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Incorporating Return on Inventory Investment into Joint Lot-Sizing and Price Discriminating Decisions: A Fuzzy Chance Constraint Programming Model

Reza Ghasemy Yaghin[1], Seyed Mohammad T. Fatemi Ghomi[2] ∗, Seyed Ali Torabi[3]

1 .Clothing Engineering and Management Group, Department of Textile Engineering, Amirkabir University of Technology, Tehran, Iran
2. Department of Industrial Engineering, Amirkabir University of Technology, Tehran, Iran
3. Department of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran

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Abstract

Coordination of market decisions with other aspects of operations management such as production and inventory decisions has long been a meticulous research issue in supply chain management. Generally, changes to the original lot-sizing policy stimulated by market prices may impose remarkable deviation revenue throughout the supply and demand chain system. This paper examines how to set the channel prices and the lot-sizing quantities so that the potential maximal return on investment is gained under a differential pricing scenario involving a number of possibilistic constraints to deal with market-segmented price setting, marketing and lot-sizing decisions, concurrently. The model aims to maximize return on inventory investment (ROII). To solve the model, a fuzzy solution approach based on the novel credibility measure is developed. An efficient and tuned search procedure using particle swarm optimization is tailored to reach the solutions of the resultant non-linear crisp model. An illustrative example is also studied to demonstrate the practicability of the proposed mathematical model and its solution approach.

Keywords

Price differentiation, Production lot size, Revenue management, Fuzzy optimization, Credibility measure.

∗ **Corresponding Author, Email:** fatemi@aut.ac.ir

Introduction

Channel pricing is one of the most important aspects of marketing decisions and Revenue Management (RM). In the traditional retail channel, retailers sell products to customers directly, under a single price setting. This had been a common pricing until the commencement of revenue management. In recent years, with the rapid development of the revenue management approaches such as differential pricing and dynamic pricing, retailers are increasingly accustomed to selling products via different channels. This paper develops a new price differentiation model aiming to maximize the Return On Investment (ROI) under uncertainty. ROI is a widely utilized performance measure in business investment evaluations (Li et al., 2008).

Segmenting market has been one of the serious conditions for an efficient application of pricing and RM by which marketers try to set different prices in each sub-market. In order to achieve greater profitability, price differentiation is an effective way for retailers (Phillips, 2005). There exists a number of real-life examples applying price differentiation policy from service and retail industries (e.g., hotels and airlines). Many retailers differentiate their potential market by leading customers to different segments, such as e-tailing versus traditional retailing, where firms set higher prices in their retail channel while offering lower prices for online purchasers. Market segmentation naturally increases revenues and hence profits; however, the price difference between the market segments stirs a part of customers to immigrate to other sell channels.

Literature Review

The literature shows that integrated lot-sizing and price setting models (Kim & Lee, 1998; Lee, 1993; Esmaeili, 2009; Abad, 1988, Wang et al., 2015), and price differentiation (Sen & Zhang, 1999; Zhang & Bell, 2007; Zhang et al., 2010) have been studied separately. Furthermore, profit maximization is usually employed as the objective function when designing and analyzing inventory models. The most powerful and useful marketing measure is the return on investment that relates the total investment made to the total generated return (Lenscold, 2003). Wang et al. (2012) integrate demand, purchasing, production, and transportation to enhance coordination between financial and physical activities in a supply chain. The use of return on investment (ROI) maximization is also popular in inventory systems' optimization (see Schroeder & Krishnan, 1976; Rosenberg, 1991). Otake et al. (1999), and Otake and Min (2001) propose an ROI maximization model with lot size and setup cost reduction investment and quality improvement policies, respectively. Recently, Li et al. (2008) develop a joint inventory and capital investment model with setup and quality considerations under ROI maximization.

In the context of multi-objective optimization, some papers studied the ROI maximization with profit in order to determine ordering policy. They employed ROI and profit as performance criteria in designing and analyzing inventory systems. Wee et al. (2009) propose a joint replenishment model under joint profit and ROI maximization. Ghasemy Yaghin and Fatemi Ghomi (2012) considered profit and ROI maximization in inventory-marketing problems. They also presented the joint pricing and lot-sizing model in order to improve service aspects of retail industries. Ghasemy Yaghin et al. (2013) extended a bi-objective inventory-marketing planning model to determine ordering and pricing policies with multiple demand classes.

The related literature lacks a JPLM problem with ROI maximization in a market-segmented environment under uncertainty. Most of the previous works address deterministic cases of the JPLM problem under return on inventory investment (ROII) objectives. To the best of our knowledge, there is no research work involving the uncertainty approaches in the area of ROII. Estimating parameters using the statistical models normally suffers from different issues (Sadjadi et al., 2010). Consequently, we have to estimate the parameters of market environment and demand functions subjectively based on both the current incomplete data and the decision makers' experiences. Zhou et al. (2008) and Zhoa et al. (2012) focus on pricing decisions with fuzzy customer demands. In this way, the theory of fuzzy sets can be applied to formulate production-marketing decisions (Sadjadi et al., 2010).

To the best of our knowledge, few studies have considered integrating ROII optimization in a joint pricing and lot-sizing problem in a fuzzy environment. They employed ROI and profit as performance criteria in designing and analyzing inventory systems (i.e., multiple objective environment). Wee et al. (2009) propose a joint replenishment model under joint profit and ROI maximization. Ghasemy Yaghin and Fatemi Ghomi (2012) consider profit and ROI maximization in inventory-marketing problems. They also present the joint pricing and lot-sizing model in order to improve service aspects of retail industries. Ghasemy Yaghin et al. (2013) extend a bi-objective inventory-marketing planning model to determine ordering and pricing policies with multiple demand classes under uncertainty. Features of the publications surveyed in this section are summarized in Table 1.

Table 1. Review of Some Existing Models.

Reference	Time dependent demand	Replenishment rate	Uncertain environment	Pricing	Marketing aspects	Multiple demand classes	ROII	Credibility measure	Chance constrained programming
Abad (1988)		Infinite		√	√				
Kim and Lee (1998)		Infinite		√					
Otake and Min (2001)		Infinite					√		
Zhang and Bell (2007)		Infinite		√		√			
Torabi and Hassini (2008)	√	-	√						
Esmaeili (2009)		Infinite		√	√				
Wee et al. (2009)	√	Infinite	√				√		
Ghasemy Yaghin and Fatemi Ghomi (2012)	√	Finite	√	√	√		√		
Bera et al. (2012)	√	Infinite	√						
Ghasemy Yaghin et al. (2013)	√	Finite	√	√	√	√	√		
Wang et al. (2015)		Finite		√					
Ghasemy Yaghin et al. (2015)	√	Finite	√	√	√	√		√	
Feng et al. (2017)	√	Infinite		√					
Ghasemy Yaghin (2018)		-		√	√	√			
This paper	√	Finite	√	√	√	√	√	√	√

This paper studies a fuzzy chance constraint model under possibilistic constraints to determine a joint lot-sizing and marketing plan for a manufacturer who faces demand from two or more market segments. The presentation of a possibility measure to our model is useful to express the decision maker's attitude and satisfaction under uncertainty. The concerned model accounts for inherent uncertainty in different parameters such as demand functions, marketing and inventory related parameters.

On the other hand, as a continuance of Ghasemy Yaghin et al.'s paper (2013), using a fuzzy chance constraint programming approach, we apply a possibility measure optimization model and propose a solution algorithm in order to defuzzify the uncertain model. Therefore, the summary of significant contributions of the paper can be presented as follows. First, it introduces a practical fuzzy chance constraint model for jointly making some major inventory, marketing and price differentiation related decisions with ROII maximization for a manufacturer including satisfaction degree of the Decision Maker (DM). Second, a credibility measure-based fuzzy procedure is incorporated into the developed model to handle the inherent ambiguity in marketing and inventory decisions. In fact, a two-stage policy is taken into account. In the first phase, a credibility measure is applied to tackle uncertain parameters in the objective function. Then, in the second phase, possibility constraints with a confidence level of DM are handled to convert them into crisp ones. Altogether, the abovementioned properties differentiate this paper from those existing in the related literature.

The rest of the paper is organized as follows. Problem description and formulation are presented in the next section. In Section 4, through developing efficient defuzzification strategies, the original fuzzy model is transformed to an equivalent defuzzified model. Section 5 elaborated the meta-heuristic algorithm to solve the resulting crisp nonlinear model within a reasonable computation time. Numerical results are presented in Section 6. Finally, concluding remarks and future research directions are given in the last section.

Model Description

The manufacturer fabricates a non-perishable item, while inventory shortages are not allowed. The planning horizon is assumed to be infinite by which a demand rate is defined for the product. The finite replenishment rate is greater than the demand rate. The supply chain system aims to maximize ROII as the main optimization criterion. Each market demand is assumed to be general, such as the models proposed by Kim and Lee (1998), Lee (1993), and Abad (1988). In each market segment, the demand response curve is an improved version of the general demand function proposed by Esmaeili (2009), and can be affected by price, marketing expenditure and time. Sub-markets are not independent and there is a demand cannibalization among segments. Price in market i is greater than that of $i+1$ ($i=1,2,..,n-1$). For example, demand function in the first sub-market is as following (Ghasemy Yaghin et al., 2013):

$$D_1\left(P_1,P_2,M\right)=D_1\left(P_1\right)B\left(M\right)-\tilde{\gamma}\left(P_1-P_2\right)$$

where $\tilde{\gamma}$ stands for fuzzy demand cannibalization rate, the part of higher-willingness to pay customers who find a way to purchase at the lower price, $D_1\left(P_1,P_2,M\right)$ is demand rate of the first market which is affected by price and marketing expenditure (units/period), P_1 and P_2 present selling prices in the first and second markets, M is marketing expenditure per unit ($/unit) and $B(M)$ is the function of marketing expenditure. It is noteworthy that manufacturers usually have insufficient and imprecise knowledge about the cannibalization rate; where they can be estimated based on both the available historical data and experience of manufacturers or decision makers. Also, there are many cases (especially in the marketing environment) where we face a newly produced product and there is no historical data about the demand cannibalization rate for the decision maker. Obviously, they cannot even apply statistical methods to estimate the parameters. Therefore, we take the fuzzy demand function into account in order to formulate customer demand. In reality, the aforementioned function is a generalization of the Zhang and Bell's (2007) function and which was considered by Ghasemy Yaghin et al. (2013).

Various stochastic modeling techniques have been successfully applied in inventory and ordering problems with randomness (Eynan & Kropp, 2007). However, probability functions obtained from historical data are not always reliable or available. As emphasized by Mula et al. (2010), the fuzzy modeling provides a useful tool to support the inventory and marketing research when the dynamics of the inventory environment limit the specification of the model objectives, constraints and parameters. Moreover, important input parameters (e.g., consumer demands and costs) are naturally imprecise because of incompleteness and/or unavailability of historical data over the product's life cycle. Fuzzy modeling is related to flexible or fuzzy constraints modeled by fuzzy sets. To do so, we have to estimate the problem parameters subjectively based on current insufficient data and the decision maker's experience.

Furthermore, triangular fuzzy numbers are considered to formulate each fuzzy parameter. As Driankov et al. (1996) mention, the triangular shape of the membership function has been a predominant function and is most common in the literature due to its functional description and economic parametric. This means that with the help of triangular possibility distribution, the output value will have a degree of membership 1 in place of intervals as it would be the case with some other types of possibility distributions (Driankov et al., 1996). In addition, the triangular possibility distribution is the most common distribution for modeling the imprecise nature of the ambiguous parameters, because of its computational efficiency and simplicity in data acquisition (Zimmermann, 1978; Torabi & Hassini, 2008). The symmetric triangular fuzzy parameter is shown in Figure 1. The fuzzy number is determined by a spread w_c and a center c_2.

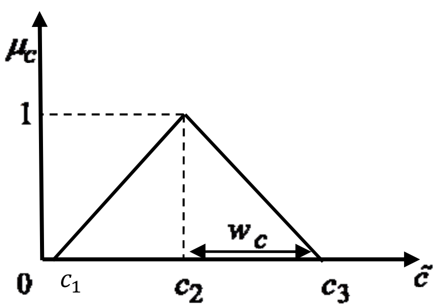

Figure 1. **Triangular possibility distribution of a fuzzy parameter**

Satisfaction degree of DM has been provided by a confidence level. It is clear that we should involve a high confidence level to come with the guarantee of the model constraints. However, because of the nature of the uncertain inputs and the restriction of the outputs, to find an operation policy with a 100% guarantee for complying with the constraints is impossible. Moreover, satisfaction degree is an appropriate safety margin which is set by the decision maker.

Mathematical Formulation with Chance Constraints

The manufacturer's problem is the differential pricing, determination of marketing expenditure, and purchasing quantities in a ROII maximization. As they have been addressed by Ghasemy Yaghin et al. (2013), within the inventory cycle time $\left[0,T\right]$, the fuzzy average profit, $\widetilde{AP}$, including sales revenue, marketing cost, production cost, ordering cost and holding cost would be derived:

$$\begin{aligned}\widetilde{AP}\left(T,P_1,..,P_n,M\right)= \quad & \frac{1}{T}\left\{\sum_{i=1}^{n}P_iD_i\times\left(\tilde{a}T-\frac{\tilde{b}}{2}T^2\right)-\tilde{c}_0R\lambda\right.\\ & -\sum_{i=1}^{n}MD_i\times\left(\tilde{a}T-\frac{\tilde{b}}{2}T^2\right)-\tilde{F} \\ & \left.-c_h\left[\frac{R}{2}\lambda^2+\frac{1}{2}\sum_{i=1}^{n}D_i\times\left(\tilde{a}T^2-\frac{2}{3}\tilde{b}T^3-2\tilde{a}T\lambda+\tilde{b}T^2\lambda\right)\right]\right\}\end{aligned} \tag{1}$$

where P_i is the price of the i-th sub-market, (indexed from 1 to n), $\tilde{a}$ and $\tilde{b}$ are fuzzy parameters of time function, D_i stands for final demand rate which is affected by price and marketing expenditure in the i-th market, that is $D_i\left(P_{i-1},P_i,P_{i+1},M\right)$. We use D_i for the i-th market instead of $D_i\left(P_{i-1},P_i,P_{i+1},M\right)$. The fuzzy production cost per unit (\$/unit) is shown by $\tilde{c}_0$. The production rate per period (units/period) and production time in cycle time T are denoted by R and λ. $\tilde{F}$ and $\tilde{c}_h$ are fuzzy setup cost (\$/setup) and fuzzy holding cost per unit (\$/unit), respectively.

The fuzzy average inventory investment, AI, is given by

$$AI\left(T,P_1,..,P_n,M\right)= \frac{\tilde{c}_p}{T}\left[\frac{R}{2}\lambda^2+\frac{1}{2}\sum_{i=1}^{n}D_i\times\left(\tilde{a}T^2-\frac{2}{3}\tilde{b}T^3-2\tilde{a}T\lambda+\tilde{b}T^2\lambda\right)\right] \tag{2}$$

Where $\tilde{c}_p$ is the fuzzy purchasing price per unit (\$/unit). Then the fuzzy return on inventory investment, $ROII$, is

$$ROII\left(T,P_1,..,P_n,M\right)=AP\left(T,P_1,..,P_n,M\right)/\,AI\left(T,P_1,..,P_n,M\right)$$

As it is calculated by Tersine (1994), λ is $\sum_{i=1}^{n}D_i\times\left(2\tilde{a}T-\tilde{b}T^2\right)/\,2R$.

As Ghasemy Yaghin et al. (2013) state the fuzzy average return on inventory investment is as follows:

$$\begin{aligned}\widetilde{ROII}(T,P_1,\dots,P_n,M)=&\left\{\sum_{i=1}^{n}P_1D_1\times\left(\tilde{a}T-\frac{\tilde{b}}{2}T^2\right)-\frac{\tilde{c}_0}{2}\sum_{i=1}^{n}D_i\times\left(2\tilde{a}T-\tilde{b}T^2\right)\right.\\&-\sum_{i=1}^{n}MD_i\times\left(\tilde{a}T-\frac{\tilde{b}}{2}T^2\right)-\tilde{F}-\tilde{c}_h\left[\frac{(\sum_{i=1}^{n}D_i\times\left(2\tilde{a}T-\tilde{b}T^2\right))^2}{8R}\right.\\&+\frac{1}{2}\sum_{i=1}^{n}D_i\times\left(\tilde{a}T^2-\frac{2}{3}\tilde{b}T^3-\tilde{a}T\frac{\sum_{i=1}^{n}D_i\times\left(2\tilde{a}T-\tilde{b}T^2\right)}{R}\right.\\&\left.\left.\left.+\tilde{b}T^2\frac{\sum_{i=1}^{n}D_i\times\left(2\tilde{a}T-\tilde{b}T^2\right)}{2R}\right)\right]\right\}/\,\tilde{c}_p\left[\frac{(\sum_{i=1}^{n}D_i\times\left(2\tilde{a}T-\tilde{b}T^2\right))^2}{8R}\right.\\&+\frac{1}{2}\sum_{i=1}^{n}D_i\times\left(\tilde{a}T^2-\frac{2}{3}\tilde{b}T^3-\tilde{a}T\frac{\sum_{i=1}^{n}D_i\times\left(2\tilde{a}T-\tilde{b}T^2\right)}{R}+\right.\end{aligned} \tag{3}$$

$$+\tilde{b}T^2\frac{\sum_{i=1}^{n}D_i\times\left(2\tilde{a}T-\tilde{b}T^2\right)}{2R}\Bigg)\Bigg]$$

ROII, as an extensively used economic performance measure coping with product inventories, involves joint profit and investment indices. Profit function is the difference of the sales revenue and the costs. For further clarification, total revenue is the selling prices per unit multiplied by the sales quantity, and the profit is obtained by subtracting the total cost (i.e., marketing cost, production cost, ordering cost and holding cost) from the revenue. The average investment is the average inventory investment. Mathematically, the average inventory investment is obtained by multiplying purchasing price per unit by average inventory.

As a result, we formulate our fuzzy decision problem as a chance constrained programming model which is:

$$M\tilde{a}x \quad ROII\left(T,P_1,..,P_n,M\right)$$

$$Pos\left(\sum_{i=1}^{n}MD_i\times\left(\tilde{a}T-\frac{\tilde{b}}{2}T^2\right)\leq\tilde{M}^{max}\right)\geq\eta$$

Subject to

$$Pos(D_1\left(P_1,P_2,M\right)<0)=0$$

$$Pos(D_i\left(P_{i-1},P_i,P_{i+1},M\right)<0)=0 \qquad i=2,..,n-1 \qquad (4)$$

$$Pos(D_n\left(P_{n-1},P_n,M\right)<0)=0$$

$$T>0,M\geq 0$$

$$P_i\geq 0 \qquad \forall i$$

The objective function is to maximize the ratio of the average profit over the average investment. In the objective function, some of the coefficients including $\tilde{a}$, $\tilde{b}$, $\tilde{F}$, $\tilde{\gamma}$, $\tilde{c}_p$, $\tilde{c}_h$ and $\tilde{c}_0$ are imprecise. The abbreviation *Pos* represents possibility measure in order to consider the chance constraints in the proposed model and it is clarified in Section 4.2. The first constraint means that the marketing cost must be lower than the maximum available budget assigned to marketing activity, $\tilde{M}^{max}$. Akin to chance constrained programming with stochastic input parameters, in the fuzzy situation, this constraint will be satisfied by minimum chance (i.e., confidence level) η that is set by the decision maker. Or a point is feasible if and only if the possibility measure of the set $(.)$ is at least η . The second, third and fourth constraints enforce the non-negativity of demand functions. Finally, the non-negativity of decision variables is shown by $T > 0, M \geq 0$ and $P_i \geq 0, \forall i$.

The Proposed Credibility-Based Fuzzy Optimization Model

Recalling the previously resultant objective function and constraints, we are tackling a fuzzy non-linear programming model with possibilistic constraints (FNLPPC) in which some critical input parameters such as market demands, cannibalization rate in market segmentation and inventory costs are represented as fuzzy parameters in the form of possibility distributions. Accordingly, a possibilistic programming approach is applied to convert the original FNLPPC model into an equivalent crisp non-linear fractional programming model (NLFP) while taking into account the marketing uncertainties.

To solve this model, we develop a new two-phased fuzzy approach. In the first phase, the objective function is transformed into an equivalent auxiliary crisp one by applying an efficient credibility-based defuzzification process. Then, in the second stage, we apply a hybridization of the Inuiguchi and Ramik (2000) and Maity (2011) methods. The efficiency of the proposed method is also examined using a similar technique as that of Maity (2011).

The Equivalent Auxiliary Crisp Objective

- Several approaches have been presented in the literature to tackle the possibilistic models including the ill-known coefficients in both objective function and constraints (e.g., Liu & Liu, 2002;

Torabi & Hassini, 2008). Here, we propose an efficient possibilistic method to transform the proposed possibilistic programming model into an equivalent auxiliary crisp model. Notably, we apply our proposed method due to its several advantages as follows:

- The method is computationally efficient to solve the fuzzy problems because it does not increase the number of objective functions and inequality constraints.
- The method is based on the novel fuzzy mathematical concepts such as expected value of fuzzy numbers and credibility measure.

In addition, there are some ill-defined parameters in the objective function. For a given fuzzy number *A*, credibility measure of *A* is considered as defined by Liu and Liu (2002):

$$Cr(A) = \frac{1}{2}\left(1 + Pos(A) - Pos\left(A^c\right)\right) \qquad A \in 2^R \tag{5}$$

Let $\tilde{c} = (c_1, c_2, c_3)$ be a triangular fuzzy number, k a crisp number and R a real number set. $Pos\left(A^c\right)$ represents the complement of the possibility measure. Then, according to Liu and Iwamura (1998), and Mandal et al. (2011), possibility and credibility measures (i.e., *Pos* and *Cr*) are as follows:

$$Pos(\tilde{c} \geq k) = \begin{cases} 1, & k \leq c_2 \\ \dfrac{c_3 - k}{c_3 - c_2}, & c_2 \leq k \leq c_3 \\ 0, & k \geq c_3 \end{cases} \tag{6}$$

$$Cr\left(\tilde{c} \geq k\right) = \begin{cases} 1, & k \leq c_1 \\ \dfrac{c_2 - \rho k}{c_2 - c_1} - \dfrac{\left(1-\rho\right)k}{c_2 - c_1}, & c_1 \leq k \leq c_2 \\ \dfrac{\rho\left(c_3 - k\right)}{c_3 - c_2}, & c_2 \leq k \leq c_3 \\ 0, & k \geq c_3 \end{cases} \tag{7}$$

$$Cr\left(\tilde{c} \leq k\right) = \begin{cases} 0, & k \leq c_1 \\ \dfrac{\rho\left(k - c_1\right)}{c_2 - c_1}, & c_1 \leq k \leq c_2 \\ \dfrac{c_3 - \rho c_2 - \left(1-\rho\right)k}{c_3 - c_2}, & c_2 \leq k \leq c_3 \\ 1, & k \geq c_3 \end{cases} \tag{8}$$

where, DM's degree of optimism is considered by $\rho \in \left[0,1\right]$ so that a larger value of ρ indicates a higher degree of optimism. For a normalized fuzzy set, the expected value of the fuzzy variable $\tilde{c}$ is calculated by:

$$E\left(\tilde{c}\right) = \int_0^{\infty} Cr\left(\tilde{c} \geq r\right)dr - \int_{-\infty}^{0} Cr\left(\tilde{c} \leq r\right)dr \tag{9}$$

Following Mandal et al. (2011) the following lemma is proven:

*Lemma***.** The expected value of fuzzy number $\tilde{c}$ is as follows:

$$E\left(\tilde{c}\right) = \frac{1}{2}\left[\left(1-\rho\right)c_1 + c_2 + \rho c_3\right], \qquad 0 < \rho < 1 \tag{10}$$

Therefore, to solve the equivalent possibilistic model, we apply the credibility measure and consider triangular fuzzy numbers, the expected value of fuzzy number $\tilde{c}$, $E\left(\tilde{c}\right)$, as $\frac{1}{2}\left[\left(1-\rho\right)c_1 + c_2 + \rho c_2\right]$. As a result, applying this method for the equivalent possibilistic model, we would compute an auxiliary defuzzified average inventory investment as follows:

$$ROII\left(T,P_1,..,P_n,M\right)=\Bigg\{\sum_{i=1}^{n}P_iD_i\times\left(\frac{1}{2}\left[(1-\rho)a_1+a_2+\rho a_3\right]T-\frac{1}{4}\left(\left[(1-\rho)b_1+b_2+\rho b_3\right]\right)T^2\right)$$

$$-\frac{1}{4}\left(\left[(1-\rho)c_{01}+c_{02}+\rho c_{03}\right]\right)\sum_{i=1}^{n}D_i\times\left(\left((1-\rho)a_1+a_2+\rho a_3\right)T-\frac{1}{2}\left((1-\rho)b_1+b_2+\rho b_3\right)T^2\right)$$

$$-\sum_{i=1}^{n}MD_i\times\left(\frac{1}{2}\left((1-\rho)a_1+a_2+\rho a_3\right)T-\frac{1}{4}\left((1-\rho)b_1+b_2+\rho b_3\right)T^2\right)-\frac{1}{2}\left((1-\rho)F_1+F_2+\rho F_3\right)$$

$$-\frac{1}{2}\left((1-\rho)c_{h1}+c_{h2}+\rho c_{h3}\right)\Bigg[\frac{\left(\sum_{i=1}^{n}D_i\times\left(\left((1-\rho)a_1+a_2+\rho a_3\right)T-\frac{1}{2}\left((1-\rho)b_1+b_2+\rho b_3\right)T^2\right)\right)^2}{8R}$$

$$+\frac{1}{2}\sum_{i=1}^{n}D_i\times\left(\frac{1}{2}\left((1-\rho)a_1+a_2+\rho a_3\right)T^2-\frac{1}{3}\left((1-\rho)b_1+b_2+\rho b_3\right)T^3\right.$$

$$-\frac{1}{2}\left((1-\rho)a_1+a_2+\rho a_3\right)T\frac{\sum_{i=1}^{n}D_i\times\left(\left((1-\rho)a_1+a_2+\rho a_3\right)T-\frac{1}{2}\left((1-\rho)b_1+b_2+\rho b_3\right)T^2\right)}{R}$$

$$\left.+\frac{1}{2}\left((1-\rho)b_1+b_2+\rho b_3\right)T^2\frac{\sum_{i=1}^{n}D_i\times\left(\left((1-\rho)a_1+a_2+\rho a_3\right)T-\frac{1}{2}\left((1-\rho)b_1+b_2+\rho b_3\right)T^2\right)}{2R}\right)\Bigg]\Bigg\}$$

$$\Bigg/\ 0.5\left((1-\rho)c_{p1}+c_{p2}+\rho c_{p3}\right)\Bigg[\frac{\left(\sum_{i=1}^{n}D_i\times\left(\left((1-\rho)a_1+a_2+\rho a_3\right)T-\frac{1}{2}\left((1-\rho)b_1+b_2+\rho b_3\right)T^2\right)\right)^2}{8R}$$

$$+\frac{1}{2}\sum_{i=1}^{n}D_i\times\left(\frac{1}{2}\left((1-\rho)a_1+a_2+\rho a_3\right)T^2-\frac{1}{3}\left((1-\rho)b_1+b_2+\rho b_3\right)T^3\right.$$

$$-\frac{1}{2}\left((1-\rho)a_1+a_2+\rho a_3\right)T\frac{\sum_{i=1}^{n}D_i\times\left(\left((1-\rho)a_1+a_2+\rho a_3\right)T-\frac{1}{2}\left((1-\rho)b_1+b_2+\rho b_3\right)T^2\right)}{R}$$

$$+\frac{1}{2}\left((1-\rho)b_1+b_2+\rho b_3\right)T^2\frac{\sum_{i=1}^{n}D_i\times\left(\left((1-\rho)a_1+a_2+\rho a_3\right)T-\frac{1}{2}\left((1-\rho)b_1+b_2+\rho b_3\right)T^2\right)}{2R}\Bigg] \quad (11)$$

Treating Possibilistic Constraints

In the presence of some possibilistic constraints in the model, we use the generalized version of Inuiguchi and Ramik (2000) to defuzzify the possibilistic constraints. Under a possibility distribution μ_{A_1} of a possibilistic variable a_1, possibility measures of the event that a is in a fuzzy set A_2 are defined as follows (Maity, 2011):

$$Pos\left(\tilde{a}_1 * \tilde{a}_2\right)=\left\{sup\left(min\left(\mu_{A_1}(x),\mu_{A_2}(y)\right)\right),x,y\in R,x*y\right\} \quad (12)$$

where $*$ is any of the relations $\leq$, $\geq$, $=$. The dual relationship of possibility and necessity requires that:

$$Nec\left(\tilde{a}_1 * \tilde{a}_2\right)=1-Pos\left(\overline{\tilde{a}_1 * \tilde{a}_2}\right) \quad (13)$$

We embed the generalized idea of Inuiguchi and Ramik (2000) for the fuzzy Model (11) with possibilistic constraints. The constraints of (11) have been converted as follows:

$$1-Nec(\sum_{i=1}^{n}MD_i\times\left(\tilde{a}T-\frac{\tilde{b}}{2}T^2\right)>\tilde{M}^{max})\geq\eta \quad (14)$$

$$\sum_{i=1}^{n}MD_i\times\left(a_2T-\frac{b_2}{2}T^2\right)+\left(\eta-1\right)\sum_{i=1}^{n}MD_i\times\left(w_a+w_b\right)$$
$$\geq\frac{1}{2}\left[\left(1-\rho\right)M_1^{max}+M_2^{max}+\rho M_3^{max}\right] \quad (15)$$

For non-negativity of the demand functions, the remaining constraints are as follows:

$$D_1\left(P_1,P_2,M\right)\geq 0$$
$$D_i\left(P_{i-1},P_i,P_{i+1},M\right)\geq 0 \qquad i=2,..,n-1$$
$$D_n\left(P_{n-1},P_n,M\right)\geq 0 \qquad (16)$$

Eventually, the resultant defuzzified non-linear fractional model is obtained as follows:

In short, the following steps present the proposed solution methodology to solve the original FNLPPC model.

Step 1: Define suitable triangular possibility distributions for the imprecise input data and formulate the original FNLPPC model for the JPLM problem.

Step 2: Transform the original fuzzy return on inventory investment into the equivalent crisp objective by credibility measures for imprecise parameters.

Step 3: Given the minimum chance level for constraints, convert the fuzzy constraints into the corresponding crisp ones, and formulate the auxiliary crisp NLFP model.

$$Max \qquad ROII\left(T, P_1, .., P_n, M\right)$$

Subject to

$$\begin{aligned}
&\sum_{i=1}^{n} MD_i \times \left(a_2 T - \frac{b_2}{2} T^2\right) + (\eta - 1)\sum_{i=1}^{n} MD_i \times (w_a + w_b) \\
&\qquad \geq \frac{1}{2}\left[(1-\rho) M_1^{max} + M_2^{max} + \rho M_3^{max}\right] \\
&D_1(P_1) B(M) - \gamma_3 (P_1 - P_2) \geq 0 \\
&D_i(P_i) B(M) + \gamma_3 (P_{i-1} - P_i) - \gamma_3 (P_i - P_{i+1}) \quad i = 2, .., n-1 \\
&D_n(P_n) B(M) + \gamma_3 (P_{n-1} - P_n) \geq 0 \\
&T > 0, M \geq 0 \\
&P_i \geq 0 \qquad \forall i
\end{aligned} \tag{17}$$

Solution Methodology: A Meta-Heuristics Algorithm

The formulation given in Equation (17) is a highly constrained non-linear fractional programming model. This characteristic causes the model to be hard enough to be solved by an exact method. Generally, there are no direct optimization algorithms to come with the guarantee of global solution if an optimization problem includes non-convex terms (Porn et al., 1999). Therefore, a search-based heuristic algorithm is required to solve the model. Eberhart and Kennedy (1995) introduced Particle Swarm Optimization (PSO) as a kind of population based stochastic optimization techniques and evolutionary algorithms, like the genetic algorithm.

There are some different research works applying particle swarm optimization to deal with mathematical programming models such as parameter estimation in regression models (De-los-Cobos-Silva et al., 2013), health care literature (Lopez et al., 2012), redundant mobile robots (Huang et al., 2015), and supply chains (Ghasemy Yaghin et al., 2013). We briefly introduce PSO from (Dye & Hsieh, 2010; Eberhart & Kennedy, 1995; Poli et al., 2007) and modify it for our non-linear model. Interested readers can refer to Eberhart and Kennedy (1995) and Poli et al. (2007) for PSO details.

In this paper, the PSO algorithm is used to solve the complicated non-linear programming models. The general form of non-linear optimization problem at hand is as follows:

$$Max\, ROII\left(T,\boldsymbol{P},M\right)$$

Subject to:

$$h_j\left(T,\boldsymbol{P},M\right)\le 0, j=1,..,m \tag{18}$$

$$\boldsymbol{P}^L \le \boldsymbol{P} \le \boldsymbol{P}^U$$

$$M^L \le M \le M^U$$

$$T^L \le T \le T^U$$

where $\boldsymbol{P}$ is the vector of price decision variables, $h_j\left(T,\boldsymbol{P},M\right)\le 0, j=1,..,m$ are m inequality constraints and $h_j\left(T,\boldsymbol{P},M\right), j=1,..,m$ are continuous functions on R^n. In addition, $\boldsymbol{P}^L,M^L,T^L$ and $\boldsymbol{P}^U,M^U,T^U$ are the lower and upper bounds of the decision variables.

Initialization

In PSO, at first a set of random particles (population) is initialized in the search space represented by a d-dimensional vector. Birds are particles and the position of each particle gives a potential solution to the problem. A particle moves with velocity. The *i*-th individual $(x_0, y_0)^i$ of the initial population and the *i*-th velocity of particles are randomly chosen as follows:

$$(x_0, y_0)^i = x^L + \Phi_1\left(x^U - x^L\right) \tag{19}$$

$$\left(v_0^x, v_0^y\right) = x^L + \Phi_2\left(x^U - x^L\right) \tag{20}$$

where Φ_1, Φ_2 are uniform random variables over the interval (0, 1). Note that we let $X \equiv (T, \boldsymbol{P}, M)$.

Handling the Objective Function and Constraints

The fitness function, Equation (21), evaluates the performance of each solution. Penalty terms are used to modify the constrained optimization model to unconstrained ones. Hence, the equivalent models can be generally obtained as follows:

$$\varphi(T,\boldsymbol{P},M) = ROII(T,\boldsymbol{P},M) + \mu_{penl}\sum_{j=1}^{m}\left\{max\left\{0, h_j(T,\boldsymbol{P},M)\right\}\right\}^2 + \left\{max\left\{0, \boldsymbol{P} - \boldsymbol{P}^U\right\}\right\}^2 \tag{21}$$
$$+\left\{max\left\{0, \boldsymbol{P}^L - \boldsymbol{P}\right\}\right\}^2 + \left\{max\left\{0, M - M^U\right\}\right\}^2 + \left\{max\left\{0, M^L - M\right\}\right\}^2$$
$$+\left\{max\left\{0, T - T^U\right\}\right\}^2 + \left\{max\left\{0, T^L - T\right\}\right\}^2$$

where μ_{penl} is a positive large number, known as a penalty number. It is apparent that no penalty will occur if a constraint is satisfied. In other words, if a constraint is not satisfied the penalty term is realized. Eqatation (21) is applied to evaluate the fitness of particles in a population. Therefore, the problem becomes

$$Max\,\varphi(T, \boldsymbol{P}, M) \tag{22}$$

In fact, the technique transforms a constrained problem into a sequence of unconstrained problems. The constraints are placed into the objective function via a penalty parameter such that any violation of the constraints is penalized.

Updating Position and Velocity

Every particle in the swarm is described by position and velocity. The velocity of each particle determines the change of its position and flying direction. To find the optimal solution, each particle regulates its flying according to its own flying experience (cognition part) and its companions' flying experience (social part). The first one is the previous best position of each particle and is called *pbest*. The second one is the previous best position attained by any particle in the swarm and is called *gbest*.

In other words, a particle moves towards its best previous position and towards the best particle. The following Equation represents the velocity update of each particle during every iteration.

$$V_i^{t+1} = w \times V_i^t + cw_1 \times r_1 \times (pbest_i^t - X_i^t) + cw_2 \times r_2 \times (gbest_i^t - X_i^t) \quad (23)$$

and

$$X_i^{t+1} = X_i^t + V_i^{t+1} \quad (24)$$

where

V_i^t	Velocity of the *i*-th particle at the *t*-th iteration
X_i^t	Current position of the *i*-th particle at the *t*-th iteration
$pbest_i^t$	Previous best position of the *i*-th particle so far at the *t*-th iteration (*pbest*)
$gbest_i^t$	Previous best position attained by any particle in swarm at the *t*-th iteration (*gbest*)
cw_1, cw_2	Positive constant weight factors
r_1, r_2	Random numbers between 0 and 1which are uniformly distributed
w	Inertia weight

Acceleration coefficients are cw_1 and cw_2 that determine the distance of the movement of each particle in a single iteration. These coefficients are based on the influence of cognition experience and social part. Usually, cw_1 and cw_2 are selected in the range of

$[1.5,2.5]$.

The influence of the previous flight direction on the current position is controlled by the inertia weight. It causes convergence performance of the proposed algorithm. In order to reduce the impact of the inertia weight, the following expression is used similar to Ghasemy Yaghin et al.'s (2012) paper:

$$w^{t+1} = w^{t}\left(1 - t/N\right) \tag{25}$$

where N is the maximum number of iterations and t is the current iteration.

The velocity of particles is limited to lie within $[-V_{max}, +V_{max}]$. This leads to particles just searching the defined space. In PSO, V_{max} is usually set at 4. So, we have $V \in [-4,+4]$. The search algorithm of the particle swarm optimization is set as follows:

Step 0. Let population size $N = 10000$, particles $n^{p} = 50$, $\mu_{penl} = 10^{15}$, $cw_1 = 1.5, cw_2 = 2.5$, $V_{max} = 4$ and k=1

Step 1. Create a population of particles

Step 2. Evaluate the fitness of all particles based on objective function (21), that is $\varphi(T, \boldsymbol{P}, M)$

Step 3. Find and keep pbest

Step 4. Find and keep gbest

Step 5. Update the velocity of each particle according to (23)

Step 6. Update the position of each particle according to (24)

Step 7. Terminate if the standard deviation (i.e., $\left|\varphi_{k+1}(T, \boldsymbol{P}, M) - \varphi_{k}(T, \boldsymbol{P}, M)\right| < 10^{-5}$) is satisfied, otherwise k=k+1 and go to Step 2

Numerical Study

An illustrative example is provided to show the applicability of the approach. A focal company desires to determine lot size, prices and marketing expenditure in two market segments. Table 2 presents the input parameters.

Table 2. Input Parameters

Inventory costs	$\tilde{c}_0 = (20, 25, 30)$	$\tilde{c}_h = (17, 20,$	$\tilde{c}_p = (80, 85,$	$k_0 = (470, 500, 53$	
Demand parameters	$\tilde{a} = (75, 80, 85)$,	$\tilde{b} = (10, 14, 1$	$\lambda = 0.73$	$\tilde{\gamma} = (0.5, 1, 1.5)$	
Other parameters	$\tilde{M}^{\max} = (560000, 680000, 8000$			$R = 88500$	

Besides, we assume $D_1(P_1) = 35000 - 15P_1$, $D_2(P_2) = 500000 - 40P_2$ and $B(M) = M^{\lambda}$. DM sets the minimum chance $\eta = 0.8$. Of particular interest, we have examined the efficiency of the proposed method using a similar technique to that used by Maity (2011). According to Liu and Iwamura (1998) and Maity (2011), the constraints are reduced to the following constraints.

$$\begin{aligned}
&\frac{M_1^{max} - M(\sum_{i=1}^{n} D_i \times \left(a_1 T - \frac{b_1}{2} T^2\right)}{M(\sum_{i=1}^{n} D_i \times (a_2 - a_1) T - \frac{(b_3 - b_2)}{2} T^2) + M_3^{max} - M_2^{max}} \geq \eta \\
&D_1(P_1) B(M) - \gamma_1 (P_1 - P_2) \geq 0 \\
&D_i(P_i) B(M) + \gamma_1 (P_{i-1} - P_i) - \gamma_1 (P_i - P_{i+1}) \geq 0 \qquad i = 2, .., n-1 \\
&D_n(P_n) B(M) + \gamma_1 (P_{n-1} - P_n) \geq 0
\end{aligned} \tag{26}$$

At last, the auxiliary defuzzified NLFP model is given as follows:

$$Max \;\; ROII(T, P_1, .., P_n, M)$$

$$\text{Subject to} \quad \frac{M_1^{max} - M(\sum_{i=1}^{n} D_i \times \left(a_1 T - \frac{b_1}{2} T^2\right)}{M(\sum_{i=1}^{n} D_i \times (a_2 - a_1) T - \frac{(b_3 - b_2)}{2} T^2) + M_3^{max} - M_2^{max}} \geq \eta \tag{27}$$

$$D_1(P_1)B(M) - \gamma_1(P_1 - P_2) \geq 0$$

$$D_i(P_i)B(M) + \gamma_1(P_{i-1} - P_i) - \gamma_1(P_i - P_{i+1}) \geq 0 \qquad i = 2,..,n-1$$

$$D_n(P_n)B(M) + \gamma_1(P_{n-1} - P_n) \geq 0$$

$$T > 0, M \geq 0$$

$$P_i \geq 0 \qquad \forall i$$

The fuzzy programming model is taken into consideration to solve the equivalent crisp models. The following results are yielded by applying the proposed algorithm:

Table 3. Optimal Solutions for the Numerical Example

	The proposed method	Maity method
M	0.7	0. 4
P_1	1883.8	2007.7
P_2	1264.0	1051.6
T	2	1.4
$ROII$	5.5320	4.6500

To compare the two fuzzy methods (i.e., the proposed method and Maity method), they are compared in different η-levels. As it can be seen from Table 4, the proposed method gives more appropriate solutions than the Maity method, especially the objective function values. Particularly, in high η- level (i.e., 0.9– 0.95), the solutions found by the Maity method have objective function values less than our proposed method. In the low η-level (i.e., 0.6–0.7), the difference between the values is not significantly meaningful. Additionally, our proposed method could find balanced solutions between prices of channels, that is to say that the difference between prices is small and the Maity method needs to pay more attention to more sensitive submarkets because of the high considerable differences between market prices.

Table 4. The Summary of Test Results According to Different η - Level

$\eta -$ *level*	The proposed method			Maity method		
	P_1	P_2	$ROII$	P_1	P_2	$ROII$
0.6	1857.6	1263.4	5.7516	1861.2	1124.2	5.5474
0.7	1868.6	1263.7	5.6378	1815.3	1080.9	5.2690
0.8	1883.8	1264.0	5.5320	2007.7	1051.6	4.6500
0.9	1909.3	1264.6	5.4334	1974.6	1013.9	4.5915
0.95	1900.0	1264.4	5.3859	1851.3	946.3	4.1981

Table 5 summarizes the impact of increasing cannibalization rate on expected return on inventory investment and confirms the point that market segmentation and differential pricing must be used in place to limit cannibalization from high priced to low priced market segments by a manufacturer wishing to improve ROII. Therefore, it is deducible that the proposed approach is sensitive to $\tilde{\gamma}$ value. The results from Table 5 are completely consistent with basic findings of cannibalization in the lot-sizing literature (see Zhang and Bell 2007).

Table 5. Results of Sensitivity Analysis on $\tilde{\gamma}$ Value

$\gamma =$	(0.5,1,1.5)	(2.5,5,7.5)	(7.5,10,12.5)	(17.5,20,22.5)	(27.5,30,32.5)	(37.5,40,42.5)	(47.5,50,52.5)
P_1	1883.8	1770.9	1697.3	1602.1	1555.8	1508.5	1493.3
P_2	1264.0	1302.9	1320.9	1337.3	1350.0	1343.8	1350.0
$ROII$	5.5320	5.3005	5.0839	4.8353	4.7025	4.6173	4.5622

Parameter Tuning

Setting of parameter values plays an important role in the efficiency of the meta-heuristic algorithms. Particle swarm optimization has four parameters to be tuned which are population size, particles and positive constant weight factors (cw_1, cw_2). The Response Surface Methodology (RSM) method is implemented to tune the parameters of the algorithm. In this regard, the codes ‘‘-1’’, ‘‘0’’, and ‘‘+1’’ are applied for the low, medium (center), and high levels of the independent variables, respectively. Table 6 shows the parameter levels of the PSO algorithm.

Table 6 .Levels of Factors in RSM Application

Parameters	Range	Low	Medium	High
N	1000-10000	1000	5500	10000
n^p	50- 250	50	150	250
cw_1	1- 3	1	2	3
cw_2	1- 2.5	1	1.75	2.5

RSM is a collection of mathematical and statistical techniques useful for the modeling and analysis of problems in which a response of interest (return on inventory investment) is influenced by several variables (PSO parameters), and the objective is to optimize this response (Montgomery, 2001). The parameters with different levels are calibrated using Response Surface Methodology (RSM) with Design Expert software.

Table 7 summarizes the obtained response and the optimum values of the parameters of the algorithms using RSM. These values are the best fitness value for each obtained problem with respect to proposed points.

Table 7. Best Fitness Values

Parameters	Optimum value
N	10000
n^p	50
cw_1	1.5
cw_2	2.5

Concluding Remarks

To cope with the issue of uncertainty in integrated lot-sizing and price setting models, a fuzzy non-linear fractional model with possibilistic constraints was proposed in this paper. The considered integrated lot-sizing and price setting models include both marketing and inventory decisions as well as return on inventory investment (ROII) maximizing concurrently to avoid the sub-optimalities resulting from the individual policies. To solve the proposed fuzzy non-linear programming with the possibilistic constraints (FNLPPC) model, a fuzzy solution approach is proposed by combining the Inuiguchi and Ramik (2000) and Maity (2011) methods. To the best of our knowledge, this work is one of the original research studies applying possibilistic programming approach for the fuzzy pricing and lot-sizing problem based on considering market segmentations under uncertainty through optimizing return on inventory investment and the literature considering this approach in JPLM is still scarce. This paper contributes to the joint pricing and lot-sizing literature by revealing ROII maximization as an objective function called ROII pricing and fuzzy formulation approaches in the lot-sizing and price differentiation problems.

Regarding the importance of uncertainty and variability in price discrimination and market environment, many possible future directions can be defined in this area. We believe that further examinations are needed to investigate qualitative implications to generalize other demand functions, even if this may potentially require more complex technical skills. Moreover, solving the resulting non-linear model by a new efficient algorithm should be useful in obtaining good solutions. Thus, one of the appealing future research works of this paper is extending exact efficient heuristics algorithm. One can analyze the proposed approach in order to solve the multiple objective optimization model such as Ghasemy Yaghin et al. (2013) and develop a fuzzy multiple objective chance constraint model in order to determine differential pricing and ordering policies for the manufacturer. Developing the supply chain network under consideration in this paper in order to coordinate inventory decisions between supply chain entities would be an interesting area in real world problems. Our study has taken myopic customers into account, and an interesting avenue would be to involve strategic customers by the theory of game’s mathematical formulation. Last but not least, additional fuzzy measurements of variability such as variance or entropy of fuzzy sets need to be considered in the pricing-inventory model and enhance the proposed methodology.

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