K}]$ is stiffness matrix of the system; $\{\mathbf{u}\}$ is displacement vector; $\{\dot{\mathbf{u}}\}$ is velocity vector;

$\{\ddot{\mathbf{u}}\}$ is acceleration vector; $\{\mathbf{f}\}$ is the applied load vector.

In this method, the step by step integration is applied directly to achieve the solution for the original equations of motion of the system. For solving this equation, it is needed to $[\mathbf{C}]$. Damping matrix of the system can be resulted from sum of the stiffness and mass matrices of the system. In many researches, similar classical damping system said as Rayleigh damping is applied for both structural and fluid domains leading to a classical damping for the whole scheme. Alpha and Beta damping are applied to indicate Rayleigh damping constants α and β. The damping matrix within the fluid domain $[\mathbf{C}]$ contains two parts which are because of impulsive and convective components of the stored fluid. In fact without these, it is not possible to do an accurate time history analysis (Kianoush and Ghaemmaghami, 2011). It should be noted that under free oscillations, the motion of free liquid surface decays due to damping forces created by viscous boundary layers. From this point of view, evaluation of damping characteristic for a fluid–tank system needs more considerations. However, due to lack of sufficient data in this field, the classical damping scheme is used in the Finite Element model. Considering impulsive and convective parts of liquid domain, damping matrix can be given as Eq. (2) (Ghaemmaghami, 2010).

$$[\mathbf{C}] = \alpha[\mathbf{M}] + \beta[\mathbf{K}] \qquad (2)$$

in which α and β are computed by Rayleigh damping method. α is defined based on the natural frequency of the primary sloshing mode and calculates for the damping because of sloshing on the free surface of the tank liquid. β is defined based on the primary frequency of the tank and simulates the damping because of the impulsive part. They

can be found according to the following equation (Moslemi & Kianoush, 2012).

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{2\omega_i\omega_j}{\omega_j^2 + \omega_i^2} \begin{bmatrix} \omega_j & -\omega_i \\ -\dfrac{1}{\omega_j} & \dfrac{1}{\omega_i} \end{bmatrix} \begin{bmatrix} \xi_i \\ \xi_j \end{bmatrix} \tag{3}$$

As offered by API 650, damping ratios of 2% and 0.5% are specified for the impulsive and convective parts of steel tanks, respectively, while as offered by ACI 350.3-06 (2006), damping ratios of 5% and 0.5% are specified for the impulsive and convective parts of concrete tanks. Two impulsive and convective frequencies are achieved from the modal analysis in FE software according to maximum effective mass or they can be obtained by analytical solutions. According to the above equations, one of the most important issues is obtaining of impulsive and convective frequencies in time history analysis. This importance is due to the fact that damping matrix is a crucial factor for solution of multi degree of freedom systems subjected to external dynamic forces. It should be noted that this study only covers anchored tanks, with an emphasis on the behavior of the cylindrical tanks. As a result, all the modeled tanks are ground-supported and the base of the tanks is assumed to be fixed. In Figure 1, convective and impulsive masses are shown. Impulsive mass is in the lower height than convective mass.

Finite Element Method

In this paper, a FE software i.e. ANSYS (2014) as a general purpose computer code is utilized to perform modal analyses. Modal analysis can find the various periods of a structure according to naturally resonate by using the overall mass and stiffness. These periods of vibration can be used to assess dynamic behavior of the structure. In Figure 2 the FE model of one of the tanks is shown.

As evident, these tanks have regular element shape. It should be mentioned that fluid element at a boundary should not be attached directly to the structural elements but should have separate coincident nodes that are coupled only in the direction normal to the interface. The materials of the tank shell are assumed isotropic and linearly elastic. This is also worthwhile to mention that in the bottom of all the tanks, the liquid is coupled with the cylindrical body which makes the results more and more accurate. In fact, this subject requires tremendous attention because of the fact that the cylindrical body of the tanks is also fixed along its another area.

The shell is modeled using shell element (Shell 63). This element has 6 degree of freedoms at each node and it has three translational and three rotational degrees of freedoms. It has both the bending as well as the membrane capacity. It has four nodes and variable thickness can be input at the nodes, giving a shell with smoothly varying thickness. The fluid inside the cylindrical tank is modeled using fluid element (Fluid 80). This element is a three dimensional solid element having three translational degree of freedom system at each node. This element has the ability to model contained fluids giving the result of hydrostatic pressure, hydrodynamic pressure as well as fluid structure interaction. The fluid elastic modulus of the element (in the input) should be taken as the bulk modulus of the fluid.

Modal analysis is done to obtain natural frequencies of these structures. Determination of these parameters in the first step can be very useful in interpreting the behavior of the tank. In addition, this analysis can be a starting point for other analysis, such as response spectrum or time history analyses. Convective and impulsive modes are integral modes which have the maximum effective mass and they should be taken into account for the dynamic analysis.
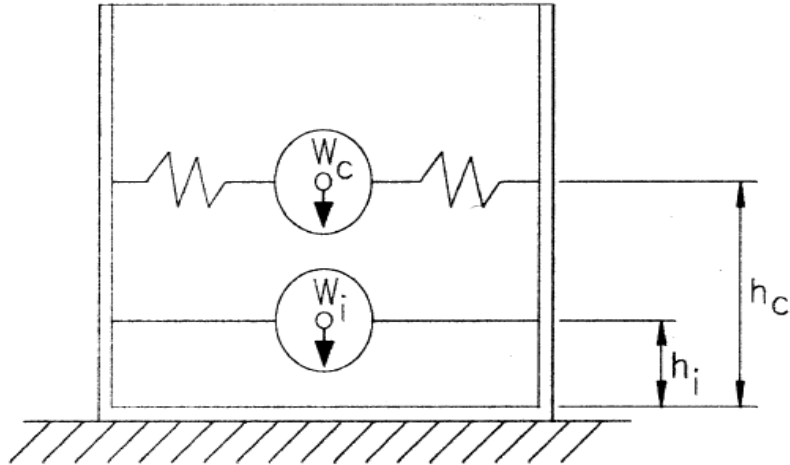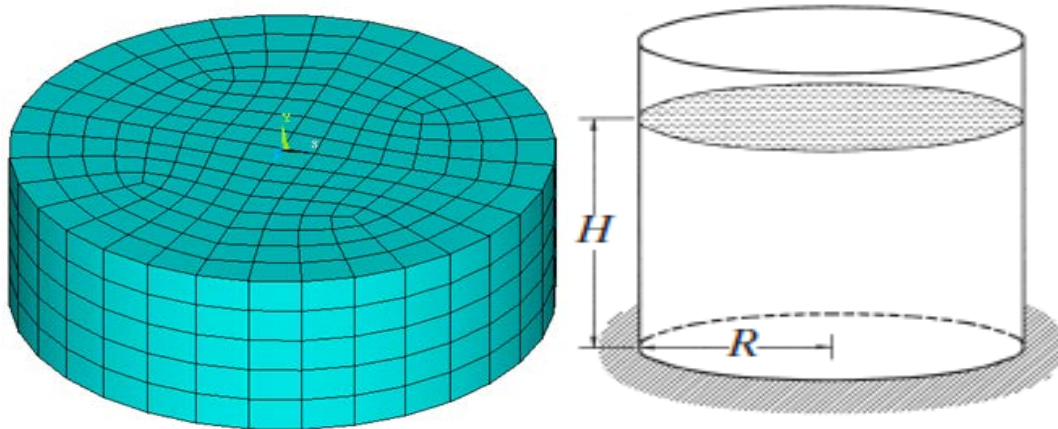
Fig. 1. Convective and impulsive mass

Fig. 2. FE modeling of tank

According to the maximum effective mass in modal analysis two modes of impulsive and convective are extracted and these modes amount would be set in Eq. (3). In modal analysis, Lanczos solver is used. In this type of solver all the natural frequencies and mode shapes are found within a specified range of frequencies. It should be noted that this system is the most effective and precise method to find the natural frequencies. Figure 3 shows both fundamental frequencies of tank no. 3. It can be seen from Figure 3(b) that the convective frequency is only related to the movement of liquid, while in Figure 3(a) both shell and liquid are moved with each other.

Convergence studies were carried out and the FE meshes which have the regular and precise meshing was found to provide good results for the purpose of the present study. Since many tanks are modeled in this study, the details of tanks including size of meshes are not mentioned. However, the number of elements for each tank is shown in Table 4. It must be pointed out that the studies of Yazdanian et al. (2016b) and Kianoush and Ghaemmaghami (2011) showed that the values of frequencies and other dynamic responses do not depend extremely on the size and the numbers of elements after a specified range of size and elements number. However, to show the validity of presented FE method, a comparison between the results of Barton and Parker (1987) with the current FE method is conducted. For this purpose,

two fundamental frequencies of tank which were obtained by Barton and Parker (1987) are compared with the current FE method. It is obvious that current FE method and Barton and Parker results are in excellent agreement. This further verifies the validity of the proposed FE technique. In addition, further validation can be found in the Yazdanian et al. (2016b) study.

Another significant validation of FE model of cylindrical tanks that is considered in this study is related to comparison of the proposed FE method with experimental method. For this purpose, the fundamental sloshing (convective) frequency of two tanks which are obtained previously using experimental method by Jaiswal et al. (2008) are compared with the proposed FE method. For this purpose, two tanks are modeled using the proposed FE method and then the convective frequencies of the tanks are

obtained. The liquid considered was water, for which, bulk modulus and mass density were taken as 2.18×109 N/m^2 and 1000 Kg/m^3 respectively. For the glass, the elastic modulus, mass density and Poisson's ratio were considered as 1.173×109 N/m^2, 955 Kg/m^3 and 0.35 respectively. It should be noted that the thickness has no effect on the convective frequencies and it will be approved in this study in the next sections.

API and EURO 8 Method

API 650 is presented the following equation for calculating the value of convective period.

$$T_c = 1.8 \, K_s \sqrt{D} \tag{4}$$

$$K_s = \frac{0.578}{\sqrt{\tanh(\frac{3.68H}{D})}} \tag{5}$$

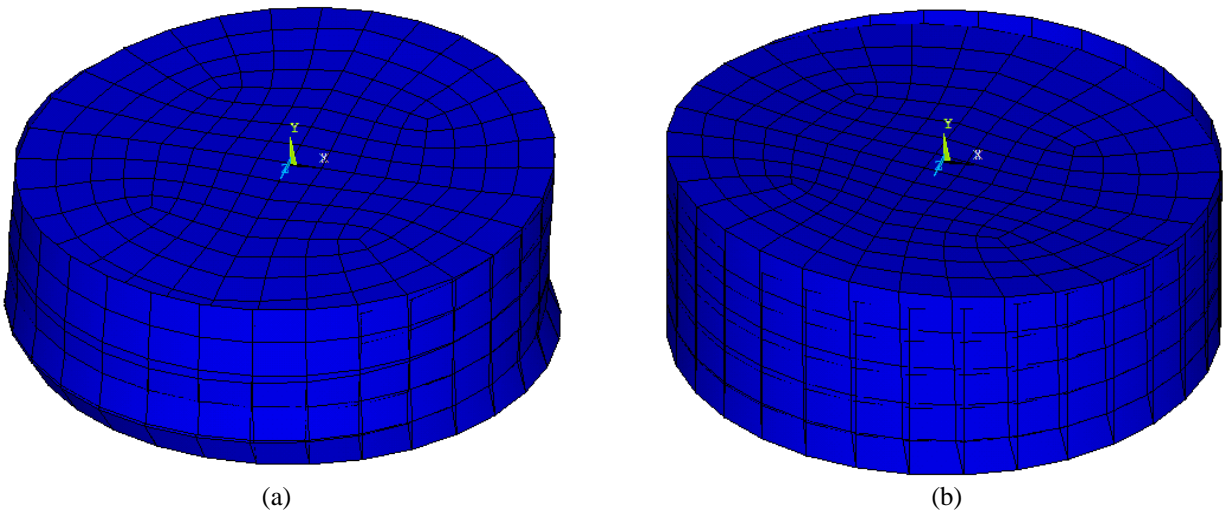(a)   (b)
Fig. 3. Fundamental mode shapes of tanks; a) impulsive; b) convective

Table 1. Comparisons fundamental frequencies calculated with the present study and with Barton and Parker (1987)

Type of Study	Impulsive	Convective
Current Study	11.5	0.29
Barton and Parker Study	11.5	0.29

Table 2. Fundamental sloshing frequencies of liquid

Depth of Liquid (mm)	Experimental	FEM
50	2.07	2.05
100	2.3	2.27

where T_c is the natural period of the convective mode (s), D is nominal tank diameter (m) and H is maximum design product level (m) and K_s is sloshing factor. According to EURO 8, the natural period of convective response can be taken from following equation which is suggested by Malhotra et al. (2000):

$$T_c = C_c \sqrt{r} \tag{6}$$

in which C_c is a coefficient for obtaining convective periods and can be obtained from Figure 4 and r is radius of the tank.

A simplified method was presented in the API and EURO 8 to determine the impulsive period. This method was taken from Malhotra et al. (2000):

$$T_i = (\frac{1}{\sqrt{2000}})(\frac{C_i H}{\sqrt{\frac{t_u}{D}}})(\frac{\sqrt{\rho}}{\sqrt{E}}) \tag{7}$$

in which, T_i is impulsive period, C_i is a coefficient for determining of impulsive period and can be obtained from Figure 4, t_u is the wall thickness, ρ is density and E is elasticity module. Furthermore, Malhotra et al. (2000) suggested values of C_i and C_c as a table which can be used instead of Figure 4.

RESULTS AND DISCUSION

Characteristics of the both selected tanks and their containers in details are shown in Tables 3 and 4, respectively. For comparison and investigation of the impact of height to radius (*H/R*) ratios, all these eight tanks are modeled with the same volume and weight, while all these tanks have different (*H/R*) ratios, with ranging from 0.3 to 3. These tanks are selected for covering all the (*H/R*) ratios which was presented by Malhotra et al. (2000). It is highly unlikely to find a real tank in a (*H/R*) ratio which is not covered in this research. In addition, tank no. 1 is selected to investigate the parameter of viscosity. For this purpose, type of liquid is considered to be changed in three different liquids and other parameters are considered similar to each other. Furthermore, in Table 4 the number of both fluid and shell elements is shown.

In the first step, static analysis is carried out and in this stage the tanks were analyzed based on their weight and the hydrostatic pressure of internal fluid. This analysis can be used in compound loading, and also it can be used as a criterion for the evaluation of constructed model. The generated hydrostatic pressure causes annular tensile stresses along the tank wall. The values of displacement and hydrostatic pressure obtained from static analysis can be found in Table 5. It can be observed that both structural responses are increased as the ratio of *H/R* is increased.
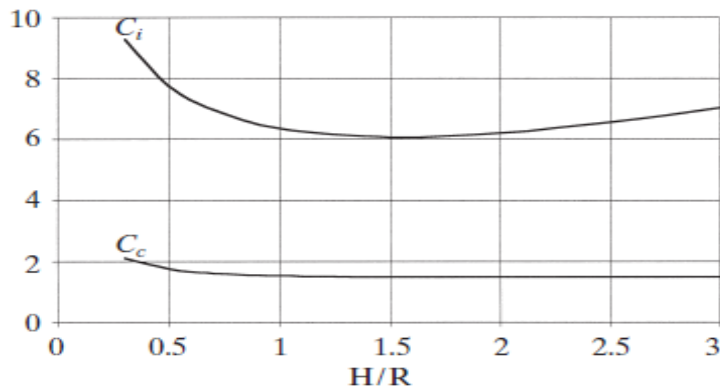

Fig. 4. Impulsive and convective coefficients (Malhotra et al., 2000)

142

Effect of Height to Radius (H/R)

In Table 6 other impulsive and convective parameters are shown. These parameters are height (h_c and h_i) and mass ratio (W_C/W_L and W_i/W_L) which convective and impulsive forces are exerted respectively. As it can be observed the height which convective force is exerted is higher than the impulsive height.

This was previously shown by Housner (1963). According to Table 6 which was suggested by Malhotra et al. (2000), the values of convective and impulsive mass and height can be obtained according to (H/R) ratio. There can be found a height and mass for each ratio.

Table 3. Material properties used in the analysis

Materials Properties	Density (kg/m³)	Poisson's Ratio	Young's Modulus (Gpa)	Bulk's Modulus (Gpa)
Steel	7850	0.3	210	-
Fluid (water)	1000	-	-	2.07
Fluid (oil)	800	-	-	1.62
Fluid (gasoline)	680	-	-	1.03

Table 4. Characteristics of the selected tanks

Tank No.	H/R	H (m)	R (m)	Thickness (m)	Number of Elements Fluid	Number of Elements Shell
1	0.3	2.427	8.09	0.01	1728	624
2	0.5	3.414	6.828	0.01	1500	500
3	0.7	4.272	6.103	0.01	960	352
4	1	5.419	5.419	0.01	2100	580
5	1.5	7.101	4.734	0.01	1920	512
6	2	8.602	4.301	0.01	1920	512
7	2.5	9.98	3.993	0.01	2688	640
8	3	11.272	3.755	0.01	2880	672

Table 5. Output of static analysis

Tank No.	H/R	Hydrostatic Pressure (MPa)	Displacement (m) $\times 10^{-3}$
1	0.3	0.021	0.70
2	0.5	0.030	0.76
3	0.7	0.038	0.78
4	1	0.049	0.82
5	1.5	0.066	0.85
6	2	0.079	0.86
7	2.5	0.094	0.88
8	3	0.100	0.89

Table 6. Comparison of impulsive and convective parameters

Tank No.	H/R	Convective Height h_c/H_L	Convective Height h_c (m)	Convective Mass W_C/W_L	Convective Mass W_C (ton)	Impulsive Height h_i/H_L	Impulsive Height h_i (m)	Impulsive Mass W_i/W_L	Impulsive Mass W_i (ton)
1	0.3	0.521	1.264	0.824	412	0.4	0.971	0.176	88
2	0.5	0.543	1.854	0.7	350	0.4	1.366	0.3	150
3	0.7	0.571	2.439	0.586	293	0.401	1.71	0.414	207
4	1	0.616	3.338	0.452	226	0.419	2.27	0.548	274
5	1.5	0.69	4.899	0.314	157	0.439	3.117	0.686	343
6	2	0.751	6.46	0.237	118.5	0.448	3.854	0.763	381.5
7	2.5	0.794	7.924	0.19	95	0.452	4.51	0.81	405
8	3	0.825	9.299	0.158	79	0.453	5.106	0.842	421

In this study, the values of these parameters are obtained using Malhotra table. It can be seen from the table that an increase in the ratio of H/R, leads to an increase and decrease in the values of W_i and W_c, respectively. It must be pointed out that all the tanks have the same volume and weight, while the values of convective and impulsive weight are different.

It can be seen from the Figure 7 and Table 7 that an increase in the value of *H/R* leads to an increase in the values of convective frequencies in all three methods. The maximum values of convective frequencies are obtained for the tank with (*H/R*) = 3, while the highest values of impulsive frequencies is seen in the tank with (*H/R*) = 0.3. It must be pointed out that 10 times increase in the H/R leads to well above double increase in convective frequencies. It can be observed that an increase in the (*H/R*) ratio leads to a decrease in the values of impulsive

frequencies. A more detailed look reveals that this decrease was just below 2.4 times in the case of impulsive frequencies. The reason why the values of impulsive frequencies are decreased is because of the fact that both convective and impulsive frequencies of tanks are dependent on each other. Both these frequencies differed with the liquid level and when *H/R* ratio is increased, the level of liquid are increased in the tanks. Then, the values of convective frequencies are increased and as a result the values of impulsive frequencies are decreased. In fact, they have opposite ratio with each other. This is because of the fact that both these frequencies have a different effective mass and they are divided according to their effective mass. It can be seen that convective frequencies are closer together in both FE method and analytical methods than those obtained for impulsive frequencies through these methods.
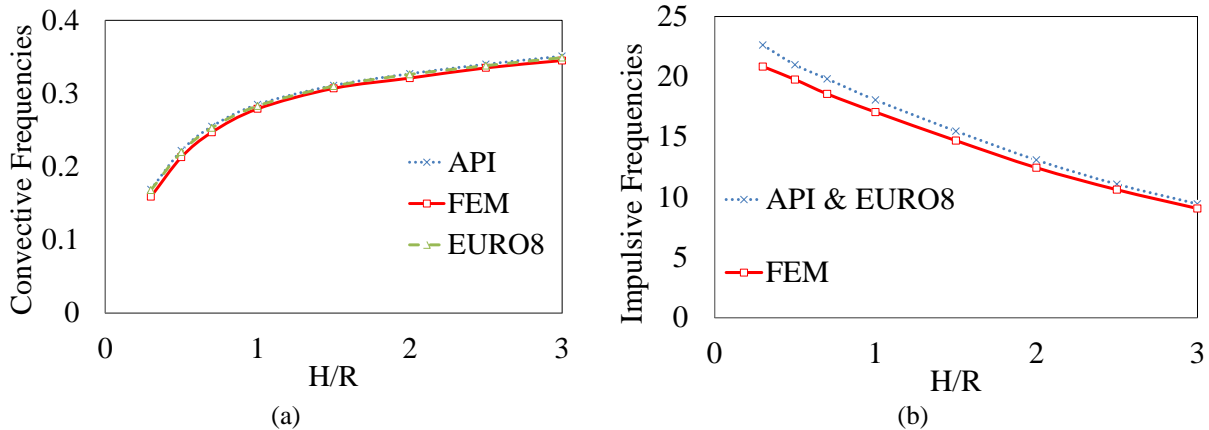

Fig. 5. Comparison of the effect of (*H/R*) on frequencies (Hz); a) convective; b) impulsive

Table 7. Comparing of impulsive and convective frequencies (effect of (*H/R*))

Tank No.	H/R	Convective Frequencies (Hz)			Impulsive Frequencies (Hz)		
		API	EUROCODE8	FEM	API	EUROCODE8	FEM
1	0.3	0.169	0.168	0.159	22.621	22.621	20.841
2	0.5	0.222	0.22	0.213	20.987	20.987	19.76
3	0.7	0.255	0.253	0.247	19.814	19.814	18.567
4	1	0.285	0.283	0.279	18.062	18.062	17.059
5	1.5	0.311	0.310	0.307	15.478	15.478	14.691
6	2	0.327	0.326	0.321	13.081	13.081	12.444
7	2.5	0.340	0.338	0.335	11.077	11.077	10.619
8	3	0.351	0.349	0.345	9.437	9.437	9.062

Furthermore, it is found that the highest values of convective frequencies are obtained under API method and the lowest are obtained under FE method. However, the average difference among all these three methods is nearly one percent. In addition, impulsive frequencies obtained from API and EURO 8 methods are higher than FE method.

Effect of Viscosity

h_c and h_i and mass ratio (W_C/W_L, W_i/W_L) which convective and impulsive forces are exerted respectively and presented in Table 8. As it can be observed the height exerted by the convective force is more than the impulsive height. It can be seen from the table that a decrease in the amount of viscosity leads to a corresponding decrease in the values of W_i and W_c, while the amount of viscosity has no effect on h_i and h_c responses.

Figure 8 and Table 9 present the values of convective and impulsive frequencies. It can be seen from the results that the highest values of convective frequencies are seen in the API method. Also, the lowest values of convective frequencies are obtained for FE method. In addition, it can be clearly seen from Figure 8 that viscosity has no effect on the values of convective frequencies in all methods. The maximum values of impulsive frequencies are obtained for API and EURO 8 method and the minimum values are seen in FE method. It can be concluded from Figure 8 and Table 9 that the maximum values of impulsive frequencies are obtained in tanks which are filled with gasoline and the minimum is obtained for a tank which is filled with water. Therefore, the results show that a decrease in the viscosity leads to a rise in the values of impulsive frequencies. In other words, the values of impulsive frequencies are decreased by increasing in the density parameter, when the density increased by 1.47 times, the impulsive frequencies faced 17 percent reduction. It is more marked with results for convective frequencies, since their

values remained unchanged with changes of density. It must be pointed out that the convective frequencies depend only on dimension of liquid and the other parameters such as weight do not play any role. Conversely, impulsive frequencies are function of parameters such as weight and other conditions of tanks body. This is due to the fact that impulsive frequencies are those which are related to both liquid and cylindrical shell. In other words, liquid moves in unison with the tanks body in impulsive frequency, while convective frequency is one that undergoes sloshing motion and does not depend highly on tanks body and the overall weight of tanks. The other key factor which can be clearly seen in both Table 9 and Figure 8b is related to discrepancy in impulsive frequencies obtained from different methods. However, this is below 10 percent. The integral factor which pose this discrepancy is because of the fact that codes do not consider bulk's module while, in FE software this factor is taken into account.

Effect of Thickness

Since the thickness parameter has a great deal of importance in designing of cylindrical tanks, in this study a tank (no. 3) with radius of 6.103 and height of 4.272 is selected for study on effect of thickness on frequencies. Shell thickness of tank no. 3 is assumed with four different ratios (0.08, 0.01, 0.012, and 0.014). In Table 10, other impulsive and convective parameters are shown. It can be seen from this table that the convective mass part of the liquid is higher than impulsive part.

Table 11 and Figure 9 show that tanks with different thicknesses have the same convective frequencies and the values of thickness have no effect on convective values, while an increase in the ratios of thicknesses lead to increase in the values of impulsive frequencies. This is due to the fact that when the thickness of tanks increased the

shell body of tanks which has to move with fluid part of tank shows more strength against movement. As a result it is understandable why the values of impulsive frequencies are increased with the rise of thickness. The highest values of impulsive frequencies are obtained for tank with thickness of 14 mm. It can be seen that convective frequencies are closer together in both FE method and analytically method than impulsive frequencies. The frequency values of FE are less than API and EURO 8 codes in both convective and impulsive. The highest values of convective frequencies are obtained for API code, while the values of impulsive frequencies in both analytical methods (API and EURO 8) are the same.

Table 8. Comparing of impulsive and convective parameters

Liquid Type	Convective				Impulsive			
	Height		Mass		Height		Mass	
	h_c/H_L	h_c (m)	W_C/W_L	W_C (ton)	h_i/H_L	h_i (m)	W_i/W_L	W_i (ton)
Water	0.521	1.264	0.824	412	0.4	0.971	0.176	88
Oil	0.521	1.264	0.824	349	0.4	0.971	0.176	61
Gasoline	0.521	1.264	0.824	300	0.4	0.971	0.176	52

Table 9. Comparing of impulsive and convective frequencies (effect of viscosity)

Liquid Type	Convective Frequencies (Hz)			Impulsive Frequencies (Hz)		
	API	EUROCODE8	FEM	API	EUROCODE8	FEM
Water	0.169	0.168	0.159	22.621	22.621	20.841
Oil	0.169	0.168	0.159	25.29	25.29	23.135
Gasoline	0.169	0.168	0.159	27.43	27.43	24.88

Table 10. Comparing of impulsive and convective parameters

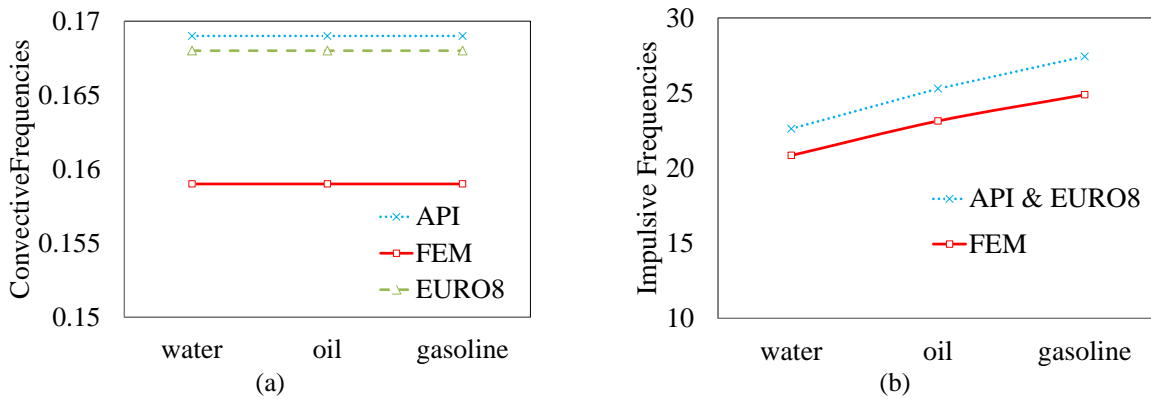| H/R	Convective				Impulsive			
	Height		Mass		Height		Mass	
	h_c/H_L	h_c (m)	W_C/W_L	W_C (ton)	h_i/H_L	h_i (m)	W_i/W_L	W_i
0.7	2.439	0.586	0.586	291	0.401	1.71	0.414	205.6

Fig. 6. Comparison of the effect of viscosity on frequencies (Hz), a) convective, b) impulsive

Table 11. Comparing of impulsive and convective frequencies

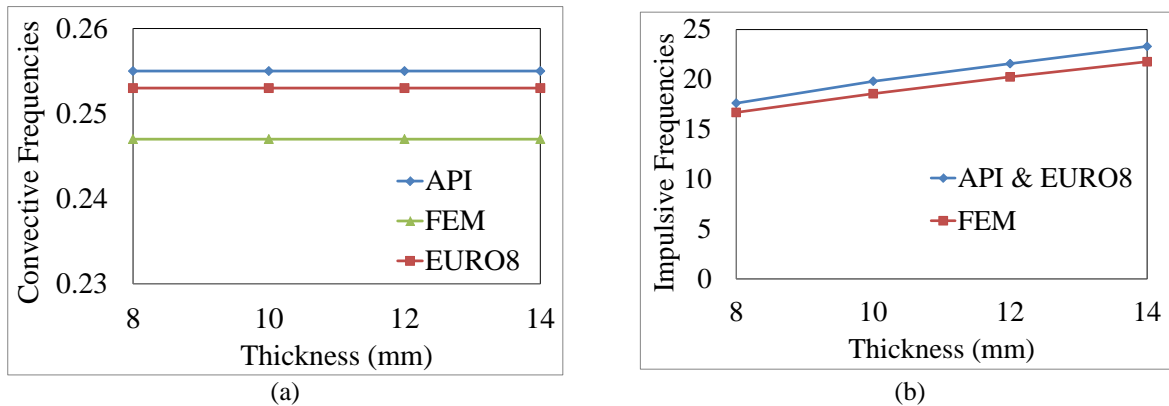| Thickness (mm)	H/R	Convective Frequencies (Hz)			Impulsive Frequencies (Hz)		
		API	EUROCODE8	FEM	API	EUROCODE8	FEM
8	0.7	0.255	0.253	0.247	17.62	17.62	16.684
10	0.7	0.255	0.253	0.247	19.814	19.814	18.567
12	0.7	0.255	0.253	0.247	21.581	21.581	20.248
14	0.7	0.255	0.253	0.247	23.31	23.31	21.776

Fig. 7. Effect of thickness on frequencies (Hz); a) convective; b) impulsive

CONCLUSIONS

In this paper, two fundamental frequencies are obtained using FE, API and EURO 8 methods. To compare the tanks with different (H/R) ratios, all these tanks are modeled with the same volume and weight. In addition, it should be noted that different (H/R) ratios between 0.3 and 3 are chosen. Furthermore, to investigate the effect of liquid type, one of the tanks is filled with three different liquids. Four different thicknesses have been taken into account to investigate the effect of thickness variation. Based on this study, the following observations and conclusions can be drawn:

1- It is resulted that convective frequencies are so close together in FE, API and EURO 8 methods. The discrepancy of convective frequencies between FEM and analytical codes is nearly one percent, while this discrepancy is more marked for impulsive frequencies, with almost five percent average difference.

2- Moreover, the results showed that the height which convective force is exerted is higher than the impulsive height. This was also shown by Housner (1963). It can be resulted that an increase in liquid height leads to an increase and decrease in the value of impulsive and convective mass. Also, a decrease in the amount of viscosity leads to a decrease in the values of W_i and W_c. But the amount of viscosity has no effect on h_i and h_c.

3- This study showed that by increasing the level of liquid, convective frequencies are increased. Also impulsive frequencies are decreased by increasing the level of liquid. This was seen in FE, API and Euro8 methods.

4- This study showed that liquid viscosity has no effect on convective frequencies because of all the convective responses remains unchanged. It was observed that a 1.47 times increase in the viscosity leads to a 17 percent decrease in the value of impulsive frequencies.

5- Results of this study showed that the value of thickness has no effect on the values of convective frequencies. It was shown that an increase in the ratios of thicknesses leads to increase in the value of impulsive frequencies. To be more precise, an increase of 1.5 times in values of thickness led to values of impulsive frequencies rise by 30 percent.

REFERENCES

ACI Committee 350.3-06. (2006). *Seismic design of liquid-containing concrete structures (ACI 350.3-06) and commentary (ACI 350.3R-06),* Farmington Hills (MI, USA), American Concrete Institute.

Amiri, M. and Sabbagh-Yazdi, S.R. (2012). "Influence of roof on dynamic characteristics of dome roof tanks partially filled with liquid", *Thin-Walled Structures*, 50(1), 56-67.

ANSYS, Inc. (2014). *ANSYS release 15.0 Documentation*, USA,

API Standard 650. (2008). *Welded steel tanks for oil storage,* 11th Ed., American Petroleum Institute, Washington, D.C.

Barton, D.C. and Parker, J.V. (1987). "Finite Element analysis of the seismic response of anchored and unanchored liquid storage tanks", *Earthquake Engineering and Structural Dynamics*, 15(3), 299-322.

Bayraktar, A., Sevim, B., Altunışık, A. and Türker, T. (2010). "Effect of the model updating on the earthquake behavior of steel storage tanks", *Journal of Constructional Steel Research*, 66(3), 462-469.

Chen, J.Z. and Kianoush, M.R. (2005). "Seismic response of concrete rectangular tanks for liquid containing structures", *Canadian Journal of Civil Engineering*, 32, 739-52.

Chopra, A.K. (2000). *Dynamics of structures: Theory and applications to earthquake engineering*, 2nd Ed., Prentice-Hall.

Epstein, H.I. (1976). "Seismic design of liquid storage tanks", *Journal of the Structural Division, ASCE*, 102(9), 1659-1673.

Eskandari-Ghadi, M., Rahimian, M., Mahmoodi, A., and Ardeshir-Behrestaghi, A. (2013). "Analytical Solution for Two-Dimensional Coupled Thermoelastodynamics in a Cylinder", *Civil Engineering Infrastructures Journal*, 46(2), 107-123.

Eurocode 8. (2003). *Design of structures for earthquake resistance, Part 1: General rules seismic action and general requirements for structures, Part 4: Silos, tanks and pipelines,* European Committee for Standardization.

Ghaemmaghami, A. (2010). "Dynamic time-history response of concrete rectangular liquid storage tanks", *Dissertation Abstracts International*, 72(08).

Ghateh, R., Kianoush. M.R. and Pogorzelski, W. (2015). "Seismic response factors of reinforced concrete pedestal in elevated water tanks", *Engineering Structures*, 87, 32-46.

Ghateh, R., Kianoush, M.R. and Pogorzelski, W. (2015). "Seismic response factors of reinforced concrete pedestal in elevated water tanks", *Engineering Structures*, 87, 32-46.

Gnitko, V., Marchenko, U., Naumenko, V. and Strelnikova, E. (2011). "Forced vibrations of tanks partially filled with the liquid under seismic load", *In Proceedings of XXXIII Conference "Boundary Elements and other mesh reduction methods", WIT Press, Transaction on Modeling and Simulation,* pp. 285-296.

González, E., Almazán, J., Beltrán, J., Herrera, R. and Sandoval, V. (2013). "Performance of stainless steel winery tanks during the 02/27/2010 Maule Earthquake", *Engineering Structures,* 56, 1402-1418.

Hosseinzadeh, N., Kazem, H., Ghahremannejad, M., Ahmadi, E. and Kazem, N. (2013). "Comparison of API650-2008 provisions with FEM analyses for seismic assessment of existing steel oil storage tanks", *Journal of Loss Prevention in the Process Industries*, 26(4), 666-675.

Housner, G.W. (1963). "The dynamic behavior of water tanks", *Bulletin of the Seismological Society of America*, 53(2), 381-387.

Haroun, M.A. and Tayel, M.A. (1985). "Response of tanks to vertical seismic excitations", *Earthquake Engineering and Structural Dynamic*, 13, 583-595.

Hariri Ardebili, M.A., Mirzabozorg, H., Kolbadi, S. and Mahdi, S. (2013). "Are there any differences in seismic performance evaluation criteria for concrete arch dams?", *Civil Engineering Infrastructures Journal,* 46(2), 233-240.

Jaiswal, O.R., Kulkarni, S. and Pathak, P. (2008). "A study on sloshing frequencies of fluid-tank system", *In Proceedings of the 14th World Conference on Earthquake Engineering,* pp. 12-17.

Kazem, H., and Mehrpouya, S. (2012). "Estimation of sloshing wave height in broad cylindrical oil storage tanks using numerical methods", *Journal of Structural Engineering and Geo-Techniques,* 2(1), 55-59.

Kianoush, M.R. and Chen, J.Z. (2006). "Effect of vertical acceleration on response of concrete rectangular liquid storage tanks", *Engineering Structures,* 28(5), 704-15.

Kianoush, M.R. and Ghaemmaghami, A.R. (2011). "The effect of earthquake frequency content on the seismic behavior of concrete rectangular liquid tanks using the finite element method incorporating soil–structure interaction", *Engineering Structures*, 33(7), 2186-2200.

Korkmaz, K.A., Sari, A., and Carhoglu, A.I. (2011). "Seismic risk assessment of storage tanks in Turkish industrial facilities", *Journal of Loss*

*Prevention in the Process Industrie*s, 24(4), 314-320.

Livaoglu, R., Cakir, T., Dogangun, A., and Aytekin, M. (2011). "Effects of backfill on seismic behavior of rectangular tanks", *Ocean Engineering*, 38(10), 1161-1173.

Maekawa, A., Shimizu, Y., Suzuki, M. and Fujita, K. (2010). "Vibration test of a 1/10 reduced scale model of cylindrical water storage tank", *Journal of Pressure Vessel Technology*, 132(5), 051801.

Malhotra, P.K., Wenk, T. and Wieland, M. (2000). "Simple procedure for seismic analysis of liquid-storage tanks", *Structural Engineering International*, 10(3), 197-201.

Moslemi, M. and Kianoush, M.R. (2012). "Parametric study on dynamic behavior of cylindrical ground-supported tanks", *Engineering Structures*, 42, 214-230.

Ormeño, M., Larkin, T. and Chouw, N. (2015). "Evaluation of seismic ground motion scaling procedures for linear time-history analysis of liquid storage tanks", *Engineering Structures,* 102, 266-277.

Ranjbar, M.M., Bozorgmehrnia, S. and Madandoust, R. (2013). "Seismic behavior evaluation of concrete elevated water tanks", *Civil Engineering Infrastructures Journal*, 46(2), 175-188.

Ruiz, D.P. and Gutiérrez, S.G. (2015). "Finite Element methodology for the evaluation of soil damping in LNG tanks supported on homogeneous elastic half space", *Bulletin of Earthquake Engineering*, 13(3), 755-775.

Ruiz, R.O., Lopez-Garcia, D. and Taflanidis, A.A. (2015). "An efficient computational procedure for the dynamic analysis of liquid storage tanks", *Engineering Structures,* 85, 206-218.

Sezen. H., Livaoglu, R. and Dogangun, A. (2008). "Dynamic analysis and seismic performance evaluation of above-ground liquid containing tanks", *Engineering Structures*, 30, 794-803.

Shahmardani, M., Mirzapour, J. and Tariverdilo, S. (2014). "Dynamic response of submerged vertical cylinder with lumped mass under seismic excitation", *International Journal of Engineering, Transactions A: Basics*, 27(10), 1547-1556.

Tedesco, J.W. (1982). "Vibrational characteristics and seismic analysis of cylindrical liquid storage tanks", *Dissertation Abstracts International Part B: Science and Engineering [DISS. ABST. INT. PT. B- SCI. & ENG.],* 43(3).

Virella, J.C., Godoy, L.A. and Suárez, L.E. (2006). "Fundamental modes of tank-liquid systems under horizontal motions", *Engineering Structures*, 28(10), 1450-1461.

Nicolici, S. and Bilegan, R.M. (2013). "Fluid structure interaction modeling of liquid sloshing phenomena in flexible tanks", *Nuclear Engineering and Design*, 258, 51-56.

Yazdanian, M., Razavi, S.V. and Mashal, M. (2016a) "Seismic analysis of rectangular concrete tanks by considering fluid and tank interaction", *Journal of Solid Mechanics*, 8(2), 435-445.

Yazdanian, M., Razavi, S.V. and Mashal, M. (2016b). "Study on the dynamic behavior of cylindrical steel liquid storage tanks using finite element method", *Journal of Theoretical and Applied Vibration and Acoustics,* 2(2), 144-165.