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Axially Forced Vibration Analysis of Cracked a Nanorod

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ABSTRACT

This study presents axially forced vibration of a cracked nanorod under harmonic external dynamically load. In constitutive equation of problem, the nonlocal elasticity theory is used. The Crack is modelled as an axial spring in the crack section. In the axial spring model, the nonrod separates two sub-nanorods and the flexibility of the axial spring represents the effect of the crack. Boundary condition of the nanorod is selected as fixed-free and a harmonic load is subjected at the free end of the nanorod. Governing equation of the problem is obtained by using equilibrium conditions. In the solution of the governing equation, analytical solution is presented and exact expressions are obtained for the forced vibration problem. On the solution method, the separation of variable is implemented and the forced vibration displacements are obtained exactly. In the open literature, the forced vibration analysis of the cracked nanorod has not been investigated broadly. The objective of this study is to fill this blank for cracked nanorods. In numerical results, influences of the crack parameter, position of crack, the nonlocal parameter and dynamic load parameters on forced vibration responses of the cracked nanorod are presented and discussed.

1. Introduction

The using of nano structures is increasing in the engineering applications from day to day. Nano structures are used various applications, such as actuators, atomic microscopes, electro-mechanical devices. The experimental investigations of the nano structures are still difficult problems in today. So, the molecular dynamic simulation is used for the nanostructural analysis. However, this method has high computational cost. So, approximate models continuum models are preferred in the researches such as the nonlocal continuum theories. The nonlocal continuum theories consist of size effect in contrast with classical continuum theory. Although, this models do not gives realistic results in contrast with molecular dynamic simulation, it can be obtained determined results in the restricted conditions.

In the last decade, vibration, stability and static behavior of the nano structure have been investigated within nonlocal continuum theories in the literature at large (Eringen [1,2], Toupin [3], Lam et al. [4], Mindlin [5,6], Yang et al. [7], Park and Gao [8], Hasanyan et al. [9], Loya et al. [10], Civalek et al. [11], Reddy [12,13], Hasheminejad et al. [14], Liu and Reddy [15], Ansari et al. [16], Wang et al. [17], Asghari et al. [18], Belkorissat et al. [19], Akgöz and Civalek [20,21], Karličić et al. [22], Kocatürk and Akbaş [23], Sedighi et al. [24], Al-Basyouni et al. [25], Şimşek [26], Chaht et al. [27], Akbaş [28,29], Arda and Aydogdu [30],Eren and Aydogdu [31], Uzun et al. [32], Arda and Aydogdu [33], Zargaripoor et al. [34], Ke et al. [35], Kordani et a. [36], Zakeri et al. [37], Ebrahimi and Shafiei [38], Ebrahimi et al. [39], Ahouel et al. [40], Aissani et al. [41], Bellifa et al. [42],Hadji et al. [43], Hosseini et al. [44,45], Hadi et al. [46,47], Akbaş [48], Shishesaz et al. [49], Moradi et al. [50], Nejad et al. [51]).

In investigation of cracked nano-elements in open literature as follows; Hasheminejad et al. [52] investigated free vibration of cracked nanobeams with surface effects. Loya et al. [53] examined free vibration analysis of Euler-Bernoulii nanobeams based on nonlocal elasticity theory. Roostai and Haghpanahi [54] investigated free vibration results of nanobeams with multi cracks by using nonlocal elasticity theory. Liu et al. [55] presented vibration responses of the cracked micro cantilever beams under electrostatic forces. Torabi and Nafar Dastgerdi [56] studied free vibration of cracked Timoshenko nanobeams by using nonlocal elasticity theory.Wang and Wang [57] presented free vibration cracked Timoshenko nanobeams with surface

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energy. Tadi Beni et al. [58] investigated effects of the cracks on vibration characterises of the nanobeams by using couple stress theory. Yaylı and Çerçevik [59] examined dynamics of nano beams with crack by analytically. Stamenkovic´ et al. [60] studied forced vibration of single-walled carbon nanotubes with magnetic effects. Peng et al. [61] presented energy rate for crack nano beams. Akbaş [62] presented effects of the cracks on the static displacements of the microbeams based on couple stress theory by analytically. Akbaş [63] studied free vibration of cracked micro beams by using finite element method and couple stress theory. Akbaş [64,65] investigated forced vibration analysis of cracked nano/micro beams based Euler-Bernoulii beam theory by using couple stress theory. Hsu et al. [66] investigated effects of the crack on axial vibration of nanobeams based nonlocal elasticity theory. Rahmani et al. [67] investigated torsional vibration of nanobeams with crack. Sourki and Hoseini [68] presented vibration analysis of cracked microbeams based couple stress theory.

In the literature survey, the forced vibration studies of the nanorods with crack have not been investigated at large. The novelty in this paper is to investigate longitudinal forced vibration of cracked cantilever nanorod and to fill this blank for cracked nanorods. In effects of crack, crack section is modelled as an axial spring which separate two sub-nanorods. Governing equation of problem is solved by analytically with using the separation of variable procedure. The explicit forced vibration displacements are obtained in domain time by analytically. The effects of nonlocal, dynamic load and crack parameters on forced vibration responses of cracked nanorod are presented and discussed.

2. Theory

Figure 1 shows a cantilever cracked circular nanorod subjected to dynamically point force (*P(t)*). The load is subjected at the free end of nanorod. In figure 1, *L* and *D* indicate the length and diameter of the nanarod, respectively. The crack depth indicates as *a* and the location of the crack from fixed support indicates as L_1.

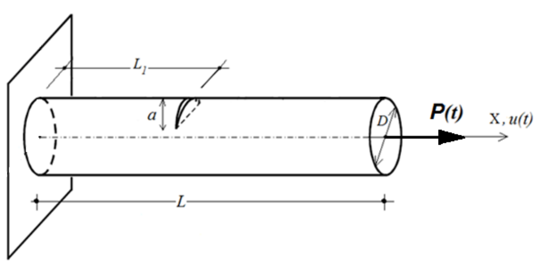

Figure 1. A cantilever cracked circular nanorod subjected to dynamically point load.

By using the nonlocal elasticity theory, constitutive equation of the problem is given (Eringen [1,2]);

$$\sigma_{xx} - \mu \frac{d^2\sigma_{xx}}{dx^2} = E\,\epsilon_{xx} \tag{1}$$

where, σ_{xx} and ϵ_{xx} are nonlocal normal stress and strain, respectively. E and μ are Young's modulus and nonlocal parameter, respectively. where $\mu = (e_0 a)^2$, e_0 indicates length scale parameter. By using equilibrium of forces in axially direction, the equation of motion is expressed as follows;

$$E\frac{\partial^2 u(x,t)}{\partial x^2} - \rho\frac{\partial^2 u(x,t)}{\partial t^2} + \rho\mu\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u(x,t)}{\partial t^2}\right) = 0 \tag{2}$$

where, ρ and u are mass density and axial displacement function, respectively. By simplifying equation 2, the following equation is obtained.

$$c^2\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\partial^2 u(x,t)}{\partial t^2} + \mu\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u(x,t)}{\partial t^2}\right) = 0 \tag{3}$$

where

$$c^2 = \frac{E}{\rho} \tag{4}$$

In the crack model, the crack section is modelled as linear elastic spring. In the axial spring model, the nonrod seperates two sub-nanorods and the flexibility of the axial spring represents the effect of the crack as shown figure 2.

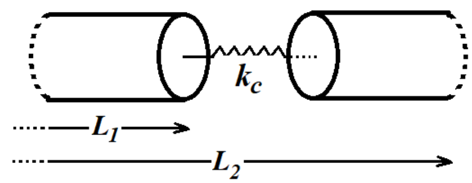

Figure 2. Axial spring model in crack section.

In figure 2, k_c indicates the axially stiffness coefficient of the crack. Because of the crack, the nanobeam is formed as two sub portion. So, two different displacement functions and equation of the motion are rewritten;

$$c^2\frac{\partial^2 u_1(x,t)}{\partial x^2} - \frac{\partial^2 u_1(x,t)}{\partial t^2} + \mu\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_1(x,t)}{\partial t^2}\right) = 0 \quad , \quad 0 \le u_1 \le L_1 \tag{5a}$$

$$c^2\frac{\partial^2 u_2(x,t)}{\partial x^2} - \frac{\partial^2 u_2(x,t)}{\partial t^2} + \mu\frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u_2(x,t)}{\partial t^2}\right) = 0, \; L_1 \le u_2 \le L \tag{5b}$$

where, u_1 and u_2 are the axial displacement functions of the first portion (left side of crack) and second portion (right side of crack). The boundary conditions of the problem are given as follows;

$$u_1(0,t) = 0, \quad \frac{\partial u_1(L_1,t)}{\partial x} = \frac{\partial u_2(L_1,t)}{\partial x},$$
$$\frac{k_c(u_2(L_1,t) - u_1(L_1,t))}{EA} = \frac{\partial u_1(L_1,t)}{\partial x}, \quad \frac{\partial u_2(L,t)}{\partial x} = \frac{P(t)}{EA} \tag{6}$$

where, A indicates the area of the cross section. The external dynamically load ($P(t)$) is considered a harmonic function;

$$P(t) = P_0 sin(\Omega t) \tag{7}$$

where, P_0 and Ω are the amplitude and frequency of load, respectively. To solve the forced vibration problem, the solution (u_p) of equation (5) for the forced vibration problem is solved by using the separation of variable;

$$u_{p1}(x,t) = U_{p1}(x) sin(\Omega t), \qquad 0 \le u_1 \le L_1 \tag{8a}$$

$$u_{p2}(x,t) = U_{p2}(x) sin(\Omega t), \qquad L_1 \le u_1 \le L \tag{8b}$$

Substituting Eq. (8) into equation (5) gives following equations of motion:

$$\left(c^2\frac{d^2 \mathrm{U_{p1}(x)}}{dx^2} + \Omega^2 \mathrm{U_{p1}(x)} - \mu\,\Omega^2\frac{d^2 \mathrm{U_{p1}(x)}}{dx^2}\right)\sin(\Omega t) = 0,$$

$$0 \le u_1 \le L_1 \quad (9a)$$

$$\left(c^2 \frac{d^2 \mathrm{U_{p2}(x)}}{dx^2} + \Omega^2 \mathrm{U_{p2}(x)} - \mu\, \Omega^2 \frac{d^2 \mathrm{U_{p2}(x)}}{dx^2}\right) \sin(\Omega t) = 0,$$

$$\mathrm{L_1} \le u_1 \le \mathrm{L} \quad (9b)$$

After the simplifying expression (9), the following equation is obtained as follows:

$$\left(\frac{d^2 \mathrm{U_{p1}(x)}}{dx^2} + \gamma^2 \mathrm{U_{p1}(x)}\right) = 0, \quad 0 \le u_1 \le L_1 \quad (10a)$$

$$\left(\frac{d^2 \mathrm{U_{p2}(x)}}{dx^2} + \gamma^2 \mathrm{U_{p2}(x)}\right) = 0, \quad \mathrm{L_1} \le u_1 \le \mathrm{L} \quad (10b)$$

where

$$\gamma^2 = \frac{\Omega^2}{c^2 - \mu\, \Omega^2} \quad (11)$$

By implementing the boundary conditions in the equation (10) for clamped-free boundary conditions, the $U_{p1}(x,t)$ and $\mathrm{U_{p2}(x,t)}$ is obtained as follows:

$$U_{p1}(x,t) = B_1 \sin(\gamma x) \sin(\Omega t), \quad 0 \le u_1 \le L_1 \quad (12a)$$

$$U_{p2}(x,t) = (A_2 \cos(\gamma x) + B_2 \sin(\gamma x)) \sin(\Omega t), \; L_1 \le u_1 \le L \quad (12b)$$

where

$$B_1 = \frac{-2\,\mathrm{P_0\, k_{cr}}}{EA\gamma\, \alpha}, \qquad A_2 = \frac{-2\,\mathrm{P_0}\,(\cos(\gamma L_1)^2)}{\alpha},$$

$$B_2 = \frac{-\,\mathrm{P_0}\,(2\,\mathrm{k_{cr}} + EA\gamma \sin(2\gamma L_1))}{EA\gamma\, \alpha}, \quad (13)$$

where

$$\alpha = EA\gamma \sin(\gamma L) - 2\,\mathrm{k_{cr}} \cos(\gamma L) + EA\gamma \sin(\gamma L - 2\gamma L_1) \quad (14)$$

The flexibility coefficient of the crack (G_{cr}) is given as follows:

$$G_{cr} = \frac{1}{k_{cr}} \quad (15)$$

The dimensionless quantities are given as follows:

$$\eta = \frac{e_0 a}{D}, \;\; \bar{\Omega} = \sqrt{\frac{\rho\, D^2}{E}}\,\Omega, \;\; \lambda = \frac{L}{D}, \;\; \bar{U} = \frac{U_p}{L}, \;\; \bar{G}_{cr} = \frac{G_{cr}\, EA}{D},$$

$$\bar{L}_{cr} = \frac{L_1}{L} \quad (16)$$

where η and $\bar{\Omega}$ indicate the dimensionless nonlocal parameter and dimensionless the frequency of the dynamic load, respectively. λ is the aspect ratio, $\bar{U}$ is dimensionless the longitudinal displacement, $\bar{G}_{cr}$ is dimensionless flexibility coefficient of crack and, $\bar{L}_{cr}$ is crack location ratio.

3. Examples

In numerical examples, effects of dimensionless nonlocal parameter, dimensionless the frequency of the dynamic load, the dimensionless flexibility coefficient and location of crack on dynamic displacements of cracked nanorod are investigated. In the numerical study, the material of the nanorod is considered as epoxy (E=1,44 GPa, $\rho = 1600\, kg/m^3$). The diameter of the nanorod is taken as D=1nm. The length of the nanorod is selected according to the aspect ratio (λ).

In order to verify this study, some results of Hsu et al. [66] and Singh [69] are compared with obtained results of present study in table 1. In the comparison study, the first three vibration eigenvalues of clamped-free cracked rod are compared in theses of [59] and [66]. In the comparison study, the following parameters are used; $\bar{\mathrm{L}}_{\mathrm{cr}} = 0.202$, $\bar{\mathrm{G}}_{\mathrm{cr}} = 0.1144$, $\eta = 0.01$, $\lambda = \sqrt{\frac{\rho\, \mathrm{L}^2}{\mathrm{EA}}}\,\omega$. Where, ω and λ dimensional and dimensionless vibration eigenvalues. It is seen from table 1 that the results of present study are good agreement of the results of Hsu et al. [66] and Singh [69].

Table 1. Comparison study: the first three dimensionless vibration eigenvalues of clamped-free cracked rod.

Mode Number	λ		
	Present	Ref. [66]	Ref. [69]
1	1,4278	1,4278	1,4278
2	4,5576	4,5576	4,5576
3	7,8540	7,8540	7,8540

In figure 3, relationship between the dimensionless displacements and the dimensionless frequency of the dynamic load ($\bar{\Omega}$) is presented for different values of dimensionless flexibility coefficients of the crack ($\bar{G}_{cr}$) for the aspect ratio $\lambda = 20$, amplitude of dynamic load is taken as $P_0 = 1$ nN, the dimensionless nonlocal parameter $\eta = 0.01$ and the crack location ratio $\bar{L}_{cr} = 0.5$. It is stated that the dimensionless amplitude displacements ($\bar{U}_m$) are calculated at the free end of the cracked nanorod in all figures.

Figure 3 displays that increasing of dimensionless flexibility coefficients of crack yields to increase the dimensionless displacements naturally. The dynamic responses the cracked nanorod change with increase of the $\bar{G}_{cr}$. Also, the resonance frequency change considerably with increase of $\bar{G}_{cr}$ parameter. The resonance case can be seen in the vertical asymptote lines in all figures. Increasing of the $\bar{G}_{cr}$ parameter yields to decreasing the resonance frequency.

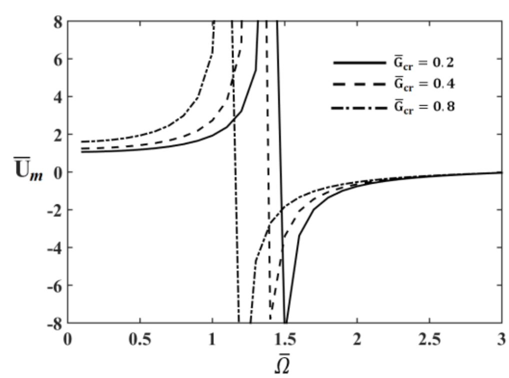

Figure 3. The relationship between dimensionless displacements and dimensionless frequency of load for different values dimensionless flexibility coefficients of crack.

In figure 4, relationship between the dimensionless displacements and the dimensionless frequency of dynamic load ($\bar{\Omega}$) is presented for different values of the crack location ratio ($\bar{L}_{cr}$) for $\lambda = 20$, $\bar{G}_{cr}$=0.3, $\eta = 0.01$and $P_0 = 1$ nN.

As seen from figure 4, the displacements and dynamic responses of the cracked nanorod change considerably. With increase of the crack location ratio, in other words, crack location get closer to fixed end (left end), dynamic displacements increase as it expected. Also, the resonant frequencies change with different values of $\bar{L}_{cr}$. With the crack get closer to fixed end, the resonant frequencies of the nanorod decrease significantly. It is concluded from figures 3 and 4 that the crack flexibility and location parameters have important role on forced dynamic responses of cracked nanorod.

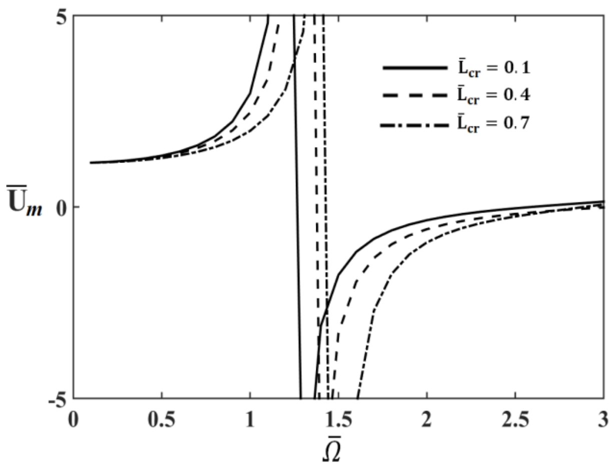

Figure 4. Relationship between dimensionless displacements and dimensionless frequency of the load for different values crack location ratios.

Figure 5 shows the relationship between the dimensionless displacements and the dimensionless nonlocal parameter (η) for various values of $\bar{G}_{cr}$ for $\lambda = 20$, $\bar{L}_{cr} = 0.5$ and $\bar{\Omega} = 3$. In a similar manner, effects of the crack location ratio on the $\bar{U}_m$-η relation is plotted in figure 6 for $\bar{G}_{cr} = 0.3$, $\lambda = 20$ and $\bar{\Omega} = 3$.

As seen from figures 5 and 6, resonant frequencies change considerably with increasing of the nonlocal parameter. With increasing dimensionless nonlocal parameter to resonance point from zero, the difference among of results in the flexibility coefficients increases. In a similar way, the difference among of the crack location ratios increases in the nonlocal parameters of the resonance region. The nonlocal parameter is very effective to change the effects of crack on dynamic responses of nanorods.

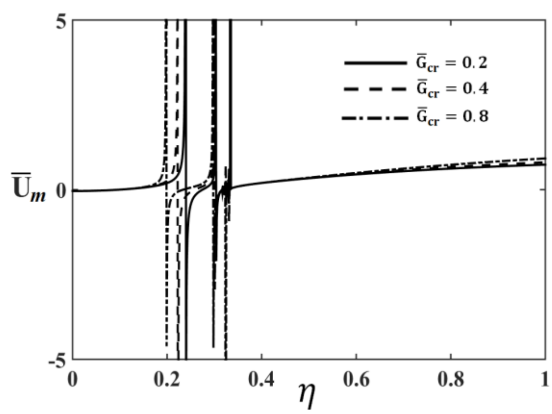

Figure 5. The relationship between dimensionless displacements and dimensionless nonlocal parameter for different values the dimensionless flexibility coefficients of the crack.

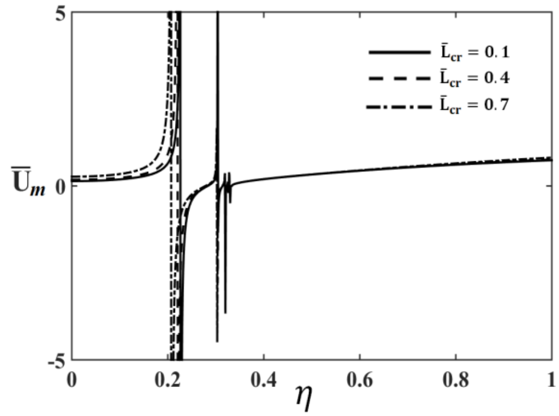

Figure 6. The relationship between dimensionless displacements and dimensionless nonlocal parameter for different values the crack location ratios.

In figure 7, time (t)- dimensionless displacement relation is presented for different values of $\bar{G}_{cr}$ for $\lambda = 20$, $\bar{L}_{cr} = 0.2$, $\eta = 0.01$ and $\bar{\Omega} = 20$. Figure 7 display that the flexibility coefficients of the crack change dynamic responses of the nanorods considerably.

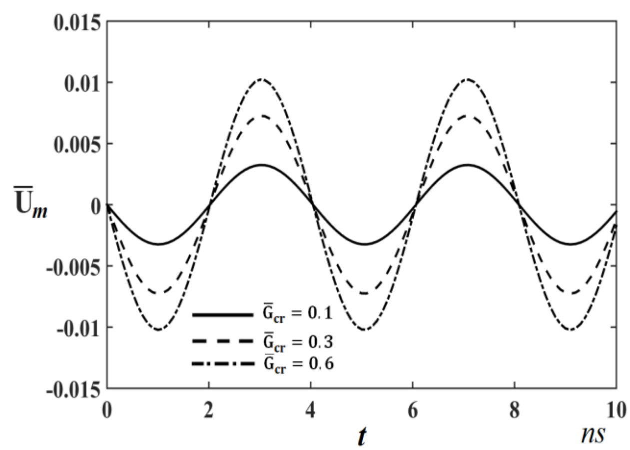

Figure 7. Time responses of the cracked nanorod for for different values the dimensionless flexibility coefficients of the crack.

4. Conclusion

Longitudinal forced vibration problem of a cracked nanorod is investigated and its formulations are derived by using nonlocal elasticity theory. The Governing equation of problem is solved by analytically within using separation of variable procedure and the dynamic displacements are obtained domain of time. In the crack section, the effect of the crack is modelled as an axial spring. The influences of nonlocal, dynamic load and crack parameters on forced vibration results of cracked nanorod are examined and discussed. It is concluded from the numerical results that nonlocal parameter is very effective in crack behavior. With changing the nonlocal parameter, forced vibration responses and resonant frequencies of cracked nanorods change significantly. Also, dynamic responses of nanorods change with increasing the dimensionless flexibility coefficients of the crack and crack location ratios considerably. The resonant frequencies change significantly with different values of crack location ratios. The nonlocal parameter play important role in the resonant region for cracked nanorods.

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