JCAMECH

Vol. 48, No. 2, December 2017, pp 217-224

DOI: 10.22059/jcamech.2017.241673.185

Prediction of Temperature distribution in Straight Fin with variable Thermal Conductivity and Internal Heat Generation using Legendre Wavelet Collocation Method

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Received: 13 Sep. 2017, Accepted: 3 Oct. 2017

Abstract

Due to increasing applications of extended surfaces as passive methods of cooling, study of thermal behaviors and development of mathematical solutions to nonlinear thermal models of extended surfaces have been the subjects of research in cooling technology over the years. In the thermal analysis of fin, various methods have been applied to solve the nonlinear thermal models. This paper focuses on the application of Legendre wavelet collocation method to the prediction of temperature distribution in longitudinal rectangular fin with temperature-dependent thermal conductivity and internal heat generation. The numerical approximations by the method are used to carry out parametric studies of the effects of the model parameters on the temperature distribution in the fin. The results show that the thermal performance of the fin is favoured at low values of thermogeometric parameter and internal heat generation decreases the performance of the fin. The results can serve as verification of the solutions of other methods of analysis of the component.

Keywords: Legendre Wavelet collocation method, Longitudinal rectangular fin, Temperature distribution, Variable thermal conductivity, Variable internal heat generation.

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1. Introduction

The production of high powered equipment in recent times requires effective cooling systems to avoid thermal breakdown of the equipment. In such quest and demand, fins have been used as media for passive effective cooling technology. The wide areas of applications of fins for heat transfer enhancement in thermal components and devices has made the subject area an interesting research area over many years. Moreover, the investigation into the effects of the inherent nonlinearities in the developed thermal models (due to its temperature-dependent thermal properties) on the thermal performance of the passive devices has attracted a large number of research works. In practice, various types of fins with different geometries are used, but the simplicity of its design, ease of construction and manufacturing process, has made rectangular fins to be widely applied in heat-transfer equipment. Also, for ordinary fins problem, the thermal properties of the fin and the surrounding medium (thermal conductivity and heat transfer coefficient) are assumed to be constant, but if large temperature difference exists within the fin, typically, between tip and the base of the fin, the thermal conductivity and the heat transfer coefficient are not constant but temperature-dependent. Therefore, while analyzing the fin, effects of the temperature-dependent thermal properties must be taken into consideration. In carrying out such analysis, the thermal conductivity may be modeled for such and other many engineering applications by power law and by linear dependency on temperature while the heat transfer coefficient can be expressed as power law for which the exponents represent different phenomena as reported by Khani and Aziz [1], Ndlovu and Moitsheki [2]. Nonlinearity results in the thermal model because of thedependency of thermal conductivity and heat transfer coefficient on temperature and consequently, the model become very difficult to solve analytically. In other to solve the nonlinear model, different numerical and approximate analytical techniques have been employed. In one of the earliest work, Aziz and Enamul-Huq [3] and Aziz [4] applied regular perturbation expansion to study a pure convection fin with temperature dependent thermal conductivity. Method of successive approximation was adopted by Campo and Spaulding [5] to predict the thermal behaviour of uniform circumferential fins. Chiu and Chen [6] and Arslanturk [7] employed Adomian Decomposition Method (ADM) to determine the temperature distribution in a convective fin with variable thermal conductivity. Ganji [8] solved the same problem with the aid of the homotopy perturbation method which was originally proposed by He [9]. The Adomian decomposition method was also utilized by Chowdhury and Hashim [10] to predict the temperature distribution of straight rectangular fin with temperature dependent surface flux for all possible types of heat transfer while Rajabi [11] applied homotopy perturbation method (HPM) to calculate the efficiency of straight fins with temperature-dependent thermal conductivity. Also, Mustapha [12] adopted homotopy analysis method (HAM) to find the efficiency of straight fins with temperature-dependent thermal conductivity. Meanwhile, Coskun and Atay [13] utilized variational iteration method (VIM) for the analysis of convective straight and radial fins with temperature-dependent thermal conductivity. In a study of comparative analysis of methods of solution, Languri *et al.* [14] applied both variation iteration and homotopy perturbation methods for the evaluation of efficiency of straight fins with temperature-dependent thermal conductivity while Coskun and Atay [15] applied variational iteration method to analyse the efficiency of convective straight fins with temperature-dependent thermal conductivity. Atay and Coskum [16] employed variation iteration and finite element methods to carry out comparative analysis of power-law-fin type problems. Domairry and Fazeli [17] used homotopy analysis method to determine the efficiency of straight fins with temperature-dependent thermal conductivity. Chowdhury *et al.* [18] investigated a rectangular fin with power law surface heat flux and made a comparative assessment of results predicted by HAM, HPM, and ADM. Khani *et al*. [19] used Adomian decomposition method (ADM) to provide series solution to fin problem with a temperature-dependent thermal conductivity while Moitsheki *et al.* [20] applied the Lie symmetry analysis to provide exact solutions of the fin problem with a power-law temperature-dependent thermal conductivity while Hosseini *et al.* [21] applied homotopy analysis method to generate approximate but accurate solution of heat transfer in fin with temperature-dependent internal heat generation and thermal conductivity. The application of differential transform method (DTM) to solve differential equations without linearization, discretization or no approximation, linearization restrictive assumptions or perturbation, complexity of expansion of derivatives and computation of derivatives symbolically. Other researchers such as Joneidi *et al.* [22], Moradi and Ahmadikia [23, 24] and the method was also used by Mosayebidorcheh *et al.* [25], Ghasemi *et al.* [26], Ganji and Dogonchi [27] have also adopted DTM to solve the fin problem. However, the search for the arbitrary value that will satisfy the second boundary condition necessitated the use of Maple or Mathematica software and such could result in additional computational cost in the generation of solution to the problem. This drawback is not only peculiar to DTM, other approximate analytical methods such as HPM, HAM, ADM and VIM also required additional computational cost and time for the determination of such auxiliary parameters in their procedures of implementation [28]. Also, most of the approximate analytical methods give accurate predictions only when the nonlinearities are weak, they fail to predict accurately for strong nonlinear models. Also, the methods often involved complex mathematical analysis leading to analytic expression involving a large number terms and when such methods as HPM, HAM, ADM and VIM are routinely implemented, they can sometimes lead to erroneous results as observed by Fernandez [29], Aziz and Bouaziz [30]. In practice, approximate analytical solutions with large number of terms are not convenient for use by designers and engineers [28]. In other to reduce the computation cost and time in the analysis of nonlinear problems Legendre collocation method was put forward and it has been adopted to solve different nonlinear equations. The ease of use, simplicity and fast rate of convergence have in recent times made these methods gain popularity in nonlinear analysis of systems and they have been applied to nonlinear problems in heat transfer analysis of fins [31-35]. The wavelet collocation method is mathematically very simple, easy and fast. It is an efficient and powerful in solving wide class of linear and nonlinear differential equations. In recent times, the method has gained the popularity and reputation of being a very effective tool for many practical applications. From the computational simulation point of view, it has been established that the numerical approximate solution provided by the method is much closer to the exact solutions in many practical applications of the method. Therefore, this paper focuses on the application of Legendre wavelet collocation method to the prediction of temperature distribution in longitudinal rectangular fin with temperature-dependent thermal conductivity and internal heat generation. The numerical approximations by the method are used to carry out parametric studies of the effects of the model parameters on the temperature distribution in the fin. The results of obtained by LWCM are in excellent agreements with exact analytical solutions (for the linear model) and the direct numerical solutions (for the nonlinear model).

2. Problem

Consider a straight fin of length l that is exposed on both faces to a convective environment at temperature, T_∞ and with heat transfer co-efficient, h and internal heat generation, q_{in} shown in Fig.1.

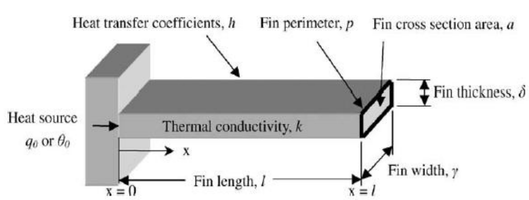

Fig. 1 Schematic diagram of a longitudinal rectangular fin

For the steady state heat transfer in fin with uniform temperature of the medium surrounding the fin and that of the base. Assuming there is no contact resistance where the base of the fin joins the prime surface, the fin thickness is small compared with its height and length, for the temperature-dependent thermal conductivity and internal heat generation, the governing equation for the heat transfer in the fin is given by

$$\frac{d}{dx}\left[k_a[1+\lambda(T-T_\infty)]\frac{dT}{dx}\right] - \frac{hP(T-T_\infty)}{k_a A_c} + \frac{q_a}{k_a}[1+\psi(T-T_\infty)] = 0 \tag{1}$$

The following dimensionless parameters are used to non-dimensionalize Eq. (1)

$$X=\frac{x}{l},\quad \theta=\frac{T-T_\infty}{T_b-T_\infty},\quad M^2=\frac{PhL^2}{A_c k_a},\quad Q=\frac{q_a}{A_c k_a(T_b-T_\infty)},\quad \gamma=\psi(T_b-T_\infty),\quad \beta=\lambda(T_b-T_\infty) \tag{2}$$

The dimensionless governing differential eq. (3) and the boundary conditions were arrived at

$$\frac{d}{dX}\left[(K(\theta)]\frac{d\theta}{dX}\right] - M^2\theta + M^2 Q(1+\gamma\theta) = 0 \tag{3}$$

The boundary conditions are

$$X=0,\quad \theta=1 \qquad X=1,\quad \frac{d\theta}{dX}=0 \tag{4}$$

Where

$$K(\theta)=1+\beta\theta \tag{5}$$

On substituting eq. (5) into eq. (3), we have

$$\frac{d^2\theta}{dX^2}+\beta\theta\frac{d^2\theta}{dX^2}+\beta\left(\frac{d\theta}{dX}\right)^2 - M^2\theta + M^2 Q(1+\gamma\theta) = 0 \tag{6}$$

3. Method of Analysis: Legendre Wavelet Collocation Method

Wavelets: Continuous wavelet are defined by the following formula [31-35]

$$\psi_{a,b}(X)=|a|^{\frac{-1}{2}}\psi\left(\frac{X-b}{a}\right), a,b\in R, a\neq 0 \qquad (7)$$

Where a and b are dilation and translation parameters, respectively. The Legendre wavelets defined on the interval (0, 1) is given by

$$\psi_{n,m}(X)=\begin{cases}\sqrt{(m+1/2)}2^{k/2}p_m\left(2^kX-\hat{n}\right), & \frac{\hat{n}-1}{2^k}\leq\frac{\hat{n}-1}{2^k}\\ 0 & otherwise\end{cases} \qquad (8)$$

Where m=0,1,…,M-1 and n=1,2,...2^{k-1}. $P_m(x)$ is the Legendre polynomial of order m

$$P_0(X)=1, P_1(X)=X,$$
$$P_{m+1}(X)=\frac{2m+1}{m+1}XP_m(X)-\frac{m}{m+1}P_{m-1}(X) \qquad (9)$$
$$m=1,2,3......,M-1.$$

A function f(x) defined in domain [0, 1] can be expressed as

$$f(X)=\sum_{n=1}^{\infty}\sum_{m=0}^{\infty}c_{n,m}\psi_{n,m}(X) \qquad (10)$$

where $c_{n,m}=<f(X),\psi_{n,m}(X)>$ in which <…> denotes the inner product

Taking some terms in infinite series, we can write Eq. (10) as

$$f(X)=\sum_{n=1}^{2^{k-1}}\sum_{m=0}^{M-1}c_{n,m}\psi_{n,m}(X)=C^T\psi(X) \qquad (11)$$

Where C and $\psi(X)$ are M x1 matrices given by

$$C=\left[c_1,0,c_1,1,.....,c_1,M-1,c_2,1,......,c_2,M-1,c_{2^{k-1}},1,...,c_{2^{k-1}},M-1\right]^T$$
$$\psi(X)=\begin{bmatrix}\psi_1,_0(X),\psi_{1,1}(X),...,\psi_{1,M-1}(X),\psi_{2,0}(X),......,\\ \psi_{2,M-1}(X),........,\psi_{2^{k-1}},_0(X)\psi_{2^{k-1},1}(X),....,\psi_{2^{k-1}},_{M-1}(X)\end{bmatrix}^T \qquad (12)$$

(i) Property of the product of two Legendre wavelets

If E is a given wavelets vector, then we have the property

$$E^T\psi\psi^T=\psi^T\hat{E} \qquad (13)$$

(ii) Operational matrix of integration: The integration of wavelets $\psi(X)$ which is defined in Eq. (8) can be obtained as

$\int_0^X\psi(s)ds=P\psi(X), X\in[0,1]$ *Where P is $2^{k-1}M$* x *$2^{k-1}M$,* the operational matrix of integration is given by

$$P=\frac{1}{2}\begin{pmatrix} 1 & \frac{1}{3} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \frac{-1}{\sqrt{3}} & 0 & \frac{1}{\sqrt{15}} & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \frac{-1}{\sqrt{15}} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & & & & \vdots \\ \vdots & \vdots & \vdots & & \ddots & & & \vdots \\ \vdots & \vdots & \vdots & & & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & \cdots & \ddots & \frac{\sqrt{2M-3}}{(2M-3)\sqrt{2M-1}} \\ 0 & 0 & 0 & \cdots & \cdots & \cdots & \cdots & \frac{-\sqrt{2M-3}}{(2M-3)\sqrt{2M-1}} \end{pmatrix} \quad (14)$$

3.1 Legendre Wavelet Collation Method

Let

$$\theta''(X)=C^T\psi(X) \quad (15)$$

Integrating Eq. (15) with respect to x from 0 to x, we have

$$\theta'(X)=\theta'(0)+C^TP\psi(x) \quad (16)$$

Put $X=1$ in (15), we have

$\theta(0)=\theta'(1)-C^TP\psi(1)$, since $\theta'(1)=0$, we obtain

$$\theta'(X)=-C^TP\psi(1)+C^TP\psi(X) \quad (17)$$

Again, integrating above equation, with respect to X from 0 to X, we obtain

$$\theta(X)=1-C^TP^2\psi(1)+C^TP^2\psi(X) \quad (18)$$

Substituting, $\theta''(X)$, $\theta''(X)$ and $\theta''(X)$ in Eq. (6), we arrived at

$$\begin{aligned}&(1+\beta\theta)C^T\psi(X)+\beta\left(C^TP\psi(X)\right)^2\\&-\left(M^2-M^2Q\gamma\right)\left\{1-C^TP\psi(1)+C^TP\psi(X)\right\}\\&+M^2Q=R\left(X,c_1,c_2,\ldots..,c_n\right)\end{aligned} \quad (19)$$

On expanding Eq. (19), we have

$$\begin{aligned}&\left[1+\beta\left\{1-C^TP^2\psi(1)+C^TP^2\psi(X)\right\}\right]C^T\psi(X)\\&+\beta\left\{C^TP\psi(X)\right\}^2\\&-\left(M^2-M^2Q\right)\left\{1-C^TP^2\psi(1)+C^TP^2\psi(X)\right\}\\&+M^2Q=R\left(X,c_1,c_2,\ldots..,c_n\right)\end{aligned} \quad (20)$$

The system of Eq. (20) is solved using Newton-Raphson method

4. Results and Discussion

The prediction of the temperature distribution and the effects of nonlinear thermal conductivity term, β is shown in Fig. 2. The figure displayed that the nonlinear thermal conductivity term has significant effect on the thermal performance of the fin. Also, it is shown that the thermal performance of the fin decreases as the nonlinear term increase. This is for the positive value of the nonlinear term. However, for the negative values of the nonlinear term, the thermal performance of the fin increases as the nonlinear term increases.

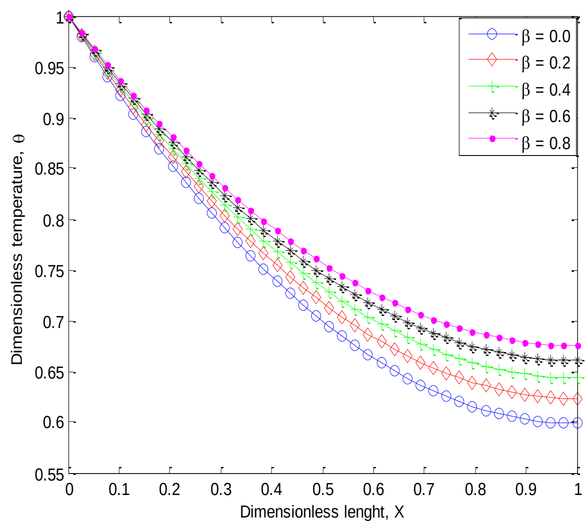

Fig.2 Dimensionless temperature distribution in the fin when M=2, Q=0.2, γ=0.5

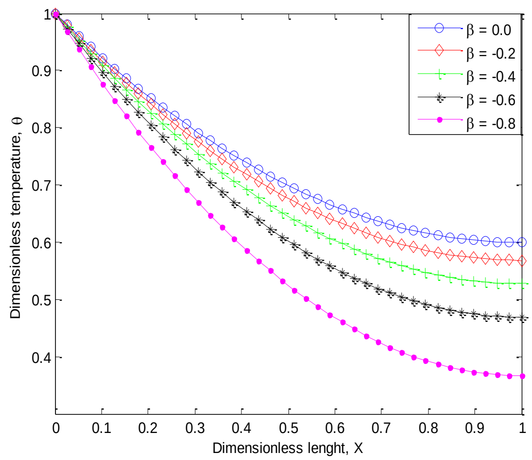

Fig.3 Dimensionless temperature distribution in the fin when M=2, Q=0.2, γ=0.5

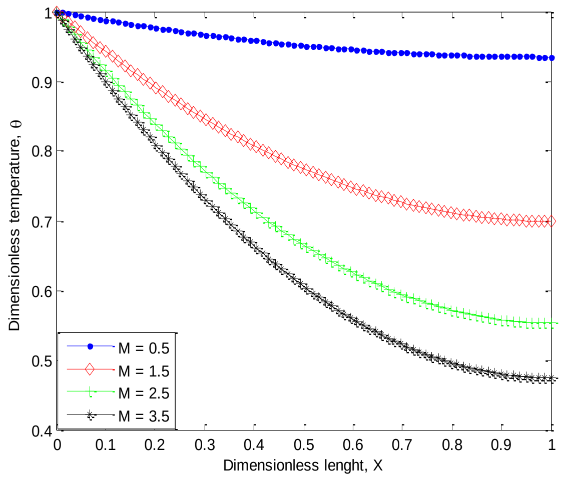

Fig.4 Dimensionless temperature distribution in the fin when β=0.2, Q=0.2, γ=0.4

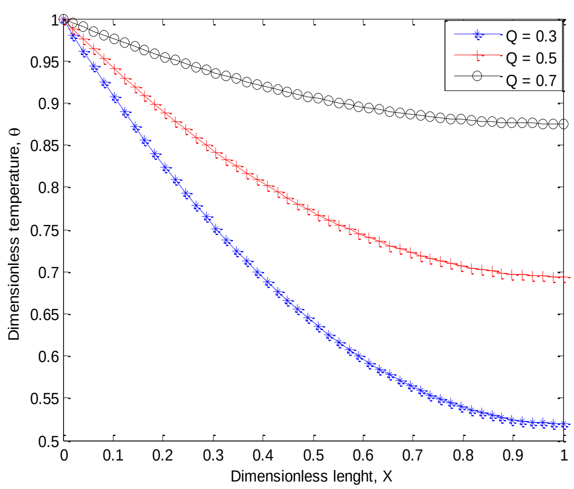

Fig.5 Dimensionless temperature distribution in the fin parameters when β=0.2, M=2, γ=0.2

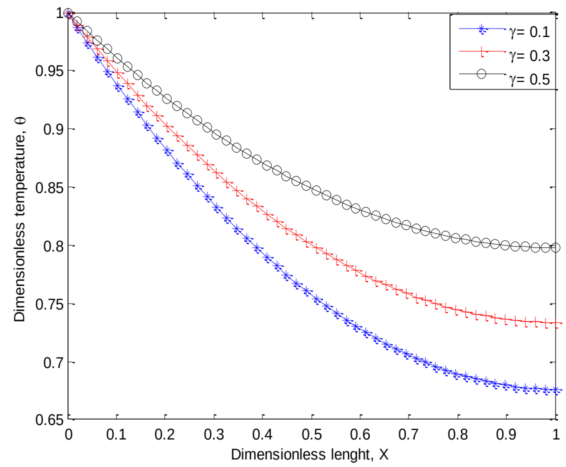

Fig..6 Dimensionless temperature distribution in the fin parameters when M=2, β=0.3, Q=0..3

Figs. 3 show the effects of thermo-geometric/conduction-convection parameter on the temperature distribution in the fin. The figure depicts that as the conduction-convection parameter increases, the rate of heat transfer through the fin increases as the temperature in the fin drops faster as depicted in the figures. The profile has steepest temperature gradient at lower value of the conduction-convection term, but its much higher value gotten from the lower value of thermal conductivity than the other values of in the profiles produces a lower heat-transfer rate. Therefore, the thermal performance or efficiency of the fin is favoured at low values of convective parameter since the aim is to minimize the temperature decrease along the fin length, where the best possible scenario is when temperature of the fin is the same s the base temperatur everywhere. It must be pointed out that a small value of M correspond to a relatively short and thick fins of poor thermal conductivity and high value of M implies a long fin or fin with low value of thermal conductivity. Since, the thermal performance or efficiency of the fin is favoured at low values of convective fin parameter, very long fins are to be avoided in practice [28]. It should be noted that a small value of M corresponds to a relatively short and thick fin of poor thermal conductivity and a high value of M implies a long fin or fin with low value of thermal conductivity. Since, the thermal performance or efficiency of the fin is favoured at low values of thermo-geometric fin parameter, very long fins are to be avoided in practice. A compromise is reached for one-dimensional analysis of fins $0 < Bi < 0.1$. When the Biot number is greater than 0.1, two dimensional analysis of the fin is recommended as one-dimensional analysis predicts unreliable results for such limit [28]. The effects of internal heat generation parameter on the temperature distribution are depicted in Figs. 4 and

5 while Fig. 6 shows the effects of internal heat generation on the fin thermal performance at different parameter, M. From the figures, as the internal heat generation parameter increases the temperature gradient of the fins decreases. This is because, as the rate of internal heat generation within fin increase, the thermal performance of the fin decreases.

Table 1shows the comparison of results. The good agreement between the results of the other methods and the results of the present study verifies the accuracy of the Legendre wavelet collocation method.

5. Conclusion

In this work, the temperature distributions in a longitudinal straight fin have been predicted using Legendre wavelet collocation method. The numerical approximations by the method are used to carry out parametric studies of the effects of the model parameters on the temperature distribution in the fin. The results show that the thermal performance of the fin is favoured at low values of thermogeometric parameter and internal heat generation decreases the performance of the fin. The results can serve as verification of the solutions of other methods of analysis of the component.

Table 1: Comparison of results

X	NM [6]	ADM [28]	GMWR	LWCM (The present study)
0.0	0.648054	0.648054	0.648054	0.648054
0.1	0. 651297	0.651297	0.651297	0.651297
0.2	0.661059	0.661059	0.661059	0.661057
0.3	0. 677436	0.677436	0.677436	0.677436
0.4	0.700594	0.700594	0.700594	0.700594
0.5	0.730763	0.730763	0.730763	0.730763
0.6	0.768246	0.768246	0.768246	0.768246
0.7	0.813418	0.813418	0.813418	0.813418
0.8	0.866731	0.866731	0.866730	0.866731
0.9	0. 928718	0.928718	0.928718	0.928718
1.0	1.000000	1.000000	1.000000	1.000000

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