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Genetic-Fuzzy Data Envelopment Analysis Model for Evaluating Financial Institutions Relative Productivity in a Fluctuating Economic Market

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Abstract

This paper presents a Genetic Algorithm Fuzzy Data Envelopment Analysis (GA-FDEA) model that caters for optimal selecting of economic indicators for the measurement of relative productivity and performance of financial institutions. Imprecise or uncertain data of financial institutions due to varying monetary policies and market risk were retrieved from Nigeria Stock Exchange Commission and evaluated. It was observed that GA-FDEA provides better results than the conventional DEA. The findings provide economic barometers for ascertaining the viability of these institutions toward bringing the expected growth of these institutions and the nation at large.

Keywords: Fuzzy; Data Envelopment Analysis; Finance; Genetic Algorithm.

Introduction

Globally, competitions among indigenous firms and financial industries are not only with internal organizations but also with external firms for optimum service qualities and performances. Assessing the significance and influence of service qualities of financial institutions are key indicators for describing economic growth and stability of any country. The government and private sector rely on them for accessing credit facilities, understanding monetary policies channels and safe deposit house for customer's valuables.

Several researchers have employed various econometric theories, models and performance indicators to measure performance of financial sectors. In most cases, financial institutions such as banks have adopted different methodologies and personalized measurement ratios to measure performance and efficiency. Using ratios and other traditional approaches are very common with each having their shortfalls and cost. Also comparing results are extremely difficult with similar/dissimilar companies operating in the same sector [1]. In Nigeria, despite the numerous economic reforms in the financial sector, most financial institutions are faced with operational challenges while some (especially banks) unable to recover from the global collapse of the financial market share. The introduction of Treasury Single Account (TSA) has significantly affected most bank capital base. Recent economic recession also affected banks optimum business performance and productivity. To address this salient economic problem, government, private sector and academic professionals are interested in identifying and employing processes and measures that will influence optimal productivity or efficiency of financial

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industries. Two handy tools employed or used are Regression Analysis (RA) and Data Envelopment Analysis (DEA).

Efficiency measurement and analysis involves interactions of several input and output parameters or quantities during production processes. Using RA to measure efficiency is limited to several input and single output parameters. For multiple inputs and outputs interaction, several frameworks and methodologies have been deployed. Emrouznejad and De Witte [2] proposed COOPER framework for business efficiency and performance measurement, consumer confidence, and analysis of large statistical databases with much emphases on non-parametric methodologies.

Efficiency evaluation of financial institutions using non-parametric methods such as Data Envelopment Analysis (DEA) provides a fair measurement considering effective utilization of multiple economic indicators (inputs and outputs) simultaneously when compared to Stochastic Frontier Analysis (SFA)[3,4]. Selecting appropriate indicators for DEA evaluation was demonstrated by Madhanagopal and Chandrasekaran [5]. Previous DEA studies assume uniform reduction and increase in input and output parameters respectively to attain higher efficiency but failed to consider the risk factors or uncertainties in the money market which significantly affect assets investment and business policy formulations [6]. Therefore, selection of key indicators and normalization of input and output parameters to reduce noise and produce better prediction is very crucial [7].

Fuzzy Data Envelopment Analysis (FDEA)

Data Envelopment Analysis (DEA) is a non-parametric alternative to traditional growth accounting and it is suitable for analyzing productivity convergence based on frontier production functions [8]. The use of this approach requires the assumption about Returns To Scale (RTS) and changes in Total Factor Productivity (TFP) and they are decomposed into changes in the efficiency of production and technological changes. Also, DEA methodology is used for evaluating the relative efficiency of operating entities of similar nature, i.e. of decision-making units (DMUs) that use the same multiple inputs and produce the same multiple outputs [8]. The original question in DEA-literature focus on measuring each unit's efficiency in production compared to sample peers [9]. DEA was proven to be an effective tool for performance evaluation and benchmarking since it was first introduced [9,10]. Two approaches that have been emphasized in basic DEA models are *input-orientated* (i.e. maximizes proportional inputs reduction while holding outputs constant) and *output-orientated (i.e* maximizes the proportional outputs increase while keeping inputs constant) [11,12]. Emrouznejad and De Witte [24] added that DEA is a complex process requiring various tools to identify the appropriate set of inputs/outputs and select a suitable model.

Fuzzy theory have been applied to cater for uncertainty and imprecision in observations/experiment. It influenced structuring complex information into meaningful and semantically sound entities which are central to all and supporting decision-making activities [13,14]. Finding appropriate membership functions of fuzzy efficiency scores when some observations are fuzzy numbers and ranking of DMUs were presented in [15,16]. Fuzzy set theory allows linguistic data to be used directly within the DEA models.

In the work of Lertworasirikul and colleagues [17], FDEA was described as a tool to compare the performance of a set of activities or organizations under uncertain environment. The FDEA realistically represent real-world problems than the conventional DEA models.

FDEA models have gained wider applications in industries and researchers have adopted four major approaches (tolerance approach; the α-level based approach; the Fuzzy ranking approach; and the Possibility approach) for dealing with FDEA model [18]. The α-level based approach is the most commonly used to treat imprecision in FDEA model [15-20]. The main idea consists to convert the FDEA model into a pair of parametric programs in order to find the lower and upper bounds at an α-level of the membership functions of the efficiency scores.

Wanke and colleagues proposed FDEA *α*-level models with bootstrap truncated regression to measure the impact of each model on the efficiency scores and to identify the most relevant contextual variables on efficiency [21]. Also, Wanke et al. [22] adopted fuzzy TOPSIS and regression approach to capture vagueness in the relative efficiency over time and identify key indicators that predict efficiency of banks in Brazil, Russia, India, China, and South Africa. Adopting Kazemi and Alimi model [23], all inputs, outputs and decision variables were defined as fuzzy numbers. The technique proposed in this work evaluates the performance of Decision Making Units (DMUs) by using fuzzy theory to translate input-output parameters and generalizes the efficiency of DMU (input vs output ratio).

Methodological Background

In a DEA model, n indicates the number of DMUs to be evaluated; $j=1,\ldots,n$; m and s are the inputs and outputs respectively for each DMUs; i=1,..m and r=1,..,s. A DMU_j utilizes X_{ij} of input i and produces y_{rj} of output r.λ_j is the weight assigned by the linear program, θ is the efficiency calculated, s_i and s_r are the input and output slacks; ε is a non-Archimedian element defined which is smaller than any positive real number [24,25]. Applying fuzzy theory, DMU_j produces a fuzzy nonzero output vector $\tilde{y}_j = (\tilde{y}_{1j}, \tilde{y}_{2j}, \dots, \tilde{y}_{sj})^t \geq 0$ using a fuzzy nonzero input vector $\tilde{x}_j = (\tilde{x}_{1j}, \tilde{x}_{2j}, \dots, \tilde{x}_{rj})^t \geq 0$where the superscript "t" indicates the transpose of a vector. Then, the fuzzified Constant Return of Scale (CRS) model with fuzzy coefficients for assessing *DMU_j* Technical efficiency (TE) was formulated as shown in equation 1 [23,26,27].

$$\text{Min } \theta + \varepsilon \left[\sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+ \right]$$

Subject to

$$\sum_{j=1}^{n} \tilde{x}_{ij}\lambda_j + S_i^- = \theta \tilde{x}_{i0}, \qquad i=1,2,\dots m$$

$$\sum_{j=1}^{n} \tilde{y}_{rj}\lambda_j - S_r^+ = \tilde{y}_{i0}, \qquad r=1,2,\dots s$$

$$S_i^-, S_r^+, \lambda_j \geq 0, \qquad j=1,2,\dots,n$$

Equation 1

Source: Zerafat et al., [27]

This research presents a novel FDEA approach that anchors on using optimization search algorithm-Genetic algorithm to select best variables. The uncertain variables are normalized before transforming into fuzzy number using Triangular Fuzzy membership function. The transformed data (inputs and outputs) are evaluated to determine the efficiency scores of various financial institutions.

Procedures/Modules of proposed method

This research methodology consists of five steps with each step addressing specific features as explained below.

Step 1: **Capturing of Economic Variables**- The economic variables (inputs and outputs) was captured from financial reports of various financial institutions as published annually in stock market. The financial institutions used in this study comprise of commercial banks, mortgage banks, microfinance banks and major insurances companies. The choice of these institutions was based on the ground of having a complete appraisal of different types of financial institutions within a single context as they in one way or the other affect the economic growth of a nation. The rule of thumb proposed by Cooper, et al. [10] was used to determine the sample size.

Step 2: **Selection of Best subset (parameters)**- Genetic Algorithm as one of the most efficient optimization search algorithm was used to search through the data set and provide the best subset or variables among the input and outputs that will contribute the most in the efficiency measure. GA consists of three components (i) Chromosomes (ii) fitness function and (iii) genetic operators i.e. mutation and crossover [28]. Searching appropriate subset from the large dataset was described in Cadima, et al.,[29] and implemented by Madhanagopal and Chandrasekaran [5] using RM coefficient. RM is a concept that indicates the weighted average of the multiple correlations between each principal component of the full data and k subset variable [5]. It highlights the criterion's nature as a weighted average of the squares of the multiple correlations (r_m) [29]. The RM coefficient lies between 0 and 1 and the more it approaches 1, the higher the proportion of explained variance and vice visa.

The RM coefficient is given in equation 2.

$$RM = corr(X, P_k X) = \sqrt{\frac{tr(X^i P_k X)}{tr(X^i X)}} = \sqrt{\frac{\sum_{i=1}^{p} \lambda_i (r_m)_i^2}{\sum_{j=1}^{p} \lambda_j}} = \sqrt{\frac{tr([S^2]_{(k)} S_k^{-1})}{tr(S)}} \qquad \text{Equation 2}$$

Where:

Corr=correlation matrix; tr=trace of matrix; X is the full data matrix; $S = \frac{1}{n} X^i X$ is p x

p covariance or correlation matrix of the full data set; K denotes the index set of k variable in the variable subset; P_k is the orthogonal projection matrix of the subspace spanned by a given k-variable subset; S_k is the k x k principle submatrix of matrix S which results from retaining the rows/columns whose indices belong to k; $[S^2]_{(k)}$ is the k x k principle submatrix of S^2 obtained by retaining the rows/columns associated with set k; λ_i stands for the i-th largest eigenvalue of the covariance or correlation matrix defined by X; r_m stands for the multiple correlation between i-th principle component of the full data set and the k variable subset.

Step 3: **Fuzzification of data**- Extracted subsets from step 2 are transformed into fuzzy number using Triangular Membership Function (TMF). The data collected therefore need to be normalizing to reduce decision errors by computing z score for each variable in the data set [7,30]. TMF consist of triple values (π^l, π^m, π^u) where π^l and π^u are the left and right sides (at

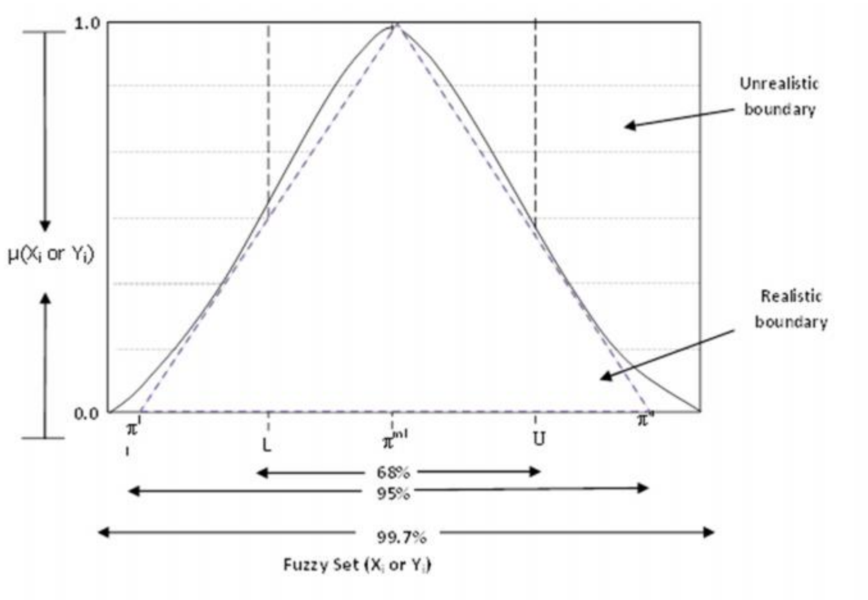

Figure 1. Triangular Membership function and Normal Distribution

99% confidence interval) respectively of a particular selected variable and π^m is the centroid or mean value of the variable. The three and the alpha cut (**α**-cut) value control the upper bound (U) and lower bound (L) of the TMF of each selected variable from step 2. The alpha cut introduces various level of risk and this will allow the model to accommodate uncertain or dynamic economic influence set by risk factor and varying money policies. See Equation 3and 4 for upper and lower bound computation.

$L = \pi^l + \alpha(\pi^m - \pi^l)$ Equation 3

$U = \pi^u + \alpha(\pi^u - \pi^m)$ Equation 4

Inputs (Xs) and outputs (Ys) are estimated using the TMF. Figure 1 exhibits both symmetric triangular and normal distribution. The larger the spread (standard deviation) the more it tend to bell or normal distribution. The smaller the spread, the more it tends towards a symmetric triangular distribution which is very usefully in risk management and changing market prices evaluation. The spread can also be control by the money policy put in place by government. As you can see, a variable can assume any position and will affect the performance of the institution. This step was implemented using FuzzyNumbers r package.

$$\mu(\text{x or y}) = \begin{cases} 0 & \text{if} \quad x < \pi^1 \\ \left(\frac{x-\pi^1}{\pi^2-\pi^1}\right) & \text{if} \quad \pi^1 \le x \le \pi^2 \\ 1 & \text{if} \quad \pi^2 \le x \le \pi^3 \\ \left(\frac{x-\pi^3}{\pi^4-\pi^3}\right) & \text{if} \quad \pi^3 \le x < \pi^4 \\ 0 & \text{if} \quad \pi^4 < x \end{cases} \qquad \text{Equation 5}$$

Step 4: DEA computation- The efficiency scores comprising technical, pure technical and scale efficiency are computed using various alpha-cut values (**α**-cut). The alpha cut approach was adopted due to it intuitive appeal to estimate DMU efficiency values. **α**-cut is assumed to have value between 0 and 1. If **α**-cut=0, it means there is a high risk of volatility of the crisp data and it lies in 99% confidence interval. When **α**-cut=1, this is zero risk volatility, therefore the crisp data is stable and lies within the 68% confidence interval. This step was implemented using the Benchmarking r package [4].

Step 5: Ranking of DMUs. In this step, the efficiency scores of each DMU are ranked using Fuzzy Ranking Index (FRI) [31]. The bigger the FRI, the better the rank for the DMUs. In evaluating performance, a DMU is fully (100%) efficient if and only if both the efficiency score equals 1 and all slacks is equal to zero. While a DMU whose efficiency score equals 1 but not all it slacks equals zero is weakly efficient [32].

An application in Nigeria Stock Exchange Commission (SEC)

The source of data used in this research was financial reports of 23 financial institutions (Table 1) obtained from Nigeria Stock Exchange Commission (SEC) that have actively published their financial reports since last four years (2013-2016). Other criteria for selecting these institutions include huge investment in ICT driven banking operations and having over 20 branches nationwide.

Table 1. List of Decision Making Units (DMU)

DMU	Institution Name	Type	2013	2014	2015	2016
1	Sterling Bank Plc	Commercial Bank	1	1	1	1
2	Access Bank	Commercial Bank	1	1	1	1
3	Ecobank Nigeria	Commercial Bank	1	1	1	1
4	First City Monument Bank	Commercial Bank	1	1	1	1
5	Skye Bank	Commercial Bank	1	1	1	0
6	Union Bank of Nigeria	Commercial Bank	1	1	1	1
7	United Bank for Africa	Commercial Bank	1	1	1	1
8	Unity Bank Plc.	Commercial Bank	1	1	1	1
9	Wema Bank	Commercial Bank	1	1	1	1
10	Zenith Bank	Commercial Bank	1	1	1	1
11	Guaranty Trust Bank	Commercial Bank	1	1	1	1
12	Fidelity Bank Nigeria	Commercial Bank	1	1	1	1
13	First Bank of Nigeria	Commercial Bank	1	1	1	1
14	Diamond Bank	Commercial Bank	1	1	1	1
15	Stanbic IBTC Bank Nigeria Limited	Commercial Bank	1	1	1	1
16	NPF Microfinance Bank Plc	Microfinance Bank	1	1	1	0
17	FORTIS Microfinance Bank Plc	Microfinance Bank	1	1	1	0
18	Abbey Mortgage Bank Plc.	Mortgage Bank	1	1	1	1
19	Infinity Trust Mortgage Bank Plc	Mortgage Bank	1	1	1	1
20	Aiico Insurance Plc	Insurance Company	1	1	1	1
21	African Alliances Plc	Insurance Company	1	1	1	0
22	Africa Prudential Registrars Plc	Insurance Company	1	1	1	0
23	Prestige Assurance Ltd	Insurance Company	1	1	1	1

In most efficiency studies of financial institutions, the two main approaches for selecting inputs and outputs parameter are production and intermediation approach. Production approach defines bank activities as production of services and views banks as using physical inputs(labour and capital) to provide deposits and loans. It is more suitable for the analysis of bank branch efficiency. Intermediation approach views banks as the intermediating funds between savers and investors. This approach is more suitable for evaluating various and different institutions within the same domain [5,30] and it will be adopted in this research.

Micheal [3] expressed that inputs and outputs selection is a major challenge, economic analysts encounter in modelling bank or financial institution's efficiency. Since DEA results are influenced by number of observations, this research observed the convention suggested by Cooper et al. [10]. In fact there are 92 observations (23 DMUs * 4year) which satisfy a minimum number greater than 3*(inputs + outputs).

Results and Discussion

Model formulation and Selection of the best Inputs and Outputs parameter.

Selecting appropriate parameters (inputs and outputs)

Table 2. Input and output parameters

	Input Variable Name	Code
1	Capital (cash and cash equivalent)	CAP
2	Fixed asset (properties and equipments)	FIA
3	Total Expenses (Interest expenses +other expenses e.g. Personnel wages/salaries)	TE
	Output Variable Name	**Code**
4	Gross Loan and advances(loans granted to customers)	GLA
5	Total Income(Operating Income + other income + Net Interest Income)	TI
6	Investment(Financial investment in subsidiaries)	INV
7	Net profit (Profit of the year after tax deductions)	NEP

Table 3. Model results of RM best subsets and selections of best inputs and outputs

MODELS	Var.1	Var.2	Var.3	Var.4	Var.5	Var.6	Var.7	RM best values
I	CAP	FIA			TI			0.878854
II	CAP	FIA			TI		NEP	0.95644
III		FIA	TE		TI	INV	NEP	0.976757
IV		FIA	TE	GLA	TI	INV	NEP	0.994354

for any performance evaluation model is very crucial because it influences the decision making process (Table 2). The best RM values of all parameters were computed using Genetic Algorithm with RM coefficient (see equation 2) using Subselect R package. All model RM values approach 1 showing higher variance among the parameters. The GA-FDEA model will be evaluated based on seven key indicators that form four models (Table 3).

Empirical Results

To determine the efficiency score, the FDEA was implemented using R statistical package. Tables 4 to 7 show the various efficiency scores of both CRS-DEA and GA-FDEA from 2013 to 2016.

In Table 4, Model 1; 2, 1 and 20 DMUs were perfectly efficient (PE), weakly or partially efficient (WE) and not efficient (NtE) respectively using the Constant Return on Scale DEA (CRS-DEA). Using Genetic Algorithm-Fuzzy DEA (GA-FDEA), 6 and 17 DMUs has PE and NtE respectively. For Model 2; 3 and 20 DMUs were PE and NtE respectively for CRS-DEA. In GA-FDEA 4 and 19 DMUs were PE and NtE respectively. For Model 3; 3 and 20 DMUs were PE and NtE respectively for CRS-DEA while 6 and 17 DMUs were PE and NtE respectively for GA-FDEA. For Model 4; 12 and 11 DMUs were PE and NtE respectively for CRS-DEA while 8 and 15 DMUs were PE and NtE respective for GA-FDEA.

In Table 5, Model 1; 3 and 20 DMUs were PE and NtE respectively using the CRS-DEA while Using GA-FDEA, 6 and 17 DMUs were PE and NtE respectively. Model 2 and 3 were the same with the previous year. Model 4; 9 and 14 DMUs were PE and NtE respectively for CRS-DEA while 6 and 17 DMUs were PE and NtE respective for GA-FDEA.

In Table 6, Model 1; 4 and 19 DMUs were PE and NtE respectively using the CRS-DEA while Using GA-FDEA, 6 and 17 DMUs were PE and NtE respectively. Model 2; 3 and 20 DMUs were PE and NtE respectively for both CRS-DEA and GA-FDEA. Model 3; 4 and 19 DMUs were PE and NtE respectively for CRS-DEA while 5 and 18 DMUs were PE and NtE respectively for GA-FDEA. Model 4: 8 and 15 DMUs were PE and NtE respectively for CRS-DEA while 7 and 16 DMUs were PE and NtE respective for GA-FDEA.

In Table 7, Model 1; 3 and 15 DMUs were PE and NtE respectively using the CRS-DEA while Using GA-

Table 4. DMU efficiency scores of CRS DEA and GA-FDEA for 2013

				Model1			Model2			Model 3			Model 4		
	CRS DEA			GA-FDEA			GA-FDEA			GA-FDEA			GA-FDEA		
	eff	S	Rk	eff	s	Rk	Eff	s	Rk	eff	s	Rk	eff	s	Rk
DMU1	1.00	0	22	1.00	0	20.5	0.51	1	11	1.00	0	20.5	1.00	0	19.5
DMU2	0.12	0	4	0.72	0	9	0.38	0	7	0.72	0	9	0.62	1	9
DMU3	0.18	0	8	0.59	1	5	0.25	0	2	0.59	1	5	0.61	1	8
DMU4	0.43	1	18	0.69	0	8	0.34	0	5	0.69	0	8	0.63	1	10
DMU5	0.49	1	19	0.68	0	7	0.35	0	6	0.68	0	7	0.61	1	7
DMU6	0.33	0	15	0.55	0	3	0.49	0	10	0.55	0	3	0.52	1	2
DMU7	0.14	0	6	0.78	0	10	0.42	1	9	0.78	0	10	0.65	1	12
DMU8	0.20	0	9	1.00	0	20.5	0.64	1	13	1.00	0	20.5	1.00	0	19.5
DMU9	0.29	0	12	0.79	1	11	0.79	0	14	0.79	1	11	0.87	1	15
DMU10	0.29	1	13	0.57	1	4	0.14	1	1	0.57	1	4	0.41	1	1
DMU11	0.26	0	11	0.29	1	2	0.29	1	4	0.29	1	2	0.58	1	3
DMU12	0.13	0	5	0.67	0	6	0.40	0	8	0.67	0	6	0.60	1	6
DMU13	0.17	0	7	0.26	1	1	0.26	1	3	0.26	1	1	0.59	1	5
DMU14	0.22	0	10	0.99	1	16	0.99	0	18	0.99	1	16	0.58	1	4
DMU15	0.35	0	16	0.93	0	14	0.55	0	12	0.93	0	14	0.66	1	13
DMU16	0.53	0	20	0.98	1	15	0.98	1	17	0.98	1	15	1.00	0	19.5
DMU17	1.00	1	22	1.00	0	20.5	1.00	0	21.5	1.00	0	20.5	0.74	1	14
DMU18	0.03	0	1	0.89	1	13	0.89	1	16	0.89	1	13	1.00	0	19.5
DMU19	0.05	0	2	1.00	0	20.5	1.00	0	21.5	1.00	0	20.5	1.00	0	19.5
DMU20	0.41	0	17	1.00	0	20.5	1.00	0	21.5	1.00	0	20.5	1.00	0	19.5
DMU21	0.31	0	14	0.99	1	17	0.99	1	19	0.99	1	17	1.00	0	19.5
DMU22	1.00	0	22	1.00	0	20.5	1.00	0	21.5	1.00	0	20.5	1.00	0	19.5
DMU23	0.07	0	3	0.81	1	12	0.81	1	15	0.81	1	12	0.64	1	11
Perfect Efficiency	2			6			4			6			8		
Weak Efficiency	1														
Not Efficient	21			17			19			17			15		

eff=efficiency scores, s=slack(0-no slack, 1- slack is present),Rk=ranking

Table 5. DMU efficiency scores of CRS DEA and GA-FDEA for 2014

	CRS DEA			**Model1**			**Model2**			**Model 3**			**Model 4**		
				GA-FDEA			GA-FDEA			GA-FDEA			GA-FDEA		
	eff	S	Rk	eff	s	Rk	Eff	s	Rk	eff	s	Rk	eff	s	Rk
DMU1	1.00	0	22	1.00	0	20.5	0.91	0	18	1.00	0	20.5	0.87	1	17
DMU2	0.10	0	3	0.57	1	6	0.57	0	7	0.57	1	6	0.45	1	3
DMU3	0.22	1	9	0.49	1	5	0.46	0	4	0.49	1	5	0.45	1	4
DMU4	0.50	0	18	0.62	0	8	0.61	0	9	0.62	0	8	0.53	1	12
DMU5	0.48	0	16	0.87	1	15	0.87	0	16	0.87	1	15	0.51	1	11
DMU6	0.45	0	15	0.81	1	14	0.81	0	14	0.81	1	14	0.46	1	5
DMU7	0.16	0	7	0.63	1	9	0.63	0	10	0.63	1	9	0.49	1	10
DMU8	0.32	0	12	1.00	0	20.5	1.00	0	21.5	1.00	0	20.5	1.00	0	20.5
DMU9	0.34	1	14	0.96	1	17	0.96	0	19	0.96	1	17	0.71	1	15
DMU10	0.26	1	11	0.49	1	4	0.41	1	2	0.49	1	4	0.47	1	8
DMU11	0.19	1	8	0.48	1	3	0.48	0	6	0.48	1	3	0.45	1	2
DMU12	0.23	0	10	0.64	1	10	0.64	0	11	0.64	1	10	0.47	1	9
DMU13	0.14	0	6	0.58	1	7	0.58	0	8	0.58	1	7	0.46	1	6
DMU14	0.50	0	17	0.64	1	11	0.64	0	12	0.64	1	11	0.47	1	7
DMU15	0.83	0	19	0.88	1	16	0.88	0	17	0.88	1	16	0.59	1	13
DMU16	0.34	1	13	0.72	1	13	0.72	1	13	0.72	1	13	1.00	0	20.5
DMU17	1.00	0	22	1.00	0	20.5	1.00	0	21.5	1.00	0	20.5	0.38	1	1
DMU18	0.05	0	2	0.42	1	2	0.42	1	3	0.42	1	2	1.00	0	20.5
DMU19	0.04	0	1	1.00	0	20.5	1.00	0	21.5	1.00	0	20.5	1.00	0	20.5
DMU20	0.14	0	5	0.35	1	1	0.35	1	1	0.35	1	1	0.73	1	16
DMU21	0.96	0	20	1.00	0	20.5	0.81	1	15	1.00	0	20.5	1.00	0	20.5
DMU22	1.00	0	22	1.00	0	20.5	1.00	0	21.5	1.00	0	20.5	1.00	0	20.5
DMU23	0.13	0	4	0.67	1	12	0.46	1	5	0.67	1	12	0.65	1	14
Perfect Efficiency	3			6			4			6			6		
Weak Efficiency															
Not Efficient	20			17			19			17			17		

eff=efficiency scores, s=slack(0-no slack, 1- slack is present),Rk=ranking

Table 6. DMU efficiency scores of CRS DEA and GA-FDEA for 2015

	CRS DEA			**Model1**			**Model2**			**Model 3**			**Model 4**		
				GA-FDEA			GA-FDEA			GA-FDEA			GA-FDEA		
	Eff	s	Rk	eff	s	Rk	Eff	s	Rk	eff	s	Rk	eff	s	Rk
DMU1	0.18	1	16	0.95	1	15	0.62	0	13	0.95	1	15	0.66	1	13
DMU2	0.09	1	10	0.45	1	4	0.20	1	5	0.45	1	4	0.47	1	6
DMU3	0.08	1	8	0.72	1	11	0.67	0	14	0.72	1	11	0.71	1	14
DMU4	1.00	0	21.5	1.00	0	21	1.00	0	22	1.00	0	21	1.00	0	20
DMU5	0.05	1	4	0.47	1	5	0.05	1	1	0.47	1	5	0.47	1	3.5
DMU6	0.07	0	5	0.70	1	9	0.33	1	9	0.70	1	9	0.59	1	12
DMU7	0.09	1	9	0.42	1	3	0.20	1	4	0.42	1	3	0.48	1	9
DMU8	0.15	1	15	1.00	0	21	0.50	1	11	1.00	0	21	1.00	0	20
DMU9	0.07	1	6	0.78	1	12	0.52	1	12	0.78	1	12	0.56	1	11
DMU10	0.09	1	11	0.37	1	2	0.15	1	2	0.37	1	2	0.48	1	7
DMU11	0.12	0	14	0.61	1	7	0.29	1	7	0.61	1	7	0.47	1	2
DMU12	0.10	1	12	0.56	1	6	0.27	1	6	0.56	1	6	0.47	1	3.5
DMU13	0.20	1	17	0.34	1	1	0.18	0	3	0.34	1	1	0.48	1	8
DMU14	1.00	0	21.5	0.64	1	8	0.32	1	8	0.64	1	8	0.47	1	5
DMU15	0.12	1	13	0.70	1	10	0.34	1	10	0.70	1	10	0.54	1	10
DMU16	0.27	1	18	0.97	1	16	0.97	1	17	0.97	1	16	1.00	0	20
DMU17	1.00	1	21.5	1.00	0	21	1.00	0	22	1.00	0	21	1.00	0	20
DMU18	0.04	0	2	0.98	1	17	0.98	1	18	0.98	1	17	1.00	0	20
DMU19	0.01	0	1	1.00	0	21	1.00	0	22	1.00	0	21	1.00	0	20
DMU20	0.08	0	7	0.85	1	13	0.84	1	15	0.85	1	13	0.34	1	1
DMU21	1.00	0	21.5	1.00	0	21	0.98	1	19	1.00	0	21	1.00	0	20
DMU22	0.72	1	19	1.00	1	18	1.00	1	20	1.00	1	18	0.81	1	16
DMU23	0.05	1	3	0.95	1	14	0.95	1	16	0.95	1	14	0.76	1	15
Perfect Efficiency	4			6			3			5			7		
Weak Efficiency															
Not Efficient	19			17			20			18			16		

eff=efficiency scores, s=slack(0-no slack, 1- slack is present),Rk=ranking

FDEA, 4 and 14 DMUs were PE and NtE respectively. Model 2; 3 and15 DMUs were PE and NtE respectively for both CRS-DEA and GA-FDEA. Model 3; 3 and15 DMUs were PE and NtE respectively for CRS-DEA while 4 and 14 DMUs were PE and NtE respectively for GA-FDEA. Model 4; 7 and 11 DMUs were PE and NtE

Table 7. DMU efficiency scores of CRS DEA and GA-FDEA for 2016

				Model1			**Model2**			**Model 3**			**Model 4**		
	CRS DEA			GA-FDEA			GA-FDEA			GA-FDEA			GA-FDEA		
	Eff	s	Rk	eff	s	Rk	eff	s	Rk	eff	s	Rk	eff	s	Rk
DMU1	1.00	1	17	0.78	1	12	0.65	0	12	0.78	1	12	0.92	1	12
DMU2	0.20	0	10	0.31	0	8	0.25	0	8	0.31	0	8	0.50	1	3
DMU3	0.14	0	5	1.00	0	16.5	1.00	0	17	1.00	0	16.5	1.00	0	15.5
DMU4	1.00	0	17	1.00	0	16.5	1.00	0	17	1.00	0	16.5	1.00	0	15.5
DMU6	0.15	0	7	0.38	0	10	0.33	0	10	0.38	0	10	0.59	1	7
DMU7	0.17	0	8	0.26	1	5	0.20	0	5	0.26	1	5	0.63	1	8
DMU8	0.27	0	11	0.36	1	9	0.33	1	11	0.36	1	9	0.71	1	9
DMU9	0.14	0	6	0.40	0	11	0.30	1	9	0.40	0	11	1.00	0	15.5
DMU10	0.28	0	12	0.24	0	2	0.19	0	2	0.24	0	2	0.45	1	2
DMU11	1.00	0	17	0.28	1	7	0.22	0	6	0.28	1	7	0.40	1	1
DMU12	0.19	0	9	0.21	0	1	0.17	1	1	0.21	0	1	0.50	1	4
DMU13	0.43	0	14	0.27	0	6	0.24	0	7	0.27	0	6	0.53	1	5
DMU14	0.07	0	2	0.25	0	3	0.20	1	4	0.25	0	3	0.57	1	6
DMU15	0.32	0	13	0.26	1	4	0.20	1	3	0.26	1	4	0.74	1	10
DMU18	0.07	0	3	0.97	1	13	0.97	1	14	0.97	1	13	1.00	0	15.5
DMU19	0.02	0	1	1.00	0	16.5	1.00	0	17	1.00	0	16.5	1.00	0	15.5
DMU21	0.59	0	15	1.00	0	16.5	0.91	1	13	1.00	0	16.5	1.00	0	15.5
DMU23	0.11	0	4	0.98	1	14	0.98	1	15	0.98	1	14	0.86	1	11
Perfect Efficiency	3			4			3			4			6		
Weak Efficiency															
Not Efficient	15			14			15			14			12		

eff=efficiency scores, s=slack(0-no slack, 1- slack is present),Rk=ranking

respectively for CRS-DEA while 6 and 12 DMUs were PE and NtE respective for GA-FDEA.

Validation

The Spearman test of correlation (*rs*) was employed to verify the results of proposed models. Correlation coefficient describes both the strength and the direction of the relationship between two variables. Its value range from −1 to +1, where 1 is total positive correlation, −1 is total negative correlation and 0 is no correlation. The Spearman correlation coefficient is often used to evaluate relationship between ranked variables rather than the raw data. In this study, Spearman (*rs)* test was employed to compare proposed GA-FDEA models with results of CRS-DEA.Table 8 and 9 represent the Spearman test between proposed GA-FDEAmodels and CRS-DEA model and their p-values respectively. To measure the Spearman correlation coefficient, the efficiency scores of DMUs of all models were ranked (see the RK column in Table 4 to 7). The *rs* for GA-FDEA models and CRS-DEA lies between -0.25 to 0.60.

In 2014, Model1, 2 and 3 have strong positive relationship and model 4 have a very weak or relationship with CRS-DEA model. In 2016, all models show weak negative relationship with CRS-DEA. In 2013 and 2015, all models show weak relationship with CRS-DEA.

The results are not significant at the 5% level except in 2014. Therefore, we conclude that the proposed models have no relationship with CRS-DEA approach.

In the result of this study, the GA-FDEA approach outperforms the conventional DEA in terms of efficiency performance scoring. In all these models, input variable (FIA) and output variables (TI and NEP) are strong indicators that are very important in determining multi-financial institutions performance.

Table 8. Spearman Correlation Coefficients of CRS DEA and four GA-FDEA models

Correlation matrix (Spearman)				
	CRS DEA			
Models	2013	2014	2015	2016
GA-FDEA1	0.286	0.601	0.218	-0.103
GA-FDEA2	0.177	0.581	0.175	-0.181
GA-FDEA3	0.286	0.601	0.218	-0.103
GA-FDEA4	0.171	0.043	0.176	-0.246

Table 9. p value of Spearman Correlation Coefficients of CRS DEA and four GA-FDEA models

p-values (Spearman):				
	CRS DEA			
Models	2013	2014	2015	2016
GA-FDEA1	0.185	0.003	0.317	0.684
GA-FDEA2	0.417	0.004	0.423	0.470
GA-FDEA3	0.185	0.003	0.317	0.684
GA-FDEA4	0.433	0.846	0.419	0.323

Values in bold are different from 0 with a significance level alpha=0.05

Also, ranking of the bank shows efficient banks were ranked higher for each year. The institutions should put in place adequate measures that will minimize their huge capital allocation on fixed assets and concentrate on how to generate more income which will influence their net profit. Government on the other hand should relax their monetary policies so that money market can be robust by putting measures that will reduce the exchange rate since this will affect the FIA.

Conclusion

This research objective was to introduce an optimized GA-FDEA approach for selecting best model(s) with key economic indicators and measure the relative efficiencies of financial institutions using these models. Past researchers on this subject area employ parameters or variables without observing their degree of relatedness. The GA approach for selecting variable removes the biasness and relatedness among economic indicators such that only variables that contribute significantly were considered before evaluation. The alpha cut Fuzzy logic approach caters for the uncertainty in DEA efficiency measure of some parameters since in real world economic indicators cannot assume fixed values. Financial reports of twenty three (23) finance institutions were capture and translated to fuzzy numbers using symmetric triangular fuzzy membership function. GA-FDEA model was able to intelligently discriminate DMUs where efficiency is very sensitive to varying inputs and outputs. The GA-FDEA demonstrated several robust and reliable models that measure efficiency across similar/dissimilar institutions and surpasses the results of the traditional CRS-DEA. We conclude by affirming that GA-FDEA will produce better results that will help both financial institutions and government to moderate their policies so as to bring the necessary growth in the economy.

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