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Prediction of blasting fragmentation using the mutual information and rock engineering system; case study: Meydook copper mine

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ABSTRACT

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One of the key outcomes of blasting in mines is rock fragmentation that profoundly affects downstream expenses. In fact, size prediction of rock fragmentation is the first step towards the optimization of blasting design parameters. This paper attempts to present a model to predict rock fragmentation using Mutual Information (MI) in Meydook copper mine. Ten parameters are considered to influence fragmentation. On the other hand, Rock Engineering System (RES) is employed in order to compare different models. To validate the results, six blasting scenarios were selected and the results were compared. The coefficient of correlation (R²), Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) were used to assess the performance of presented models. The R², RMSE and MAE values for 30 blasting cycles were calculated to be 0.81, 10.7, and 9.02 for MI model, and 0.75, 11.87, and 9.61 for RES, implying the better capability of MI model to predict fragmentation.

Keywords : Blasting, Mutual Information, Fragmentation, Rock Engineering System

1. Introduction

Activities related to rock engineering such as underground opening excavation, surface excavation and tunnel construction interfere with initial state of the ground causing a permanent interaction between rock mass characteristics and the structure itself [1]. Rock fragmentation as one of the most important aspects of blasting, has been an important topic among researchers [2]. This is because inappropriate fragmentation may significantly increase different mining expenses such as secondary drilling and blasting costs. Downstream expenses such as loading and hauling, on the other hand, may cause rock crushing machinery to operate inefficiently [3]. The parameters affecting fragmentation can be classified into two categories. The first category includes controllable parameters such as pattern design of blasting and explosives while the second category is composed of those parameters which may not be under control such as geomechanical characteristics of rock mass [4].

Due to fragmentation, size prediction is the first step towards blast optimization process to reach desirable fragmentation [5]. In underground and surface activities, where blasting process is the key part of production cycle, the rock mass blastability should be greatly analyzed. However, since there are numerous parameters affecting rock blastability, application of analytical methods seems to be challenging.

Rock Mass Rating (RMR) was presented by Bieniawski in 1976, and is currently agreed to be one of the most globally comprehensive and commonly-used methods of rock mass classification [6]. Hudson in 1992 presented the Rock Engineering System (RES) where every individual parameter is considered as a two-variable system [7]. RES possesses a variety of applications in different rock engineering activities such as stability analysis of underground openings [8], geo-hazard due to TBM tunneling [9], and system assessment for rock mass blastability. Lu and Latham in 1999 used RES to predict size distribution of blasting-induce fragmentation. In terms of blastability, their classification categorizes rock mass into five groups of very easy, easy, medium, hard, and very hard [10]. Faramarzi et al. used RES to predict fragmentation based on interaction matrix [11].

In order to form the interaction matrix, for RES method using experts view, a variety of expert perspectives are used. Obviously, experiences and knowledge of each expert differ from those of others; therefore, the results highly depend on experts and may change if the group of experts change. This is considered to be a weakness of the method.

Several empirical and artificial intelligent methods are widely used to predict blast fragmentation [11-13]. However, empirical methods usually consider a few number of parameters [11]. In these methods, it is not easy to involve many variables. On the other hand, most intelligent methods need large number of data to result acceptable accuracy [12-13]. In this paper, we have proposed the Mutual Information (MI) method as a new approach to overcome both aforementioned shortcomings. The mutual information is the information that two random variables have in common [14]. One of the most applicable performance criteria for feature selection is be mutual information [15]. In addition, mutual information is employed to determine how dependent two random parameters are [16]. The great advantage of mutual information is to decrease the amount of randomness of a parameter due to the other random parameter. Using this characteristic one can determine the weight factor of each parameter from the data that significantly increases the analysis accuracy. Therefore, application of MI method eliminates possible inaccuracies resulted from expert's perspective and can incorporate unlimited number of parameters for available data.

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2. General geology of Meydook copper mine

Meydook mine is located 42 kilometers north east of Shahrebabak district on the copper - rich belt in Kerman province, Iran. It is geologically considered as part of the Alp-Himalaya orogeny. Meydook copper ore mine has a reserve of 171 million tons at an average grade of 0.83%. In Meydook mine, the waste material is generally composed of altered andesite and the ore is mostly porphyritic diorite [17].

Currently, the mine pit is ??? m deep, and Emolan and cordtex are used in benches lower than the ground water level, and ANFO and Nonel system are used in dry areas. The extraction process is carried out in an open pit with 15-meter high benches and a slope of 70°. Figure 1 shows a collapsed boulder in Meydook mine which causes adverse problems in current blasting operations.

2.1. Data acquisition

Data of 36 blast cycles from Meydook mine were used in this research. The data from 30 blast cycles were used as the models input data and the other six datasets were employed to validate the results. Uniaxial compressive strength and density parameters of different rock types were adopted from a report prepared by Marefvand in 2012 [18]. Rock Quality Designation (RQD) and stemming for each blast were derived from geological investigation of each blast in the mine. Parameters such as joint spacing, joint persistence, joint plane orientation ratio to bench face, burden, ratio of boreholes spacing to their diameters, specific charge and 80% passing size (D_{80}) were directly measured and/or surveyed in mine site. In order to survey the joints, dominant joints were determined at first, and then, their characteristics were specified through selecting a scan line using compass and measuring tape. Data

for each block were independently surveyed and measured as the rock type changed.



Fig. 1. A sample of boulder collapse problem in Meydook mine.

Image processing was used to plot the grain size distribution as well as D_{80} . Two balls of 23 cm diameter were used for scaling the images. Considering the size of blasting blocks, 20 images on average were taken from each bench face, and were analyzed by Split-Desktop software. Figure 2 presents a typical image analysis along with grain size distribution graph for blast block 2615-680 (bench 2615 and blasting cycle of 680). In addition, Table 1 provides the statistical description of 30 selected blasts performed in Meydook mine. It should be noted that the rock type varies throughout the mine in different zones.



Fig 1. Image analysis for blast block 2615-683.

Table 1. Main parameters for 30 selected blasts in Meydook copper mine.

Demonstern	Descenden descelation and write	Course had	Minimum meeting and makes	Maximum measured	Standard
Parameter	Parameter description and unit	Symbol	Minimum measured value	value	deviation
P1	Uniaxial compressive strength (MPa)	UCS	13.56	150.43	56.68
P ₂	Joint persistency (m)	Р	1.5	13	3.02
P3	Rock Quality Designation	RQD	10	45	10.76
P4	Joint Spacing (m)	JS	0.1	3.5	0.94
P5	Density (t/m ³)	ρ	2.25	2.68	0.18
P ₆	Specific charge (kg/m ³)	q	0.26	0.5	0.07
P ₇	Burden (m)	В	5	6.5	0.58
P ₈	Stemming (m)	\mathbf{S}_{t}	4	9	1.30
P ₉	ratio of boreholes spacing to their diameters	S/D	34.44	52.49	4.23
P ₁₀	Joint plane orientation ratio to bench face	JPO	-	-	-
P ₁₁	80% passing size (cm)	D ₈₀	7.65	92.88	24.6

3. Rock engineering system

Rock engineering system was first introduced by Hudson in 1992 to deal with intricate engineering problems, and is one the most powerful methods in rock engineering. Interaction matrix is the core of rock engineering system. In this matrix, the main parameters are arranged along the main diagonal elements of a matrix and the interrelations between pairs of parameters are identified in off-diagonal elements. There are variety of methods to quantify the interaction matrix such as 0-1 binary method, Expert Semi-Quantitative (ESQ) and Continuous Quantitative Coding (CQC) [7]. The most common method is ESQ in which interaction intensity is denoted by the values from 0 (no interaction) to 4 (critical interaction). In the interaction matrix, the sum

of values in a row (
$$c_{p_i} = \sum_{j=1}^n \mathrm{I}_{ij}$$
) is called "Cause" value and the sum of

values in a column ($_{E_{p_i}} = {\displaystyle \sum_{i=1}^{n}} l_{ij}$) is called "Effect" value [19]. Therefore,

the more the value of C+E is for a parameter, the more important that parameter will be in the system, implying that there is significant interaction between this parameter and the system. It is noted that there is no limitation on the number of parameters in rock engineering system. For example, n by n matrix is used to define a system with n parameters. Figure 3 shows an interaction matrix.



Figure 2. Interaction matrix in RES [7]

C+E value may be used as weight factor of parameters (a_i) as shown in equation 1 [7].

$$a_{i} = \frac{(C_{i} + E_{i})}{(\sum_{i=1}^{n} C_{i} + \sum_{i=1}^{n} E_{i})} \times 100$$
(1)

3.1. Fragmentation prediction using rock engineering system

The main goal of using RES in vulnerability index, presented by Benardos for the first time, was to detect hazardous sections of ground affecting the performance of a TBM [9]. In this paper, a similar method is applied to predict fragmentation. As mentioned earlier, a variety of parameters can affect fragmentation. Considering previous researches in this field, ten parameters were selected as the main items for Meydook copper mine [1, 2, 5, 10]. Regarding interaction matrix, the effect of every parameter is rated from 0 (no interaction) to 4 (critical interaction). The rating system is presented in Table 2. Table 3 shows interaction matrix used for RES, it should be noted that the interaction matrix is finally evaluated and completed by blasting experts. Weight factor of each parameter is then calculated using equation 1. Table 4 shows 'Cause' and 'Effect' values and calculate 'ai' for each parameter.

Coding Description				
0	No interaction			
1 Weak interaction				
2 Medium interaction				
3 Strong interaction				
4 Critical interaction				

Table 3. The interaction matrix for the parameters affecting fragmentation

					-				0	
	UCS	Р	RQD	JS	ρ	q	В	\mathbf{S}_{t}	S/D	JPO
UCS	P_1	1	2	2	0	3	4	4	2	0
Р	0	P_2	2	1	0	1	1	2	1	1
RQD	0	1	P_3	1	0	3	2	2	2	0
JS	0	2	4	P_4	0	3	3	2	2	2
ρ	3	1	2	0	P_5	1	1	1	1	0
q	0	0	0	0	0	P_6	2	2	2	0
В	0	0	0	0	0	3	P_7	0	3	0
S_t	0	0	0	0	0	3	3	P_8	2	0
S/D	0	0	0	0	0	2	3	1	P ₉	0
JPO	0	2	2	2	0	2	3	1	2	P_{10}

parameter	C value	E value	$(\sum_{i} C_{i} + \sum_{i} E_{i})$	$a_{i} = \frac{(C_{i} + E_{i})}{(\sum_{i=1}^{10} C_{i} + \sum_{i=1}^{10} E_{i})} \times 100$
P_1	18	3	212	9.9
P ₂	9	7	212	7.55
P ₃	11	12	212	10.85
P_4	18	6	212	11.32
P ₅	10	0	212	4.72
P ₆	6	21	212	12.74
P ₇	6	22	212	13.21
P_8	8	15	212	10.85
P ₉	6	17	212	10.85
P ₁₀	14	3	212	8.02

Table 4. Weight factor of different parameters for RES met	thod
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Blast Quality Index (BQI), a parameter presented in this research, is calculated using equation 2.

$$BQI = 100 - \sum_{i=1}^{n} a_i \frac{Q_i}{Q_{max}}$$
⁽²⁾

Where a_i is weight factor of *ith* parameter (%), Q_i is the rating of *ith* parameter and Q_{max} represents the maximum value of rating assigned to *ith* parameter, shown in Table 5. This table shows scoring for a different range of values for each parameter which are proposed based on judgment and previous researches [20-21]. Scoring of the parameters is based on the way they influence the fragmentation. Five classification groups, each scored from 0 to 4, are considered so that 0 is referred to as poor fragmentation and 4 is referred to as desirable fragmentation. BQI always ranges between 0 and 100. The high value of BQI represents poor fragmentation whereas low values refer to as desirable fragmentation.

Table 5. Proposed ranges for the parameters effective in fragmentation [20-21]

		Rating (Q _i)	0	1	2	3	4
1	UCS (MPa)	Value	≥150	150-100	100-50	50-25	25-1
2	P (m)	Value	< 2	2-5	5-8	8-12	12≤
3	RQD	Value	≥60	60-40	40-25	25-15	<15
4	JS (m)	Value	≥2	2-0.6	0.6-0.4	0.4-0.2	<0.2
5	ρ (t/m³)	Value	≥2.5	2.5-2.3	2.3-2	2-1.6	<1.6
6	q (kg/m³)	Value	0.2-0.3	0.3-0.4	0.4-0.5	0.5-0.6	0.6-0.7
7	B (m)	Value	≥8	6-8	4-6	2-4	<2
8	S _t (m)	Value	≥10	10-8	8-6	6-3	<3
9	S/D	Value	≥50	50-40	40-30	30-20	<20
10	IPO	Value	HOR	DOF	SNF	SAF	DIF

HOR = Horizontal, DOF = Dip out of face, SNF = Strike normal to face, SAF = Strike at an angle acute to face, DIF = Dip into face

 D_{80} variation has a close relationship with BQI. A linear regression between D_{80} and BQI shows a promising correlation as the following equation.

$$D_{80} = 1.6(BQI) - 44.73$$
 , $R^2 = 0.75$ (3)

4. Information theory

Information theory is based on probability theory and statistics. Information theory often concerns itself with measures of information of the distributions associated with random variables [22]. It was initially developed by Claude E. Shannon to find essential limits on signal processing and communication operations such as data compression. Important quantities of information are entropy, a measure of information in a single random variable, and mutual information, a measure of information in common between two random variables [23].

Let X be a discrete random variable with probability mass function of $P(x) = Pr[X=x], x \in \chi$, then the entropy function is defined as follows:

$$H(x) = -\sum_{x \in \chi} p(x) \cdot \log p(X)$$
(4)

If the base of the logarithm is b, we denote the entropy as $H_b(x)$. For example, if the base of the logarithm is e or 2, the entropy is then measured in terms of nat or bit, respectively. Nat and bit are units of information entropy. It should be noted that entropy is a function of X distribution and it does not depend on assigned values to X. Therefore the results do not depend on selected units for parameters.

In fact, the fundamental concept of entropy in information theory is defined as how random a signal or an event is. Entropy, known also as Shannon entropy, expresses randomness as a mathematical quantity.

Joint entropy H(x,y) for two random variables x and y is in relationship with Joint distribution P(x,y) as defined in equation 5:

$$H(x, y) = -\sum_{x \in \chi} \sum_{y \in \gamma} p(x, y) \log p(x, y)$$
(5)

Where two random variables are independent, i.e. p(x,y)=p(x).p(y), equation 5 can be rewritten as equation 6 [24]:

$$H(x, y) = H(x) + H(y)$$
 (6)

Furthermore, entropy function can be generalized beyond two random variables as stated in equation 7 [24]:

$$H(x, y, z) = -\sum_{x \in \chi} \sum_{y \in \gamma} \sum_{z \in Z} p(x, y, z) \log p(x, y, z)$$
(7)

4.1. Mutual information

As mentioned earlier, entropy is a measure to show randomness of a variable. Similarly, conditional entropy is defined as H(x | y) that measures the residual information in X, while the values of Y are known. Equation 8 shows the chain rule of conditional entropy.

$$H(X, Y) = H(X) + H(Y|X)$$
 (8)

If two variables x and y are independent, resulting in H(x|y)=H(x), then knowing Y has no effect on remaining information in X. Reduction in the uncertainty of X due to knowing Y value is referred to as mutual information [24]. In fact, mutual information is defined as the amount of information in common between two random variables. Mutual information for two random variables is defined as equation 9.

$$I(X;Y) = H(X) - H(X|Y) = H(X) + H(Y) - H(X,Y)$$
(9)

Equation 9 can be also expressed in another form as follows:

$$I(X;Y) = \sum_{x,y} p(x,y) \log \frac{p(x,y)}{p(x).p(y)}$$
(10)

MI method has numerous advantages such as having unaffected interpretation of variables in terms of uncertainty reduction, the capability of measuring nonlinear relationship among variables, the applicability for multi-variable random variables [15].

As mentioned earlier, MI is a measure of dependence of two random variables. Two variables are independent if MI is 0, meaning that any increase in mutual information raises the dependence between two variables. When MI values are known, every MI value is then normalized with the summation of all MI values to calculate the weight factor for each parameter, as stated in equation 11.

$$a_{i} = \frac{MI_{i}}{\sum_{i=1}^{n} MI_{i}}$$
(11)

Figure 4 demonstrates the relationship between H(x), H(y), H(x,y), H(x|y), H(y|x) and I(x;y) expressed in Venn diagram. As shown in this Figure, mutual information confirms the amount of common information between X and Y [24].



Figure 3. Relationship between entropy and mutual information [23]

4.2. Prediction of fragmentation using mutual information

In order to predict the fragmentation, equation 2 is used along with the rating system that was explained for RES. The only change here is the way a_i is calculated. Weight factor of each parameter (a_i) is calculated using MI with no dependency on expert's opinion. Table 6 shows a_i for each parameter and the calculation of BQI for a selected blast in block 2540-302 in Meydook copper mine. Equation 12 shows linear regression between D_{80} and BQI using the data from 30 blast cycles. It can be seen that the coefficient of correlation (R^2) for this equation is higher than that of similar equation for RES model. It is clear that BQI in this equation is calculated based on MI model. Figure 5 depicts the comparison between weight factor of each parameter calculated by RES and MI models.

Table 6. Corresponding BQI for blast block 2540-302 in MI model, Meydook copper mine										
Parameter	UCS	Р	RQD	JS	ρ	q	В	St	S/D	JPO
Value of parameter	68.76	11	25	0.45	2.61	0.3	5.5	6	45.93	HOR
Q _i /Q _{max}	0.5	0.75	0.5	0.5	0	0.25	0.5	0.5	0.25	0
Weight factor of parameters (a_i)	16.24	13.83	12	14.97	12.25	9.89	4.22	5.53	5.68	5.39
BQI=59.25										

$$D_{80} = 1.24(BQI) - 23.17$$
, $R^2 = 0.81$ (12)



Figure 4. Comparison between weight factor of parameters in MI and RES

5. Comparison and evaluation of models proposed for fragmentation prediction

Six blast cycles were considered to evaluate the proposed models i.e. MI and RES. Measured and predicted values of D₈₀ using both models are presented in Table 7 for six blast cycles which were considered for evaluation. There are several ways to compare the models; R², RMSE (Root Mean Square Error) and MAE (Mean Absolute Error) have been used as the comparison criteria by many authors. MAE and RMSE can be used together to diagnose the variation in the errors in a set of forecasts. RMSE will always be larger or equal to MAE; the greater difference between them, the greater the variance in the individual errors in the sample [25]. In general, application of RMSE is more appropriate than MAE when model errors follow a normal distribution. [25]. Shapiro-wilk test revealed that the errors of both models follow normal distribution. However both criteria were used for comparison. The R², RMSE and MAE values for 30 blast cycles were respectively calculated as 0.81, 10.7, and 9.02 from MI model, and 0.75, 11.87, and 9.61 RES model (Table 8). It clearly shows that MI model yields less error and manifests stronger relation between BQI and D₈₀ compared to that of RES model. Moreover, 6 blast cycles were also considered to evaluate the proposed models. The RMSE and MAE values for MI models are 8.51 and 7.55, respectively, confirming acceptable accuracy and efficiency of MI model. Also, the greater difference between RMSE and MAE in RES model shows a higher inconsistent error size. Figure 6 presents a comparison between predicted and measured D₈₀ values for six blast cycles. As shown in this figure, all models are generally able to predict D₈₀ with an acceptable accuracy. MI model has better prediction in some cases while in some other cases RES is more accurate. Among these two models, it is advised to use MI since it directly uses the data sets to calculate weight factor of each parameter.

6. Conclusions

In this paper, Mutual Information (MI) and RES models were used to predict fragmentation. In Meydook copper mine, 36 blast cycles were chosen, six of which were considered to validate the models. Although there is no limitation in the number of input parameters for both models, MI model is expected to show better accuracy since the weight factor of each parameter is derived from direct interpretation of data. R², RMSE and MAE were used to validate the results obtained from MI and RES models. Using the results of 30 blast cycles, the R², RMSE and MAE values were calculated as 0.81, 10.7, and 9.02 for MI and 0.75, 11.87, and 9.61 for RES model, confirming that MI model possesses a higher accuracy level. Finally, six blast cycles were also used to validate the results. For validating the datasets, RMSE and MAE values for MI model were respectively 8.51 and 7.55, showing a promising level of accuracy.

Table 7. D_{80} predicted by models MI and RES for six blast cycles, Meydook copper mine

Blast number	Measured D ₈₀ (cm)	Predicte	ed D80 (cm)
		RES	МІ
31	21.05	30.24	25.01
32	35.92	33.81	29.66
33	49.75	42.84	39.39
34	44.31	48.11	48.10
35	58	44.17	43.17
36	49.66	45.67	43.53

Table 8. Comparison between MI and RES Models in Meydook copper mine						
	Model	R ²	RMSE	MAE		
For 30 Blast cycles	MI	0.81	10.7	9.02		
FOI 50 Blast cycles	RES	0.75	11.87	9.61		
For six validation blast	MI	-	8.51	7.55		

7.73

6.63

RES

cycles







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