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Robust Discretionary Monetary Policy under Cost-Push Shock Uncertainty of Iran's Economy

Fatemeh Labafi Feriz[1], Saeed Samadi[*2]
Khadijeh Nasrullahi[3], Rasul Bakhshi Dastjerdi[4]

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Abstract

There is always uncertainty about the soundness of an economic model's structure and parameters. Therefore, central banks normally face with uncertainty about the key economic explanatory relationships. So, policymaker should take into account the uncertainty in formulating monetary policies. The present study is aimed to examine robust optimal monetary policy under uncertainty, by a cost-push shock to the Iran's economy. For this purpose, three new-Keynesian Phillips curve equations are used, and robust discretionary optimal monetary policy is formulated by employing Hansen and Sargent robust control approach (2002). In all three curve equations, robust discretionary monetary policy is more aggressive comparing to the rational expectations. Considering the last period inflation rate in New- Keynesian Phillips curve, the degree of aggressiveness of robust monetary policy reduces, and with reducing the weight of the last period inflation rate, more reduction in the degree of aggressiveness of monetary policy is observed. On one hand, in all three models, with increasing the weight of inflation in the loss function of monetary policymakers, robust monetary policy is still more aggressive than the monetary policy under certainty. On the other hand, the degree of aggressiveness of monetary policy decreases, while the expected loss increases.
Keywords: Cost-Push Shock Uncertainty, Discretionary, New-Keynesian Phillips Curve, Robust Control, Robust Optimal Monetary Policy.
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1. Introduction

Monetary policy is a concept or a general understanding of the

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1. Department of Economics, University of Isfahan, Isfahan, Iran (f.labafi@ase.ui.ac.ir).
2. Department of Economics, University of Isfahan, Isfahan, Iran (Corresponding Author: samady@ase.ui.ac.ir).
3. Department of Economics, University of Isfahan, Isfahan, Iran (khadijh@ase.ui.ac.ir).
4. Department of Economics, University of Isfahan, Isfahan, Iran (r.bakhshi@ase.ui.ac.ir).

capacities and capabilities of the monetary policy-making institution and its impact on the main economic variables. Although the main task of this institution is controlling the price levels, but keeping up the level of economic activities is its other main task (Dargahi, 2010). An appropriate monetary policy formulating is very important to achieve the desired economic goals. In fact, to achieve these aims, a monetary policymaker changes the policy instruments affecting economic activities and prices (Shuetrim and Thompson, 2000).

Future events, shocks and economic fluctuations, actual performance of the economy, market response to the central bank's current policy, market expectations of central bank’s future policy and data limitations are five sources of uncertainty that affect monetary policy formulation (Onatski and Stock, 2002). Uncertainty of future events is called shock uncertainty, which means that unpredictable shocks that affect economy, can also be effective on monetary policies formulations. The remaining cases are considered as model–parameter uncertainties.

The model–parameter uncertainty can be raised from a variety of sources. All models are approximate; therefore they have approximation error. On one hand, sound economic relationships may change over time. In this respect, approximate models are not an exception. On the other hand, if economy is stable over time (for example, it is not affected by any shock case), parameters of approximation model may change too over time, due to the specified error. It must be noted that if a model is correctly specified, its parameters have an estimated nature. Therefore, they may be subjects to econometric estimation errors.

As there is uncertainty about the soundness of the structure and parameters of an economic model, central banks normally deal with uncertainty about cause and effect relationships in economy; furthermore uncertainty leads to disagreement about the effects of monetary policies (Traficant, 2013). In other words, in most cases, uncertainty is so great that it is thought that the effects of policy decision on economy are ambiguous (Onatski and Williams, 2003). Therefore, paying attention to the uncertainty that affects monetary policies formulation has a great importance.

Brainard (1967) was the first that studied the results of uncertainty over monetary policy. He denoted that if monetary policymaker is uncertain about the monetary instruments' effectiveness exists in economy; the optimal monetary policy in such circumstances is more conservative than those circumstances where uncertainty is not considered. This policy guideline is known as "Brainard conservatism principle". Before Brainard, Tinbergen (1952) and Thiel (1958) suggested that the central bank can ignore uncertainty, and consider a policy that has been adopted if all things were certain which is called "certainty equivalence principle". Brainard showed that "certainty equivalence principle" did not hold for long time, and for uncertainty complicated specifications. Specifically, if uncertainty was about the model–parameters, the central bank would not act in such a way that there was no uncertainty. About thirty years later, this conclusion was described by Alan Blinder (1997) as "Brainard uncertainty principle".

As mentioned above, Brainard (1967) was the first one that evaluated the results of uncertainty about the monetary policy. In his article, Brainard showed that the optimal policy under uncertainty is significantly different from optimal policy under certain. He initially took into consideration the circumstances where policymaker has only one objective, and one policy instrument. He assumed that policymaker focuses only on a target variable of y, and y is linearly dependent on policy instrument (P) and different exogenous variables (u). As a coefficient, a shows the response y to a policy instrument.

$$y = aP + u \tag{1}$$

In these circumstances, policymaker faces with two types of uncertainty:

1. While adopting a policy decision, policymaker is uncertain about the effects of exogenous variables u on y . This may be either an indication of his inability in full prediction of exogenous variables value, or the response of y to them.
2. While adopting a policy decision, Policymaker is uncertain about the response of y to any specified policy activity. He may consider an expected value for a coefficient; but he is aware that

the real response y to policy may fundamentally be different from its expected value.

These two types of uncertainty emphasize that policymaker cannot ensure that the value of y will achieve its target value y^*. However, these two have different applications for policy-making. The first type has no effect on policy-making. This means that policymaker must act based on the expected values. In fact, he should act in such a way that if everything had been certain, he would have done (Certainty Equivalence) (Brainard, 1967). Brainard believed that the assumption that all uncertainties are the first type is a cause of describing absolute equal treatment by Theil[1].

In the second type of uncertainty, Brainard denoted that if policymaker is uncertain about the effectiveness of monetary instruments that exist in the economy, the optimal monetary policy in these circumstances is more conservative than those circumstances where uncertainty is not considered. This means response to both inflation and output gap; while considering model–parameter uncertainty, is smaller than those circumstances in which model–parameter uncertainty is not considered. In other words, in model–parameter uncertainty, the monetary instruments of the economy should be altered in lesser extent comparing to those circumstance where there is no uncertainty. This policy guideline is entitled "Brainard conservatism principle". Then, in his studies, Blinder came to the conclusion that model uncertainty could be important for policy-making, and in particular, the model uncertainty may make monetary authorities more conservative.

After Blinder's study (1997), many papers studied the application of monetary policy in structural uncertainty. Many of them have used Bayesian approach. In this method, the expected value of central bank's loss function is minimized, presuming a specified prior distribution for structural parameters. The major results of studies that have used the Bayesian approach indicate that the monetary policy response to shocks, compared to a circumstance where there is no

1. Before Brainard, Tinbergen (1952) and Thiel (1958) suggested that the central bank could ignore uncertainty and adopts a policy that would have been taken if the circumstances were certain. This is called "Certainty Equivalence Principle".

uncertainty, is weaker (Similar to the results obtained by Blinder, 1997; Levine and Williams, 2003). In this regard, Rudebusch (2001), Clarida (1999), and Chow (1975) shall be pointed out.

As mentioned before, in the Bayesian approach, a specified prior distribution is considered for structural parameters which are, in fact, at odds with the definition of uncertainty. Uncertainty is defined as for the situations where probabilities are unknown. In fact, by considering specified prior distribution for model–parameters, uncertainty is reduced to risk. Therefore, according to the limitations of the Bayesian approach in considering the prior distribution function for model–parameters, recent studies have tended to use the alternative methods which are called “robust optimal control”.

By using these methods, the robust optimal monetary policy is, in fact, a policy that can minimize the worst loss that may occur due to parameter misspecification (Traficante, 2013). In his research, in order to study the parameter uncertainty effects on monetary policy, Giannoni (2002) formed a robust min-max optimization which was applied with a simple rule. He achieved conflicting results, compared to the results obtained by Brainard (1967). In fact, he concluded that monetary policymaker should have stronger response to the inflation under uncertainty. In general, in the studies conducted using these methods, results emphasize on a monetary policy that is more aggressive than a monetary policy adopted in certainty equivalence (Levine and Williams, 2003). In this regard, the studies conducted by Onatski and Williams (2003), Soderstrom (2000), Traficante (2013), and Onatski and stock (2002) can be mentioned.

In the present paper, we examine the monetary policy under uncertainty employing Hansen and Sargent robust control approach (2002), and Giordani and Soderlind (2004). In Hansen and Sargent approach, the purpose of robust planners, similar to the model of rational expectations, is minimizing the loss function with respect to the law of motion of the economy (approximation model). But despite of rational expectations, they are not certain about the accuracy of the approximation model. In their approach, they use a min-max method in which, the planner creates the worst-case model in the collection (through maximizing expected loss), and finally he chooses the rule that can minimize this loss. In fact, the aim of robust control is to formulate a policy that acts well, even in the circumstances where the approximation model is very different from the actual one (Giordani and Soderlind, 2004). It should be noted that the Bayesian approach has been employed in the only study on monetary policy under uncertainty for Iran's economy. Thus, the present study is important because of applying robust optimal control about Iran's economy.

The remainder of this paper is organized as follows. In section 2, the literature review is presented, and in section 3, the robust optimal control is explained. Section 4 explains the model. In section 5, monetary policy under uncertainty is illustrated. At last, conclusion is presented in section 6.

2. Literature Review

In this field, there is no study examining the impact of uncertainty on formulating the monetary policy using optimal control for Iran's economy. The only study is Zarra-Nezhad et al. (2012) that has examined the effect of uncertainty on monetary policy for Iran's economy using Bayesian method. They found that a policymaker that employs a control approach under uncertainty sets interest rates less aggressively to react against fluctuations in inflation or in output gap than in the case of certainty. In the main result, policy performance can be improved if the discretionary policymaker implements an optimum policy in the model. When there is uncertainty about the inflation persistence, it is optimal for policymakers to respond more aggressively to shocks than if the parameter were known with certainty, since they avoid bad outcomes in the future.

Some studies examined the effect of uncertainty on formulating the monetary policy in other countries, including Shuetrim and Thompson (1999) in which they have studied the results of parametric uncertainty on monetary policy in a simple model for the Australia's economy. In this paper, the optimal monetary policy was calculated and compared with or without taking into account the uncertainty. In this study, Ellis model (1998) is used, and it contains five estimated relations between key variables in Australian economy. In fact, in this paper, through the creation of Brainard model with multi-period horizon and multivariable model, it is concluded that unlike the results of Brainard, parameter uncertainty leads to a larger policy activity compared to more types of shocks. This increased activity is primarily the result of uncertainty about the persistence of shocks to the economy.

In his article, Sack (2000) seeks to examine the issue whether the uncertainty discussed by Brainard is an important aspect to explain the behavior of the Federal Reserve Board. He extracted a structural model of the economy using five vector auto regression (VAR). This model is first estimated based on ordinary least squares (OLS), and it is assumed that the point estimation of autoregressive vectors provides correct and logical values, and offers a “certainty equivalent” policy rule through minimizing the loss function of policymaker by using these estimations. In the next phase, parametric errors, estimated by ordinary least squares would be used as tools of uncertainty for each parameter, and the new optimal rule would be obtained. This rule is compared with “certainty equivalence” policy rule. Sack came to the conclusion that the optimal policy rule under uncertainty is much closer to the actual behavior of the Federal Reserve funds rate, compared to an optimal policy that has ignored the uncertainty.

Giannoni (2002) generally suggested a method based on the features of zero-sum game between the two players to derive robust optimal monetary policy rule in circumstances, where there were uncertainty about the parameters of the model. Uncertainty considered in this paper, is about the slope of the Phillips curve and slope of Euler equation for the output, which provides a robust min-max optimization by using a simple rule. The study is about the US economy.

Onatski and Stock (2002) sought to test a monetary policy in a two-equation macroeconomic model, where policymaker recognizes that the model is an approximate one, and there is uncertainty about the approximate amount. They tended to employ robust control min-max approach to achieve a robust monetary policy, rather than using the conventional Bayesian approach. In most cases, robust monetary policy, in the absence of uncertainty, is more aggressive than optimal policy.

In an article, Levine and Williams (2003) studied the problem of parameter uncertainty in micro-founded model, and reported that the purpose of the central bank was to maximize the households’ welfare. In other words, the central bank has a loss function that its weights are directly related to the structural parameters depth of the model. In their study, not only the uncertainty is true about the economy's dynamic behavior, but it is also true for each variable weight in the loss function. They employed both Bayesian and robust control approach in this study. As far as the Bayesian approach was applied, they came to the conclusion that uncertainty about the weights of loss function led to overcoming the classical consequences of ignoring uncertainty. Similarly, in the case of robust control method, it was found that the robust control policy may be quantitatively or even qualitatively different from circumstances where the loss function is assumed to be constant.

In their article, Onatski and Williams (2003) found that different assumptions about uncertainty may lead to various robust policy recommendations. Therefore, they used new methods for analyzing uncertainty about the model–parameters, specification lag, shock serial correlation and the effects of actual data in a coherent structure. They applied Bayesian and min-max methods to obtain optimal monetary policy, and found that that time recommendations about the use of aggressive robust policy rules was likely the result of too much emphasis on uncertainty about the economy's dynamics in low frequencies.

In his article, Tillman (2009) explored the optimal monetary policy, when there is uncertainty about the cost channels. In fact, the cost channel of money transfers is described as the supply side effect of interest rates on firms' expenditures. Tillman (2009) employed min-max approach to get optimal monetary policy approach. He showed that uncertainty about cost channels could lead to decline of the policy-making status.

In his study, Traficante (2013) obtained the optimal robust monetary policy in a new-Keynesian model with uncertainty about price rigidity and the cost-push shocks correlation. In fact, his paper focused on the uncertainty in supply sides relations of economy. He discussed that uncertainty about the degree of price rigidity would affect the awareness of the central bank about the slope of the aggregate supply and relative weight assigned to targets in the loss function. In his paper, uncertainty has been discussed theoretically, and he finally came to the conclusion that under discretion, the central bank response to uncertainty about the degree of price rigidity and cost-push shock autocorrelation is more aggressive. In addition, if the central bank uses the optimal discretionary robust balance with the Taylor's rule, the interest rates under uncertainty will show less aggressive responses to inflation.

Medeiros et al. (2016) investigated the Central Bank of Brazil's non-linear response function, resulted by these policymakers' uncertainty about the outcomes of the output gap and inflation. They found that uncertainty in the slope of the Phillips curve caused the nonlinear behavior of the Central Bank of Brazil.

Applying the methods of the studies mentioned above, we have tried to evaluate the effect of uncertainty on formulating the monetary policy, based on a case study of Iran's economy; so, we have applied a structural relationship adopted with Iran's economy.

3. Robust Optimal Control

The robust optimal control is a very important part of random control, and in some respects, it is the most real version of optimal control theory. In situations where the law of state development is not exactly known, but the system must be controlled, the robust control is used. In this method, a set of scenarios are considered, and the worst possible condition is controlled. In these circumstances, the best policy for the worst-case scenario is the robust control (Yannacopoulos, 2013). In this approach, robust optimal monetary policy actually minimizes the worse possible loss that could occur due to parameter misspecification (Traficant, 2013).

In the present paper, we use Hansen and Sargent robust control approach (2002), and Giordani and Soderlind (2004). In this approach, Hansen and Sargent offered a robust solution for monetary policy commitment, and for backward looking models. Then, Giordani and Soderlind expanded their approach for forward-looking models, discretionary, and the simple rule of monetary policy. As it was already mentioned, in the approach of Hansen and Sargent, the objective of robust planner, similar to the mode of rational expectations, is minimizing the loss function based on the law of motion of the economy (approximation model). But unlike rational expectations, they are not certain about the accuracy of the approximation model.

Technically, robustness makes it possible to switch the problem of maximization to the proper specification of min-max problem. For this purpose, Hansen and Sargent considered a two-agent problem which follows a policy chosen by the planner, and is the result of an equilibrium outcome of a two-person game that chooses a model from the available set. In this game, a fictitious evil agent, which aims to maximize the planner's loss, is assumed, and the planner chooses the policy function. It should be noted that the evil agent is only a metaphor for the cautionary behavior of the planner, and he shares in the approximation model and loss function of the planner. Also, it should be mentioned that the evil agent seeks maximization rather than minimization. Hansen and Sargent described this problem as a zero-sum game, and therefore, they considered single loss function. Giordani and Soderlind (2004) improved the framework of robust control problem in order to explicate the forward-looking models (It was suggested by Hansen and Sargent to explain the backward-looking models).

The problem was formulated by Hansen and Sargent as the followings:

$$\underset{\{u\}_0^\infty}{min}\ \underset{\{v\}_1^\infty}{max}\ E_0 \sum_{t=0}^{\infty} \beta^t \left(x_t' Q x_t + u_t' R u_t + 2 x_t' U u_t\right) \tag{2}$$

$$x_{t+1} = A x_t + B u_t + C(\epsilon_{t+1} + v_{t+1}) \tag{3}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t v_{t+1}' v_{t+1} \leq \eta_0 \tag{4}$$

In this problem, (2) is the loss function, and (3) is law of motion of the economy. x_t is the variable vector consisting of predetermined variables of x_{1t}, and forward-looking variables of x_{2t}. u_t is the planner's control vector, and v_{t+1} is the evil agent's control vector. η_0 is the misspecification. The standard rational expectation dynamic control problem corresponds to $\eta_0 = 0$. In this case, the planner minimizes the loss function (2) by using the control vector u_t subject to the law motion of the economy (3) and $v_{t+1} = 0$.

By embedding constraint (4) in loss function (2) we have:

$$\underset{\{u\}_0^\infty}{min}\ \underset{\{v\}_1^\infty}{max}\ E_0 \sum_{t=0}^{\infty} \beta^t \left(x_t' Q x_t + u_t' R u_t + 2 x_t' U u_t - \theta v_{t+1}' v_{t+1}\right) \tag{5}$$

$$x_{t+1} = A x_t + B u_t + C\left(\epsilon_{t+1} + v_{t+1}\right) \tag{6}$$

Parameter θ summarizes the central bank attitude towards wrong specification of model to adopt its policy. The small amounts of this parameter refer to the great misspecifications. We have $0 < \theta < \infty$, and the set of available models is determined for the evil agent. The smaller θ is an indication of the powerful evil agent, and $\theta = \infty$ corresponds to the rational expectations solution.

Misspecifications distort the approximation model by modifying errors. Given the constraint (4) that emphasizes on the imposed evil agent and its embedment in the loss function (2), it is concluded that the selection of policy functions for v_{t+1} involves a wide range of dynamic misspecifications, including wrong parameters (v_{t+1} is a linear function of x_t), Autocorrelation errors (v_{t+1} is a linear function of x_t lags) and nonlinear functions (v_{t+1} is a nonlinear function of x_t).

Forward-looking models introduce another player, as a private sector that forms the expectations. Forward-looking models are shown by the following linear law of motion:

$$\begin{bmatrix} x_{t+1} \\ E_t x_{2t+1} \end{bmatrix} = A \begin{bmatrix} x_t \\ E_t x_{2t} \end{bmatrix} + Bu_t + C(\epsilon_{t+1} + v_{t+1}) \tag{7}$$

where the x_{1t} is the predefined variable and x_{2t} is the forward-looking vector. Others are defined as the previous section, but the matrix is $C = \begin{bmatrix} C_1 \\ O_{n_2 * n_1} \end{bmatrix}$.

To introduce robustness in forward-looking models, we need to decide whether private sector expectations are standard or robust. If expectations are considered robust, approximation model of the private sector, loss function and degree of robustness must be decided. It should be noted that Giannoni’s study (2002) is very similar to Hansen and Sargent by their min-max method. Giannoni (2002) and Onatski (2000) studied uncertainty in the forward-looking models under commitment. They assumed that there is no uncertainty in the private sector; but the private sector knows that approximation model is correct, and is aware of the loss function of planner, and the robustness degree. But Hansen and Sargent assumed that the private sector and the planner are equally share the same loss function, approximation model and the robustness degree. In his study, Sims (2001) suggested that min-max optimization, as a modeling tool for the private sector, is appropriate for the private sector rather than the central bank.

Policymaker maximizes the loss function (5) with subject to the constraint (7). u_t and v_{t+1} are the solutions for this optimization that the state variables linear function obtains.

$$u_t = -F_u \quad , \qquad \begin{bmatrix} u_t \\ v_{t+1} \end{bmatrix} = -\begin{bmatrix} F_u \\ F_v \end{bmatrix} x_t \tag{8}$$

Equilibrium dynamics of the model are obtained by combining the policy function with the law of economy motion (6). Most researchers have focused on two cases:

1. Worst case model which defines the economic behavior when the policymaker is fully concerned. In this case, the evil agent is fully active. By putting policy function (8) in the law of motion (7), the following is obtained:

$$x_{t+1} = (A - BF_u - CF_v)x_t + C\epsilon_{t+1} \tag{9}$$

2. Approximation model, a reference model, which is determined in the equation (8), $F_v = 0$. It should be noted that in this case, the policy is still robust, and F_u is similar to the worst case model. In fact, the approximation model is solved by using robust control method; but the evil agent is assumed to be zero.

Comparison of dynamics of these two types of models provides information about the misspecification that the planner is concerned about.

4. Model

As mentioned in the earlier sections, in order to use the robust control approach, Central Bank’s loss function and the law of motion of the economy must be specified. In various studies, Phillips and Euler equations have been considered as the reference economic equations. Therefore, we present these equations and three variables in the following sections.

In his study, Tavakkolian (2012) introduced three new-Keynesian

Phillips curves, which along with Euler equation and the monetary policy rule, form dynamic stochastic general equilibrium models. The aim of his study was to choose a model that is closer to the Iranian economy. In this study, the central bank's policy instrument is the rate of money growth. In the present study, we use three types of Phillips curves, presented in the study conducted by Tavakkolian, along with Euler equation to achieve a robust monetary policy.

Models which are used in the present study are as follows:

Model 1:

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - E_t \pi_{t+1}) + \varepsilon_{1,t} \tag{10}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \varepsilon_{2,t} \tag{11}$$

$$m_t = \eta_x x_t - \eta_i i_t \tag{12}$$

$$\mu_t = m_t - m_{t-1} + \pi_t \tag{13}$$

Model 2:

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - E_t \pi_{t+1}) + \varepsilon_{1,t} \tag{14}$$

$$\pi_t = (1-\varphi)\beta E_t \pi_{t+1} + \kappa x_t + \varphi \pi_{t-1} + \varepsilon_{2,t} \tag{15}$$

$$m_t = \eta_x x_t - \eta_i i_t \tag{16}$$

$$\mu_t = m_t - m_{t-1} + \pi_t \tag{17}$$

Model 3:

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - E_t \pi_{t+1}) + \varepsilon_{1,t} \tag{18}$$

$$\pi_t = \left(\frac{\beta}{1+\beta}\right) E_t \pi_{t+1} + \kappa x_t + \left(\frac{1}{1+\beta}\right) \pi_{t-1} + \varepsilon_{2,t} \tag{19}$$

$$m_t = \eta_x x_t - \eta_i i_t \tag{20}$$

$$\mu_t = m_t - m_{t-1} + \pi_t \tag{21}$$

π is the rate of inflation, x is the output gap, i is the interest rate,

m is the amount of money, μ is the money growth[1], ε_1 is the demand-push shock, and ε_2 is the cost-push shock. Demand-push shock ε_1 and cost-push shock ε_2 are defined as a first-order autocorrelation process[2].

$$\varepsilon_{1,t+1} = \rho_1 \varepsilon_{1,t} + \epsilon_{1,t+1} \tag{22}$$

$$\varepsilon_{2,t+1} = \rho_2 \varepsilon_{2,t} + \epsilon_{2,t+1} \tag{23}$$

It should be noted that new-Keynesian Phillips equations of Model 1 and Model 2 are different from each other in terms of taking into account the inflation rate of the last period, and the difference between Models 2 and 3 is in the higher weight given to the inflation rate of the past period rather than to the expected inflation rate (inflation rate of next period).

In the present paper, we assume that the central bank adopts monetary policy under discretion. In fact, the central bank refuses to get caught with the future policy, and therefore, cannot affect the private sector's expectations of future inflation. The central bank's loss function is as follows, where λ_π , λ_x and λ_μ are the weights of inflation, output gap and money growth, respectively[3].

$$l = E_t \sum_{t=0}^{s} \beta^s \left(\lambda_\pi \pi_t^2 + \lambda_x x_t^2 + \lambda_\mu \mu_t^2 \right) \tag{24}$$

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1. As it is obvious, Tavakkolian has used a rule based on which the rate of money growth is calculated, rather than using the Taylor rule.
2. κ is the slope of the Phillips curve, acquired from $(1-\omega)(1-\beta\omega)/\omega$.
3. The central bank loss function expressed in this article, is different from the loss function expressed in foreign studies in terms of the money growth rate. To study the use of money growth variable rather than the interest rates, please see Walsh (2010) Chapter II, and Shahmoradi and Sarem (2013). This relationship can be derived as follows:
$M_t V_t = P_t Y_t$
Assuming that M_t is the money quantity, V_t is the money velocity, P_t is the price, and Y_t is the product in the economy. Then we have in the equilibrium:
$M_{t+1} V_{t+1} = P_{t+1} Y_{t+1}$
On the other hand, we can write:
$M_t(1+m)V_t(1+g_v) = P_t(1+\pi)Y_t(1+r)$
Or, according to the first equation, the following relation can be derived:
$(1+m)(1+g_v) = (1+\pi)(1+r)$
Assuming $g_v = 0$, in the equilibrium, $\mathrm{m} = i$.

5. Monetary Policy under Uncertainty

With reference to what was mentioned, we employ Hansen and Sargent approach to take into account the uncertainty about the approximation model. The state–space of a model includes misspecifications statements as follows:

$$A_0 \begin{bmatrix} x_{t+1} \\ E_t x_{2t+1} \end{bmatrix} = A_1 \begin{bmatrix} x_t \\ E_t x_{2t} \end{bmatrix} + Bu_t + C(\epsilon_{t+1} + v_{t+1}) \tag{25}$$

A_0, A_1 and B are matrices of model–parameters. C is a vector that measures the effect of the error terms vector v_{t+1}. x_{1t} is the vector of predetermined variables, and x_{2t} is the vector of forward-looking ones. u_t is the control variable of the planner, and v_{t+1} is the control vector of evil agent. It is assumed that misspecification has been limited as follows (η_0 is the potential misspecification).

$$E_0 \sum_{t=0}^{\infty} \beta^t v'_{t+1} v_{t+1} \leq \eta_0 \tag{26}$$

Policymaker maximizes the loss function of L_t subject to the above-mentioned constraint. Hansen and Sargent (2002), and Giordani and Soderlind (2004) showed that the problem could be formulated as follows:

$$\min_{\{u\}_0^{\infty}} \max_{\{v\}_1^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left(L_t - \theta v'_{t+1} v_{t+1}\right) \tag{27}$$

θ is the detection error probability. To calculate θ, Hansen and Sargent adapted a detection error probability approach based on the idea that the models in the set should not be easy to distinguish with the available data. This method is based on whether the true data are gathered through the application of an approximation model or the worst case model, and for a given size of the sample, what signifies the probability of making the wrong choice between two models based on compliance with appropriate example.

$$\pi(\theta) = \text{probability } (L_A > L_W | W)/2 + \text{probability } (L_W > L_A | A)/2 \tag{28}$$

L_A and L_W are the values of likelihood of the approximating and worst case model, respectively. Hansen and Sargent suggest the range of 10–20 percent for error detection.

Then, the equations system defined in section 5 is specified in the form of state–space.

Model 1:

$$A_0 = \begin{bmatrix} 10 & 0 & 00 & 0 \\ 01 & 0 & 00 & 0 \\ 00 & \eta_i & 10 & 0 \\ 00 & 0 & 10 & 0 \\ 00 & -1/\sigma & 01 & 1/\sigma \\ 00 & 0 & 00 & \beta \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad A_1 = \begin{bmatrix} \rho_1 & 0 & 00 & 0 & 0 \\ 0 & \rho_2 & 00 & 0 & 0 \\ 0 & 0 & 00 & \eta_x & 0 \\ 0 & 0 & 01 & 0 & -1 \\ -1 & 0 & 00 & 1 & 0 \\ 0 & -1 & 00 & -\kappa & 1 \end{bmatrix} \quad C = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad n_1 = 4\,, n_2 = 2 \tag{29}$$

Model 2:

$$A_0 = \begin{bmatrix} 100 & 0 & 0 & 0 & 0 \\ 010 & 0 & 0 & 0 & 0 \\ 001 & 0 & 0 & 0 & 0 \\ 000 & \eta_i & 1 & 0 & 0 \\ 000 & 0 & 1 & 0 & 0 \\ 000 & -1/\sigma & 0 & 1 & 1/\sigma \\ 000 & 0 & 0 & 0 & (1-\varphi)\beta \end{bmatrix} B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} A_1 = \begin{bmatrix} \rho_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \eta_x & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -\varphi & 0 & 0 & -\kappa & 1 \end{bmatrix} C = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} n_1 = 5\,, n_2 = 2 \tag{30}$$

Model 3:

$$A_0 = \begin{bmatrix} & & & 0 & 0 \\ 100 & 0 & 0 & 0 & 0 \\ 010 & 0 & 0 & 0 & 0 \\ 001 & 0 & 0 & 0 & 0 \\ 000 & \eta_i & 1 & 0 & 0 \\ 000 & 0 & 1 & 1 & 1/\sigma \\ 000 & -1/\sigma & 0 & & \\ 000 & 0 & 0 & 0 & \left(\frac{\beta}{1+\beta}\right) \end{bmatrix} B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} A_1 = \begin{bmatrix} & & 0 & 0 & & & \\ \rho_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \eta_x & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -\left(\frac{1}{1+\beta}\right) & & 0 & 0-\kappa & 1 \end{bmatrix} C = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} n_1 = 5\,, n_2 = 2 \tag{31}$$

The loss function of planner Lt is as follows based on the Eq (24):

$$Q = \begin{bmatrix} 0_{n_1 \times n_1} & 0_{n_1 \times n_2} & \\ & \lambda_x & 0 \\ 0_{n_2 \times n_1} & 0 & \lambda_\pi \end{bmatrix} \quad R = \lambda_\mu \;\; U = 0_{(n_1+n_2)\times 1} \tag{32}$$

The calibrated values of the parameters are shown in Table 1:

Table 1: Values of Calibrated Parameters

Parameter	Definition	Model 1	Model 2	Model 3
β	Discount rate	0.9695	0.9636	0.9636
φ	Measure of forward-looking behavior in determining the price	-	0.7006	0.7006
σ	Depreciation rate	1.4656	1.4733	1.4733
ω	The degree of nominal rigidity	0.5080	0.5001	0.5001
η_x	The coefficient of the output gap in equation to the quantity of money	0.0863	0.0896	0.0896

Table 1: Values of Calibrated Parameters

Parameter	Definition	Model 1	Model 2	Model 3
η_i	The coefficient of interest rates in equation to the quantity of money	0.6846	0.6843	0.6843
λ_x	The weight of output gap in the loss function of policymaker	0.5	0.5	0.5
λ_π	The weight of inflation in the loss function of policymaker	1	1	1
λ_μ	The weight of money growth rate in the loss function of policymaker	0.1–1	0.1–1	0.1–1

5.1 Cost-Push Shock

In this section, the impact of cost-push shock uncertainty on robust optimal monetary policy, in three models described in the previous section, is examined, and it is compared with conventional monetary policy, which is the same monetary policy for rational expectations solution. Figures 1, 2 and 3 show the impulse response functions of output gap and inflation variables of the cost-push shock. Discretionary monetary policy along with rational expectations solution, have been shown by the blue line, and approximation solution as well as the worst case model solution, have been shown by the green and red lines, respectively.

- Model 1:

Figure 1 shows the impulse response functions of output gap and inflation variables to the cost-push shock in the model 1. As seen in the diagram, the output gap in solving the worst case model (solving by robust optimal control method) has been dropped, more than solving the rational expectations model. Thus, robust monetary policy must be more aggressive than the conventional monetary policy. On one hand, the cost-push shock uncertainty leads to more inflation than the rational expectations model solution. On the other hand, it is more persistent to represent the high rate of inflation for an extended period and its slow adjustment. Then, as far as the inflation is concerned, the monetary policy is more aggressive in solving robust optimal control. As indicated in Table 2, the monetary policy instrument in rational expectations solution is equal to 0.38, while it is equal to 0.40 in robust optimal control solution which indicates a more aggressive robust monetary policy.

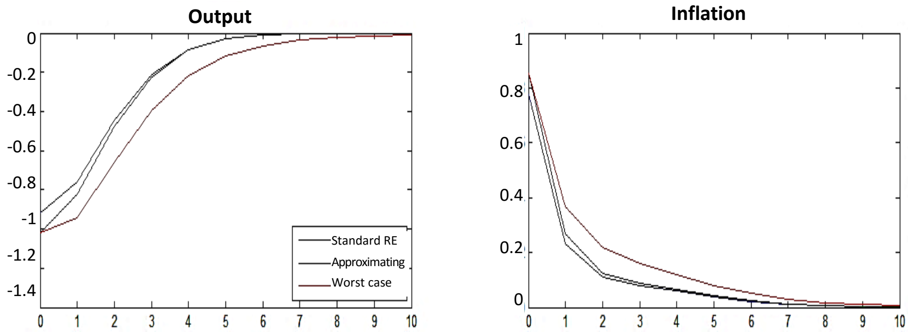

Figure 1: The Impulse Response Functions of Output Gap and Inflation Variables of the Cost-Push Shock in Model 1

- Model 2:

Figure 2 shows the impulse response functions of output gap and inflation to the cost-push shock in the model 2. In this model, like the model 1, the response of output gap and inflation to the cost-push in the worst case solution is more than rational expectations solution, and the coefficient of robust monetary policy is more than rational expectations monetary policy, indicating that it is more aggressive. In this model, unlike the model 1, the inflation initially increases less in response to cost-push shock in the worst case solution, compared to the rational expectations solution. But then, it increases to a higher level which reflects the higher inflation in future periods. Comparison the monetary policy rate in rational expectations solution (which is equal to 0.17) with robust optimal control solution (which is equal to 0.19) shows that in uncertainty, the robust monetary policy is more aggressive.

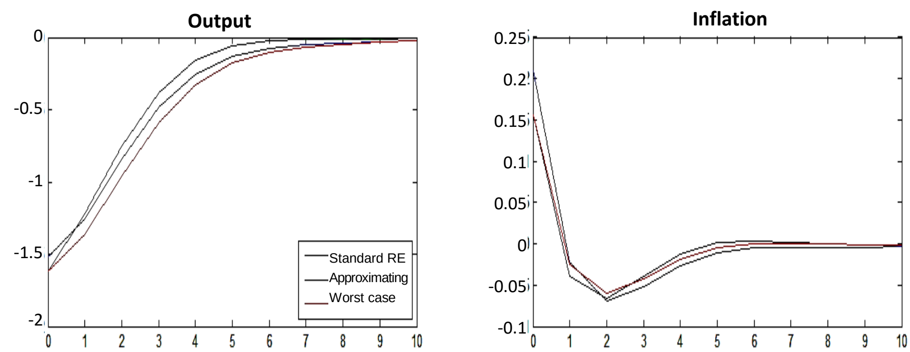

Figure 2: Impulse Response Functions of Output Gap and Inflation Variables to the Cost-Push Shock in Model 2

- Model 3:

Figure 3 shows the impulse response functions of output gap and inflation of the cost-push shock uncertainty. The results of this model are similar to the model 2 results. The only difference is that the inflation rate in both methods of robust optimal control solution and rational expectations solution increases more than the inflation rate in model 2. But the output decline is less than the one in the model 2. In this model, an aggressive monetary policy is recommended as well.

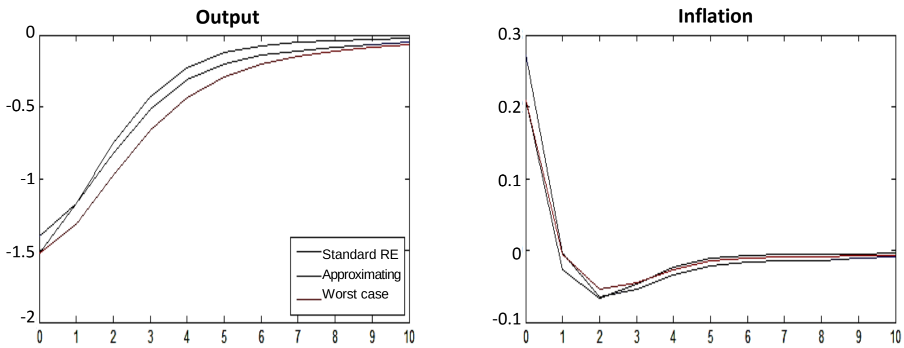

Figure 3: The Impulse Response Functions of Output Gap and Inflation Variables to the Cost-Push Shock in the Model 3

As is clear in Table 2, with the presence of last period inflation on the Phillips curve, and with reduction in the weight of past inflation compared to the future inflation in this equation, the degree of aggressiveness of robust monetary policy declines. Furthermore, taking into account the past period inflation in the Phillips curve, the losses in rational expectations solution increases compared to the model 1; but in the worst case solution, and in a case in which the past inflation has more weight than the expected inflation, the loss is more than the loss in the absence of past inflation in the Phillips curve; although the opposition is true for the model 3. Therefore, when policymaker gives more importance to the past inflation than the future inflation in the Phillips curve, in rational expectations solution and in robust optimal control solution, losses increase. In a case where policymaker gives more weight to the future inflation, losses in the worst case solution are less than losses in the rest of the models.

Table 2: Parameters of Optimal Instrument Rules and Expected Loss for Cost-Push Shock ($\lambda_\pi = 1$, $\lambda_y = 0.5$, $\lambda_\mu = 0.5$ and $p(\theta)$ =25%)

	Model 1	Model 2	Model 3
Θ	54.35	61.72	56.81
Rule of rational expectations monetary policy	0.38	0.17	0.08
Rule of robust monetary policy	0.40	0.19	0.1
Expected loss in rational expectations solution	65.56	72.70	67.08
Expected loss in robust control solution	69.16	74.07	68.83

Table 3: Parameters of Optimal Instrument Rules for Cost-Push Shocks in Different Weights of Inflation in the Policymaker Loss Function ($\lambda_\pi = [0.1 - 1]$, $\lambda_y = 0.5$, $\lambda_\mu = 0.5$ and $p(\theta)$= 20%)

λ_π **Model 1**	**0.1**	**0.5**	**1**
Θ	46.30	54.95	54.35
Rule of rational expectations monetary policy	0.82	0.58	0.38
Rule of robust monetary policy	0.86	0.60	0.40
Expected loss in rational expectations solution	44.51	56.10	65.56
Expected loss in robust control solution	47.98	59.55	69.16
λ_π **Model 2**	**0.1**	**0.5**	**1**
Θ	48.45	56.82	61.72
Rule of rational expectations monetary policy	0.14	0.15	0.17
Rule of robust monetary policy	0.16	0.17	0.19
Expected loss in rational expectations solution	70.18	71.33	72.70
Expected loss in robust control solution	72.23	73	74.07
λ_π **Model 3**	**0.1**	**0.5**	**1**
Θ	56.82	56.82	56.81
Rule of rational expectations monetary policy	0.007	0.04	0.08
Rule of robust monetary policy	0.03	0.06	0.1
Expected loss in rational expectations solution	63.41	65.12	67.08
Expected loss in robust control solution	65.51	67.05	68.83

Another question in the field of monetary policy is that, by varying the degree of importance of inflation in the policymaker's loss function, if the monetary policy changes. According to Table 3, in the model 1, with increasing inflation weight in the policymaker's loss function, the rule of robust monetary policy is still more aggressive than the monetary policy under certainty. But with the increased $\boldsymbol{\lambda_\pi}$, the aggressiveness of monetary policy declines, while the expected loss increases. In models 2 and 3, in which the last period inflation rate is considered in the Phillips curve, with increasing $\boldsymbol{\lambda_\pi}$, the monetary policy is more aggressive in the worst case solution, than the rational expectations solution, and the expected loss increases; but compared with the model 1, the aggressiveness degree of monetary policy increases.

6. Conclusion

Monetary policymaker needs to formulate appropriate monetary policy in order to achieve the policy objectives. Thus, he should employ an appropriate economic model. Considering the fact that even properly formulated economic models may face with unpredictable shocks, taking into account the shocks uncertainty in the formulating of monetary policy, is of great importance. For this purpose, we examined formulation of a monetary policy under cost-push shock uncertainty for three models of new-Keynesian Phillips curve. In all three models, robust discretionary monetary policy is more aggressive than rational expectations monetary policy. By regarding the inflation rate of the last period in the new-Keynesian Phillips curve, the aggressiveness of robust monetary policy declines, and with decreasing weight of the last period inflation, compared to the future inflation, the aggressiveness of monetary policy declines more. In all three models, with increasing weight of inflation in the policymaker's loss function, the robust monetary policy becomes more aggressive comparing to the monetary policy under certainty, and the aggressiveness of monetary policy declines, while the expected loss increases.

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