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Dynamic Competitive Supply Chain Network Design with Price Dependent Demand and Huff Utility Function

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Abstract

This paper develops a two-stage model to consider a franchise/franchisee environment in which supply chains are simultaneously entering the untapped market to produce either identical or highly substitutable products and give franchise to franchisees. Customer demand is elastic, price dependent and customer utility function is based on Huff gravity rule model. The supply chains, in the first stage, shape their networks and set the market prices based on dynamic games. The franchisees, in the second stage, specify their attractiveness levels and set the locations of their retailers in simultaneous games. Possibility theory was also applied to cope with uncertainty. Finally, we applied our model to a real world problem, discussed the results, conducted some sensitivity analyses, and gained some managerial insights.

Keywords

Bi-level programming, simultaneous games, Nash equilibrium, dynamic competitive supply chain network design, Wilson algorithm.

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Introduction

Competition in business is slowly changing from "firms against firms" to "supply chains versus supply chains"; based on the literature; (Farahani et al., 2014), markets are full of different brands like Nike, Adidas, Nachi, Koyo, TTO, Nokia, SAMSUNG, Apple, Kia, Hyundai, GM, Volvo, Renault, and so on that mostly have some plants and distribution centers to produce and distribute their products to the retailers where the customers can buy the products directly. In this model, they have a semi-integral Supply Chain (SC) in which the retailers are working individually but the plants and distribution centers are working together as an integrated part of the chain. This structure can be matched with the customers' utility function and they think where firstly to select their famous brand then will choose the suitable retailers to patronize their demand. For example, the authors ask a lot of people who want to buy a cellphone and almost all of them agreed that if they want to buy a cellphone, firstly, they select their famous brand mainly based on the brand reputation and prices, then after selecting the most preferred brand, they select a suitable franchisee to buy the cellphone. This example can be adapted to a lot of industries and shows that customers have two-stage utility functions, firstly they choose their famous brand and then their franchisees; so we consider this two-stage approach as our main assumption in the rest of the paper.

Also nowadays, most of the chains design their network structure and set the market price then use local retailers as their franchisees to serve the demands. By this way, they reduce their costs and also make some job opportunities, but also they will face the questions like: What is their equilibrium network structure? What is their equilibrium price? How many market shares can they obtain? What is the equilibrium attractiveness and locations of the franchisees? The aim of this paper is to find the solutions to these questions.

Competitive Supply Chain Network Design (CSCND) considers the impact of competitive markets in designing the network structure of a chain to improve its future competitiveness (see Farahani et al., 2014, for a review on CSCND).

CSCND problems have three main decisions: Strategic, tactical and operational decisions. Based on these decisions, the related literature of this subject can be categorized into two sub-fields such as: Competitive location problems and competitive supply chain problems in which the former usually concentrates on strategic decisions like location and the latter mostly concentrates on operational decisions like pricing. On the other hand, competition, in general, is classified into three different types as: Static competition, dynamic competition, and competition with foresight.

Moreover, in each type of competition, customer utility function and customer demand are two essential factors which shape the structure of a competition. Hotelling (1929) and Huff (1964, 1966) are the most commonly used customer utility function and price-dependent demand and inelastic demand are the most commonly used customer demand functions in the literature.

The existing literature considers different criteria for elastic demand like service levels (Boyaci & Gallego, 2004), prices (Bernstein & Federgruen, 2005; Anderson & Bao, 2010), price and service level (Tsay & Agrawal, 2000; Xiao & Yang, 2008), price and distance (Fernandez et al., 2007), distance (Plastria & Vanhaverbeke, 2008; Godinho & Dias, 2013; Godinho & Dias, 2010), distance and one or more attractiveness attributes (Aboolian et al., 2007) that are mostly modeled according to 0-1 (all or nothing) rule based on Hotelling's (1929) utility function. On the other hand, inelastic demand (Kucukaydın et al., 2011; Kucukaydın et al., 2012, Fahimi et al., 2017a) is mostly modeled according to Huff (1964, 1966). Definitely, customers have different criteria like quality, price, brand image, service level and etcetera to choose a SC and patronize their demand to the convenient retailers and do their purchasing. As our mentioned example in cellphone market customers have two-stage approach, but all the mentioned articles consider one step utility function for the customers that cannot be applied to our described environment, so we assume the customers have two-stage utility function and define our approach to model this behavior.

Three kinds of competitions can be found in the SC competition literature: Horizontal competition, a competition between firms of one tier of a SC; vertical competition, a competition between the firms of different tiers of a SC; and SC versus SC, a competition between SCs.

Most of the franchise/franchisee problems are put into competitive location problems. Kucukaydın et al. (2011) presented a franchise/franchisee problem in which a franchise entered a market with existing franchises that belonged to a competitor and wanted to shape his network by locating some new facilities and set the attractiveness of the facilities where the competitor could react to his entrance by adjusting the attractiveness of the existing facilities of his own. Kucukaydın et al. (2012) follows the introduced problem by Kucukaydın et al. (2011), they consider the same franchise/franchisee problem with this difference that the existing competitor can also open some new facilities as new franchises or close or adjust the attractiveness of the current franchises; also, they use Huff utility function with inelastic customer demand. Godinho and Dias (2010) presented a franchise/franchisee problem in which two competitors simultaneously enter the distance dependent market with elastic demand and want to shape their network and maximize profits while they also should maximize social welfare and propose an algorithm to solve the introduced problem. Following the their prior work, Godinho and Dias (2013) introduced another franchise/franchisee problem in which the franchisor defined the potential locations and rule of the game, in fact, the paper considers preferential rights and overbidding which means that one competitor has preferential right over another one in the same situation.

Watson, Dada, Grünhagen, and Wollan (2016) employed organizational identity theory to explain when the franchisor desires to select specifically franchisees that have the potential for entrepreneurial behavior. Badrinarayanan et al. (2016) offer a parsimonious framework of the antecedents of brand resonance in franchising relationships. Shaikh (2016) proposes a comprehensive conceptualization of the concept of fairness in the context of franchisor–franchisee relationship. In CSCND problem, we can mention the following works: Rezapour and Farahani (2010), Rezapour et al. (2011), Rezapour and Farahani (2014), Rezapour et al. (2014), Rezapour et al. (2015), Fallah et al. (2015), Fahimi et al. (2017a), and Fahimi et al. (2017b).

Contributions

In this paper, we turn to the essential issue of CSCND problem by assuming a two-stage customer behavior utility function. Our modeling and solution approaches are similar to Fahimi et al. (2017a) and Fahimi et al. (2017b). Our main contributions are:

- ✓ Our modeling approach that is inspired from our customer utility function driven from a real market, we assume a two-stage customer behavior utility function.
- ✓ Our parameters that are known as fuzzy numbers instead of convex functions which make them more practical.
- ✓ Our solution approach that is based on bi-level programming, differential system, enumeration method and Wilson algorithm.
- ✓ Our definition of quality that is based on discrete scale.

According to our mentioned example in the cellphone market, we model the customer behavior by two stages, firstly each customer selects a brand (SC) to patronize it based on the price and brand reputation, and next he/she chooses different franchisees to buy from them. Up to our knowledge, this point of view is novel and did not appear in the previous literature. Turning our view to the player's side, we consider n supply chains simultaneously enter the untapped market. In stage one, the SCs shape their networks and set market price in dynamic competition; in stage two, each supply chain gives franchises to m_n competing and independent franchisees. There is a high tight interaction between the SCs and their franchisees whereby the SCs specify the market price and the network to satisfy the franchisees' needs, which essentially impacts their profits.

Actually we propose a two-stage solution approach to solve the model. Stage one is related to SC's problem and constructed based on bi-level programming, differential system and Wilson algorithm. Stage two is related to franchisee's model in which by the help of enumeration method the problem is convexified and solved.

Table 1. Characteristic of the Relavant Works

Authors(s)(year)	Pre-determined component				Modeling framework				Integration degrees			Type of considered game		competitive characteristics
	SC tiers	Competition type	parameters		Bi-level	Multi-level	Customer utility function		full	semi	decentralized	dynamic	Stackelberg	
			certain	uncertain			Hoteling based	Huff based						
Rezapour and Farahani (2010)	4	Duopoly					✓					✓		Price
Godinho and Dias (2010)	2	Duopoly					✓				✓	✓		Distance
Kucukaydın et al. (2011)	2	Duopoly						✓			✓		✓	Quality, distance
Rezapour et al. (2011)	4	Duopoly					✓						✓	
Kucukaydın et al. (2012)	2	Duopoly						✓			✓		✓	Quality, distance
Godinho and Dias (2013)	2	Duopoly					✓				✓	✓		Distance
Rezapour and Farahani (2014)	4	Oligopoly					✓					✓		Price, service level
Rezapour et al. (2014)	4	Oligopoly						✓				✓		Price, distance
Rezapour et al. (2015)	4	Duopoly					✓					✓		Price
Fallah et al. (2015)	4	Duopoly					✓							Price
Fahimi et al. (2017a)	3	Duopoly					✓				✓	✓		Quality, distance
Fahimi et al. (2017b)	3	Oligopoly					✓				✓	✓		Price
This paper	4	Oligopoly					✓	✓				✓		Price, quality, distance

To clarify the primary contributions of this paper in relation to the existing literature, Table 1 summarizes the characteristics of the relevant published models, including those of the current paper. The remainder of this paper is organized as follows: Section 2 describes the problem; Section 3 presents the solution approach; Section 4 presents the numerical results and discussions; and Section 5 discusses the conclusions.

Problem Definition

In this section, we first describe the problem environment and then formulate the problem faced by the SCs, their independent and competing franchisees. n SCs are planning to enter the competitive markets in which no rival has previously existed. The SCs are centralized and have two different tiers named according to the plants and DC levels. They produce the same or highly substitutable products and sell them to customers via m_n independent and competing franchisees. They are set to shape their networks (set the plants and DC locations) and market price and award franchises to the franchisees. Figure 1 shows the problem environment. SCs shape their networks based on a dynamic game relating to specified market shares. Next, they give franchises to m_n franchisees, paying attention to the fact that customers patronize their demand to the franchisees by a probability related to the franchisees' attractiveness. In other words, customers first select the chain based on brand imaging and price and according to 0-1 rule; second, they choose to patronize their demand to the franchisees according to the franchisees' attractiveness (in this step, each franchisee has a chance to be selected according to Huff's gravity rule model).

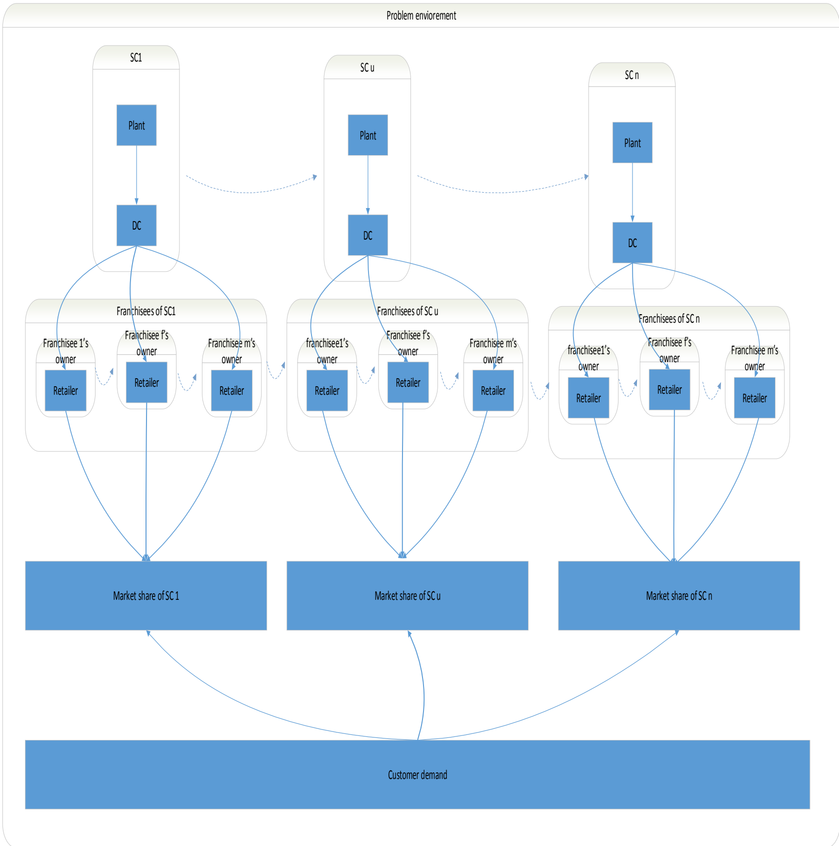

Figure 1. Problem environment

The total profit and market shares of each supply chain are dependent on their prices and the paths that they choose to satisfy the markets. The paths are based on the opened plants and the DCs of the chains. The total profits of the franchisees are also highly dependent on the prices and paths defined by the chains as well as the attractiveness of the franchisees' facilities. This definition shows that there are two stages by two different games in our proposed environment: The first one is a simultaneous game between the chains and pertains to shaping the network structures and the price specifying the equilibrium market price with respect to the fact that the prices are strictly related to the SC's opened paths (opened plants and DCs). The second game is between the franchisees, which is aimed at specifying the equilibrium qualities and distances by paying attention to the fact that m_n franchisees enter the market at the same time, thus, the second game is also a simultaneous game and will take place after the SC's game. Now, we can introduce the stages as follows:

Stage 1. SC selection

According to 0-1 rule and based on the price and reputation, customers choose one SC to patronize their demand. In this step, we use linear demand function.

Assume there are l demand points indexed by k and n incoming SCs indexed by u, then u th SC has s_u potential locations for opening plants indexed by e_u and m_u potential locations for opening DCs indexed by i_u correspondingly. So, the demand functions for u'th SC in market k can be defined as follows, similar to Tsay and Agrawal (2000):

$$d_k^{(u')}(P_k) = \tilde{\alpha}_{u'}\tilde{d}_k - \tilde{\delta}P_k^{(u')} + \tilde{\beta}\sum_{\substack{u=1\\u\neq u'}}^{n}\left(P_k^{(u)} - P_k^{(u')}\right) \qquad (1)$$

$\tilde{d}_k$ is the potential market size (if all prices were zero), $\tilde{\alpha}_{u'}$ is related to SC u' brand reputations, $\tilde{\alpha}_{u'}\tilde{d}_k$ is related on the basis of demand for SC u' if all prices were set to zero. Since demand cannot be negative, we assume:

$$\tilde{\alpha}_{u'}\tilde{d}_k - \tilde{\delta}P_k^{(u')} \succ \tilde{\beta}\sum_{\substack{u=1\\u\neq u'}}^{n}\left(P_k^{(u)} - P_k^{(u')}\right) \qquad (2)$$

Stage 2. Franchisee selection

In this step, the customers in each chain patronize their demands to the franchisees of the chain based on the Huff gravity-based rule, so each franchisee has a chance to be selected by the customers. Imagine that SC u has f_u franchisees and each franchisee has m_{f_u} potential retailers indexed by j_{f_u}, if the franchisee opens a retailer at site j_{f_u}, with $d^2_{j_{f_u}k}$ as the Euclidian distance between the retailer j_{f_u} and customer k, and with a quality level of $a_{j_{f_u}}$, so, the attractiveness of this facility for customer k is given by $\frac{a_{j_{f_u}}}{d^2_{j_{f_u}k}}$. By utilizing the gravity-based rule, the total attractiveness of franchisee f_u for customer k by the newly-opened retailers is given by $\sum_{j_{f_u}} \frac{a_{j_{f_u}}}{d^2_{j_{f_u}k}}$. Then the probability $Atr_{j_{f_u}k}$ that customer k visits facility j_{f_u} of franchisee f_u (based on all opened retailers in all franchisees of SC u) is expressed as $Atr_{j_{f_u}k} = \frac{\frac{a_{j_{f_u}}}{d^2_{j_{f_u}k}}}{\sum_{j_{f_u}} \sum_{f_u} \frac{a_{j_{f_u}}}{d^2_{j_{f_u}k}}}$ .Therefore, the revenue of franchisee f_u is as follows $\sum_{j_{f_u}} \sum_{k} m^{(u)} (P_k^{(u)} d_k^{(u)} (P_k) Atr_{j_{f_u}k})$. By a similar fashion, we can calculate the total revenue of other franchisees.

The following assumptions, parameters, and variables are used to model the introduced problems:

Assumptions

- ✓ The candidates' plant locations are known in advance.
- ✓ The candidates' DC locations are known in advance.
- ✓ There are no common potential locations between the chains.
- ✓ The demand of each customer market is concentered at discrete points.
- ✓ Demand is elastic and price dependent.
- ✓ Customer utility function is based on Huff gravity rule model.
- ✓ Products are either identical or highly substitutable.

Parameters

$\tilde{f}_{e_u}$	Fixed cost of opening a plant at location e for SC u
$\tilde{g}_{i_u}$	Fixed cost of opening a DC at location i for SC u
$\tilde{s}_{e_u}$	Unit production cost at plant e for SC u
$\tilde{c}_{e_u i_u}$	Unit transportation cost between plant e and DC i for SC u
$\tilde{h}_{i_u}$	Unit holding cost at DC i for SC u

$\tilde{f}_{j_{f_u}}$	Fixed cost of opening retail j for franchisee f at SC u
$\tilde{c}_{j_{f_u}}$	Unit attractiveness cost for retail j for franchisee f at SC u
$\tilde{h}_{j_{f_u}}$	Unit holding cost at retailer at location j for franchisee f at SC u
$\tilde{c}_{i_u j_{f_u}}$	Unit transportation cost between DC i and retailer j for franchisee f at SC u
$\tilde{c}_{j_{f_u} k}$	Unit transportation cost between retailer at location j for franchisee f at SC u and customer k
$d^2_{j_{f_u} k}$	Euclidian distance between retailer at location j for franchisee f at SC u and customer k
$P^{(1)}_{e_u}$	Number of opened plants for SC u
$P^{(2)}_{i_u}$	Number of opened DCs for SC u
$P_{j_{f_u}}$	Number of opened retailers for franchisee f at SC u
$m^{(u)}$	Percent of marginal profit for SC u

Decision variables

$y^{(1)}_{e_u}$	$\begin{cases} 1 \ if \ SC \ u \ opens \ a \ plant \ in \ location \ e \\ 0 \ otherwise \end{cases}$
$y^{(2)}_{i_u}$	$\begin{cases} 1 \ if \ SC \ u \ opens \ a \ DC \ in \ location \ i \\ 0 \ otherwise \end{cases}$
$y_{j_{f_u}}$	$\begin{cases} 1 \ if \ franchisee \ f \ in \ SC \ u \ opens \ a \ retailer \ in \ location \ j \\ 0 \ otherwise \end{cases}$
$x_{e_u i_u}$	Quantity of product shipped from plant e to DC i
$x_{i_u j_{f_u}}$	Quantity of product shipped from DC i to retailer at location j for franchisee f at SC u
$x_{j_{f_u} k}$	Quantity of product shipped from retailer at location j for franchisee f at SC u to customer k
$a_{j_{f_u}}$	Quality level of retailer at location j for franchisee f at SC u

The following model represents the problem of SC u:

$$P_{SCu} = \max Z_{SCu} = \sum_{i_u}\sum_{e_u}\sum_{k} P_k^{(u)} m^{(u)} (x_{e_u i_u}) y^{(1)}_{e_u} y^{(2)}_{i_u} - \qquad \forall u \qquad ((3)$$

$$\left(\begin{array}{l} \sum_{e_u} \tilde{f}_{e_u} y^{(1)}_{e_u} + \sum_{i_u} \tilde{g}_{i_u} y^{(2)}_{i_u} + \sum_{e_u}\sum_{i_u} \tilde{s}_{e_u} x_{e_u i_u} y^{(1)}_{e_u} y^{(2)}_{i_u} + \\ \sum_{e_u}\sum_{i_u} \tilde{c}_{e_u i_u} x_{e_u i_u} y^{(1)}_{e_u} y^{(2)}_{i_u} + \sum_{e_u}\sum_{i_u} (\frac{\tilde{h}_{i_u}}{2})(x_{e_u i_u}) y^{(2)}_{i_u} y^{(2)}_{i_u} \end{array} \right)$$

s.t

$$\sum_{i_u}\sum_{e_u} x_{e_u i_u} y_{e_u}^{(1)} y_{i_u}^{(2)} = \sum_{k} d_k^{(u)}(P_k) \qquad \forall u \tag{4}$$

$$\sum_{e_u} x_{e_u i_u} y_{e_u}^{(1)} = \sum_{f_u}\sum_{j_u} x_{i_u j_{f_u}} y_{i_u}^{(2)} \qquad \forall i_u \tag{5}$$

$$\sum_{e_u} y_{e_u}^{(1)} = P_{e_u}^{(1)} \tag{6}$$

$$\sum_{i_u} y_{i_u}^{(2)} = P_{i_u}^{(2)} \tag{7}$$

$$x_{e_u i_u}, x_{i_u j_{f_u}}, P_k^{(u)} \geq 0, y_{e_u}^{(1)}, y_{i_u}^{(2)} \in \{0,1\} \tag{8}$$

Term 3 represents the objective function of SC u, which includes profits captured by selling the product to the franchisees minus the fixed cost of opening plants and DCs, the production cost of plants, the transportation cost between plants and DCs, and the holding cost at DCs. Constraint 4 ensures that all the demands of the customers are satisfied by the opened plants and DCs. Constraint 5 is related to flow balance; Constraints 6 and 7 ensure that only $P_{e_u}^{(1)}, P_{i_u}^{(2)}$ plants and DCs are opened; and Constraint 8 is related to binary and non-negativity restrictions on the corresponding decision variables.

The problem of franchisee f in SC u:

$$P_{f_u}: \max Z_{f_u} = \sum_{k}\sum_{j_u}\left(1-m^{(u)}\right)P_k^{(u)} x_{j_{f_u}k} y_{j_{f_u}} - \qquad \forall u, f_u \tag{9}$$

$$\left(\begin{array}{l} \sum_{i_u}\sum_{j_u}\left(\frac{\tilde{h}_{j_{f_u}}}{2}\right)x_{i_u j_{f_u}} + \sum_{k}\sum_{j_u}\tilde{c}_{j_{f_u}k} x_{j_{f_u}k} \\ +\sum_{j_u}\tilde{c}_{j_{f_u}} a_{j_{f_u}} y_{j_{f_u}} + \sum_{j_u}\tilde{f}_{j_{f_u}} \end{array}\right) y_{j_{f_u}}$$

$s.t$

$$x_{j_{f_u}k} = d_k^{(u)}(P_k)\frac{\frac{a_{j_{f_u}}}{d_{j_{f_u}k}^2} y_{j_{f_u}}}{\sum_{j_{f_u}}\sum_{f_u}\frac{a_{j_{f_u}}}{d_{j_{f_u}k}^2} y_{j_{f_u}}} \qquad \forall u, f_u, j_{f_u} \tag{10}$$

$$\sum_{j_u} y_{j_{f_u}} = P_{j_{f_u}} \tag{11}$$

$$\sum_{i_u} x_{i_u j_{f_u}} y_{i_u}^{(2)} = \sum_{k} x_{j_{f_u} k} y_{j_{f_u}} \qquad \forall u, f_u, j_{f_u} \tag{12}$$

$$x_{j_{f_u} k}, a_{j_{f_u}} \geq 0, y_{j_{f_u}} \in \{0,1\} \tag{13}$$

Term 9 represents the objective function of franchisees in SC u, which includes the profits from selling the product to the customers minus the fixed cost of opening and setting the quality level of the facilities, the holding cost at the retailers, and the transportation cost between the retailers and customers. Constraint 10 ensures that each opened retailer satisfies the level of demand from customers; Constraint 11 specifies the number of opened retailers; Constraints 12 is related to balance flow; Constraints 13 is related to binary and non-negativity restrictions on the corresponding decision variables.

Solution Approaches

In this section, we present the solution approaches to our two-stage dynamic competitive supply chain network design. Our solution approaches are similar to Fahimi et al. (2017a) and Fahimi et al. (2017b). We also, based on the proposed modeling approach, categorize the problem into two distinct stages. In the first stage, the SCs set the market prices and shape their networks. In the second stage, the franchisees select their optimum locations and attractiveness to maximize their profits. The proposed algorithm is as follows:

Stage 1. SC selection

1- Consider the whole strategies for the SCs:

1-1 Construct an empty poly-matrix by considering all pure strategies of the SCs.

2- Calculate Nash equilibrium prices and flows for all the chains in the defined strategies.

2-1 Construct the profit function in each strategy and differentiate the terms and solve equilibrium prices for all SCs simultaneously.

3- Find the best response of all the players.

3-1 Fill the empty poly-matrix with the obtained payoffs from the previous stage and find the best network structure using Wilson algorithm.

Stage 2. Franchisee selection

4- Consider the whole strategies for the franchisees:

4-1 Construct an empty poly-matrix by considering all pure strategies of the players based on locations and quality levels.

5- Calculate Nash equilibrium locations and quality levels for all the franchisees in the defined strategies.

5-1 Use enumeration method to obtain locations and quality levels for all franchisees simultaneously

6- Find the best response of all the franchisees.

6-1 Fill the empty poly-matrix with the obtained payoffs from the previous stage and find the best network structure using Wilson algorithm.

However, in our solution approaches, we introduce a step-by-step procedure in which we can reach equilibrium networks, price, location and attractiveness. Moreover, in each step, we formulate the equivalent crisp model based on the method introduced by Inuiguchi and Ramik (2000), Liu and Iwamura (1998), Heilpern (1992), and Pishvaee et al. (2012).

Stage one: SC selection

Each SC has two intrinsically different decisions. Price and location decisions in which price is operational and location is strategic cannot be decided simultaneously as they are naturally different. Also, the model should first decide about the locations and then sets the price; in addition, the variable costs that should be considered in the price are directly related to the location of facilities and production, holding and transportation costs. Therefore, to solve this problem, we use a three-step algorithm in which step one constructs a poly matrix based on location variables of the chains; step two uses bi-level programming and sets the price and assignments; and step three selects the equilibrium networks and consequently equilibrium prices with the help of Wilson algorithm (Wilson, 1971).

Step one

This step is shaped based on the location variables of the SCs, as the number of opened plants $\begin{pmatrix} s_u \\ P_{e_u}^{(1)} \end{pmatrix}$ and DCs $\begin{pmatrix} m_u \\ P_{e_u}^{(2)} \end{pmatrix}$ in each chain is known in advance, so we can construct a poly matrix by dimension equal to $\left(\begin{pmatrix} s_1 \\ P_{e_1}^{(1)} \end{pmatrix} \cdot \begin{pmatrix} m_1 \\ P_{e_1}^{(2)} \end{pmatrix}\right) * \ldots * \left(\begin{pmatrix} s_u \\ P_{e_u}^{(1)} \end{pmatrix} \cdot \begin{pmatrix} m_u \\ P_{e_u}^{(2)} \end{pmatrix}\right) * \ldots * \left(\begin{pmatrix} s_n \\ P_{e_n}^{(1)} \end{pmatrix} \cdot \begin{pmatrix} m_n \\ P_{e_n}^{(2)} \end{pmatrix}\right)$. To clarify, consider we have two incoming SCs and each one wants to open one plant and two DCs through 5 and 3 potential locations so there exist $\begin{pmatrix} 5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 15$ pure strategy so we have a bi-matrix by dimension equal to $15 * 15$ and we encountered with 225 different problems in the next step that should be solved through differential systems and mathematical optimization. Now, we can calculate the price in each strategy in the next step.

Step two

We introduce a bi-level programming here to solve the model of the SCs in each defined strategies as follows: Inner level

This step deals with the inner part of the bi-level model, which determines the equilibrium prices for the SCs. In fact, pricing decisions are highly related to the possible paths (indexed by s) in serving the market. Each path is a combination of one plant and one DC from each chain. For example, if SC u' opens a plant and DC at location $e_{u'}, i_{u'}$ then the costs of path for the chain (including production, transportation, and holding costs) is calculated as:

$$\tilde{c}_{u'}^{s} = \tilde{s}_{e_{u'}} + \tilde{c}_{e_{u'} i_{u'}} + \left(\frac{\tilde{h}_{i_{u'}}}{2}\right) \tag{14}$$

The following models are then used to maximize the profit of the SCs:

$$\pi_{SCu'} = (P_k^{(u')} - \tilde{c}_{u'}^{s})\left(\tilde{\alpha}\tilde{d}_k - \tilde{\delta} P_k^{(u')} + \tilde{\beta}\sum_{\substack{u=1\\u\neq u'}}^{n} P_k^{(u)} \right) \qquad (15)$$
$$\max\{\pi_{SCu'}\}$$

Let assume $P_k^{(u')} \geq \tilde{c}_{u'}^{s}$ then by differentiating the terms and solving equilibrium prices for all SCs simultaneously that result in equilibrium prices.

Outer level

This step deals with the outer part of the bi-level model. The mathematical model for this part is constructed as follows with respect to the fact that the opened plants and DCs are predefined in previous stage and prices here are given by the inner part.

$$P_{SCu} = \max Z_{SCu} = \sum_{i_u}\sum_{e_u}\sum_{k} P_k^{(u)*} m^{(u)} (x_{e_u i_u}) - \left(\begin{array}{l} \sum_{e_u} \tilde{f}_{e_u} + \sum_{i_u} \tilde{g}_{i_u} + \sum_{e_u}\sum_{i_u} \tilde{c}_{e_u i_u} x_{e_u i_u} + \\ \sum_{e_u}\sum_{i_u} \tilde{s}_{e_u} x_{e_u i_u} + \sum_{e_u}\sum_{i_u} (\frac{\tilde{h}_{i_u}}{2})(x_{e_u i_u}) \end{array} \right) \qquad \forall u, e_i \in \begin{pmatrix} s_1 \\ P_{e_1}^{(1)} \end{pmatrix}, i_u \in \begin{pmatrix} m_1 \\ P_{e_1}^{(2)} \end{pmatrix} \qquad (16)$$

s.t

$$\sum_{i_u}\sum_{e_u} x_{e_u i_u} = \sum_{k} d_k^{(u)}(P_k) \qquad \forall u, e_i \in \begin{pmatrix} s_1 \\ P_{e_1}^{(1)} \end{pmatrix}, i_u \in \begin{pmatrix} m_1 \\ P_{e_1}^{(2)} \end{pmatrix} \qquad (17)$$

$$\sum_{e_u} x_{e_u i_u} = \sum_{f_u}\sum_{j_u} x_{i_u j_{f_u}} \qquad \forall i_u \in \begin{pmatrix} m_1 \\ P_{e_1}^{(2)} \end{pmatrix} \qquad (18)$$

$$x_{e_u i_u}, x_{i_u j_{f_u}} \geq 0 \qquad \forall u, e_i \in \begin{pmatrix} s_1 \\ P_{e_1}^{(1)} \end{pmatrix}, i_u \in \begin{pmatrix} m_1 \\ P_{e_1}^{(2)} \end{pmatrix} \qquad (19)$$

Term 16 represents the objective function of SC u in the defined strategy and with respect to the fact that the prices here are given by the inner part. Constraint 17 is related to demand satisfactions. Constraint 18 is related to flow balance; and Constraint 19 is related to non-negativity restrictions on the corresponding decision variables.

Step three

In this step we first fill the poly matrix by the given payoffs from the previous step and then calculate equilibrium networks by the help of Wilson algorithm (see Wilson (1971) for more information).

Stage two: Franchisee selection

In the second stage, the franchisees should select the locations and set their attractiveness levels for the facilities in order to maximize their profits according to the market prices and customer demand achieved by the SCs. The franchisee's problems are formulated by a Mixed Integer Nonlinear Programming Model (MINLP) and are non-convex in terms of its attractiveness function. But with respect to the modeling structure, the only nonlinear term in the model is the attractiveness term $\dfrac{\dfrac{a_{j_{f_u}}}{d^2_{j_{f_u}k}} y_{j_{f_u}}}{\sum_{j_{f_u}} \sum_{f_u} \dfrac{a_{j_{f_u}}}{d^2_{j_{f_u}k}} y_{j_{f_u}}}$, which specifies the quality, distance, and location of opened retailers. If we can fix the attractiveness term, the remainder of the model's terms are linear. On the other hand, the number of opened retailers in each franchisee is known in advance $P_{j_{f_u}}$. The attractiveness level of the retailer is directly related to its quality level. For this purpose, we define some scenarios for quality levels; therefore, like the SC's problem, we construct a poly matrix based on the pure strategies of the franchisees $\begin{pmatrix} j_{f_u} \\ P_{j_{f_u}} \end{pmatrix}$ in each chain and also define a five-scale measurement of the quality level as 1, 2, 3, 4 and 5 which are equal to very bad, bad, average, good, and very good quality levels. So this step encountered with $\left(5^{\binom{j_{1_u}}{P_{j_{1_u}}} * \binom{j_{2_u}}{P_{j_{2_u}}} * \dots * \binom{j_{f_u}}{P_{j_{f_u}}} * \dots * \binom{j_{m_u}}{P_{j_{m_u}}}} \right) * \left(\begin{pmatrix} j_{1_u} \\ P_{j_{1_u}} \end{pmatrix} * \begin{pmatrix} j_{2_u} \\ P_{j_{2_u}} \end{pmatrix} * \dots * \begin{pmatrix} j_{f_u} \\ P_{j_{f_u}} \end{pmatrix} * \dots * \begin{pmatrix} j_{m_u} \\ P_{j_{m_u}} \end{pmatrix} \right)$ different problems that should be solved to fill the poly matrix and be able to find the Nash equilibrium locations and quality levels of the franchisees by Wilson algorithm.

We therefore used Wilson algorithm and the Nash equilibrium concept and introduced a very simple and efficient procedure to obtain the Nash equilibrium point. In the proposed method, each player has several pure strategies $\begin{pmatrix} j_{f_u} \\ P_{j_{f_u}} \end{pmatrix}$ that are defined by their quality levels for the opened facilities. With this procedure, the problem is also convexified, and we can define a poly matrix based on the opened retailers and their corresponding quality levels. By this manner, there is no need for major computational calculations. Moreover, it can be easily applied to small size problems; therefore, the equivalent model of franchisee f in SC u is as follows:

$$P_{f_u} : \max Z_{f_u} = \sum_k \sum_{j_u} \left(1 - m^{(u)}\right) P_k^{(u)} x_{j_{f_u}k} - \qquad \forall u, f_u, y \in \begin{pmatrix} j_{f_u} \\ P_{j_{f_u}} \end{pmatrix}$$

$$\left(\begin{array}{l} \sum_{i_u} \sum_{j_u} \left(\frac{\tilde{h}_{j_{f_u}}}{2}\right) x_{i_u j_{f_u}} + \sum_k \sum_{j_u} \tilde{c}_{j_{f_u}k} x_{j_{f_u}k} \\ + \sum_{j_u} \tilde{c}_{j_{f_u}} a_{j_{f_u}} + \sum_{j_u} \tilde{f}_{j_{f_u}} \end{array} \right) \tag{20}$$

$$s.t$$

$$x_{j_{f_u}k} = d_k^{(u)}(P_k) \frac{\dfrac{a_{j_{f_u}}}{d^2_{j_{f_u}k}}}{\sum_{j_{f_u}} \sum_{f_u} \dfrac{a_{j_{f_u}}}{d^2_{j_{f_u}k}}} \qquad \forall u, f_u, j_{f_u}, y \in \begin{pmatrix} j_{f_u} \\ P_{j_{f_u}} \end{pmatrix} \tag{21}$$

$$\sum_{i_u} x_{i_u j_{f_u}} = \sum_k x_{j_{f_u}k} \qquad \forall u, f_u, j_{f_u} \in \begin{pmatrix} j_{f_u} \\ P_{j_{f_u}} \end{pmatrix} \tag{22}$$

$$x_{j_{f_u}k} \geq 0, a_{j_{f_u}} \in \{1, 2, 3, 4, 5\} \tag{23}$$

Term 20 represents the objective function of franchisee f_u in SC u; Term 21 ensures that each opened retailer satisfies the level of patronized demands; Constraint 22 is related to balance flow and Term 23 is related to the quality, and non-negativity restrictions on the corresponding decision variables.

It is worth noting that as the proposed algorithm uses Wilson algorithm and enumeration method, it needs a lot of time, especially in its worst case, and is just suitable for small-scaled problems, so proposing a meta-heuristic solution by computing the complexity of the algorithm can be a good idea.

Numerical Study and Discussion

Our case study is related to two Iranian investors who want to produce their brands in the spare parts industry; in particular, they want to produce a kind of bearing used in washing machines. This market is untapped for the Iranian investors. Based on the quality of their product and the market price, they have no competitors. The two chains in this study are simultaneously entering the market, and each chain wants to open one plant and one DC from five potential locations. They also want to give franchises to two competing and independent franchisees named R_1^{SC1}, R_2^{SC1} in SC1 and R_1^{SC2}, R_2^{SC2} in SC2. Each franchisee has four potential locations and wants to open two retail points and set their quality based on the given prices to maximize its profit. There is one demand point. The demand functions of the chains are as follows:

$$0.55\tilde{d} - 0.03\tilde{d}P_1^{(1)} + 0.07\tilde{d}P_1^{(2)} \tag{24}$$

$$0.45\tilde{d} - 0.03\tilde{d}P_1^{(2)} + 0.07\tilde{d}P_1^{(1)} \tag{25}$$

The parameters are assumed to be trapezoidal fuzzy numbers. The following distributions are used to extract the required parameters (Table 2)

Table 2. Distribution of Parameters

$\tilde{f}_{e_u} \square (u(1500,2000),\ u(2000,2500),\ u(2500,3000),\ u(3000,4000))$
$\tilde{g}_{i_u}, \tilde{f}_{j_u}^{r} \square (u(900,1500),\ u(1500,2000),\ u(2000,2500),\ u(2500,3000))$
$\tilde{s}_{e_u} \square (u \square (2,2.5),\ u \square (2.5,2.75),\ u \square (2.75,3),\ u \square (3,3.5))$
$\tilde{c}_{e_u i_u} \square (u(0.9,1.5),\ u(1.5,2.1),\ u(2.1,2.5),\ u(2.5,3.12))$

$$\tilde{c}^{r}_{j_u k} \sim (u(1.5,2),\ u(2,2.5),\ u(2.5,3),\ u(3,3.5))$$

$$\tilde{h}_{i_u}, \tilde{h}^{r}_{j_u} \sim (u(1.25,1.5),\ u(1.5,1.75),\ u(1.75,2),\ u(2,2.25))$$

$$d \sim (u(9000,10000),\ u(10000,11000),\ u(11000,12000),\ u(12000,13000))$$

$$\tilde{c}^{r}_{j_u} \sim (u(900,1500), u(1500,2000),\ u(2000,2500), u(2500,3000))$$

The proposed algorithm was implemented in Matlab (2014a) and carried out on a Pentium dual-core 2.6 GHz with 2 GB RAM. In this study, we determine equilibrium prices and locations and specify how the chain should give franchises to franchisees, and the effect of marketing activities on their total profits. Dynamic competition occurred between them on the basis of location and price, and they used the prices obtained as the market price for their franchisees. The franchisees sold the product to the customers at the equilibrium prices specified by two SCs in the price competition. There is also a dynamic competition between the franchisees in terms of market shares. Table 3 shows the results of the study. According to this table, SC1 opens a plant at Location 5 and a DC at Location 2; SC2 opens at 3 and 5, and therefore, the opened path is (5,2,3,5). The remainder of results are presented in Table 3.

Table 3. Numerical Example

	Opened paths	Total market share	DC price	objSC		Retailer price	Equilibrium location	Equilibrium quality	objfranchisee
SC1	(5,2,3,5)	12029.6	7.95	32727	R_1^{SC1}	8.74	(1,3)	(2,3)	10905.29
					R_2^{SC1}		(2,4)	(3,3)	8407.12
SC2	(5,2,3,5)	16119.2	7.02	30729	R_1^{SC2}	7.72	(1,2)	(3,1)	13163.88
					R_2^{SC2}		(2,3)	(2,2)	5650.864

Discussion

We now discuss the sensitivity analysis of the equilibrium prices, market shares, total SC profit, total franchisee profit, opened paths, attractiveness levels, and equilibrium location of the retailers with respect to the effect of δ, β parameters, which are related to switching and marginal customers and represent different marketing decisions. Moreover, we discuss the situations in which the SCs have different levels of power, specifying by λ as the weighting factor to cooperate with each other; and simply we use weighted sum the objective function of the chains by the corresponding constraints of both and it is worth noting that $0 \le \lambda \le 1$, and we assumed that λ belongs to $\lambda \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$. In the franchisees' phase, we analyze the effect of SC decisions to give their franchise to just one franchisee instead of two. In addition, they can consider the situations in which the franchisees can sell the products of both chains, named in terms of common franchisees. In this case, we also analyze the effect of the existence of one to two independent franchisees on their attractiveness levels and profits.

Table 4 shows the behavior of the opened paths, total market share, DC price, total SC profit, equilibrium locations and qualities, and franchisees' total profit with respect to β. The amount of parameter β varies in the solved examples while δ is set to $0.03EV(d)$. According to figure 2, by increasing the competition intensity, the total market share of both chains increases, but the amount of expansion for SC2 is higher than that of SC1. In the case of low competition intensity, SC1 has gained more market share. According to figure 3, the DC price of both chains decreases by increasing β; in terms of low competition intensity, their difference is more than high competition intensity. Figure 4 shows the total profit of the chains; in the low amount of β, SC1 has gained greater profits than SC2. However, by increasing the amount of β, their total profit becomes similar because of similar DC prices, and the market share of SC2 increases. Figure 5 shows the behavior from total profit of the franchisees in SC1 with respect to β, which has the same patterns as the total profit of SC1.

Table 4 . The Change of the Opened Paths, Total Market Share, DC Price, Total SC Income, Equilibrium Locations and Qualities and Retailer's Total Income with Respect to β

$\delta = 0.03EV(d)$	Opened paths	Total market share	DC price	objSC		Retailer price	Equilibrium locations	Equilibrium qualities	objfranchisee	β
SC1	(5,2,3,5)	10073.60	10.97	57068.36377	R_1^{SC1}	12.07	(1,3)	(3,3)	25854.98	$\delta = 0.03EV(d)$
					R_2^{SC1}		(2,4)	(3,3)	18521.16	
SC2	(5,2,3,5)	9094.38	8.89	32308.68637	R_1^{SC2}	9.78	(1,2)	(3,1)	13066.9111	
					R_2^{SC2}		(2,3)	(2,2)	5649.061466	
SC1	(5,2,3,5	10238.64824	10.76	55952.82602	R_1^{SC1}	11.84	(1,3)	(3,3)	25166.05249	$\delta = 0.03d_k$
					R_2^{SC1}		(2,4)	(3,3)	18002.38625	
SC2	(5,2,3,5)	9465.504477	8.81	33021.29351	R_1^{SC2}	9.69	(1,2)	(3,1)	11704.67331	
					R_2^{SC2}		(2,3)	(3,2)	6032.590397	
SC1	(5,2,3,5)	10393.30557	10.57420813	54895.1531	R_1^{SC1}	11.63162895	(1,3)	(3,3)	24512.79151	$\delta = 0.03d_k$
					R_2^{SC1}		(2,4)	(3,3)	17510.37311	
SC2	(5,2,3,5)	9815.004571	8.722836642	33582.74695	R_1^{SC2}	9.595120306	(1,2)	(3,1)	12081.09246	
					R_2^{SC2}		(2,3)	(3,2)	6361.265591	
SC1	(5,2,3,5)	10537.80486	10.39938254	53882.85126	R_1^{SC1}	11.43932079	(1,3)	(3,3)	23887.42254	$\delta = 0.009d_k$
					R_2^{SC1}		(2,4)	(3,3)	17039.1829	
SC2	(5,2,3,5)	10145.43619	8.639002181	34019.35216	R_1^{SC2}	9.502902399	(1,2)	(3,1)	12382.03807	
					R_2^{SC2}		(2,3)	(3,2)	6623.307599	
SC1	(5,2,3,5)	10606.35822	10.31701587	53391.01336	R_1^{SC1}	11.34871746	(1,3)	(3,3)	23583.51877	$\delta = 0.01d_k$
					R_2^{SC1}		(2,4)	(3,3)	16810.11724	
SC2	(5,2,3,5)	10304.1712	8.59758153	34197.56551	R_1^{SC2}	9.457339683	(1,2)	(3,1)	12508.2258	
					R_2^{SC2}		(2,3)	(3,2)	6732.893052	
SC1	(5,2,3,5)	11552.34031	9.130492929	44874.1369	R_1^{SC1}	10.04354222	(1,3)	(3,3)	18312.03119	$\delta = 0.03d_k$
					R_2^{SC1}		(2,4)	(3,3)	12824.15562	

δ = 0.03EV(d)	Opened paths	Total market share	DC price	objSC		Retailer price	Equilibrium locations	Equilibrium qualities	objfranchisee	β
SC2	(5,2,3,5)	12843.27734	7.886032323	34631.57888	R_1^{SC2}	8.674635556	(1,2)	(3,1)	13117.57212	
					R_2^{SC2}		(2,3)	(3,2)	7236.613728	
SC1	(5,2,3,5)	11947.72798	8.424780795	38142.684	R_1^{SC1}	9.267258874	(1,3)	(3,3)	14132.53451	$\delta = 0.05d_k$
					R_2^{SC1}		(2,4)	(3,3)	9645.496299	
SC2	(5,2,3,5)	14665.67309	7.383880362	32841.03869	R_1^{SC2}	8.122268398	(1,2)	(3,1)	12329.23423	
					R_2^{SC2}		(2,3)	(3,2)	6511.584906	
SC1	(5,2,3,5)	12029.6	7.95	32727.11107	R_1^{SC1}	8.74	(1,3)	(2,3)	10905.29	$\delta = 0.07d_k$
					R_2^{SC1}		(2,4)	(3,3)	8407.12	
SC2		16119.2	7.02	30729	R_1^{SC2}	7.72	(1,2)	(3,1)	13163.88	
					R_2^{SC2}		(2,3)	(2,2)	5650.864	

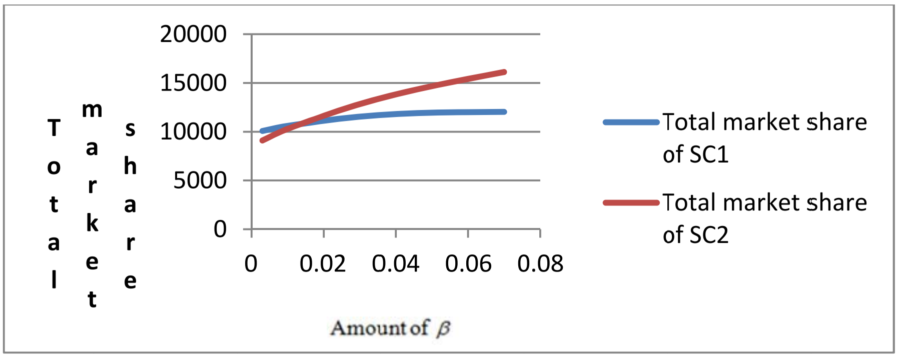

Figure 2. Behavior of total market share of SCs with respect to β

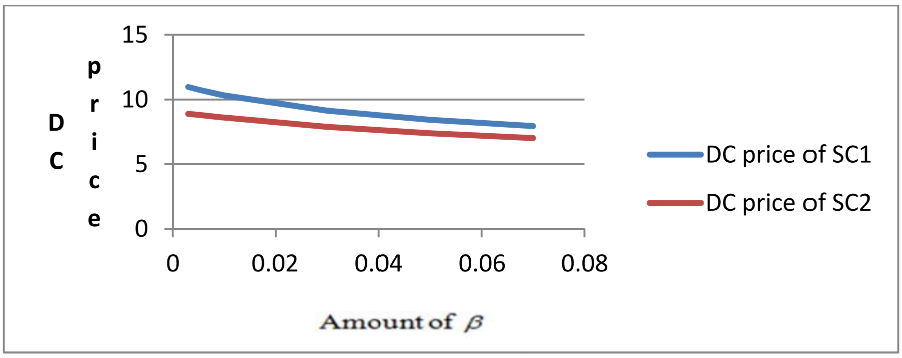

Figure 3. Behavior of DC price with respect to β

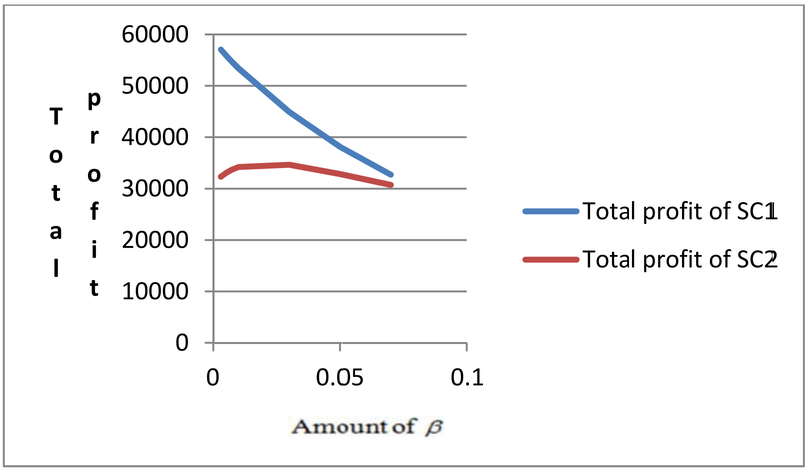

Figure 4. Behavior of total SCs profit with respect to β

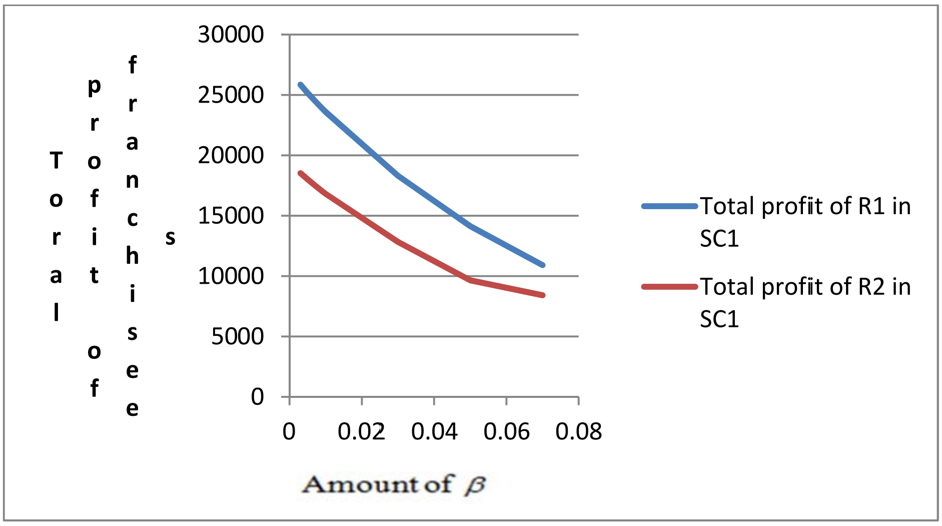

Figure 5. Behavior of total profit of franchisees in SC1 with respect to δ

Table 5 shows the behavior of the opened paths, total market share, DC price, total SC profit, equilibrium locations and qualities, and franchisees' total profit with respect to δ; the amount of parameter δ varies in the solved examples while β is set to $0.07EV(d)$. Figure 6 shows the behavior of total market share with respect to δ. According to the figure, the total market share of the chains will decrease by increasing the amount of δ; however, SC1 experiences a greater decrease in its market share than SC2. Figure 7 shows the behavior of DC prices with respect to δ, which are very similar to each other, decreasing by the increase in the amount of δ. It is observable from Figure 8 that the SCs' total profits are strictly close to each other with respect to δ, decreasing to zero by increasing δ. Figure 9 shows the behavior of the total profits of the franchisees in SC2 with respect to δ. According to this figure, at the high amount of δ, it is not profitable for the franchisees to participate in the market as their profits go below zero.

Table 5. The Change of the Opened Paths, Total Market Share, DC Price, Total SC Income, Equilibrium Locations and Qualities and Retailer's Total Income with Respect to δ

δ = 0.07EV(d)	Opened paths	Total market share	DC price	objSC		Retailer price	Equilibrium location	Equilibrium quality	objfranchisee	δ
SC1	(5,2,3,5)	25745.93956	11.90939401	177453.3519	R_1^{SC1}	13.10033341	(3,4)	(3,3)	103808.899	$\delta = 0.001d_k$
					R_2^{SC1}		(2,4)	(3,3)	72317.55712	
SC2	(5,2,3,5)	28495.21474	10.94606811	169699.9122	R_1^{SC2}	12.04067492	(1,2)	(3,3)	93452.07796	
					R_2^{SC2}		(2,3)	(3,3)	73126.97431	
SC1	(5,2,3,5)	24386.90615	11.44289412	156456.2734	R_1^{SC1}	12.58718353	(3,4)	(3,3)	90188.33852	$\delta = 0.003d_k$
					R_2^{SC1}		(2,4)	(3,3)	62724.74379	
SC2	(5,2,3,5)	27227.30455	10.48262981	149323.9415	R_1^{SC2}	11.53089279	(1,2)	(3,3)	81035.72419	
					R_2^{SC2}		(2,3)	(3,3)	62842.47724	
SC1	(5,2,3,5)	23127.35638	11.0229468	138415.2367	R_1^{SC1}	12.12524148	(3,4)	(3,3)	78483.02564	$\delta = 0.005d_k$
					R_2^{SC1}		(2,4)	(3,3)	54476.86321	
SC2	(5,2,3,5)	26059.1165	10.06563275	131851.1115	R_1^{SC2}	11.07219602	(1,2)	(3,3)	70378.91344	
					R_2^{SC2}		(2,3)	(3,3)	54014.48667	
SC1	(5,2,3,5)	21953.06767	10.64290666	122800.352	R_1^{SC1}	11.70719732	(3,4)	(3,3)	68349.73013	$\delta = 0.007d_k$
					R_2^{SC1}		(2,4)	(3,3)	47333.04659	
SC2	(5,2,3,5)	24976.41521	9.6884375	116758.7933	R_1^{SC2}	10.65728125	(1,2)	(3,3)	51227.92393	
					R_2^{SC2}		(2,3)	(3,3)	38123.93408	
SC1	(5,2,3,5)	20852.40405	10.29733622	109196.7629	R_1^{SC1}	11.32706984	(3,4)	(3,3)	59519.72117	$\delta = 0.009d_k$
					R_2^{SC1}		(2,4)	(3,3)	41104.68497	
SC2	(5,2,3,5)	23967.55285	9.345612132	103638.0913	R_1^{SC2}	10.28017335	(1,2)	(3,3)	43435.00765	
					R_2^{SC2}		(2,3)	(3,3)	31664.73598	
SC1	(5,2,3,5)	20326.61123	10.13604216	103043.72	R_1^{SC1}	11.14964638	(3,4)	(3,3)	55525.12041	δ =

δ = 0.07EV(d)	Opened paths	Total market share	DC price	objSC		Retailer price	Equilibrium location	Equilibrium quality	objfranchisee	δ
				55	R_2^{SC1}		(2,4)	(3,3)	38285.83839	
SC2	(5,2,3,5)	23487.73735	9.185654809	97713.1955	R_1^{SC2}	10.10422029	(1,2)	(3,3)	39928.01551	
					R_2^{SC2}		(2,3)	(3,3)	28757.57234	
SC1	(5,2,3,5)	12029.60419	7.9501295	32727.11107	R_1^{SC1}	8.74514245	(1,3)	(2,3)	10905.29151	$\delta = 0.03d_k$
					R_2^{SC1}		(2,4)	(3,3)	8407.118952	
SC2	(5,2,3,5)	16119.18541	7.022318052	30728.67912	R_1^{SC2}	7.724549858	(1,2)	(3,1)	13163.88316	
					R_2^{SC2}		(2,3)	(2,2)	5650.864146	
SC1	(2,1,3,5)	5918.522032	6.790482663	6949.426014	R_1^{SC1}	7.46953093	(1,3)	(1,1)	470.9622065	$\delta = 0.05d_k$
					R_2^{SC1}		(2,4)	(1,1)	-483.002	
SC2	(2,1,3,5)	10949.03853	5.87942108	6867.524437	R_1^{SC2}	6.467363188	(1,2)	(1,1)	727.3986097	
					R_2^{SC2}		(2,3)	(1,1)	-1730.79	

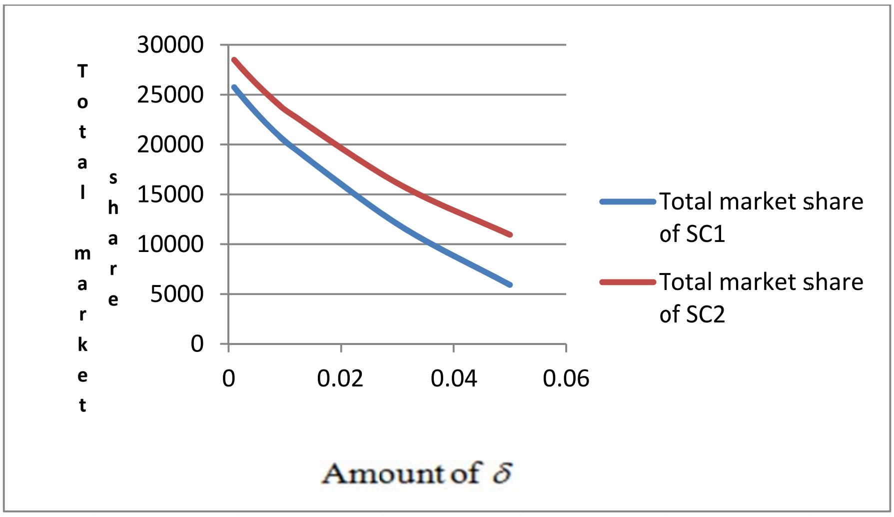

Figure 6. Behavior of total market share with respect to δ

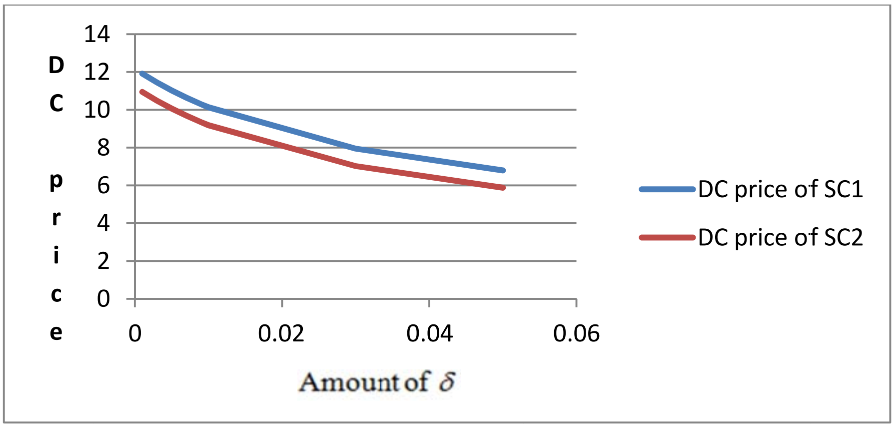

Figure 7. Behavior of DC price with respect to δ

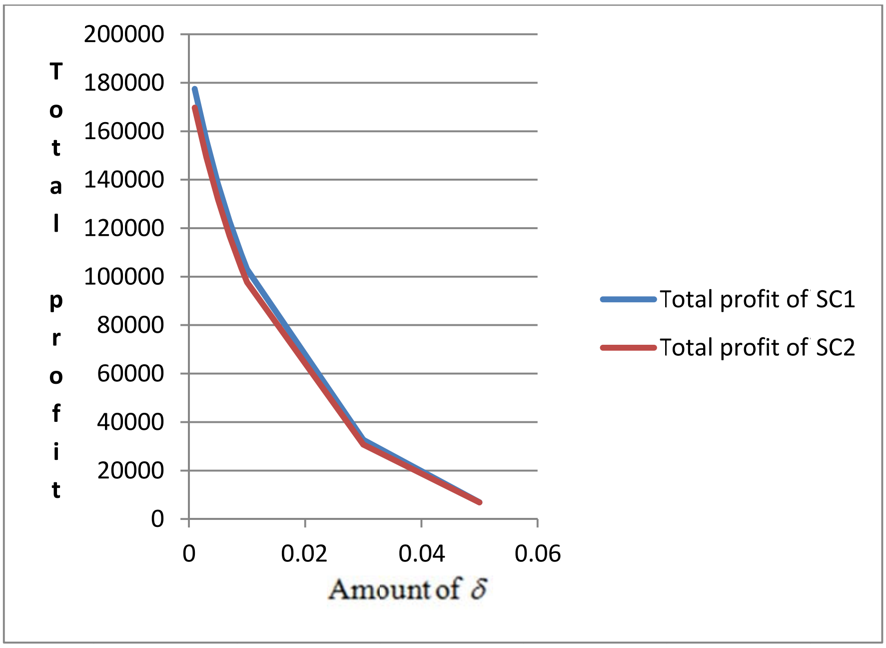

Figure 8. Behavior of total SCs profit with respect to δ

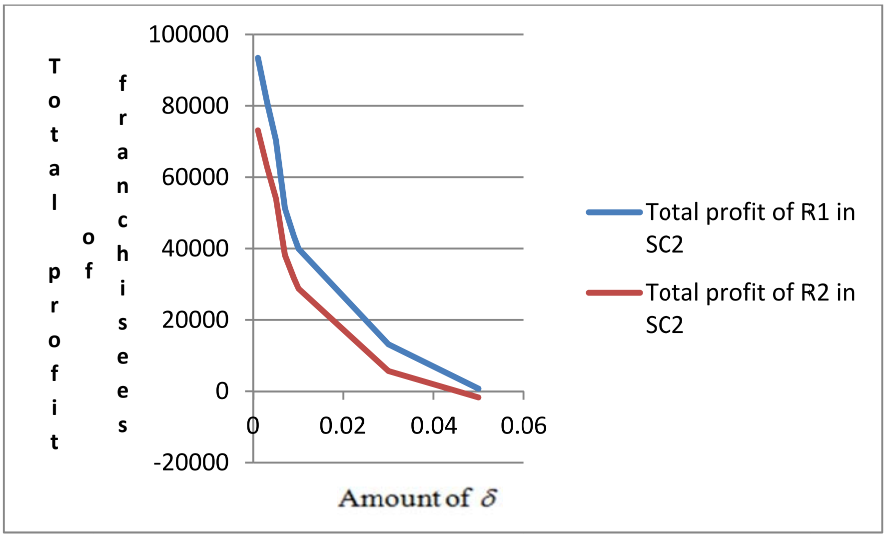

Figure 9. Behavior of total profit of franchisees in SC2 with respect to δ

As β represents competition intensity, by increasing the amount of intensity in competition, the chains were forced to decrease their price to obtain in the competition and absorb some customers, by this way, their market shares will increase, according to the demand function, but their total profits will decrease because of the lower marginal profit. On the other hand, when δ increases, the customers of the chain pay more attention on the chain price itself and in this manner, the chain is forced to decrease its price and by the same way results in decreasing the total profits (interested readers can refer Anderson and Bao (2010) for more details and mathematical proofs).

In the pricing step, the power factor has no effect on the equilibrium price because it has been omitted by the differential system. However, according to Table 6, it has this effect on the mathematical step.

In SC1, total franchisee profits in duopoly competition is 19,312; R_1^{SC1} and R_2^{SC1} total profits in monopoly competition are 35,606 and 36,925, respectively. Total franchisee profits in duopoly competition, in the case that the franchisees sell the products of both chains, if R_1^{SC1} and R_2^{SC1} served the market is 75,915; R_1^{SC1} and R_2^{SC1} total profits in monopoly competition, in the case that the franchisees sell the products of both chains, are 98,382 and 94,687, respectively. Correspondingly, for SC2, total franchisee profits in duopoly competition is 18,814; R_1^{SC2} and R_2^{SC2} total profits in monopoly competition are 36,051 and 35,482, respectively. Total franchisee profits in duopoly competition, in the case that the franchisees sell the products of both chains, if R_1^{SC2} and R_2^{SC2} served the market is 70,140; R_1^{SC2} and R_2^{SC2} total profits in monopoly competition in the case that the franchisees sell the products of both chains are 101,364 and 87,677. Obviously, the best structure for franchisees is monopoly competition, in the case that the franchisees sell the products of both chains, and the worst case is duopoly competition when the quality of the facilities is exactly vice versa. Therefore, if the SCs want to increase customer satisfaction, they should chose duopoly competition; if they want more profits, they should use some negotiating mechanism to profit from the monopoly structure (Table 7 shows these situations).

Moreover, the SCs can choose to cooperate with each other; the outcomes of this model are shown in Table 8. In this circumstance, the market share, total objective function of SCs, DC price, objective function of SC1, and objective function of Franchisee 1 and Franchisee 2 in SC1 increased by 15%, 33%, 3.7%, 55%, and 41%, respectively. Correspondingly, for SC2, they decreased by 24%, 56%, -12%, 3.7, and 4.7%, respectively.

Table 6. The Change of the Optimal Price, Market Share, SCN Structure and Total Income with Respect to Power Effect Parameter

δ = 0.07EV(d) δ = 0.03EV(d)	Opened paths	Total market share	DC price	objSC		Retailer price	Equilibrium location	Equilibrium quality	objfranchisee	λ
SC1	(5,2,3,5)	12029.60419	7.9501295	32727.11107	R_1^{SC1}	8.74514245	(1,3)	(2,3)	10905.29151	$\lambda = 0.1$; $\lambda = 0.2$; $\lambda = 0.3$; $\lambda = 0.4$; $\lambda = 0.5$;
					R_2^{SC1}		(2,4)	(3,3)	8407.118952	

δ = 0.07EV(d) δ = 0.03EV(d)	Opened paths	Total market share	DC price	objSC		Retailer price	Equilibrium location	Equilibrium quality	objfranchisee	λ
SC2	(5,2,3,5)	16119.18541	7.022318052	30728.67912	R_1^{SC2}	7.724549858	(1,2)	(3,1)	13163.88316	
					R_2^{SC2}		(2,3)	(2,2)	5650.864146	
SC1	(5,2,3,4)	14315.99604	8.323698524	45208.04785	R_1^{SC1}	9.156068376	(1,3)	(3,3)	18622.77321	$\lambda = 0.7$; $\lambda = 0.8$; $\lambda = 0.9$
					R_2^{SC1}		(2,4)	(3,3)	13205.33426	
SC2	(5,2,3,4)	11187.11156	8.08965812	8960.383436	R_1^{SC2}	8.898623932	(1,2)	(3,1)	13099.65729	
					R_2^{SC2}		(2,3)	(2,2)	5652.487948	

Table 7. Competition Intensity between Franchisees

	Duopoly competition			Monopoly competition			Duopoly competition, common			Monopoly competition, common		
	Equilibrium location	Equilibrium quality	Objective function	Equilibrium location	Equilibrium quality	Objective function	Equilibrium location	Equilibrium quality	Objective function	Equilibrium location	Equilibrium quality	Objective function
R_1^{SC1}	(1,3)	(2,3)	10905	(2,3)	(1,1)	35606	(1,3)	(3,3)	48837	(2,3)	(1,1)	98382
R_2^{SC1}	(2,4)	(3,3)	8407	(2,4)	(1,1)	36925	(1,2)	(3,3)	27078	(1,2)	(1,1)	94687
R_1^{SC2}	(1,2)	(3,1)	13163	(3,4)	(1,1)	36051	(1,2)	(3,3)	43765	(3,4)	(1,1)	101364
R_2^{SC2}	(2,3)	(2,2)	5651	(1,3)	(1,1)	35482	(2,3)	(3,3)	26375	(1,3)	(1,1)	87677

Table 8. Numerical Result in Cooperative Mode

Cooperation $\lambda = 0.5$	**Opened paths**	**Total market share**	**DC price**	**objSC**		**Retailer price**	**Equilibrium location**	**Equilibrium quality**	**objfranchisee**
SC1	(5,2,1,1)	13835	8.24	43516	R_1^{SC1}	9.07	(1,3)	(3,3)	16882
					R_2^{SC1}		(2,4)	(3,3)	11848
SC2	(5,2,1,1)	12224	7.86	13408	R_1^{SC2}	8.65	(1,2)	(3,1)	13663
					R_2^{SC2}		(2,3)	(2,2)	6064

The following managerial insights are derived from these sensitivity analyses:

- ✓ Increasing market competition is more profitable for the smaller SC, because its market expansion is greater than that of the larger SC.
- ✓ By increasing the number of factors δ, β, the total profit of both chains will decrease, and it would be more profitable for them to control the competition intensity at a low level.
- ✓ By decreasing the number of competing franchisees and allowing them to sell the products of both chains, their attractiveness level will decrease, but their profits will increase. This can make customers unhappy in the long run and decrease customer-based demand.
- ✓ Having more power has no effect on the pricing step, but it can help to gain more profits and change the network structure in the location phase.
- ✓ Cooperation in the location phase helps the smaller SC (the one with less market-based demand) to gain more profits, but the larger SC will gain more profits in a non-cooperative manner.

It is worth noting that according to the literature, duopoly is the most commonly used form of competition and in this way, we follow the literature trend. Moreover, Anderson and Bao (2010) gave mathematical proofs showing no difference between duopoly and oligopoly in terms of the behavior of market shares, prices, and total profits.

Conclusion

This paper has developed a dynamic competitive supply chain network design problem with price dependent demand and Huff utility function in which n supply chains tending to enter the untapped market and give franchises to competing franchisees. Customers are faced with a two-step decision model: At first, they chose a brand (SC) to buy based on the price according to 0-1 rule; then, they chose the retailers of the franchisees by a certain probability based on their attractiveness applying Huff gravity rule model. There are two games in this context. The first one is a dynamic game between the SCs, as the first stage, based on the location and price. After the franchisees, as the second stage, enter a simultaneous game to set their locations and attractiveness.

We converted the model of the SCs into a bi-level model in which the inner part sets the price and the outer part shapes the networks. We also used Nash's concept and Wilson algorithm to convexify the model of the franchisees and find equilibrium locations and qualities. Moreover, we used fuzzy set theory to cope with the uncertainty that the players encounter as they are all newcomers and have no precise knowledge and information about the parameters.

Finally, we applied our model and solution approach to a real world problem and discussed the sensitivity analysis of the total market share, DC price, total profit of both chains, equilibrium locations and qualities, and franchisees' total profit with respect to β,δ. We then considered the effect of SC power in the pricing and location phases and analyzed the effect of changing the competition intensity on the franchisees' attractiveness level and profits.

We concluded that by increasing the amount of β,δ, the profits of both chains will decrease and that power has no effect on the pricing step, although it can change the structure of the chains. Moreover, the best situation for the franchisees is one in which they can sell the products of both chains without any competitors. However, this is also the worse situation for customers, and it can decrease customer-based demand in the long run. Further, cooperation is helpful for the small SC, but it decreases the profits of the larger SC.

This model can be applied in many different industries as most industries prefer to have some independent and competing franchisees, such as the car, shoe, and retail industries. Moreover, the proposed model can be extended by different aspects. For example, the closed-loop, robust, or sustainable SC can be considered, or stochastic approaches can be used to handle uncertainty.

References

Aboolian, R., Berman, O., & Krass, D. (2007). Competitive facility location and design problem. *European Journal of Operational Research*, *182*(1), 40-62.

Anderson, E., & Bao Y. (2010). Price competition with integrated and decentralized supply chains. *European Journal of Operational Research*, *200*(1), 227-234.

Badrinarayanan, V., Suh, T., & Kim, K. M. (2016). Brand resonance in franchising relationships: A franchisee-based perspective. *Journal of Business Research*, *69*(10), 3943-3950.

Bernstein, F., & Federgruen, A. (2005). Decentralized supply chains with competing retailers under demand uncertainty. *Management Science*, *51*(1), 18-29.

Boyaci, T., & Gallego, G. (2004) Supply chain coordination in a market with customer service competition. *Production and Operation Management 13*(1):3–22.

Fahimi, K., Seyedhosseini, S. M., & Makui, A. (2017a). Simultaneous competitive supply chain network design with continuous attractiveness variables. *Computers & Industrial Engineering, 107,*235-250.

Fahimi, K., Seyedhosseini, S. M., & Makui, A. (2017b). Simultaneous decentralized competitive supply chain network design under oligopoly competition. *Iranian Journal of Management Studies, 10*(2), 409-434.

Farahani, R. Z., Rezapour, SH., Drezner, T., & Fallah, S. (2014). Competitive supply chain network design: An overview of classifications, models, solution techniques and applications. *Omega*, *45*, 92-118.

Fallah, H., Eskandari, H., & Pishvaee, M. S. (2015). Competitive closed-loop supply chain network design under uncertainty. *Journal of Manufacturing Systems*, *37(3)*, 649-661.

Fernandez, P., Pelegrin, B., Perez, M. D. G., & Peeters, P. H. (2007). A discrete long term location price problem under the assumption of discriminatory pricing: Formulations and parametric analysis. *European Journal of Operational Research, 179*, 1050-1062.

Godinho, P., & Dias, J. A. (2010). Two competitive discrete location model with simultaneous decisions. *European Journal of Operational Research, 207,* 1419-1432.

Godinho, P., & Dias, J. A. (2013). Two player simultaneous location game: Preferential rights and overbidding. *European Journal of Operational Research, 229,* 663-672.

Heilpern, S. (1992). The expected value of a fuzzy number. *Fuzzy Sets and Systems, 47,* 81-86.

Hotelling, H. (1929). Stability in competition. *The Economic Journal, 39*(153), 41-57.

Huff, D. L. (1964). Defining and estimating a trade area. *Journal of Marketing, 28*(3), 34-38.

Huff, D. L. (1966). A programmed solution for approximating an optimum retail location. *Land Economy, 42*(3), 293-303.

Inuiguchi, M, & Ramik, J. (2000). Possibilistic linear programming: A brief review of fuzzy mathematical programming and a comparison with stochastic programming in portfolio selection problem, *Fuzzy Sets and Systems, 111,* 3-28.

Kucukaydın, H., Aras, N., & Altınel, I. K. (2011). Competitive facility location problem with attractiveness adjustment of the follower: a bi-level programming model and its solution. *European Journal of Operational Research, 208*(3), 206-20.

Kucukaydın, H., Aras, N.,& Altınel. I. K. (2012). A leader–follower game in competitive facility location. *Computer and Operation Research, 39,* 437-448.

Liu, B., & Iwamura, K. (1998). Chance constrained programming with fuzzy parameters. *Fuzzy Sets and Systems, 94,* 227-237.

Rezapour, S. H., & Farahani, R. Z. (2010). Strategic design of competing centralized supply chain networks for markets with deterministic demands. *Advances in Engineering Software, 41*(5), 810-822.

Rezapour, S. H., Farahani, R. Z., Ghodsipour, S. H., & Abdollahzadeh, S. (2011). Strategic design of competing supply chain networks with foresight. *Advances in Engineering Software 42*(4), 130-141.

Rezapour, S. H., & Farahani, R. Z. (2014). Supply chain network design under oligopolistic price and service level competition with foresight. *Computers & Industrial Engineering, 72,*129-142.

Rezapour, S. H., Farahani, R. Z., Dullaert, W., & Borger, B. D. (2014). Designing a new supply chain for competition against an existing supply chain. *Transportation Research Part E: Logistics and Transportation Review. 67,* 124-140.

Rezapour, S. H., Farahani, R. Z., Fahimnia, B., Govindan, K., & Mansouri, Y. (2015). Competitive closed-loop supply chain network design with price dependent demand. *Journal of Cleaner Production, 93,*251-e272.

Plastria, F., & Vanhaverbeke. L. (2008). Discrete models for competitive location with foresight. *Computers & Operations Research, 35,*683–700.

Pishvaee, M. S., Ramzi, J., & Torabi, S. A. (2012). Robust possibilistic programming for socially responsible supply chain network design: A new approach. *Fuzzy Sets and Systems, 206,* 1–20.

Shaikh, A. (2016). Conceptualizing fairness in franchisor–franchisee relationship: Dimensions, definitions and preliminary construction of scale. *Journal of Retailing and Consumer Services*, *28*(Supplement C), 28-35.

Tsay, A. A., & Agrawal, N. (2000). Channel dynamics under price and service competition. *Manufacturing & Service Operations Management, 2*(4), 372-391.

Xiao, T., & Yang, D. (2008). Price and service competition of supply chains with risk-averse retailers under demand uncertainty. *International Journal of Production Economics, 114*(1), 187-200.

Wilson, R. (1971). Computing Equilibria of N-Person Games. Society for Industrial and Applied Mathematice Journal on Applied Mathematics,21(1), 80–87.

Watson, A., Dada, O., Grünhagen, M., & Wollan, M. L. (2016). When do franchisors select entrepreneurial franchisees? An organizational identity perspective. *Journal of Business Research, 69*(12), 5934-5945.