

Applicability analysis of the two-dimensional discrete wavelet transform for de-stripping of GRACE products

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Article history:

Received: 4 April 2018, Received in revised form: 19 September 2018, Accepted: 25 September 2018

ABSTRACT

Gravity field models derived from the Gravity Recovery and Climate Experiment (GRACE) satellite mission are affected by the orbital arrangement of the satellites of this mission and the spatial dependence of the gravity field coefficients, which manifest themselves in the form of north to south strips in the maps of corresponding products. One of the common ways to deal with this phenomenon is to use a Gaussian filter to reduce the contribution of higher-order coefficients and to extract realistic images of the surface-mass variability. In this paper, the applicability of the two-dimensional wavelet transforms as non-statistical methods in reducing the effect of vertical components in GRACE products, including the mentioned strips, was tested. The results for different levels of transformation, as well as for using various base functions were evaluated and then compared with the results of applying a Gaussian filter on the data of mass change derived from GRACE in Iran. The results indicate that although the two-dimensional wavelet transform can inherently be used to reduce the striping error of images, the wavelet-based de-stripping of GRACE products is at most as effective as the Gaussian filter of 100km, while previous studies showed that the application of the Gaussian filter of 350km would be the optimal radius to extract the hydrological signals in Iran. However in the case of wavelet transformation, the most important issue is the number of transformation levels such that applying the wavelet transforms at levels 3 and 4 have led to the closest results to the outputs of Gaussian filtering.

KEYWORDS

GRACE
De-stripping
Two-Dimensional
wavelet transform
Applicability analysis

1. Introduction

From a general view, the gravity field of Earth can be divided into two static and time-variable parts. Although the static part is not absolutely unchanged and experiences some variations from thousands to millions of years, such variations can be ignored. Occurring as a result of the gravity of other celestial bodies, the tide is one of the factors that makes changes in the gravity field. The non-tidal component constituting the variable part of the gravity field has mainly a hydrological, oceanic, glaciological or atmospheric source, and can inherently be considered as related to the mass movement levels in the earth (Chen, 2007).

In the past, it seemed difficult or even impossible to

directly observe the mass movement of the Earth, but with the emergence of satellite missions designed specifically for the purpose of modeling the gravity field of Earth, which have been in operation since the beginning of the current century - such as CHAMP, GRACE and GOCE - the subject has considerably changed (Drinkwater et al., 2007; Reigber et al., 2002; Tapley et al., 2004). Specifically, the GRACE satellite mission is the first dedicated gravity field mission capable of measuring the changes in distance between the twin co-orbiting low-altitude satellites along with the exact positioning of each satellite with GPS receivers as well as the non-gravitational accelerations by the accelerometers installed in the center of mass of each satellite. These

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DOI: 10.22059/eoge.2018.263789.1028

missions allowed to record and monitor the changes in the gravity field of Earth, in particular, due to the changes in the surface mass distribution at global and regional scales (Tapley et al., 2004). For this purpose, after more than a decade of observations, the products of this mission are unprecedentedly utilized in various ways to trace the global and regional changes in the mass distribution, in particular, the changes in the water resources (for example, Awange et al., 2011; Cao et al., 2015; Frappart et al., 2013; Mulder et al., 2015 and Ramillien et al., 2012 can be outlined).

Among the GRACE data available at various levels known as Level 0, 1 and 2 (Chen, 2007), the level-2 data of the mission, as presented in the form of spherical harmonic expansion coefficients of geopotential with the monthly temporal resolution, could be considered the most commonly used products in providing a picture of the temporal and spatial variations of the gravity field due to their relative ease of use. Due to the orbital arrangement of the satellites of this mission and the spatial dependency between the gravity field coefficients, this issue has led to some stripes in the north-south direction in maps representing the changes in the surface mass distribution produced by GRACE. Hence, in order to properly use these data, appropriate filtering (de-striping) has to be applied to the GRACE spherical harmonic products to remove the striping error in the corresponding maps. In the studies based on these products (Level-2 data) several methods have been used to eliminate or reduce such stripes, including the use of the Gaussian isotropic filter, by Jekeli (1981), Wahr et al. (2011) and the modified Gaussian isotropic filter by Chambers (2006), respectively. The dependency of the employed filters in the aforementioned studies on the degree of spherical harmonics is such that by increasing the degree, they approach to zero and their application leads to a decrease in the contributing coefficients.

In some studies, the non-isotropic filters are utilized to encounter the striping error (Guo et al., 2010 and Han et al., 2005). These filters depend on both the degree and the order of spherical harmonics. Fathollah Zadeh et al. (2013) introduced a method in which the striping error is eliminated using the gravity gradient tensor. They showed that some components of the gravity gradient tensor that were derived from the differentiation of the gravity potential function along the latitude could effectively eliminate the striping error of the GRACE data. Fathollah Zadeh et al. (2016) presented a method based on the application of spherical wavelet transform on the GRACE level-2 data to reduce the striping error and compared the obtained results with the Gaussian filtering, concluding that the application of spherical wavelet transforms leads to more desirable results. Some other notable methods to deal with the striping error of the GRACE spherical harmonic products include Wiener optimal filtering (Sasgen et al., 2006), Statistical filtering (Davis et al., 2008) and decorrelation along with Gaussian

filtering (Swenson & Wahr 2006).

Similar to what happens in the GRACE gravity maps, the striping error is observed in most remote sensing images. Rezaei (2015) showed that by applying the discrete wavelet transform on the hyperspectral images and then, the Fourier filtering of vertical component, the quality of the images can be maintained while eliminating the striping error. Torres and Infante (2001) encountered the striping noise in satellite images based on the wavelet analysis, as well.

In this paper, the efficiency of the 2D discrete wavelet transform is tested as a non-statistical method for the elimination of striping errors in the mass variation data related to Iran obtained from the GRACE level-2 data, and the results for different levels of the wavelet transform and also for the application of various basis functions are compared with what resulted from applying the Gaussian filtering on the same data. For this purposes, the next sections of this paper are organized as follows. At first, in addition to a brief overview on the fundamentals of the 2D wavelet transform, we describe how this transformation is encountered with the vertical stripes. Then the desired process is introduced, the used data are described, and after applying the wavelet transform at different levels based on the selected basis functions, the results are finally presented and compared.

2. Two-dimensional discrete wavelet transform in brief

According to wavelet terminology (Burrus et al., 1997), a discrete function $f(t)$ can be represented as a weighted summation of wavelets $\psi_{j,k}(t)$, plus a coarse approximation $\phi_k(t)$ as follows

$$f(t) = \sum_{k=-\infty}^{+\infty} c_k \phi_k(t) + \sum_{k=-\infty}^{+\infty} \sum_{j=0}^{+\infty} d_{j,k} \psi_{j,k}(t) \quad (1)$$

where the wavelets $\psi_{j,k}(t)$ are constructed by translating and stretching a main producer function, known as mother wavelet $\psi(t)$

$$\psi_{j,k}(t) = \psi(2^j t - k) \quad (2)$$

and the scaling functions $\phi_k(t)$ are translated versions of a basic scaling function $\phi(t)$ as

$$\phi_k(t) = \phi_k(t - k) \quad (3)$$

The expansion coefficients c_k and $d_{j,k}$ are called the discrete wavelet transform (DWT) of the signal. In expression (3), the first part consists of the coarse information of the function and the remaining details are reflected at the second part; such that the bigger j , the more obtained details. According to the wavelet transformation, as a multiresolution analysis, introduced by Mallat (1989) and improved by Beylkin et al. (1991), the fast and simple implementations of discrete wavelet transform and its inverse can be carried out by using a highpass and a lowpass filter with impulse responses $h(n)$ and $g(n)$, respectively. These filters are related to each other as (Li, 1996)

$$g(n) = (-1)^{n-1} h(-n-1) \quad (4)$$

The decomposition and reconstruction of a uniformly sampled function $\{f_{i+1}(n), n \in \mathbb{Z}\}$, can respectively be computed as follows (Li, 1996)

$$\begin{cases} c_i(n) = \sum_k h(2n-k) f_{i+1}(k) \\ d_i(n) = \sum_k h(2n-k) f_{i+1}(k) \end{cases} \quad (5)$$

and

$$f_{i+1}(n) = \sum_k h(2k-n) c_i(k) + \sum_k g(2k-n) d_i(k) \quad (6)$$

Similar to the 1DWT, the two dimensional discrete wavelet transform (2DWT) of two dimensional functions is constructed using the above mentioned filters. In this situation, the forward and inverse 2DWT of a given 2D sequence $\{f_{i+1}(n,m), m, n \in \mathbb{Z}\}$ can respectively be written as (Li, 1996)

$$\begin{cases} c_i(n,m) = \sum_k \sum_l h(2n-k) h(2m-l) f_{i+1}(k,l) \\ d_{i,1}(n,m) = \sum_k \sum_l g(2n-k) h(2m-l) f_{i+1}(k,l) \\ d_{i,2}(n,m) = \sum_k \sum_l h(2n-k) g(2m-l) f_{i+1}(k,l) \\ d_{i,3}(n,m) = \sum_k \sum_l g(2n-k) g(2m-l) f_{i+1}(k,l) \end{cases} \quad (7)$$

and

$$\begin{aligned} f_{i+1}(n,m) = & \sum_k \sum_l h(2k-n) h(2l-m) c_i(k,l) + \\ & \sum_k \sum_l g(2k-n) h(2l-m) d_{i,1}(k,l) + \\ & \sum_k \sum_l h(2k-n) g(2l-m) d_{i,2}(k,l) + \\ & \sum_k \sum_l g(2k-n) g(2l-m) d_{i,3}(k,l) \end{aligned} \quad (8)$$

In expression (7), the details $d_{i,1}$, $d_{i,2}$ and $d_{i,3}$ are called horizontal, vertical and diagonal details, respectively, corresponding to the direction of decomposition in 2DWT

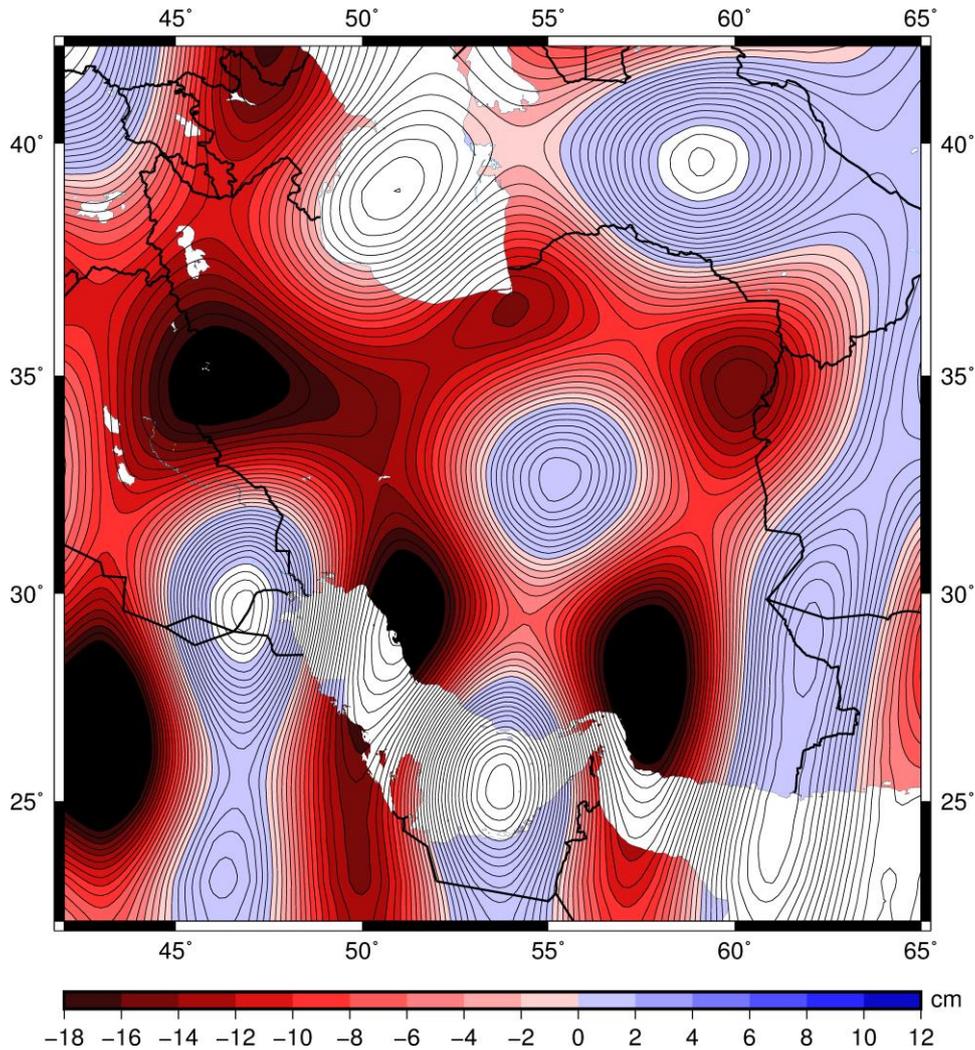


Figure 1. GRACE Total Water Storage (TWS) changes in Iran from August 2002 to April 2015 before de-striping

3. Used data, data analysis and obtained results

In this study, the GRACE 141-month, level-2 data were used between August 2002 and April 2015 which were extracted from GRACE level 2 Release 05, produced by the Center for Space Research (CSR) at the University of Texas (Bettadpur, 2012). These data were first modified in accordance with the methodology proposed by Ashraf Zadeh Afshar et al. (2016) by replacing the degree-one spherical harmonic coefficient with those computed by Swenson et al. (2008) and the lowest-degree zonal harmonic coefficient, C20, which is due to the flattening of the Earth, with those estimated from Satellite Laser Ranging (SLR) and finally by correcting the effect of the Caspian Sea using surface water changes derived from satellite altimetry observations. The result is the map of variations in the total water layer thickness equivalent to the total water storage (TWS) changes in Iran during this period as shown in Figure 1, in which the north-south stripes are evident.

Application of isotropic Gaussian filters of different radii can reduce the above stripes to some extent. Figure 2 shows the results of applying the Gaussian smoothers with radii of 100km, 350km and 750 km, respectively, to the data used in

plotting Figure 1. According to Ashraf Zadeh Afshar et al. (2016), the application of a de-stripping isotropic Gaussian filter of 350km radius could effectively extract the TWS changes in Iran.

Now, by applying the 2DWT on the monthly unfiltered total water layer thickness changes in Iran, we separate the coarse parts of the above signal from its horizontal, vertical, and diagonal details according to Eq. (7). Since the image high frequency information with vertical orientation mostly affects the obtained vertical details $d_{i,3}$ in Eq. (7), these coefficients are largely influenced by the striping phenomenon, while part of the effect of this phenomenon remains in the resulting approximation coefficients c_i . Therefore, the process of separating the approximation and the details can now be continued by reapplying the wavelet transform to the above-mentioned approximation coefficients. Each time, by filtering the vertical component of the obtained details and restoring the signal at a lower level using Eq. (8), the filtered data can be compared with the results that lead to the generation of Figure (2). In this study, the above comparison provides

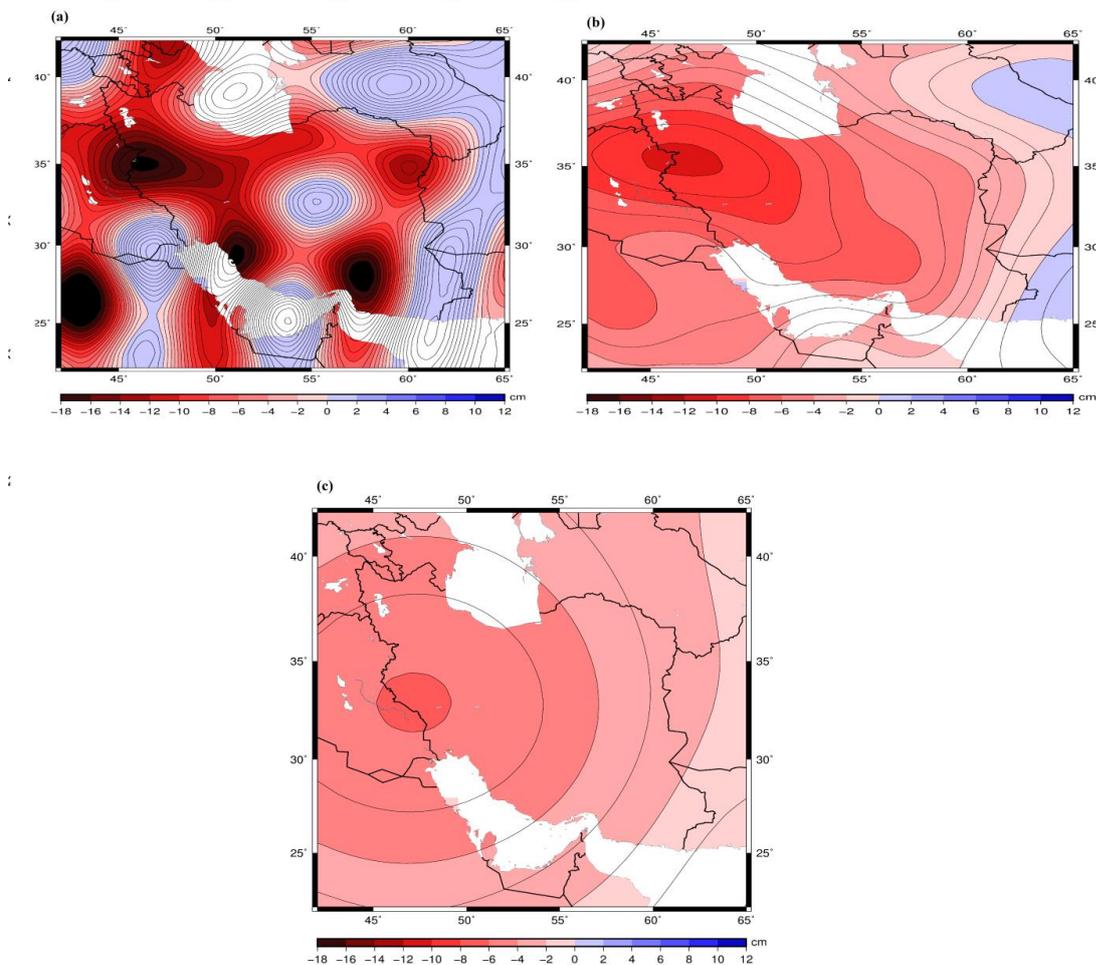


Figure 2. GRACE Total Water Storage (TWS) changes in Iran from August 2002 to April 2015 after smoothing using a de-stripping isotropic Gaussian filters of (a) 100km, (b) 350km and (c) 750km radii (Ashraf Zadeh Afshar et al., 2016)

directions for selecting the optimal wavelet basis function as well as the number of optimum levels for the transformation.

This procedure has been tested for several well-known Daubechies orthogonal wavelet functions (Daubechiesi, 1992) and for each selected basis function, the wavelet transform is applied up to level 6, as the maximum possible level that can be selected according to the number of data points and each time, the root-mean-square error (RMSE) obtained from the approximation by the wavelet transform method is estimated in comparison with their respective values from the Gaussian filtering method. These results are shown in Figure 3.

The results clearly show that, first, application of the wavelet transform up to decomposition levels 3 and 4 leads to the best results in terms of proximity to the Gaussian filtering, and its progress up to levels 5 and 6 deteriorates the results. Second, regardless of the transformation levels, it could be concluded that the issue of selecting the mother wavelet in the case of facing with the striping error will not change or improve the results. Given the above results, the application of the discrete 2D wavelet transform with a

Daubechies-10 wavelet system up to level 4 on the monthly variations in the thickness of the unfiltered water layer, and the filtering of the vertical details from these transformations result in the TWS changes shown in Figure 4.

The Comparison of Figures 2 and 4 implies that the effective de-striping of the GRACE monthly solutions cannot be carried out using the 2DWT, as the best result of the wavelet-based de-striping of GRACE maps is very similar to those of applying the Gaussian filter with radius of 100km, in which the striping errors are not completely removed. The lower inefficiency of the wavelet-based method may be associated with: a) the possible number of transformation levels or b) the effect of the strips on the 2DWT components other than the vertical details at each level. According to Figure (1) and the number of possible levels for wavelet transform which is affected by the resolution of the GRACE level-2 data, the first factor is not relevant to the above mentioned inefficiency and it seems that the second factor is the reason for the failure of the wavelet transform in this particular case.

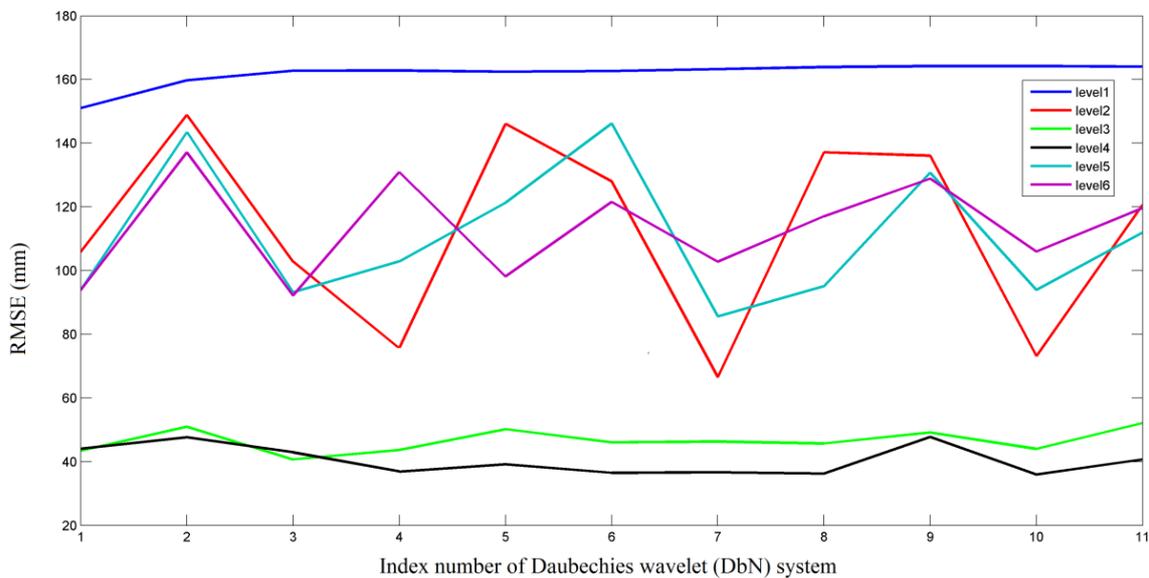


Figure 3. Root Mean Square Error (RMSE) of the wavelet-based GRACE TWS changes in Iran from August 2002 to April 2015 with respect to the corresponding values resulted from Gaussian smoothing

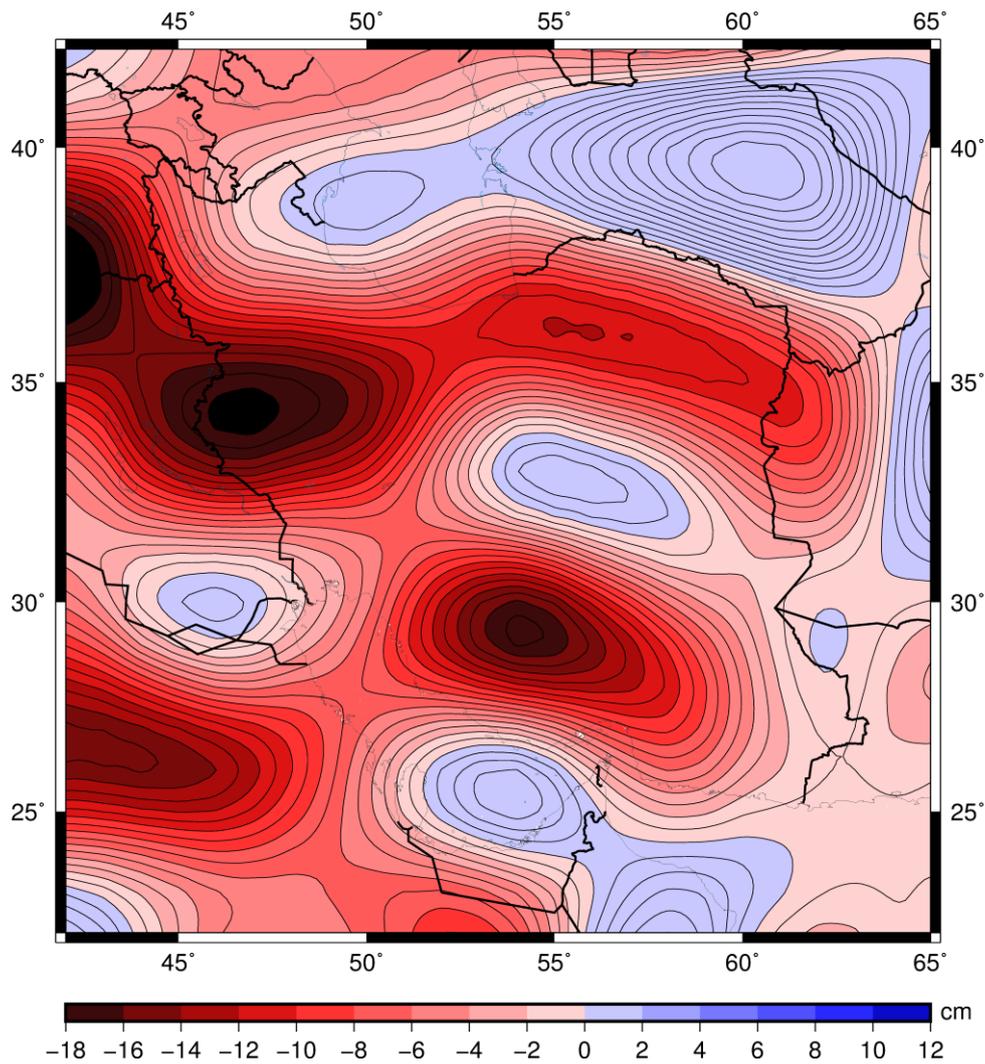


Figure 4. GRACE Total Water Storage (TWS) changes in Iran from August 2002 to April 2015 after smoothing using the 2DWT with Daubechies-10 wavelet system up to level 4

4. Conclusion

In this paper, the applicability of the 2D discrete wavelet transform facing the striping phenomenon was studied in the maps reflecting the surface mass distribution derived from the GRACE satellite mission. To this end, the process of eliminating the effect of striping error from the data corresponding to the monthly unfiltered total water layer thickness changes in Iran was carried out by applying the 2D discrete wavelet transform and filtering the vertical details at successive levels.

The inherent characteristics of the wavelet transform in terms of the diversity of base functions that can be used in the transformation as well as being a multispectral transformation affected the selected base functions. Besides, the number of optimum transformation levels in achieving the best results was tested in the process of de-striping. For this purpose, the optimal mode measurement, the least difference between the result of applying the wavelet transform, and the isotropic

Gaussian filter of 350km radius to the monthly variations in the total water storage derived from GRACE were taken into account. The results showed that applying the wavelet transform at levels 3 and 4 led to the best results in terms of decreasing the root-mean-square error, and the progress up to levels 5 and 6 deteriorated the results, probably due to the removal of some useful information in successive wavelet transforms. On the other hand, the selection of the mother wavelet is not so determinative in eliminating the striping error; however, among the selected functions, the Daubechies-10 wavelet system and continuing the transform up to level 4 leads to the closest results to the Gaussian filtering. However, the applied wavelet-based two-dimensional transformation for de-striping of the GRACE gravity maps did not lead to better results than those using the isotropic Gaussian filter of 100km radius, which was not satisfactory to extract mass change signals from GRACE. The mentioned inefficiency was probably associated to the effect of the strips on the unfiltered 2DWT constituents that

could be investigated in a modified wavelet-based procedure.

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