JSER

Journal of Solar Energy Research (JSER)

Journal homepage: www.jser.ut.ac.ir

Mathematical modeling of solar still for desalination of seawater

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ARTICLE INFO

Received: 03 Dec 2017
Received in revised form: 19 Dec 2017
Accepted: 23 Dec 2017
Available online: 27 Dec 2017

Keywords:
Solar Still; Water Desalination; Mathematical Modeling; Ordinary Differential Equations;

A B S T R A C T

Solar still is a green apparatuses for seawater desalination that can be a replace of common method of seawater vaporization using fossil fuel and condensing it, especially in Persian Gulf with hot weather which has a high radiation power of sun in summer when drinking water resources are limited. Solar still works using solar radiation that is a renewable source of energy and reduces the production of pollutant and greenhouse gases that produced using fossil fuel in a normal process. Therefore, commercialization of solar still is in direction of sustainable development. In this article, a solar still was modeled mathematically for describing the effect of parameters on performance of this device. Energy and mass balance equations were written using lumped formulation. Set of ordinary differential and algebraic equations was solved numerically using MATLAB software. Iteration method was used for estimating implicit heat transfer coefficients that are a function of temperatures. In this model effect of water depth and different daily solar profile was studied on total amount of water production. Temperature of basin water as a function of time was obtained by changing water depth, cycle number and solar flux profile. The cycle of still process was continued for several days without any brackish water make-up. The results show that at higher water depth, nightly desalinated water production continued more than lower water depth. The model indicates that at Muscat with higher solar flux, the water production and maximum of basin water temperature are greater than Shiraz with lower solar flux. It can be concluded that the potential of commercialization of solar still in an area near the sea with high solar flux is higher. But at lower initial water depth, the amount of water production at locations with lower solar flux approaches to the locations with higher solar flux.

1. Introduction

Desalination of seawater is an applicable method for producing fresh water in an area that does not have access to a river or annual average raining is low [1-4]. Available water per capita from 6042 m^3 in 1947 has fallen to 1545 m^3 in 2011, the latest studies show that global access to water in 2025 will reach 1340 m^3 and by 2050 it will reach 1140 m3 [5]. The benefits of using sunlight for desalination water are: easier construction, minimum operational and maintenance skills, and environmental compatibility. The major defects in solar water desalination systems are their low production capacity compared to industrial desalination systems (about 2-5 lit/m^2.s) [6]. it for producing fresh water is used for this purpose especially in petroleum producer countries of Persian Gulf. Vaporization of water is very energy consuming because of high latent heat of water. The area near the Persian Gulf have hot climate and the power of solar radiation in these areas are very strong especially in the summer when the natural resources of fresh water is limited. Therefore, using solar still as an environmental friendly device for producing drinking water in these areas has a potential for commercialization.

Solar still uses the power of solar radiation for evaporating seawater in a chamber that have a black basin and a glass cover. Using solar radiation as a

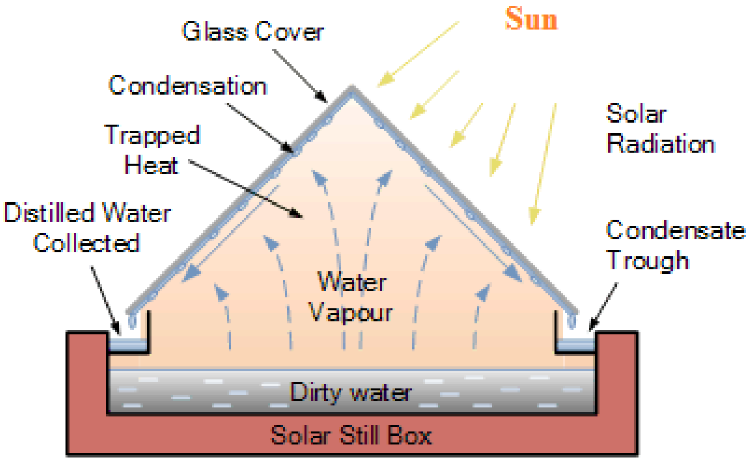


Figure 1. Solar still and its compartments

renewable source of energy converts this device to an environmental friend apparatus. Figure 1 shows a solar still and its parts.

This device has a glass cover that permits transition of the solar radiation to the basin liner which is black and suitable for absorption of solar radiation. Dirty or saline water was located at the bottom of still basin which gets warm by absorbing solar radiation and convective heat transfer of hot liner surface at the bottom of the basin. Warmed water evaporated gradually. Water vapor moves to the top of the still chamber and in contact to the glass surface, condense to liquid water. Liquid water droplets move from the center of sloped glass cover to the side of the glass roof and collects in two side channels.

Elango et al. [7] reviews thermal models for solar still. In this article, system of one basin and double slop glass was modeled and the effect of climate at two different locations near the Persian Gulf was compared together and effect of water initial depth on solar still performance was studied. Also, several days of continuous solar still process without any brackish water make-up using cyclic daily solar flux was studied for the first time.

1. Materials and Methods

Lumped formulation was used for modeling a solar still. Main assumptions of the model are listed in the following:

A. Profile of solar radiation flux is a function of daily time.

B. Convection heat transfer coefficient of surfaces to the ambient is a function of wind velocity.

C. Heat accumulation in glass cover, glass wool insulation was neglected.

D. Heat accumulation inside of water layer at the bottom of solar basin was taking into account in the model.

E. Level of water layer at the bottom of chamber was considered as a function of time.

F. Heat of solar radiation at each domain was considered using constant absorption, reflection factors.

G. Ambient temperature was considered as a function of daily time.

In the following subsections, Energy balance and water mass balance were developed. Numerical methods for solving combination of algebraic and differential equations were discussed and finally the parameters of the model and correlations for estimating heat transfer coefficients were introduced.

2.1. Energy and Mass Balance

Energy balance for water in basin follows the Equation (1) [8]:

$$\rho l S C_w \frac{dT_w}{dt} = h_3 S_s (T_b - T_W) - h_1 S (T_w - T_g) \tag{1}$$

Energy balance around solar still box is written in Equation (2) Energy balance for glass cover was presented in Equation (3) Water mass balance can be discussed by Equation (4) [8]:

$$S\tau_2 H_s = h_3 S_s (T_b - T_w) + U_{sw} S_{sw} (T_b - T_a) + U_{bw} S (T_b - T_a) \tag{2}$$

$$h_1 S (T_w - T_g) = h_2 S_g (T_g - T_a) \tag{3}$$

$$\frac{dM_w}{dt} = -m^{\cdot} = \frac{-Q_{ew}}{\lambda} \tag{4}$$

$$Q_{ew} = h_{ew}(T_w - T_g) \tag{5}$$

2.2. Numerical Method

Equation (6) and (7) were derived from Equation (2) and (1) respectively. Two ordinary differential equations (ODEs) were developed by rewriting Equation (1) and (4) in to format of Equation (8) and (9). Set of nonlinear algebraic Equations (6) and (7) and nonlinear ordinary differential equation of (8) an (9) can be solved numerically.

$$T_b = \frac{(U_{sw}S_{sw} + U_{bw}S)T_a}{h_3S_s + U_{sw}S_{sw} + U_{bw}S} + \frac{S\tau_2H_s + h_3S_sT_w}{h_3S_s + U_{sw}S_{sw} + U_{bw}S} \tag{6}$$

$$T_g = \frac{h_1ST_w + h_2S_gT_a}{h_2S_g + h_1S} \tag{7}$$

$$\frac{dT_w}{dt} = \frac{-(h_3S_s + h_1S)T_w + h_3S_sT_b}{M_wC_w} + \frac{h_1ST_g + \tau_1H_s}{M_wC_w} = f_1(T_w, M_w) \tag{8}$$

$$\frac{dM_w}{dt} = \frac{-h_{ew}(T_w - T_g)}{\lambda} = f_2(T_w, M_w) \tag{9}$$

$$m_t = \int_0^t \dot{m}dt \tag{10}$$

Accumulated produced fresh water can be calculated using integration that was indicated in Equation (10).

Considering solar flux and ambient temperature as a function of daily time and heat transfer coefficient as a function of temperatures make analytical solution of the combination of algebraic and ordinary differential set of equations impossible. Therefore, set of ordinary and algebraic Equations of (6) to (9) was solved numerically using MATLAB 2017 software [9].

Ode15s solver was used that have a step size controller algorithm inside its procedures. It can solve all ODEs more rapidly than ode45 solver and also has potential of solving stiff ODEs. This ODE solver is based on numerical differentiation formulas (NDFs) of variable order and if in its options, the backward differentiation formulas (BDFs) are turned on, it uses the Gear's method [10, 11].

Iteration method was used inside the ODE solver for estimating a heat transfer coefficient that is in implicit relation to glass and water temperature.

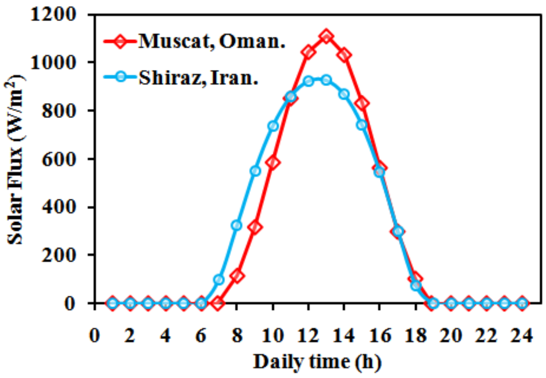


Figure 2. Daily profile of solar flux at a typical summer day in Shiraz (Iran) [8] and Muscat (Oman) [4].

2.3. Parameters and Coefficients

Values of absorption and reflection coefficients were summarized in Table 1. Surface areas of different parts of solar still are listed in Table 2. Physical parameters were summarized in Table 3. The solar daily profile of Shiraz (Iran) and Muscat (Oman) are illustrated in Figure 2. Typical summer ambient temperature daily profile was plotted in Figure 3. Radiation factors are calculated based on Equation (11) and (12):

$$\tau_1 = (1 - R_g)(1 - \alpha_g)\alpha_w \tag{11}$$

$$\tau_2 = (1 - R_g)(1 - \alpha_g)(1 - \alpha_w)\alpha_b \tag{12}$$

Table 1- Values of absorptivity and reflectivity coefficients [8]

Parameter	Value	Parameter	Value
α_g	0.05	R_g	0.1
α_b	0.8	α_w	0.8

Table 2- Areas of different parts of modeled solar still in m^2 [8]

Area	Value	Parameter	Value
S	1.00	S_s	1.12
S_g	1.04	S_{sw}	0.12

Table 3- Physical constants

Parameter	Value	Unite	Ref

V	4	m/s	[8]
λ	2450000	J/kg	
C_w	4180	$J/(kg.K)$	[12]
$ANTA$	18.3036	−	[13]
$ANTB$	3816.44	−	
$ANTC$	−46.13	−	

Heat transfer coefficients were estimated based on Equation (13) to (16). They are all in SI unites (W/(m^2.°C)).

$$h_1 = 8.71 + h_{ew} \quad (13)$$

$$h_{ew} = 4 \times \frac{P_w - P_g}{T_w - T_g} \quad (14)$$

$$h_2 = 5.7 + 3.8V \quad (15)$$

$$h_3 = 130 \quad (16)$$

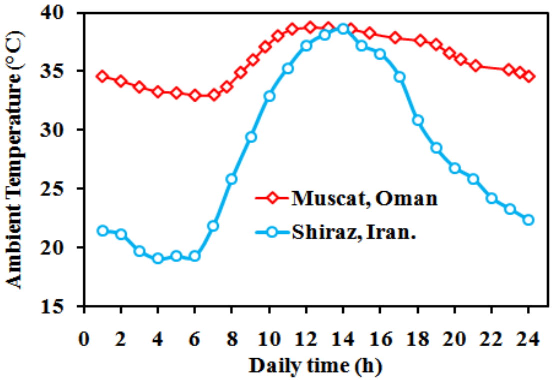


Figure 3. Daily profile of ambient temperature in a summer day in Shiraz (Iran) [8] and Muscat (Oman)[4].

Where P_w and P_g are water saturation pressure in Pa at temperature of T_w and T_g respectively. Antoine equation for water was presented in Equation (17):

$$P_i\,(Pa) = \frac{exp\left(ANTA - \dfrac{ANTB}{T_i(K) - ANTC}\right)}{760 \times 10^5} \quad (17)$$

$$i = w \; or \; g$$

3. Results and Discussions

Shiraz conditions were used as a based case for parameter studying and in some results; the two cases of Shiraz and Muscat were compared. Figure 4 shows the basin water temperature at different initial water depth for the Shiraz case study. The results show that by increasing the water depth, maximum of water temperature were shifted to lower value that can be discussed by the accumulation term of Equation (8). By decreasing the water content in the basin (M_w), the denominator of the right hand side of the Equation (8) decreases and therefore, dT_w/dt increases. It causes to reach to a higher value of maximum of water temperature at higher value of water depth. Figure 5 illustrates the accumulated water production (m_t) along the time at different initial water depths. It reveals that water production will be continued after sun set for longer time at higher amount of initial water depth in the basin.

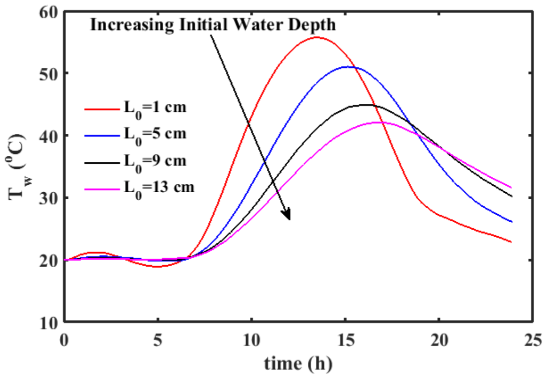


Figure 4. Effect of initial water depth on water temperature during 24h

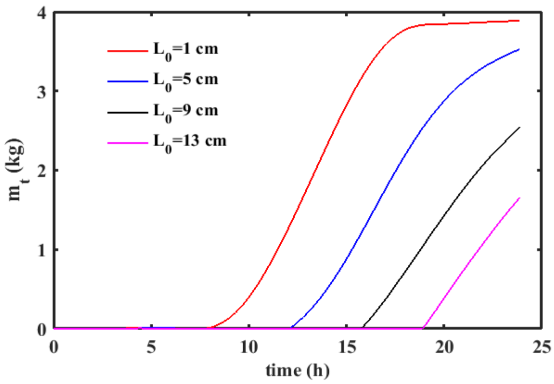


Figure 5. Effect of initial water depth on total water production during 24h

Figure 6 shows the total water production profile during seven days continuing the still process without adding any seawater make-up. At initial water depth of 2 cm, the water production is stopped before 3 days of continuous operation because of ending the water inside the basin.

Figure 7 shows the water productions in the solar still using Shiraz and Oman daily solar radiation profile. Comparing these two figures indicates that solar power has a very strong significant effect on the amount of produced water because of high water latent heat of vaporization that was a limiting step in desalination. Therefore, using solar still in an area with high solar radiation power has higher performance that makes use it more feasible for commercialization.

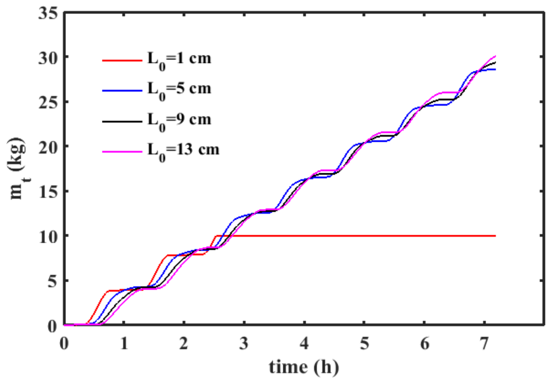
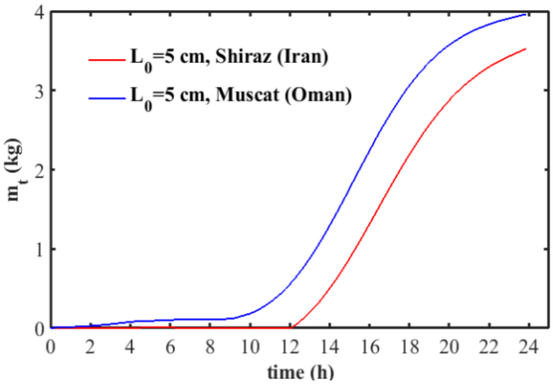
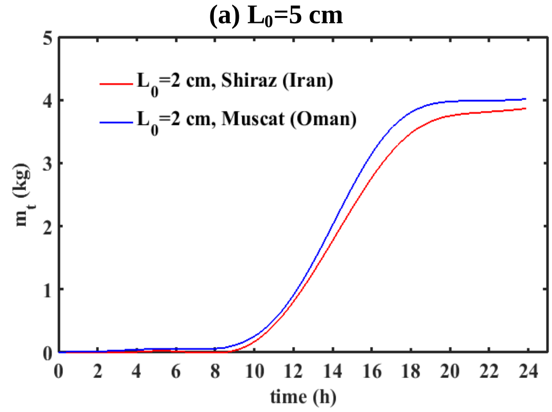


Figure 6. Effect of initial water depth on total water production during seven days of continuous process



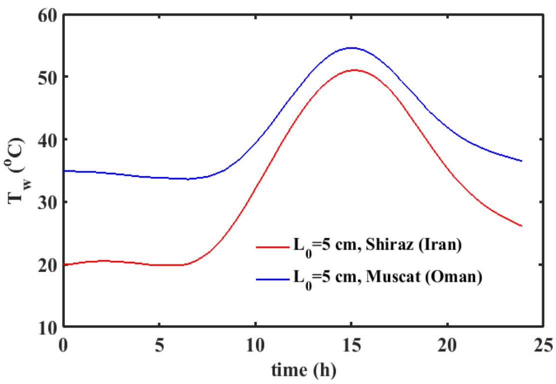
(a) L_0=5 cm



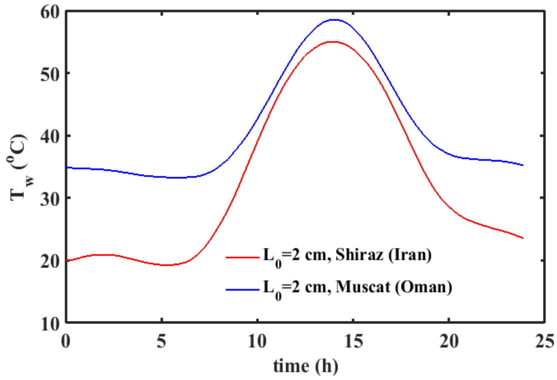
(b) L_0=2 cm

Figure 7. Comparison between Shiraz (Iran) and Muscat (Oman) water production (a) L_0=5 cm, (b) L_0=2 cm

Figure 7 (a) and (b) are related to initial water depth of 5 and 2 cm, respectively. The difference between the case of (a) and (b) in this figure shows that by decreasing the initial water depth, the water production in Shiraz approaches to the result of Muscat. That means when the solar radiation power is lower, one should select the lower initial water depth to approach to results of the location with higher solar power.



(a) L_0=5 cm



(b) L_0=2 cm

Figure 8. Comparison between Shiraz (Iran) and Muscat (Oman) water temperature (a) L_0=5 cm, (b) L_0=2 cm

Figure 8 (a) and (b) illustrate the water temperature of the basin at initial water depth of 5 and 2 cm, respectively for the two locations of Shiraz and Muscat. Comparing the Shiraz and Muscat curves in this figure shows that at Muscat the maximum of water temperature is higher than Shiraz because of higher solar power of Muscat. The higher water temperature at Figure 8 leads to higher water production at Figure 7. Comparing Figure 7 (a) and (b) shows that maximum temperature at initial water depth of 5 cm was shifted to higher value of time in comparison to 2 cm. because the multiplication of mass and heat capacity of water at higher value of L_0 is upper than lower value of L_0. Therefore the time lag of changing temperature at the maximum solar power in the noon was greater in the case with lower L_0.

Figure 9 indicates the effect of water initial depth on basin water temperature at seven days continuous process of solar still in Shiraz without any brackish water make-up. The results show that by increasing the water depth, the water temperature shifted to lower temperature at all time values and amplitudes of oscillations were damped at higher L_0. Maximums of T_w at a cycle move to lower value by increasing L_0 and shifted to higher

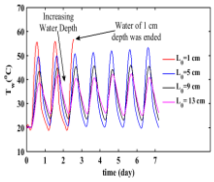


Figure 9. Effect of initial water depth on water temperature in seven days continuous cyclic process in Shiraz

time value because of lower value of (mass × heat capacity). By increasing the cycle number, the values of T_w grow that is related to decreasing the amount of water content of basin by going ahead the cycles without any make-up for brackish water.

4. Conclusion

Solar still can be used for seawater desalination as a substitute for common method of using fossil fuels for vaporization and condensation process especially in Persian Gulf with sweltering weather which has a high radiation power of sun in summer when drinking water resources are limited. Solar still was modeled using lumped formulation for parametric study and the mathematical model was solved numerically.

The results show that by decreasing the water depth, maximum of basin water temperature increases. At higher initial water depth, nocturnal water production will be continued because of higher amount of heat capacity.

Daily profiles of solar flux and ambient temperature have strong effect on the fresh water production. Comparing the results of Muscat (with higher solar flux) and Shiraz (with lower solar flux) shows that fresh water production in Muscat is higher than Shiraz. It reveals that the potential of commercialization of solar still at the area with high solar flux is higher than low solar flux. The results also show that fresh water production at Shiraz and Muscat has higher difference at high initial basin water depth and vice versa. Therefore, it can be concluded that at the locations with lower solar fluxes, decreasing the initial basin water depth can increase the fresh water production to an amount near the value of higher solar flux. In addition, this shows that the initial water depth has different optimum value at various locations with different solar flux profile.

The model results for seven days continuous process of solar still in Shiraz without any brackish water make-up shows that by increasing the cycle number, the values of basin water temperature grow. Also, amplitudes of cycle oscillations were damped at higher initial water depth because of decreasing the amount of water content of basin by proceeding the cycles with no make-up for brackish water.

Nomenclature

T_w	Water temperature (°C)
T_b	Basin liner temperature (°C)
T_g	Glass temperature (°C)
T_a	Ambient temperature (°C)
S	Still bottom area (m^2)
S_s	Total still area (m^2)
S_{sw}	Side wall area (m^2)
S_g	Glass cover area (m^2)
M_w	Water content of the basin (kg)
H_s	Solar radiation on glass cover (W/m^2)
h_1	Heat transfer coefficient from the water surface to the glass cover (W/(m^2 .°C))
h_2	Heat transfer coefficient from the glass cover to the ambient (W/(m^2 .°C))
h_3	Heat transfer coefficient from the water to the basin liner (W/(m^2 .°C))
h_{ew}	Evaporation heat transfer coefficient (W/(m^2 .°C))
τ_1	$(1 - R_g)(1 - \alpha_g)\alpha_w$

τ_2	$(1-R_g)(1-\alpha_g)(1-\alpha_w)\alpha_b$
α_g	Glass cover absorptivity
α_b	Basin liner absorptivity
α_w	Water surface absorptivity
V	Wind velocity (m/s)
λ	Latent heat of water (J/kg)
C_w	Specific heat of water (J/kg.°C)
R_g	Glass cover reflectivity
P_w	Water vapor pressure at T_w (Pa)
P_g	Water vapor pressure at T_g (Pa)
U_{sw}	Overall heat transfer coefficient from the side wall to the ambient (W/(m^2 .°C))
U_{bw}	Overall heat transfer coefficient from the bottom to the ambient (W/(m^2 .°C))
ρ	Water density (kg/m^3)
l	Water depth in the basin (m)
t	Time (s)
Q_{ew}	Evaporative heat transfer (W)

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