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A New Robust Bootstrap Algorithm for the Assessment of Common Set of Weights in Performance Analysis

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Abstract
The performance of the units is defined as the ratio of the weighted sum of outputs to the weighted sum of inputs. These weights can be determined by data envelopment analysis (DEA) models. The inputs and outputs of the related Decision Making Unit (DMU) are assessed by a set of the weights obtained via DEA for each DMU. In addition, the weights are not generally common, but rather, they are very close to zero or they are even equal to zero. This means that some major criteria will not be considered. Another problem is the similarity of the efficiency scores of efficient DMUs. However, this is not the case in reality, and the performance of the DMUs should be completely ranked. Using common weights can solve these problems completely during measuring the performance of DMUs. There are some articles in the literature to determine common weight sets (CSWs), but none of them takes into account the bootstrap approach. This paper introduces a novel, empirical and robust algorithm based on bootstrap technique to find CSWs.

Keywords
Data envelopment analysis, Common set of weights, Performance evaluations, Bootstrapping.

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Introduction

Data Envelopment Analysis (DEA) is one of the most popular methods to assess the performance of Decision Making Units (DMUs). DEA is used for the performance assessment of DMUs based on the used multiple inputs and outputs (Charnes et al., 1978). DEA classifies DMUs into two classes such as efficient and inefficient. A DMU is efficient if the efficiency score has equal to 1 and if not, it is inefficient. Generally, some of the weights are very close to zero or they are even equal to zero. This causes the DMU to ignore inputs and outputs during performance evaluation. Therefore, DMUs are not assessed with the same criteria. Another important issue concerns the ranking of DMUs. Because DEA gives 1 score to efficient DMUs, it is not possible to rank DMUs. These are the remarkable difficulties and inadequate facets of DEA.

To come through all of these problems, researchers have been doing some studies and these efforts will probably continue later. The essential ones of these are as follows. Andersen and Petersen (1993) presented a super efficiency DEA model. The difference with classic DEA models is its more than 1 efficiency score assignment for the exceedingly efficient DMUs. The principal attempt to restrict the weights was made by Thompson et al. (1990). Mecit and Alp (2013) proposed the correlation DEA model which generates bounds via correlation coefficient through the addition of new restrictions. Other references on the subject are as follows: Wong and Beasley (1990), Roll et al. (1991), Ganley and Cubbin (1992), Doyle and Green (1994), Sinuany-Stern et al. (1994), Troutt (1995), Torgersen et al. (1996), Cooper and Tone (1997), Mehrabian et al. (1999), Adler et al (2002), Angulo-Meza and Estellita Lins (2002), Kao and Hung (2005), Cook et al. (2007), Podinovski (2007), Liu and Peng (2008) and Bal and Örkcü (2011).

This paper is organized in the following manner. The concept of DEA is briefly reviewed in section 2. Section 3 presents the proposed algorithm to find CSWs with bootstrap technique. After that, two numerical examples are used in section 4 to find new CSWs with bootstrap technique. Finally, in section 5, some conclusions are drawn.

Theoretical Backgrounds

The assessments of performance are particularly concerned with assessing the activities of firms. Examples include satisfaction per unit, which is a measure stated in the form of a ratio as $Output \,/\, Input$, and is a usually used measure of efficiency.

Then, efficiency is equal to:

$$\frac{virtual\ output}{virtual\ input} = \frac{u_1 y_{10} + u_2 y_{20} + \cdots + u_s y_{s0}}{v_1 x_{10} + v_2 x_{20} + \cdots + v_m x_{m0}}$$

where u_i: weight of i[th] output, v_i: weight of i[th] input, s: the number of outputs, and m: the number of inputs. There are two methods in determining the weights of inputs and outputs for this ratio value. The first of these methods is a subjective approach, determined by the opinion of an authority. Another method is an objective approach, determined by an approach on the basis of the scientific method. The direct calculation of performance is the computation of weights using these two methods.

There are auxiliary processes in which inputs and outputs used for performance evaluation are considered as dependent or independent variables. Obtained results can be used as the score of its adoption in the ranking of DMUs after implementing some related statistical techniques such as regression line, canonical correlation analysis (Friedman and Sinuany-Stern, 1997), factor analysis and discriminant analysis (Sinuany-Stern et al., 1994; Sinuany-Stern and Friedman, 1998).

Data Envelopment Analysis

Charnes et al. (1978) introduced the CCR model in DEA for the first time. To maximize efficiency scores, the CCR model calculates the weights for each DMU that is designed to assign different weights. Input-oriented CCR model has been formulated in the following format under the assumption of constant returns to scale (CRS):

$$\begin{aligned} & max\ \theta_0 = \sum_{r=1}^{s} u_r y_{r0} \Big/ \sum_{i=1}^{m} v_i x_{i0} \\ & \sum_{r=1}^{s} u_r y_{rj} \Big/ \sum_{i=1}^{m} v_i x_{ij} \leq 1 \quad j = 1, 2, \ldots, n \\ & u_r, v_i \geq 0 \quad (r = 1, 2, \ldots, s \;\; i = 1, 2, \ldots, m) \end{aligned} \tag{1}$$

The translated version to linear programming model of input oriented CCR is as follows:

$$\begin{aligned} & \theta_0 = max \sum_{r=1}^{s} u_r y_{r0} \\ & \sum_{i=1}^{m} v_i x_{i0} = 1 \\ & \sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad j = 1, 2, \ldots, n \\ & u_r, v_i \geq 0 \quad (r = 1, 2, \ldots, s \;\; i = 1, 2, \ldots, m) \end{aligned} \tag{2}$$

where: θ_0: the efficiency score for the DMU_0, x: input, y: output, u: the output weights, v: the input weights, and $m, s,$ and n are number of inputs, outputs and DMUs, respectively.

Bootstrap Approach

Bootstrapping is a form of larger class methods, which is re-sampled from the original data set, and therefore referred to as resampling procedures. Similar to Bootstrapping, some resampling procedures, such as jackknife (Quenouille, 1949) and permutation methods (Fisher, 1935; Pitman, 1937; 1938) have a long history.

After the bootstrap was invented , the research operations on it increased exponentially. Initially, there were various academic advancements on the asymptotic coherence of the bootstrap estimates. It soon came to be accepted in the statistical community and recognized in the natural sciences (Casella, 2003). Therefore, it has been applied to an extensive class of applications such as time series analysis, nonlinear regression, error rate estimation in discriminant analysis and logistic regression.

Generally, you may be asked to estimate a parameter with an n-size sample or to specify a standard error or parametric range of privacy or to test a hypothesis about the parameter. These difficult tasks can be undertaken via bootstrap because it does not make any parametric assumptions. The bootstrap idea is simply to replace the unknown population distribution with the known empirical distribution (Efron, 1982, 1993; Chernick, 2008).

Pigeot (2001) compares the basic principles of two resampling techniques, namely the jackknife and the bootstrap. The bootstrap is more flexible than the jackknife and is currently the most popular resampling technique. Therefore, the bootstrap method is preferred to find the most optimal CSWs as an auxiliary tool in the presented algorithm in the next section.

An Algorithm to Find CSW with Bootstrap Technique

In this section, we propose a new algorithm to find a CSW vector based on bootstrap (hereafter, CSWB) technique. The new algorithm steps are as follows:

Step 1. Calculate vectors (v^*_{km}, u^*_{ks}) which are the weights of inputs and outputs respectively. The efficiency scores (θ^*_k) for each DMUs $(k = 1, \dots, n)$ are obtained by solving the linear programming model (2).

Note that the weights for the first DMU are expressed as $(u^*_{11}, u^*_{12}, u^*_{13}, \dots, u^*_{1s})$ for outputs and $(v^*_{11}, v^*_{12}, v^*_{13}, \dots, v^*_{1m})$ for inputs whereas the weights for n^{th} DMU are expressed as $(u^*_{n1}, u^*_{n2}, u^*_{n3}, \dots, u^*_{ns})$ for outputs and $(v^*_{n1}, v^*_{n2}, v^*_{n3}, \dots, v^*_{nm})$ for inputs.

Thus, when the efficiency scores of all DMUs are obtained, a weight matrix of $n \times s$ dimension is obtained for outputs while a weight matrix of $n \times m$ dimension is obtained for inputs. Furthermore, the model which contained in this step can be each of the DEA models. However, only input-oriented CCR model is presented in the current study.

Step 2. New *n*-dimensional DMUs are drawn with resampling by replacement from *n*-dimensional DMUs. *n*-dimensional bootstrap DMU sample was generated for each of the DMUs.

Step 3. Calculate the weights of inputs $\left(v_{km}^{*b}\right)$ and outputs $\left(u_{ks}^{*b}\right)$ and the efficiency scores $\left(\theta_k^{*b}\right)$ for each DMUs in bootstrap sample by solving the following linear programming model.

$$\begin{aligned}
&\boldsymbol{\theta_k^b = max \sum_{r=1}^{s} u_r^b y_{rk}}\\
&\boldsymbol{\sum_{i=1}^{m} v_i^b x_{ik} = 1}\\
&\boldsymbol{\sum_{r=1}^{s} u_r^b\, y_{rj} - \sum_{i=1}^{m} v_i^b x_{ij} \leq 0 \quad j \in \{Bootstrap\ Sample\}}\\
&\boldsymbol{u_r, v_i \geq 0 \quad (r = 1, \dots, s \;\; i = 1, \dots, m)}
\end{aligned} \tag{3}$$

where b, which is the upper index, indicates the repetition of the bootstrap. In addition, the output weights of *k*^th DMU are expressed as $u_{k1}^{*b},\ u_{k2}^{*b}, u_{k3}^{*b}, \dots, u_{ks}^{*b}$ and the input weights are expressed as $v_{k1}^{*b}, v_{k2}^{*b}, v_{k3}^{*b}, \dots, v_{km}^{*b}$ for *b*^th bootstrap repetition.

Step 4. Repeat steps 2–3 B times to obtain robust weights $\{\hat{u}_r\ \hat{v}_i; b = 1, \dots, B\}$. We repeated the procedure 2000 times to ensure appropriate accuracy. The equations (4) and (5) are used to obtain the robust weights of output and input

$$\boldsymbol{\hat{u}_r^* = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\sum_{b=1}^{B} u_{jr}^{*b}}{B}\right), \qquad r = 1, \dots, s} \tag{4}$$

$$\boldsymbol{\hat{v}_i^* = \frac{1}{n}\sum_{j=1}^{n}\left(\frac{\sum_{b=1}^{B} v_{ji}^{*b}}{B}\right), \qquad i = 1, \dots, m} \tag{5}$$

Step 5. Calculate efficiency scores for each DMU. The efficiency score for DMU_0 is calculated as follows:

$$\boldsymbol{\hat{E}_0 = \sum_{r=1}^{s} \hat{u}_r^* y_{r0} \Big/ \sum_{i=1}^{m} \hat{v}_i^* x_{i0}} \tag{6}$$

Illustrative Examples

To illustrate the important role of the CSWB method in determining a CSWs in the efficiency evaluation, we use two numerical examples in this section.

First Example: Forest districts of Taiwan

A numerical case adopted by Kao and Hung (2005) is explained in this part. In addition, the common weights for DEA model are determined. Kao and Hung (2005) demonstrated their solution method for the problem by displaying their example. Makui et al. (2008) studied the earlier example to find CSWs using their goal programming method. In this study, this example is investigated with the proposed approach and its outcomes are compared with earlier studies. Thus, the current research is concerned with the assessment of 17 forest districts of Taiwan. Each forest district has four inputs and three outputs. The detail of inputs and outputs are shown in Appendix 1-A.

Table 1. CCR scores and weights of inputs and outputs

DMUs	*CCR Score*	*Weights of Outputs*			*Weights of Inputs*			
		U1	**U2**	**U3**	**V1**	**V2**	**V3**	**V4**
1	1	0.0000	0.0000	0.0003	0.0000	0.0000	0.0203	0.0000
2	1	0.0000	0.0057	0.0000	0.0009	0.0075	0.0000	0.0000
3	1	0.0072	0.0000	0.0000	0.0027	0.0009	0.0028	0.0000
4	1	0.0000	0.0076	0.0000	0.0031	0.0085	0.0000	0.0000
5	1	0.0216	0.0000	0.0000	0.0133	0.0000	0.0090	0.0004
6	1	0.0000	0.0052	0.0000	0.0031	0.0046	0.0000	0.0008
7	1	0.0000	0.0083	0.0000	0.0000	0.0000	0.1052	0.0000
8	1	0.0005	0.0078	0.0000	0.0000	0.0121	0.0000	0.0000
9	1	0.0220	0.0000	0.0000	0.0000	0.0044	0.0171	0.0000
10	0.9403	0.0000	0.0071	0.0000	0.0000	0.0068	0.0003	0.0035
11	0.9346	0.0000	0.0048	0.0000	0.0000	0.0045	0.0002	0.0024
12	0.8290	0.0052	0.0055	0.0000	0.0000	0.0077	0.0042	0.0000
13	0.7997	0.0000	0.0043	0.0000	0.0000	0.0041	0.0002	0.0021
14	0.7733	0.0048	0.0076	0.0000	0.0000	0.0077	0.0027	0.0040
15	0.7627	0.0000	0.0056	0.0000	0.0000	0.0065	0.0032	0.0000
16	0.7435	0.0000	0.0067	0.0000	0.0010	0.0087	0.0000	0.0000
17	0.6873	0.0153	0.0000	0.0000	0.0058	0.0018	0.0061	0.0000

Note: U1: Main product, U2: Soil cons, U3:Recreation V1:Budget, V2:Initial stocking, V3:Labor Land

Note that in the CCR solution from Table 1, the weights of some DMUs are 0 or very close to 0. This implies that some inputs and outputs are not considered. This is one of the weaknesses of the DEA in performance evaluation. Important inputs and outputs are being ignored for performance evaluations. Decision makers have objections to this type of assessment.

The results obtained through the application of the five steps of the proposed CSWB algorithm are shown in Table 2. Table 2 and Figure 1.(a) show the weights of the inputs and outputs according to the B (B: number of repetitions) times (5, 25, 250, 500, 1000, 1500, 2000) mentioned in the fourth step of given algorithm. Note that except for U3 weight, all weights increase when the simulation repetitions increase. It is observed that all of the weights are zero or very close to zero, especially as shown in the first relatively few repetitions. In the last row, the estimated weights have a reasonable size.

Table 2. Common weights of CSWB algorithm for forest districts of Taiwan

Number of Repetition *(B)*	Weights of Outputs			Weights of Inputs			
	U1	**U2**	**U3**	**V1**	**V2**	**V3**	**V4**
5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
25	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001
100	0.0001	0.0001	0.0000	0.0000	0.0002	0.0003	0.0003
250	0.0004	0.0003	0.0000	0.0001	0.0005	0.0007	0.0007
500	0.0007	0.0006	0.0000	0.0002	0.0010	0.0013	0.0014
1000	0.0015	0.0011	0.0001	0.0004	0.0020	0.0026	0.0028
1500	0.0022	0.0017	0.0001	0.0006	0.0030	0.0040	0.0041
2000	0.0030	0.0023	0.0001	0.0008	0.0040	0.0053	0.0055

Table 3 shows the efficiency scores for all DMUs according to B repetitions. The discrimination power of DEA is increasing at this point and the important DMU becomes more important. It is expected to exceed the value 1 of the efficiency scores due to the increase in weights. If some values are greater than one in weights, all values can be standardized by dividing the maximum weight.

Table 3.Efficiency scores of DMUs based on CSWB algorithm for forest districts

DISTRICT (DMUS)	NUMBER OF REPETITION *(B)*						
	5	**25**	**250**	**500**	**1000**	**1500**	**2000**
1	0.6383	0.8112	0.8653	0.9533	1.0793	1.1170	1.1353
2	0.3238	0.3434	0.3570	0.3572	0.3589	0.3602	0.3611
3	0.3680	0.3762	0.3579	0.3577	0.3565	0.3557	0.3553
4	0.2442	0.2615	0.2756	0.2760	0.2781	0.2798	0.2810
5	0.3573	0.3798	0.3995	0.4006	0.4030	0.4050	0.4065
6	0.4809	0.5095	0.5525	0.5652	0.5812	0.5868	0.5907
7	0.1644	0.1771	0.1856	0.1856	0.1872	0.1885	0.1892
8	0.2851	0.3081	0.3152	0.3190	0.3256	0.3281	0.3293
9	0.3121	0.3587	0.3890	0.4074	0.4343	0.4437	0.4490
10	0.2907	0.3074	0.3191	0.3196	0.3213	0.3224	0.3232
11	0.2296	0.2380	0.2438	0.2430	0.2425	0.2423	0.2422
12	0.2348	0.2519	0.2607	0.2612	0.2629	0.2642	0.2651
13	0.2337	0.2443	0.2493	0.2490	0.2490	0.2492	0.2494
14	0.2575	0.2731	0.2827	0.2833	0.2848	0.2858	0.2865
15	0.1643	0.1740	0.1821	0.1819	0.1829	0.1837	0.1842
16	0.1223	0.1276	0.1281	0.1279	0.1282	0.1284	0.1285
17	0.1880	0.2015	0.2052	0.2056	0.2069	0.2079	0.2085

The correlation test of the scores of earlier, different methods was performed and the results are shown in Table 4. A Spearman rank correlation test (Daniel, 1990), with an r_s statistic equal to 0.4781, 0.4347, 0.4856, 0.6074, 0.4204, 0.8136 and 0.9007, shows that the ranks of some previously studied models in the literature are correlated, with $p<0.001$.

Table 4. Correlation results for forest districts

Model	**Correlation Coefficient**
CCR	47,81%
MAD	43,47%
MSE	48,56%
MAX	60,74%
Makui et al. (2008)	42,04%
Razavi et al. (2014)	81,36%
Alp (2016)	90,07%

Second example: manufacturing systems

In this example, a data set that has been studied in Shang and

Sueyoshi (1995), Li and Reeves (1999) and Lam and Bai (2011) is used. This data includes a set of twelve manufacturing systems with two inputs and four outputs. The detailed inputs and outputs are shown in Appendix 1-B.

Table 5. CCR Scores and weights for manufacturing systems

SYSTEM (DMUS)	CCR SCORE	Weights of Outputs				Weights of Inputs	
		U1	**U2**	**U3**	**U4**	**V1**	**V2**
1	1.0000	0.0238	0.0000	0.0000	0.0000	0.0588	0.0000
2	1.0000	0.0145	0.0000	0.0000	0.0146	0.0499	0.0397
3	0.9824	0.0283	0.0000	0.0162	0.0009	0.0850	0.0000
4	1.0000	0.0238	0.0000	0.0070	0.0159	0.0847	0.0273
5	1.0000	0.0000	0.0000	0.0833	0.0000	0.1053	0.0000
6	1.0000	0.0000	0.0000	0.0614	0.0420	0.2088	0.0000
7	1.0000	0.0000	0.0188	0.0312	0.0144	0.1610	0.0000
8	0.9614	0.0211	0.0040	0.0000	0.0118	0.0758	0.0262
9	1.0000	0.0000	0.0000	0.0000	0.0552	0.2725	0.0000
10	0.9535	0.0000	0.0278	0.0000	0.0000	0.0877	0.0610
11	0.9831	0.0206	0.0000	0.0000	0.0031	0.0564	0.0000
12	0.8012	0.0167	0.0000	0.0049	0.0111	0.0593	0.0191

Note: U1: Qualitative Benefits, U2: Work in Process, U3: Average number of Tardy Jobs, U4: Average Yield, V1: Capital and Operating Cost, V2: Floor Space Requirements

As in the previous example, these manufacturing systems also show that the input and output weights for some DMUs are zero in Table 5. This shows that these inputs and outputs are not considered in the computation of the DMU’s performance evaluations. As in the case of first example, the neglected inputs and outputs are no longer neglected by means of the proposed CSWB method. Table 6 and Figure 1.(b) show the weights determined by CSWB method for twelve DMUs. As the weights shown in Table 6 are around zero in the first iteration, as seen in the previous example, it appears that as the simulation repetitions increase, they become non-zero.

Table 6. Common weights based on CSWB for manufacturing systems

Number of Repetition (B)	Weights of Outputs				Weights of Inputs	
	U1	**U2**	**U3**	**U4**	**V1**	**V2**
5	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
25	0.0001	0.0000	0.0001	0.0001	0.0006	0.0004
100	0.0003	0.0001	0.0005	0.0003	0.0022	0.0018
250	0.0007	0.0003	0.0013	0.0009	0.0055	0.0044
500	0.0014	0.0007	0.0026	0.0017	0.0108	0.0088
1000	0.0027	0.0014	0.0051	0.0034	0.0212	0.0178
1500	0.0041	0.0020	0.0077	0.0051	0.0317	0.0267
2000	0.0055	0.0027	0.0102	0.0068	0.0421	0.0357

After 2000 repetitions, the efficiency scores obtained through the CSWB weights are shown in Table 7.

Table 7. Efficiency results for manufacturing systems

SYSTEM (DMUS)	NUMBER OF REPETITION *(B)*						
	5	**25**	**250**	**500**	**1000**	**1500**	**2000**
1	0.7507	0.7767	0.7748	0.7772	0.7807	0.7817	0.7822
2	0.7363	0.7613	0.7600	0.7625	0.7661	0.7673	0.7678
3	0.7614	0.7780	0.7819	0.7829	0.7841	0.7842	0.7841
4	0.8274	0.8480	0.8519	0.8539	0.8565	0.8572	0.8573
5	0.8395	0.8625	0.8687	0.8704	0.8728	0.8733	0.8733
6	0.7035	0.6976	0.6960	0.6952	0.6930	0.6921	0.6913
7	0.7397	0.7382	0.7371	0.7365	0.7347	0.7339	0.7332
8	0.7106	0.7237	0.7204	0.7214	0.7225	0.7227	0.7227
9	0.4763	0.4563	0.4462	0.4449	0.4420	0.4409	0.4400
10	0.6183	0.6235	0.6189	0.6190	0.6186	0.6182	0.6177
11	0.6900	0.7095	0.7067	0.7082	0.7105	0.7111	0.7114
12	0.6440	0.6604	0.6640	0.6653	0.6670	0.6674	0.6675

A Spearman correlation test is applied for detailed investigation of the relation among the efficiency scores of the previous three methods in the literature. According to the test results given in Table 8, the correlation between the efficiency scores of other methods is significant at the 0.01 level. Because these methods have given in similar results, our proposed approach is beneficial and will give consistent assessment results.

Table 8. Correlation results for manufacturing system

MODEL	CORRELATION COEFFICIENT
PDM Model 1 Lin et al. (2016)	98,54%
PDM Model 2 Lin et al. (2016)	98,43%
Lin et al. (2016)	98,31%

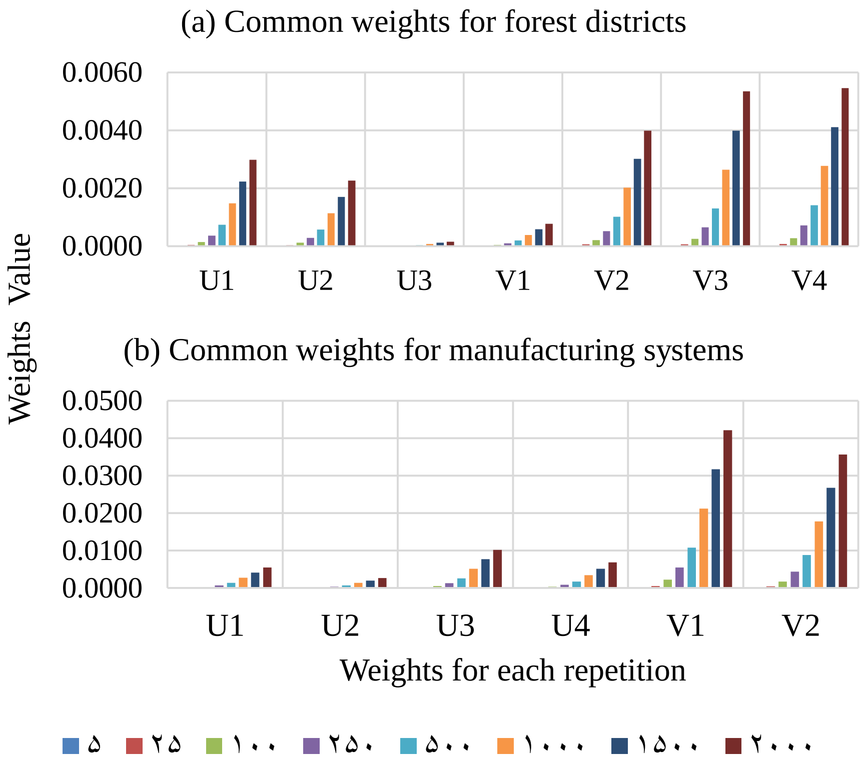

Figure 1.Common weights for forest districts and manufacturing systems

Conclusion and Discussion

The weights may change in order to maximize the relative efficiency scores in DEA. Some of the weights of their inputs and outputs could be 0 or very close to 0. Thus, they cannot be taken into account in the assessment of the performance of some DMUs. Inputs and outputs are assigned the common weights for all DMUs via the proposed robust bootstrap algorithm. In addition, all inputs and outputs will be included in the performance evaluation due to the weights greater than zero, and all of them are the same for all DMUs. The Bootstrap method is based on the law of large numbers of statistics and the central limit theory. Problematic data was imagined as a population and generated finite number of samples with replacement by Bootstrap method. All of them are possible samples. Thus, weights of inputs and outputs get more reliable and realistic. In this study, the number of resampling was set at 2000 iterations. Optimum resampling number may be studied by future studies. However, it can be said that as the number of resampling increases, weights converge to more stable values due to the law of large numbers. This is seen experimentally in Tables 2 and 5 and in Figures 1. Therefore, decision-makers were prevented from objecting to different weights of DEA models, and more balanced weights were obtained. All inputs and outputs in the performance calculations were also taken into account, too.

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APPENDIX

Table 1-A. The forest districts dataset (Kao and Hung, 2005*)*

DISTRICT	BUDGET (DOLLARS) (I1)	INITIAL STOCKING (M^3) (I2)	LABOR (PERSONS) (I3)	LAND (HA) (I4)	MAIN PRODUCT (M^3) (O1)	SOIL CONSERVATİON (M^3) (O2)	REC EATION (VISITS) (O3)
1	51.62	11.23	49.22	33.52	40.49	14.89	3155.71
2	85.78	123.98	55.13	108.46	43.51	173.93	6.45
3	66.65	104.18	257.09	13.65	139.74	115.96	0
4	27.87	107.6	14	146.43	25.47	131.79	0
5	51.28	117.51	32.07	84.5	46.2	144.99	0
6	36.05	193.32	59.52	8.23	46.88	190.77	822.92
7	25.83	105.8	9.51	227.2	19.4	120.09	0
8	123.02	82.44	87.35	98.8	43.33	125.84	404.69
9	61.95	99.77	33	86.37	45.43	79.6	1252.6
10	80.33	104.65	53.3	79.06	27.28	132.49	42.67
11	250.62	183.49	144.1	59.66	14.09	196.29	16.15
12	82.09	104.94	46.51	127.28	44.87	108.53	0
13	202.21	187.74	149.39	93.65	44.97	184.77	0
14	67.55	82.83	44.37	60.85	26.04	85	23.95
15	72.6	132.73	44.67	173.48	5.55	135.65	24.13
16	84.83	104.28	159.12	171.11	11.53	110.22	49.09
17	71.77	88.16	69.19	123.14	44.83	74.54	6.14

I: Input O: Output

Table 1-B. The manufacturing systems dataset (Shang and Sueyoshi, 1995*)*

SYSTEM	CAPITAL AND OPERATING COST (I1)	FLOOR SPACE REQUIREMENTS (I2)	QUALITATIVE BENEFITS (O1)	WORK IN PROCESS (O2)	AVERAGE NUMBER OF TARDY JOBS (O3)	AVERAGE YIELD (O4)
1	17.02	5	42	45.3	14.2	30.1
2	16.46	4.5	39	40.1	13	29.8
3	11.76	6	26	39.6	13.8	24.5
4	10.52	4	22	36	11.3	25
5	9.5	3.8	21	34.2	12	20.4
6	4.79	5.4	10	20.1	5	16.5
7	6.21	6.2	14	26.5	7	19.7
8	11.12	6	25	35.9	9	24.7
9	3.67	8	4	17.4	0.1	18.1
10	8.93	7	16	34.3	6.5	20.6
11	17.74	7.1	43	45.6	14	31.1
12	14.85	6.2	27	38.7	13.8	25.4

I: Input O: Output