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A novel computational procedure based on league championship algorithm for solving an inverse heat conduction problem

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Abstract

Inverse heat conduction problems, which are one of the most important groups of problems, are often ill-posed and complicated problems, and their optimization process has lots of local extrema. This paper provides a novel computational procedure based on finite differences method and league championship algorithm to solve a one-dimensional inverse heat conduction problem. At the beginning, we use the Crank-Nicolson semi-implicit finite difference scheme to discretize the problem domain and solve the direct problem which is a second-order method in time and unconditionally stable. The consistency, stability and convergence of the method are investigated. Then we employ a new optimization method known as league championship algorithm to estimate the unknown boundary condition from some measured temperature on the line. League championship algorithm is a recently proposed probabilistic algorithm for optimization in continuous environments, which tries to simulate a championship environment wherein several teams with different abilities play in an artificial league for several weeks or iterations. To confirm the efficiency and accuracy of the proposed approach, we give some examples for the engineering applications. Results show an excellent agreement between the solution of the proposed numerical algorithm and the exact solution.

Keywords: Inverse problem**,** League championship algorithm, Finite differences scheme, Numerical solution.

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1. Introduction

In many industrial problems it is necessary to specify the temperature inside some objects or determine the temperature in a boundary which both of them are unreachable or the temperature is very high to be measured by a thermometer. In such cases the temperature can be calculated from some measured temperature history at a fixed location that is measurable. The thermal properties in heat conduction problems has a great importance for temperature field in many engineering problems. The thermal properties, such as the heat capacity and the thermal conductivity, can be considered temperature-dependent, because these values can be changed by the variation of temperature especially for large variations. The identification of these properties and temperatures is called inverse problems and especially in this cases Inverse heat conduction problems which are known as IHCPs in the literature [1-7]

Shidfar et al. have solved a nonlinear inverse heat conduction problem to estimate unknown diffusion coefficients by using the Taylor's series expansion as one of the examples of IHCPs [5]. In another paper, Shidfar et al. have used the same method to find the unknown function in an inverse problem and it showed different uses of these methods [4]. Ebrahimi et al. have introduced a new method to solve a two dimensional inverse heat conduction problem by using the Monte Carlo method and represented a biological example to show the advantages of IHCPs in other branches of science [1]. Farnoosh and Ebrahimi have employed the same method to find diffusion coefficient as a thermal property of a one dimensional inverse heat conduction problem [2].

Luo and Yang have defined and solved an IHCP for estimation of total heat exchange factor in reheating furnace by solving an inverse heat conduction problem [3]. Furthermore Li et al. have studied the conductive heat flux from concrete to liquid nitrogen by solving an inverse heat conduction problem [6]. Li et al. have investigated a modified space marching method using future temperature measurements for transient nonlinear inverse heat conduction problem [7].

The above examples from the literature are described to show the different kinds and advantages of IHCPs.

This paper represents a numerical estimation of temperature distribution in a one dimensional inverse parabolic problem using a combined numerical-probabilistic algorithm. The proposed algorithm is based on finite differences scheme to solve directed problem and LCA algorithm to estimate the unknown coefficients via an optimization problem. As inverse problems are usually ill posed problems and hard to solve it needs to use a very stable method to discretize the partial differential equation [1-7]. The Crank-Nicolson method as a finite differences method is a far-reaching method to solve partial differential equations and it is unconditionally stable as was proven by Crank and Nicolson [8, 9]. The advantage of a semi-implicit method is that retains much of the speed of the implicit method without sacrificing all of the higher quality.

LCA method or or league championship algorithm is a stochastic meta-heuristic algorithm for optimizing numerical functions which is introduced as a sport driven algorithm. This methodology simulates some sport teams competitions in a league. New solutions are generated by metaphorical strengths, weaknesses, opportunities, threats (SWOT) analysis which is new planning of coaches for the next games. The main thing that makes a team stronger than the other is the strength of its players which is the quantity of each variable and could be changed as random variables [10].

In fact there is a very simple path to solve the IHCPs that is shown in the Fig. 1:

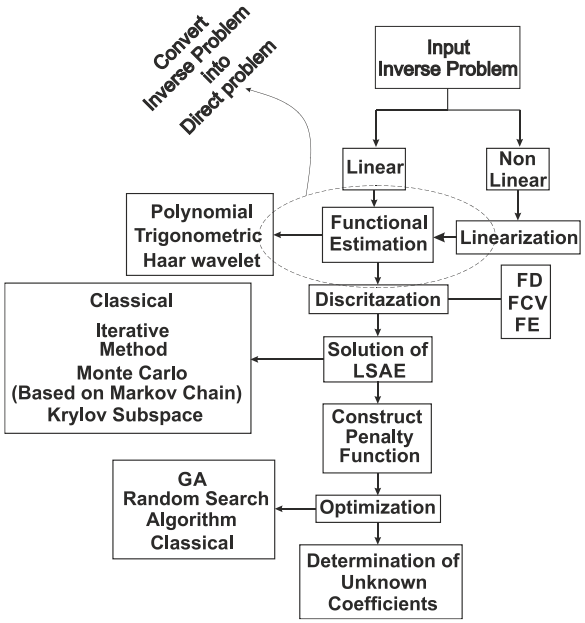

Fig. 1: *A framework for solving inverse problems*

The main part that have been changed in several ways to have different solutions is the optimization part of the chart. Some has used classic ways to optimize the numerical functions and some has used heuristic and meta-heuristic methods. Probabilistic methods as heuristic and meta-heuristic methods are faster but classic methods are more accurate than the others. You can use each of them in the case that you have much of time and currency or you need less accuracy.

Many of these methods have been used during the years to solve one-dimensional inverse heat conduction problems [2, 11-22]. In the following some of them have been named along with their benefits as a solution.

Farnoosh and Ebrahimi have introduced a numerical algorithm involving the combined use of the finite differences scheme and Monte Carlo method for estimating the diffusion coefficient in a one-dimensional nonlinear parabolic inverse problem [2]. Then they introduce a random search algorithm in Monte Carlo method for global optimization and they found that this method is performable for parallel processing.

Shivanian and Khodabandehlo have proposed and investigated an application of meshless local radial point interpolation on a one-dimensional inverse heat conduction problem [15]. Jolly and Autrique have applied a semi-analytic conjugate gradient method to a simple inverse heat conduction problem [13]. Beck and Woodbury have studied sensitivity coefficient insights, filter coefficients, and intrinsic verification of inverse heat conduction problems [11]. Duda has presented numerical and experimental verification of two methods for solving an inverse heat conduction problem [12]. Lu et al. have studied A two-dimensional inverse heat conduction problem for simultaneous estimation of heat convection coefficient, fluid temperature and wall temperature on the inner wall of a pipeline [14].

Woodbury and Beck have examined the structure of the Tikhonov regularization problem and concludes that the method can be interpreted as a sequential filter formulation for continuous processing of data [21]. Lee et al. have applied an inverse algorithm based on the conjugate gradient method and the discrepancy principle to solve the inverse hyperbolic heat conduction problem in estimating the unknown time-dependent surface heat flux in a living skin tissue from the temperature measurements taken within the tissue [18]. Pourgholi et al. have proposed a numerical algorithm combined with the genetic algorithm and least squares method to solve inverse heat conduction problems [20]. Also they used the parallel processing as a benefit of the genetic algorithm to speed up the processing time.

Movahedian and Boroomand have introduced a numerical algorithm to solve inverse and direct layered materials by expressing the solution with some series of exponential basis functions (EBFs) defined in space and time as the main idea of that presented method [19]. Some examples have approved the stability of the algorithm. Fernandes et al. have proposed a transfer function identification (or impulse response) method to solve inverse heat conduction problems [17]. That technique was based on Green's function and the equivalence between thermal and dynamic systems. The work was concluded with the application of the technique in an experimental case of temperature estimation at the tool-work-piece interface during a machining process. Cui et al. have proposed an optimization technique, using which the relaxation factor is adaptively updated at each iteration, rather than a constant during the whole iteration process [16]. One- and two-dimensional transient nonlinear inverse heat conduction problems was involved, and the instability issues occurred in their previous works were reconsidered [13].

Wu et al. have introduced an inverse algorithm based on the conjugate gradient method and the discrepancy principle to solve the inverse hyperbolic heat conduction problem with the dual-phase-lag heat transfer models [22]. Results was shown that an excellent estimation on the time-dependent pulse heat flux could be obtained. The existence and uniqueness of the solutions to this problem and also some more applications are discussed [2, 11-22] But, the numerical solutions of these problems are far from satisfactory.

2. Mathematical Formulation

In this section direct and inverse problems are introduced. In subsection 2.1 the direct problem its formulation which are concerned to determination of surface temperature in a road, is considered also in subsection 2.2 an inverse heat conduction problem which is raised from the above mentioned direct problem is introduced.

3. Direct Problem

The mathematical formulation of a one-dimensional linear parabolic problem is given as follows:

Consider that $t_f > 0$ and let $T = \{(x,t) : x \in (0,1), t(0,t_f)\}$, and to determine the function $T(x,t)$ for the initial and boundary value problem:

$$T_t(x,t) = T_{xx}(x,t), \quad \circ < x < 1, \quad \circ < t < t_f \tag{1}$$

$$T(x,\circ) = f(x), \qquad \circ \le x \le 1 \tag{2}$$

$$T(\circ,t) = p(t), \qquad \circ \le t \le t_f \tag{3}$$

$$T(1,t) = q(t), \qquad \circ \le t \le t_f \tag{4}$$

Where $f(x)$ is a known and continuous function, $p(x)$ and $q(x)$ are infinitely differentiable known functions

and t_f is the time of the last measurement of the case**.** The problem which is numbered by Eqs. (1) to (4) is a direct problem. The direct problem shown here is interested with the determination of the distribution of the temperatures on a one dimensional environment when the initial condition $f(x)$ and the boundary conditions $p(x)$ and $q(x)$ are known continuous functions. Fig. 2 shows the physics of a one dimensional heat conduction problem.

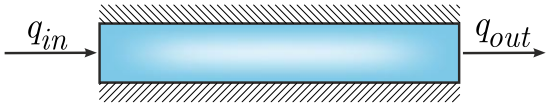

Fig. 2: *One dimensional heat conduction example*

3.1. Inverse Problem

Now, we consider the above heat conduction problem while function $q(t)$ is unknown and an over specified condition is also considered available. To specify the unknown condition $q(t)$ some additional information is needed. This information is some internal temperature measurements.

$$T(x_1,t)=s(t), \qquad 0<x_1<1, \quad \circ \le t \le t_m \tag{5}$$

It is absolutely clear that for an unknown boundary condition $q(t)$ the problem is under-determined and there is several answers. It is evident that to have a unique solution we are imposed to have additional information.

Friedman has shown that, if $f(x)$, $p(t)$ and $q(t)$ are continuous functions and the $f(0)=p(0)$ then the problem (Eqs. (1) to (5)) has a unique solution [23].

4. Numerical Algorithm

In this section the numerical algorithm which is used in this study in details is discussed. The Crank-Nicolson as a finite difference scheme is used for discretizing the problem domain is discussed in subsection 3.1.

In subsection 3.1.1 and 3.1.2 the stability, consistency and convergence of the scheme are respectively proved.

Finally in subsection 3.2 LCA method is introduced.

4.1. Finite difference method for discretizing

To discretize the problem domain the Crank-Nicolson scheme as a semi-implicit finite difference approach is employed:

$$\frac{T_i^{n+1}-T_i^n}{\Delta t}=\frac{\left[\left(T_{i-1}^{n+1}-2T_i^{n+1}+T_{i+1}^{n+1}\right)+\left(T_{i-1}^{n}-2T_i^{n}+T_{i+1}^{n}\right)\right]}{2\Delta x^2} \tag{6}$$

By letting $r=\dfrac{\Delta t}{2\Delta x^2}$ we obtain:

$$-rT_{i+1}^{n+1}+2(1+r)T_i^{n+1}-rT_{i-1}^{n+1}=rT_{i+1}^{n}+2(1-r)T_i^{n}+rT_{i-1}^{n} \tag{7}$$

So By using Eq. (7), the following linear algebraic system of equations is divided:

$$\begin{pmatrix} 2+2r & -r & 0 & 0 & 0 & 0 & 0 \\ -r & 2+2r & -r & 0 & 0 & 0 & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & -r & 2+2r & -r \\ 0 & 0 & 0 & 0 & 0 & -r & 2+2r \end{pmatrix} \begin{pmatrix} T_1^{n+1} \\ T_2^{n+1} \\ . \\ . \\ . \\ T_{N-2}^{n+1} \\ T_{N-1}^{n+1} \end{pmatrix} = \begin{pmatrix} 2-2r & r & 0 & 0 & 0 & 0 & 0 \\ r & 2-2r & r & 0 & 0 & 0 & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & r & 2-2r & r \\ 0 & 0 & 0 & 0 & 0 & r & 2-2r \end{pmatrix} \begin{pmatrix} T_1^{n} \\ T_2^{n} \\ . \\ . \\ . \\ T_{N-2}^{n} \\ T_{N-1}^{n} \end{pmatrix} + \begin{pmatrix} T_0^n+T_0^{n+1} \\ 0 \\ . \\ . \\ . \\ 0 \\ T_N^n+T_N^{n+1} \end{pmatrix} \tag{8}$$

Linear system (8) has $N-1$ unknown essential values through the boundary $x=1$.

4.1.1. Stability of the finite difference scheme

The stability of the scheme is discussed in this session. The scheme that is used is unconditionally stable. To prove stability by the Fourier series method, Von Neumann's method, it is assumed that the error function is:

$$E_i^n=e^{\hat{i}\beta i\Delta x}\xi^n \tag{9}$$

where $\xi=e^{\alpha\Delta t}$ and α in general, is a complex constant. The error could not be increased as t increases provided that:

$$|\xi| \le 1 \qquad (10)$$

By Substituting E_i^n into Eq. (2-7):

$$-re^{\hat{i}\beta(i+1)\Delta x}\xi^{(n+1)} + 2(1+r)e^{\hat{i}\beta i\Delta x}\xi^{(n+1)} - re^{\hat{i}\beta(i-1)\Delta x}\xi^{(n+1)} = re^{\hat{i}\beta(i+1)\Delta x}\xi^{n} + 2(1-r)e^{\hat{i}\beta i\Delta x}\xi^{n} + re^{\hat{i}\beta(i-1)\Delta x}\xi^{n} \qquad (11)$$

Divided by $e^{\hat{i}\beta i\Delta x}\xi^n$ leads to:

$$-re^{\hat{i}\beta\Delta x}\xi + 2(1+r)\xi - re^{-\hat{i}\beta\Delta x}\xi = re^{\hat{i}\beta\Delta x} + 2(1-r) + re^{-\hat{i}\beta\Delta x} \qquad (12)$$

Hence

$$\xi = \frac{-4r\sin^2 \beta\Delta x/2 + 2}{4r\sin^2 \beta\Delta x/2 + 2} = \frac{4}{4r\sin^2 \beta\Delta x/2 + 2} - 1 \qquad (13)$$

From Eq. (13) it is concluded that for all positive values of r the condition (10) is true. Therefore, the finite differences scheme (7) is unconditionally stable.

4.1.2. Consistency of the finite difference scheme

In this section the consistency of the scheme (7) is investigated and it is proved that the finite differential scheme is consistent with the parabolic partial differential Eq. (1).

By Taylor's expansion:

$$T_i^{n+1} = T_i^n + \Delta t(T_t)_i^n + \tfrac{1}{2}\Delta t^2(T_{tt})_i^n + \tfrac{1}{6}\Delta t^3(T_{ttt})_i^n + \ldots, \qquad 14)$$

$$T_{i+1}^{n+1} = T_i^{n+1} + \Delta x(T_x)_i^{n+1} + \tfrac{1}{2}\Delta x^2(T_{xx})_i^{n+1} + \tfrac{1}{6}\Delta x^3(T_{xxx})_i^{n+1} + \ldots \qquad (15)$$

$$T_{i-1}^{n+1} = T_i^{n+1} - \Delta x(T_x)_i^{n+1} + \tfrac{1}{2}\Delta x^2(T_{xx})_i^{n+1} - \tfrac{1}{6}\Delta x^3(T_{xxx})_i^{n+1} + \ldots \qquad (16)$$

Hence the local truncation error (LTE) at the $(i, n+1)$ mesh point:

$$\text{LTE}_i^{n+1} = (T_t - T_{xx})_i^{n+1} - \tfrac{1}{2}\Delta t(T_{tt})_i^{n+1} - \tfrac{1}{12}\Delta x^2(T_{xxxx})_i^n - \tfrac{1}{2}\Delta t(T_{tt})_i^n - \tfrac{1}{12}\Delta x^2(T_{xxxx})_i^n + \ldots \qquad (17)$$

But T is the solution of differential equation and $(T_t - T_{xx})_i^{n+1} = 0$ therefore $\text{LTE}_i^{n+1} = 0\Delta t + 0\Delta x^2$. Now as the $\Delta t \to 0$ and $\Delta x \to 0$ then $\text{LTE} \to 0$.

Consequently, the finite differences scheme (7) is consistent with the parabolic Eq. (1).

From 3.1.1 and 3.1.2 and Lax's equivalence theorem it obviously follows that the answer converges to exact solution T as Δx tends to zero [24].

4.2. League Championship Algorithm

LCA is a global optimization over a continuous search space which is a population based algorithmic framework. All like this algorithms do something like that moving a population of possible solutions into specialized areas of search space to seek the optimum. As all other algorithm a set of answers in the search space must be chosen randomly as a priori answer for the initial form of the LCA. In this method "league" is used as the sporting terminology for the population. LCA consists in evolving gradually the combination of the population in consecutive iterations, as the size of population is constant. Some times to be consistent "week" may be used in place of "iteration". Any solution of the population is one of teams (L is the number of teams and is an even number) and is considered as the team's current formation. Therefore the "team i" is the "ith member of the population" and its current formation refers to its solution. The value of each solution of the population can be calculated by the cost function and fitness value as a degree of adaptation to objective which is aimed is certain. The fitness value is the playing strength of any team formation. As all other population based algorithms this algorithm change the formation of any team in each season into a better formation to get the best formation. It is the artificial match analyses processor work to change the formation into a better one. Many of population based algorithms are Evolutionary Algorithms (EA) which during the iterations the fitness of individuals are improved [25]. As a pseudo evolutionary algorithm, selection in LCA is very greedy which replaces in each iteration the current best formation with a better team formation having a more adaptive playing strength. All the algorithm will terminate after certain number of seasons which represented by S and any season has $L-1$ week. At

last each algorithm has $S(L-1)$ iterations.

Fig. 3 shows the LCA flowchart. The method has paradigms which focuses on the relative comparison of individuals instead of their absolute fitness gains. Therefore the algorithm ensures that the win possibility for the better solution (team) is greater than the win possibility of the weaker solution (team).

Consider that any team like X has the list of players $X=(x_1,x_2,...,x_n)$ which are the coefficients of optimizing functional which should be optimized and the team arrangement in iteration t for the team number i can be shown as $X_i^t=(x_{i1}^t,x_{i2}^t,...,x_{in}^t)$. Now assume that the team k had won l so it is reasonable that if the team i wants to win l its playing style should be like k and in the other hand if k had lost team l and team i wants a win its playing style should be in the opposite direction of k. So by defining the $X_i^t-X_j^t$ we appoint to the differences between i and j. So by the Eq.s like (18) to (21) we use the treats and opportunities of the previous wins and losses to get the next best playing style:

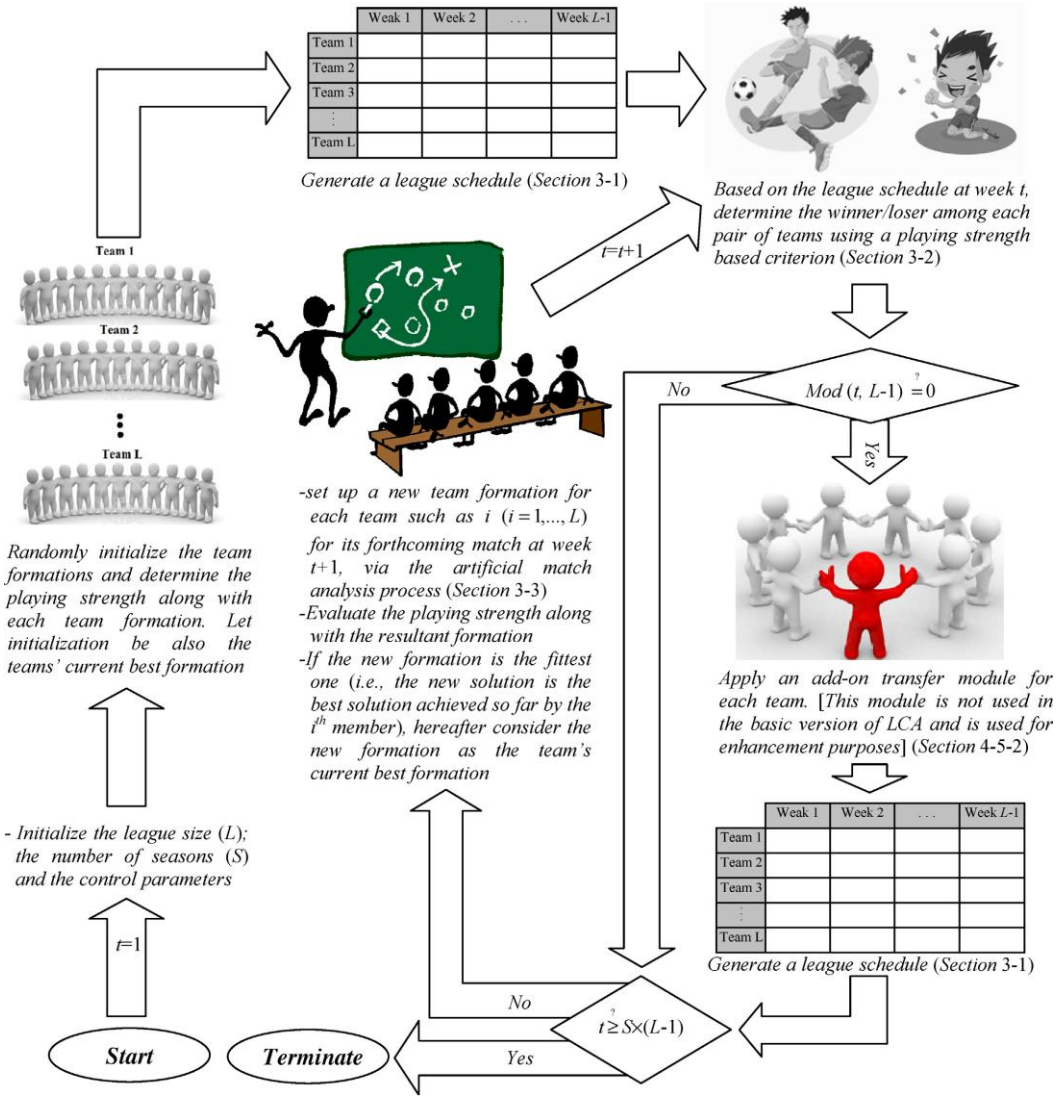

Fig. 3: *LCA flowchart [10]*

(S/T equation) $$x_{id}^{t+1}=b_{id}^t+y_{id}^t\left(\psi_1 r_{1id}\left(x_{id}^t-x_{kd}^t\right)+\psi_1 r_{2id}\left(x_{id}^t-x_{jd}^t\right)\right) \quad (18)$$

(S/O equation) $$x_{id}^{t+1}=b_{id}^t+y_{id}^t\left(\psi_2 r_{1id}\left(x_{kd}^t-x_{id}^t\right)+\psi_1 r_{2id}\left(x_{id}^t-x_{jd}^t\right)\right) \quad (19)$$

(W/T equation) $$x_{id}^{t+1}=b_{id}^t+y_{id}^t\left(\psi_1 r_{1id}\left(x_{id}^t-x_{kd}^t\right)+\psi_2 r_{2id}\left(x_{jd}^t-x_{id}^t\right)\right) \quad (20)$$

(W/O equation) $x_{id}^{t+1} = b_{id}^{t} + y_{id}^{t}\left(\psi_2 r_{1id}\left(x_{kd}^{t} - x_{id}^{t}\right) + \psi_2 r_{2id}\left(x_{jd}^{t} - x_{id}^{t}\right)\right)$ (21)

where l Is the index of the team that will play with team i at week $t+1$ and j is the index of the team that has played with team i at week t k is the index of the team that has played with team i at week t based on the league schedule r_{1id} and r_{2id} are random uniform numbers in $[0,1]$. ψ is a coefficient that control the acceleration toward the best answer and run away from the worst. b is the best choice and y is the dimension coefficient.

The Eq. (18) is based on the strengths and weaknesses those make treads or opportunities of the other teams which makes them a winner or loser and with these equations next optimizing variables of team i like $x_1, x_2, ..., x_n$ can be calculated.

Hence it is expected that the direction of the search is toward winner (better solution) and in opposition of loser (weaker solution). Like this algorithms moves the population toward the reach areas and at the same time they escape from poor local areas. Unlike most of algorithms which just go through the better solution in the search area, LCA avoids from some locals in the search space (supplied by the league schedule module).

For this reason the LCA has more speed and can work on various types of the problems with complexities and lots of local minima or maxima. Therefore in this method we can set the smaller or greater rate of changes dynamically.

5. Numerical Experiment

In this section, it is going to demonstrate some numerical results for determining T and $q(t)$ in the inverse problem mentioned Eqs. (1) to (5). Therefore the following examples are considered and the solution is obtained. Remember that in IHCPs there are two sources of estimation errors. The first is the unavoidable bias deviation (or deterministic error). The second one is the variance due to the amplification of measurement errors (stochastic error). The mean squared error or total error has the global effect of deterministic and stochastic considered in its terms [26]:

$$S = \left[\frac{1}{N}\sum_{i=1}^{N}(\bar{T}_i - T_i)^2\right]^{\frac{1}{2}} \quad (22)$$

where N is the total number of estimated values, $\bar{T}_i$ are calculated values from interpolated equation and $\bar{T}_i$ are exact values of $q(t)$.

Example 1. In an engineering experiment, when we have the set of experimental data

$T(x,\circ) = \sin \pi x + \cos \pi x, \qquad \circ \le x \le 1$

$T(\circ,t) = e^{-\pi^2 t}, \qquad \circ \le t \le 1$

and overspecified data (additional condition)

$T(0.5,t_i) = s(t_i), \quad t_i = i \times 0.05,$

$i = 1,2,...,20$

We obtain the unique exact solution:

$$q(t) = -e^{-\pi^2 t}, \qquad \circ \le t \le 1 \quad (23)$$

To give a clear overview of the present method, the above example will be considered. Therefore the unknown function $q(t)$ is defined as the following form:

$$\bar{q}(t) = a(e^{bt}) \quad (24)$$

where a and b are unknown coefficients which should be find in the optimization procedure by LCA. Now, the computation as a procedure to estimate the unknown coefficients are repeated until:

$S < 7.9 \times 10^{-7}$

when the number of teams in league was 16 and the number of competition seasons was 500 season.

The results are shown in Tables 1, 2, 3 and Fig. 4 and Fig. 5. Table 1 shows the error between the exact boundary condition $q(t)$ and The calculated boundary condition $\bar{q}(t)$.

Table 1: Comparison between exact $q(t)$ and calculated $\bar{q}(t)$ for example 1.

$t(s)$	$q(t)$	$\bar{q}(t)$	*error*	$t(s)$	$q(t)$	$\bar{q}(t)$	*error*
0.05	*-0.6104*	*-0.5888*	*0.0216*	*0.60*	*-0.0026*	*-0.0024*	*0.0002*
0.10	*-0.3727*	*-0.3576*	*0.0150*	*0.70*	*-9.99e-4*	*-9.01e-4*	*9.79e-05*
0.20	*-0.1389*	*-0.1319*	*0.0069*	*0.80*	*-3.72e-4*	*-3.32e-4*	*3.99e-05*
0.40	*-0.0192*	*-0.0179*	*0.0013*	*0.90*	*-1.38e-4*	*-1.22e-4*	*1.61e-05*
0.50	*-0.0071*	*-0.0066*	*0.0005*	*1.00*	*-5.17e-05*	*-4.52e-05*	*6.49e-06*

In Table 1 the exact $q(t)$, the calculated $\bar{q}(t)$ and the error between them for example 1 are shown. As its obvious the error rate is in the E-03 order and the calculated answer has an excellent adjustment.

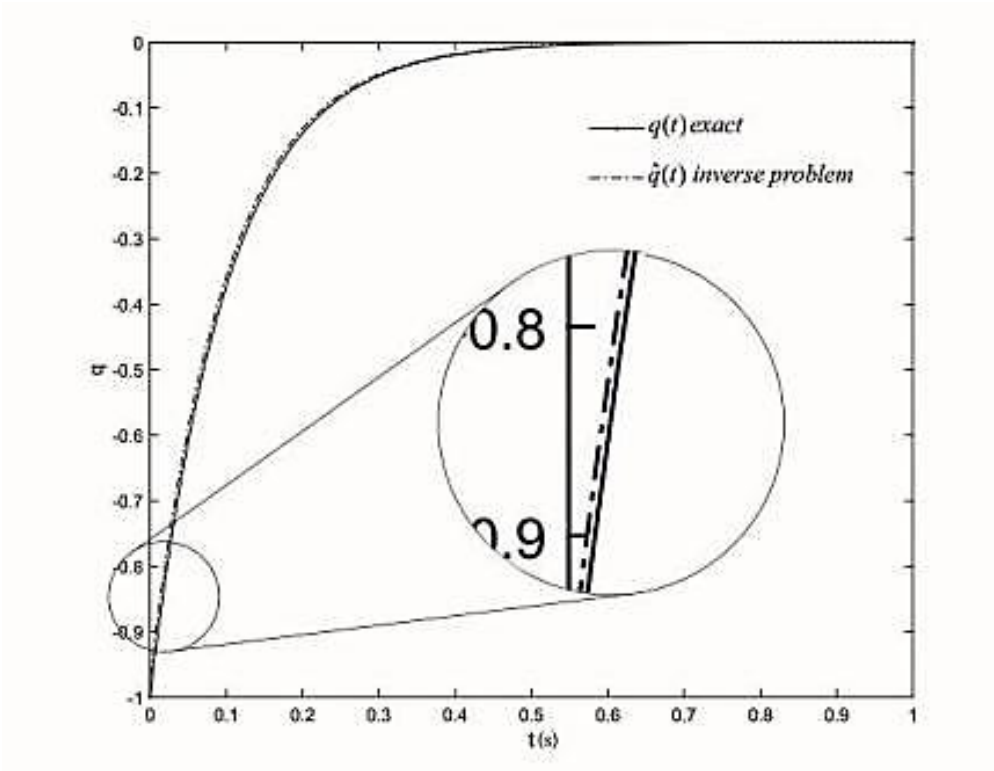

Fig. 4: *The difference between exact* q(t) *and calculated* $\bar{q}(t)$ *in example 1*

In Fig. 4 the exact q(t) and calculated $\bar{q}(t)$ curves are shown. The accuracy of the answer is shown in the this figure. The bigger circle in the Fig. 4 is the magnified view of the smaller circle in the figure and it shows the adjustment of the calculated part.

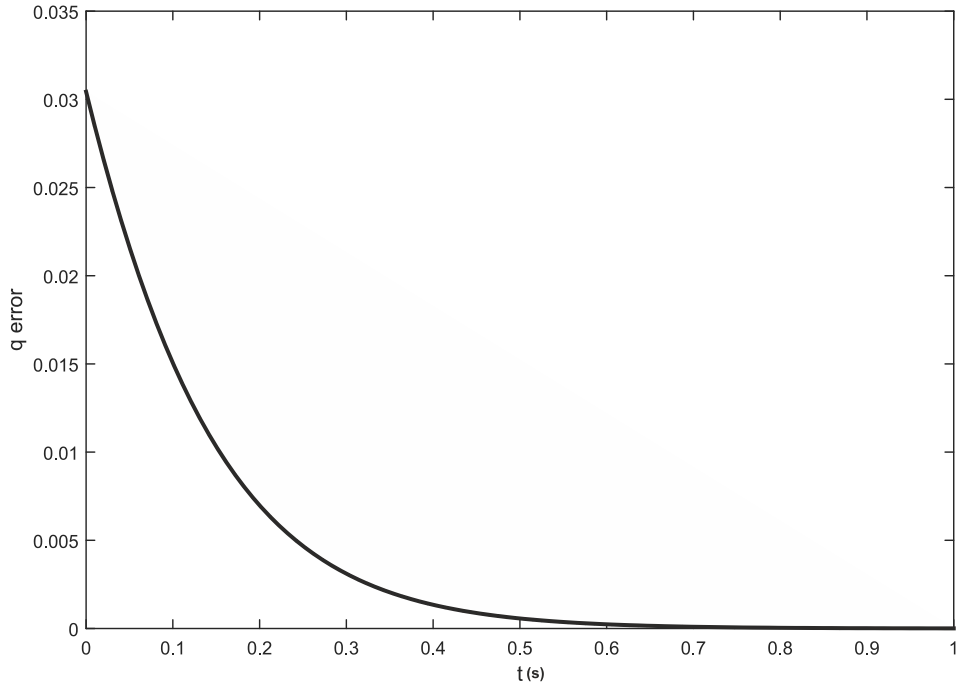

Fig. 5: *Error between the exact* q(t) *and calculated* $\bar{q}(t)$ *in example 1*

In Fig. 5 the error between the exact q(t) and the calculated $\bar{q}(t)$ is shown. This error goes down as the time goes forward.

Table 2: Comparison between the exact and calculated temperature in example 1

T Exact	$x = 0.25$	$x = 0.75$	$x = 1.00$	T Cal	$x = 0.25$	$x = 0.75$	$x = 1.00$
$t = 0.00$	1.4142	-3.67e-07	-1.0000	$t = 0.00$	1.4142	-3.67e-07	-1.0000
$t = 0.05$	0.8633	-2.24e-07	-0.6104	$t = 0.05$	0.8628	0.0015	-0.6104
$t = 0.10$	0.5270	-1.36e-07	-0.3727	$t = 0.10$	0.5275	0.0030	-0.3727
$t = 0.20$	0.1964	-5.10e-08	-0.1389	$t = 0.20$	0.1965	0.0020	-0.1389
$t = 0.40$	0.0272	-7.08e-09	-0.0192	$t = 0.40$	0.0270	0.0002	-0.0192
$t = 0.80$	0.0005	-1.36e-10	-0.0003	$t = 0.80$	0.0004	-1.32e-05	-0.0003
$t = 1.00$	7.31e-05	-1.90e-11	-5.17e-05	$t = 1.00$	6.50e-05	-4.59e-06	-5.17e-05

In Table 2 the exact temperatures and the calculated temperatures are shown. As its obvious the error rate is too tiny and the answer has a high accuracy.

Table 3: The error in calculating temperature for example 1

Time	Error in x=0.25	Error in x=0.75	Error in x=1.00
$t = 0.00$	0.0000	0.0000	0.0000
$t = 0.05$	0.0004	0.0015	2.9999E-07
$t = 0.10$	0.0004	0.0030	2.0000E-07
$t = 0.20$	9.56E-05	0.0020	0.0000
$t = 0.40$	0.0002	0.0002	1.00E-08
$t = 0.80$	3.37E-05	1.32E-05	9.99E-11
$t = 1.00$	8.11E-06	4.59E-06	1.99E-11

In Table 3 the error of calculating temperatures are shown and also the error rate is in E-03. As its obvious the error rate is too low and it makes the method accurate.

Example 2. In an engineering experiment, when we have the set of experimental data

$$T(x,\circ) = \frac{1}{2}x^2, \qquad \circ \le x \le 1$$
$$T(\circ,t) = t, \qquad \circ \le t \le 1$$

and overspecified data (additional condition)

$$T(0.5,t_i) = s(t_i), \quad t_i = i \times 0.05, \quad i = 1,...,20,$$

We obtain the unique exact solution

$$q(t) = \frac{1}{2} + t, \qquad \circ \le t \le 1 \qquad (25)$$

The unknown function $q(t)$ is defined as the following form:

$$\bar{q}(t) = a + bt + ct^2 \qquad (26)$$

where a, b and c are unknown coefficients which should be find by LCA. Now, the computation as a procedure to estimate the unknown coefficients are repeated until:

$S < 4.7 \times 10^{-5}$,

when the number of teams in league was 16 and the number of competition seasons was 500 season. The results are shown in Tables 4 to 6 and Fig. 6 and Fig. 7.

Table 4: Comparison between exact $q(t)$ and calculated $\bar{q}(t)$ in example 2

$t(s)$	$q(t)$	$\bar{q}(t)$	error	$t(s)$	$q(t)$	$\bar{q}(t)$	error
0.05	0.55	0.533101	1.69E-02	0.60	1.1	1.107372	7.372E-03
0.10	0.6	0.588802	1.11E-02	0.70	1.2	1.202698	2.698E-03
0.20	0.7	0.698108	1.89E-03	0.80	1.3	1.295228	4.772E-03
0.40	0.9	0.908332	8.33E-03	0.90	1.4	1.384962	1.5038E-02
0.50	1	1.00925	9.25E-03	1.00	1.5	1.4719	2.81E-02

In Table 4 the exact $q(t)$, the calculated $\bar{q}(t)$ and the error between them for example 2 are shown. It is clear in the table that the error rate is in the E-02 order or lower. So the calculated answer is precise.

In Fig. 6 the exact $q(t)$ and calculated $\bar{q}(t)$ curves are shown. The accuracy of the answer is shown inside the

bigger circle. The bigger circle is the magnified view of the smaller circle.

In Fig. 7 the error of calculating $q(t)$ and the $\bar{q}(t)$ is represented. The shape of error is because of the function which is estimated is in second order and the absolute amount of error is represented.

In Table 5 the calculated and the exact temperatures are represented. As its obvious the error rate is too low and the answer is accurate.

In Table 6 the error in calculating the temperatures is represented and the error rate is in E-02. As its obvious the error rate is too low and it makes the method accurate.

6. Conclusion and future direction

The presented study successfully represented a combined numerical and probabilistic algorithm involving the finite differences method in conjunction with league championship algorithm to solve a one-dimensional linear parabolic inverse problem. From the explained example it can be seen that the demonstrated numerical-probabilistic method is efficient and accurate to estimate the unknown boundary condition and the results presented here suggest that the synthesis of the LCA method provides an undertaking probabilistic approach to parabolic inverse problem of the theory of heat transfer. The advantages of this approach are not limited to, versatility, the possibility of computing the functions of interest at isolated points without computing them on massive meshes, and the opportunity of having simple scalable. The numerical experiments of the present work show LCA is preferable when one needs to have an excellent estimation of the solution.

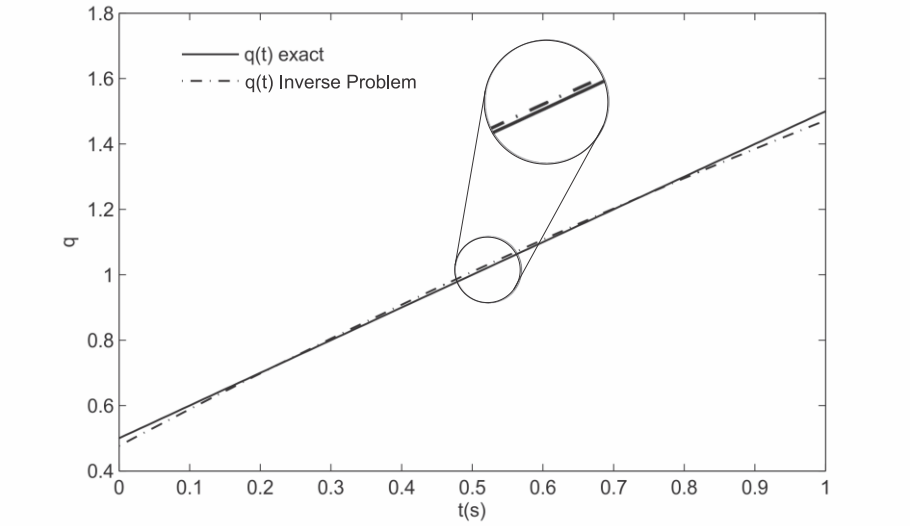

Fig. 6: *The difference between exact* q(t) *and calculated* $\bar{q}(t)$ *in example 2*

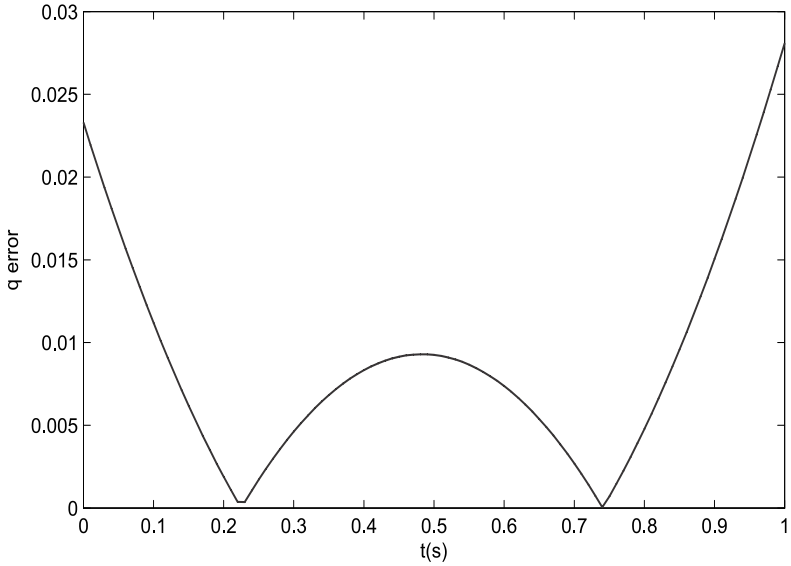

Fig. 7: *Error between the exact* q(t) *and calculated* $\bar{q}(t)$ *in example 2*

Table 5: Comparison between the exact and calculated temperature in example 2

T Exact	$x = 0.26$	$x = 0.73$	$x = 1.00$	T Cal	$x = 0.26$	$x = 0.73$	$x = 1.00$
$t = 0.00$	0.034626	0.271468	0.5	$t = 0.00$	0.034626	0.271468	0.4767
$t = 0.05$	0.087258	0.3241	0.552632	$t = 0.05$	0.084048	0.313312	0.532945
$t = 0.10$	0.137119	0.373961	0.602493	$t = 0.10$	0.135248	0.365476	0.591561
$t = 0.20$	0.236842	0.473684	0.702216	$t = 0.20$	0.234434	0.469154	0.700499
$t = 0.40$	0.436288	0.67313	0.901662	$t = 0.40$	0.436627	0.676972	0.910032
$t = 0.80$	0.83518	1.072022	1.300554	$t = 0.80$	0.83625	1.072055	1.295733
$t = 1.00$	1.034626	1.271468	1.5	$t = 1.00$	1.03184	1.257472	1.4719

Table 6: The error in calculating temperature for example 2

Time	Error in x=0.26	Error in x=0.73	Error in x=0.1
$t = 0.00$	0.000513	0.008167	0.0233
$t = 0.05$	0.001871	0.008485	0.016916
$t = 0.10$	0.002408	0.00453	0.006234
$t = 0.20$	0.000338	0.003842	0.00679
$t = 0.40$	0.00107	3.24E-05	0.000533
$t = 0.80$	0.002786	0.013996	0.020877
$t = 1.00$	0.000513	0.008167	0.0233

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