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Market Adaptive Control Function Optimization in Continuous Cover Forest Management

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Abstract

Economically optimal management of a continuous cover forest is considered here. Initially, there is a large number of trees of different sizes and the forest may contain several species. We want to optimize the harvest decisions over time, using continuous cover forestry, which is denoted by CCF. We maximize our objective function, the expected present value, with consideration of stochastic prices, timber quality variations and dynamically changing spatial competition. The problem is solved using an adaptive control function. The parameters of the control function are optimized via the first order optimum conditions based on a multivariate polynomial approximation of the objective function. The second order maximum conditions are investigated and used to determine relevant parameter ranges. The procedure is described and optimal results are derived for a general function multi-species CCF forest. Concrete examples from Germany, with beech, and from Sweden, with Norwegian spruce, are used to illustrate the methodology and typical numerical results. It is important to make market adapted harvest decisions. If the stochastic price variations are not considered when the harvest decisions are taken, the expected present value is reduced by 23%.

Keywords

Economic optimization, Multi species forests, Forest management, Stochastic processes, Adaptive optimal control.

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Introduction

Economic forest management is an interesting area from a methodological point of view. Several dynamic and stochastic processes should be considered. Market prices are very important to optimal decisions, but often rapidly change and cannot be perfectly predicted. The central question is this: what is the best way to sequentially update the information and adaptively determine the management decisions? It is important to get a perspective on the theoretically rapidly changing field of forest management. First, we start with two typical cases of the classical steady state definition of the forest management problem. Note that the advocated planning principles with this approach are completely insensitive to prices, costs and dynamic market changes. Hessenmöller et al. (2018) derive target volumes and densities for different diameter ranges of uneven-aged beech stands, with the plan to obtain a steady state distribution of tree diameters. Such forests can also be called ”continuous cover forests”, or CCF forests. They write that beech-dominated selection forests presently are covering about 10,000 hectares in the Hainich-Dün region of Thuringia, Germany. These forests have been managed in different ways during almost 1000 years. They also claim that such forests often are considered to be resilent to disturbances and resemble natural conditions.

Schütz (2006) investigates steady states and sustainability of pure beech forests. He defines demographic steady state via an invariant distribution of numbers of trees of different dimensions and writes that, if we are interested in ”real” sustainability, it is also necessary that the removal volume is equal to the observed periodic volume increment. We note that Hessenmöller et al. (2018) and Shütz (2006) focus on dynamic equilibria. Key problems are to determine ”target volumes”, ”target densities” and ”steady states”. From the point of view of economic management, we make this observation: the suggested and utilized forest management planning methods in these cases are completely insensitive to prices, costs and dynamic market changes. Still, the methods and ideas in these articles are common in practical forest management and forest research projects in several countries. However, if we are interested in economic results and market adapted forest management decisions, we cannot ignore prices, costs and other economically important parameters. It is necessary to consider the degree of predictability of future values of these parameters.

Furthermore, in a dynamic world, the economically optimal levels of adjustments of management decisions to changes in prices, that are not perfectly predictable, are important. In stochastic markets, production capacity levels, stock policies and flexibility are important to the expected profitability. Nasseri and Bavandi (2018) discuss and analyze the importance of stochastic objective function parameters in mathematical programming. They claim that stochastic objective function parameters are important in real applications. In an earlier article, Attari and Nasseri (2014) introduced FMP, Fuzzy Mathematical Programming. Then, however, stochastics and fuzziness concerned the problem of feasibility under stochastic disturbances, via fuzzy constraints. In a similar spirit, Nemati et al. (2017) advocate new investigations of dynamic prices and the effects of forecast errors on the optimal selection of management planning models.

Forest management decisions can be taken at many different levels. When the level of detail increases, the number of partial decision problems increases almost without bound. In continuous cover forestry, CCF, we may consider the management of each tree as a decision problem. Should we harvest this tree now or wait until some future point in time? Furthermore, these decision problems at the tree level are not independent. If one tree is harvested now, the available space increases for other trees in the neighborhood to continue growing. Hence, the optimal management decisions of the neighbor trees are affected. In production planning, it is often necessary to derive solutions that optimize many objective functions at the same time, which is stressed by Moradi Dalini and Noura (2018). This is quite correct and relevant to forestry and motivates the present model development. In the present paper, a harvest control function is developed. With the help of this function, all of the harvest decisions, of every tree, are optimized in such a way that the expected value of the total objective function is maximized. All of the interdependencies are considered.

With large numbers of problem dimensions, stochastic parameter changes, large numbers of nonlinearities and adaptive decisions, the problem structure makes it difficult or even impossible to utilize standardized linear and nonlinear programming methods from the field of operations research. In order to give relevant solutions to real world problems, it is necessary to let the model contain the relevant structure with respect to how different parts of the analyzed system are connected and influence each other. One way to do this is to use stochastic simulation. A very positive and wide perspective on the importance and use of simulation in production management problems is given by Shahbazi et al. (2017). Many kinds of applications of simulation are suggested, including job scheduling, ground and air transportation, resource distribution and so on. In the present paper, stochastic simulation will be used as a part of an adaptive control function optimization procedure. This approach has been developed by Lohmander (2017b; 2018b). With stochastic simulation as a subroutine, it is possible to search the best way to control the system to reach the most desirable solution, in case the following procedure is utilized: first, a stochastic simulation model of the complete system under analysis is developed. The adaptive control of this system is defined via a specified control function. General theoretical principles in the field of analysis can be used to define the functional form of the adaptive control function to be used in the system. The exact values of the optimal parameters of the control function are still unknown. Next, the complete system is simulated with a large number of alternative control function parameter value combinations. Thereafter, multidimensional regression analysis is used to determine an approximating function that gives the expected objective function value of the system as a nonlinear function of the control function parameters. Then, we maximize the value of the approximating function. From the first order optimum conditions, the optimal parameter values of the control function are determined. In case the approximating function is quadratic, the equation system of first order optimum conditions is linear. Then, the optimum is usually unique and it is possible that the approximating function can be shown to be strictly concave. If that is the case, the derived control function parameters give a maximum that is globally unique. In case the approximating function is not quadratic, but for instance cubic, the analysis is more complicated. Then, the equation system of first order optimum conditions is not linear. Still, if the equation system only contains a limited number of nonlinearities, the solutions may be calculated via elimination and analytical methods of quadratic, cubic or quartic equations. In such cases, it may be found that the approximating function is strictly concave in some region(s). Then, it may be possible to show that one of the solutions of the first order optimum conditions gives an optimum that is also a locally unique maximum. In some cases, it may be possible to show that we also have a unique global maximum. In this paper, this method will be utilized to derive optimal adaptive control functions in forest management.

Optimization problems with many dimensions, nonlinearities and stochastic disturbances are common in most sectors of the economies. Lohmander (2007), (2017a) and (2018a) presents alternative optimization methods to handle such situations in a rational way. In this paper, we focus on CCF, continuous cover forestry. Initially, there is a large number of spatially distributed trees of different sizes. The central question is this: what is the best way to adaptively control and manage such a forest? Lohmander (1992a), (1993) and (2000) shows that there are considerable option values associated with mixed forests. Single species forests give less options to adjust production to possible stochastic events. For instance, prices of different species may change. Then, it is valuable to have the option to adjust the harvest activities to these changing market conditions. Furthermore, some species may be negatively affected by pests, insects or large animals. Some species may not produce well in case the climate changes or if pollution increases. In these cases, it is valuable to be able to adaptively adjust forest production. This can easily be done if we already have several species growing in the forest. In some cases, it is possible to calculate the expected present value of forestry, conditional on the initial species mix. This has been done and Lohmander (1993) reports such results. Lohmander (2007) presents several optimization methods and typical solutions to adaptive forest management problems.

Methods

In the present study, we have the ambition to develop and describe a general analytical and numerical method to handle management decision problems of this type: we want to optimize the harvest decisions over time. We want to maximize our objective function, the expected present value. The prices of the different species are stochastic. The problem is solved using an adaptive control function. The parameters of the control function are optimized via the first order optimum conditions of an approximation of the expected objective function. The second order maximum conditions are investigated. The expected objective function is estimated via Monte Carlo simulation. In this section, a general procedure is given for a multi-species forest with trees of different sizes. In later sections, concrete examples are analyzed and discussed. Now, we consider a mixed species forest. Initially, there is a large number of spatially distributed trees of different sizes and species. We want to optimize the harvest decisions over time. We want to maximize our objective function, the expected present value. The prices of the different species are stochastic.

$$b_i(0) = b_{i0} \quad \forall i \tag{1}$$

$b_i(t)$ is the basal area of tree number i (at height 1.3 meters above ground) at time t (from $t_0 = 0$). Each period normally represents one year but other time intervals are sometimes more relevant. The initial condition is b_{i0}. $d_i(t)$ is the diameter of tree i at time t at height 1.3 meters above ground.

$$d_i(t) = 2\sqrt{\left(\frac{b_i(t)}{\pi}\right)} \tag{2}$$

$u(i,t)$ represents the control decision. If $u(i,t) = 1$, the tree i is harvested in period t. Otherwise, $u(i,t) = 0$.

$$u(i,t) \in \{0,1\} \quad \forall i,t \tag{3}$$

$$\sum_{t=0}^{T} u(i,t) \le 1 \quad \forall i \tag{4}$$

$b_i(t)$ develops according to a discrete time process. The increment, growth, G, is a function of the basal area $b_i(t)$, the species $S(i)$ and the competition, $L(i,t)$. $S(i) \in \{1,.,N\}$ where N is the total number of species. $s_m(i)$ is a ”species dummy variable” defined in (5).

$$s_m(i) = \begin{cases} 1 & if\ S(i) = m \\ 0 & if\ S(i) \ne m \end{cases} \qquad \forall i, m \in \{1,.,N\} \tag{5}$$

The level of competition of relevance to tree i at time t is denoted by $L(i,t)$. In different studies, $L(i,t)$ has been specified in different ways. Usually, $L(i,t)$ is a strictly increasing function of the size (for instance, basal area) of neighbouring trees. Furthermore, neighbouring trees that are close to tree i influence the value of $L(i,t)$ more than what more distant trees do. In Lohmander (2018b), $L(i,t)$ is the total basal area of neighbouring trees per hectare within a circle of radius 10 meters, where tree i represents the center of the circle. In Lohmander et al. (2017), $L(i,t)$ is a nonlinear function of the properties of the competitors; $L(i,t)$ decreases with the distance to the competitors and increases with the size of the competitors.

The general function $G(.)$, for $L(i,t)=0$, has been defined and presented by Lohmander (2017c). Lohmander also derived a differential equation consistent with $G(.)$ and the dynamic properties of the basal area development. Empirical estimations of the parameters of $G(.)$ with variations of $L(i,t)$ have been performed for forests with different tree species in Iran by Hatami et al. (2017).

$$b_i(t+1)=\begin{cases} b_i(t)+G\left(b_i(t),S(i),L(i,t)\right) & for\ u(i,t)=0 \\ 0 & for\ u(i,t)=1 \end{cases} \qquad (6)$$

The present value of all profits is denoted by Z. This is the discounted net value of all harvests. Hence it is a function of all harvest decisions, the rate of interest, the prices of the different species, the volumes of trees at different points in time and the harvest costs.

$$Z=\sum_{t=0}^{T}e^{-rt}\sum_{i=1}^{I}u(i,t)\left(P(S(i),t)V(S(i),b_i(t))-C(S(i),b_i(t),t)\right) \qquad (7)$$

e^{-rt} Is the discounting factor of period t with rate of interest r. $P(S(i),t)$ denotes price per cubic meter of species i in period t. $P(S(i),t)$ is a stationary variable which is stochastic at $t-n, \forall n>0$. $E\left(P(S(i),t)\right)$ Is the expected value of $P(S(i),t)$ at $t-n, \forall n>0$. In the two species case, if trees i and j belong to different species, then $P(S(i),t)$ and $P(S(j),t)$ have correlation ρ. $-1\le\rho\le1$. $V(S(i),b_i(t))$ is the volume of tree i as a function of the species and the basal area. $C(S(i),b_i(t),t))$ denotes the harvest cost of tree i . This cost is a function of species, basal area and time.

This problem is highly stochastic, multidimensional and nonlinear. Furthermore, it contains a large number of integer variables. It is necessary to define a reasonable type of adaptive control function that can be used to handle the many control decisions in a way that takes the stochastic prices and competition between trees into account. Then the parameters of the control function may be optimized. For this purpose, the following rule is suggested: first, we calculate the ”limit diameter” $D_i(t)$ of tree i at time t. The limit diameter is a function of the tree species index, the relative deviation of the price from the expected level and the competition.

$$D_i(t)=\sum_{m=1}^{N}\alpha_m s_m(i)+\alpha_P\left(\frac{P(S(i),t)-E\left(P(S(i),t)\right)}{E\left(P(S(i),t)\right)}\right)+\alpha_L L(t,i) \qquad (8)$$

In case the diameter $d_i(t)$ is larger than the limit diameter, we instantly harvest. Otherwise, we wait at least one more period before we harvest the tree.

$$u(i,t)=\begin{cases}0 & if\ \left(d_i(t)-D_i(t)\right)\left(1-\sum_{t=0}^{t-1}u(i,t)\right)\leq 0\\ 1 & if\ \left(d_i(t)-D_i(t)\right)\left(1-\sum_{t=0}^{t-1}u(i,t)\right)>0\end{cases} \quad \forall i,\quad t\in\{0,\kappa,2\kappa,...,n\kappa\} \ (9)$$

κ denotes the harvest decision interval. This is an integer $\kappa\geq 1$. n is the total number of harvest decision intervals and $n\kappa=T$, where T is the total planning horizon.

Z is a function of many things, including the stochastic price outcomes. When the control decisions are optimized, we are interested in the expected value of Z, namely $E(Z)$. We may estimate $E(Z)$ for given parameter values via Monte Carlo simulation. The average value of Z is determined based on a large number of random outcomes of the stochastic prices of the different species. The correlations may be estimated from real price series and Cholesky factorization can be used to generate the correlated price series. It is a good idea to store all of the simulated price series and to use the same set of simulated price series in every step of the control function parameter optimizations.

$$E(Z)=E\left(\sum_{t=0}^{T}e^{-rt}\sum_{i=1}^{I}u(i,t)\left(P(S(i),t)V(S(i),b_i(t))-C(S(i),b_i(t),t)\right)\right) (10)$$

In the following derivations, N is assumed to take the value 2, which is a typical case in real applications. The procedure can easily be extended to other values N.

Procedure:

Make an initial guess $\left(\alpha_{1_0},\alpha_{2_0},\alpha_{P_0},\alpha_{L_0}\right)$ of the optimal values of the parameters $\left(\alpha_1,\alpha_2,\alpha_P,\alpha_L\right)$.

Create a number, W, of alternative parameter combinations, w, such that stochastic variables $\left(\varepsilon_1,\varepsilon_2,\varepsilon_P,\varepsilon_L\right)$ are added to $\left(\alpha_{1_0},\alpha_{2_0},\alpha_{P_0},\alpha_{L_0}\right)$. The probability density functions of these stochastic variables are defined with consideration of the interesting parameter ranges. $\left(d_{j_\lambda}, d_{j_\gamma}\right)$ denote the lower (λ) and upper (γ) bounds of ε_j, $j \in \{1, 2, P, L\}$.

$d_{1_\lambda} < \varepsilon_1 < d_{1_\gamma}; d_{2_\lambda} < \varepsilon_2 < d_{2_\gamma}; d_{P_\lambda} < \varepsilon_P < d_{P_\gamma}; d_{L_\lambda} < \varepsilon_L < d_{L_\gamma}$. Normally, this means that $d_{1_\lambda} < 0 < d_{1_\gamma}; d_{2_\lambda} < 0 < d_{2_\gamma}; d_{P_\lambda} < 0 < d_{P_\gamma}; d_{L_\lambda} < 0 < d_{L_\gamma}$. Let ε_{j_w} denote the value of ε_j in parameter combination w.

$\left(\alpha_{1_w}, \alpha_{2_w}, \alpha_{P_w}, \alpha_{L_w}\right) = \left(\alpha_{1_0} + \varepsilon_{1_w}, \alpha_{2_0} + \varepsilon_{2_w}, \alpha_{P_0} + \varepsilon_{P_w}, \alpha_{L_0} + \varepsilon_{L_w}\right)$.

For each random parameter combination w, estimate the value of $E(Z) = E\left(Z\left(\alpha_{1_w}, \alpha_{2_w}, \alpha_{P_w}, \alpha_{L_w}, ...\right)\right)$.

Make a quadratic approximation $\Phi = \Phi\left(\alpha_1, \alpha_2, \alpha_P, \alpha_L\right)$ of the function $E(Z) = E\left(Z\left(\alpha_1, \alpha_2, \alpha_P, \alpha_L, ...\right)\right)$ according to the lines suggested by equations (11) – (14), via OLS, the ordinary least squares method.

It is important to be aware that cubic approximations or other functional forms may sometimes be more relevant. This is further investigated at the end of this paper.

$$\Phi = \Phi\left(\alpha_1, \alpha_2, \alpha_P, \alpha_L\right) \tag{11}$$

$$\begin{aligned}\Phi &= \Phi\left(\alpha_1, \alpha_2, \alpha_P, \alpha_L\right) \\ &= k_1\alpha_1 + k_2\alpha_2 + k_P\alpha_P + k_L\alpha_L + \\ &\quad + k_{11}\alpha_1^2 + k_{22}\alpha_2^2 + k_{PP}\alpha_P^2 + k_{LL}\alpha_L^2 + \\ &\quad + k_{12}\alpha_1\alpha_2 + k_{1P}\alpha_1\alpha_P + k_{1L}\alpha_1\alpha_L + k_{2P}\alpha_2\alpha_P + k_{2L}\alpha_2\alpha_L + k_{PL}\alpha_P\alpha_L\end{aligned} \tag{12}$$

$$\Phi(\alpha_1, \alpha_2, \alpha_P, \alpha_L) \approx E(Z(\alpha_1, \alpha_2, \alpha_P, \alpha_L, ...)) \tag{13}$$

$$\min_{k_0, k_1, ..., k_{PL}} \Psi = E\left[\left(\Phi\left(\alpha_1, \alpha_2, \alpha_P, \alpha_L; k_1, ..., k_{PL}\right) - E(Z(\alpha_1, \alpha_2, \alpha_P, \alpha_L, ...))\right)^2\right] \tag{14}$$

Use the quadratic approximation $\Phi = \Phi\left(\alpha_1, \alpha_2, \alpha_P, \alpha_L\right)$ to determine the approximately optimal values of the parameters $\alpha_1, \alpha_2, \alpha_P, \alpha_L$. The approximate optimal values can be used as new initial conditions, and

the approximation process can continue any number of iterations until the solution is considered satisfactory.
The first order optimum conditions are found in (15).

$$\begin{cases} \dfrac{d\Phi}{d\alpha_1} = k_1 + 2k_{11}\alpha_1 + k_{12}\alpha_2 + k_{1P}\alpha_P + k_{1L}\alpha_L = 0 \\ \dfrac{d\Phi}{d\alpha_2} = k_2 + k_{12}\alpha_1 + 2k_{22}\alpha_2 + k_{2P}\alpha_P + k_{2L}\alpha_L = 0 \\ \dfrac{d\Phi}{d\alpha_P} = k_P + k_{1P}\alpha_1 + k_{2P}\alpha_2 + 2k_{PP}\alpha_P + k_{PL}\alpha_L = 0 \\ \dfrac{d\Phi}{d\alpha_L} = k_L + k_{1L}\alpha_1 + k_{2L}\alpha_2 + k_{PL}\alpha_P + 2k_{LL}\alpha_L = 0 \end{cases} \quad (15)$$

We may determine the optimal parameter values via (16).

$$\begin{bmatrix} 2k_{11} & k_{12} & k_{1P} & k_{1L} \\ k_{12} & 2k_{22} & k_{2P} & k_{2L} \\ k_{1P} & k_{2P} & 2k_{PP} & k_{PL} \\ k_{1L} & k_{2L} & k_{PL} & 2k_{LL} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_P \\ \alpha_L \end{bmatrix} = \begin{bmatrix} -k_1 \\ -k_2 \\ -k_P \\ -k_L \end{bmatrix} \quad (16)$$

$|M|$ is presented in (17) and the second order maximum conditions are found in (18).

$$|M| = \begin{vmatrix} 2k_{11} & k_{12} & k_{1P} & k_{1L} \\ k_{12} & 2k_{22} & k_{2P} & k_{2L} \\ k_{1P} & k_{2P} & 2k_{PP} & k_{PL} \\ k_{1L} & k_{2L} & k_{PL} & 2k_{LL} \end{vmatrix} \quad (17)$$

$$|2k_{11}| < 0, \qquad \begin{vmatrix} 2k_{11} & k_{12} \\ k_{12} & 2k_{22} \end{vmatrix} > 0,$$

$$\begin{vmatrix} 2k_{11} & k_{12} & k_{1P} \\ k_{12} & 2k_{22} & k_{2P} \\ k_{1P} & k_{2P} & 2k_{PP} \end{vmatrix} < 0, \qquad \begin{vmatrix} 2k_{11} & k_{12} & k_{1P} & k_{1L} \\ k_{12} & 2k_{22} & k_{2P} & k_{2L} \\ k_{1P} & k_{2P} & 2k_{PP} & k_{PL} \\ k_{1L} & k_{2L} & k_{PL} & 2k_{LL} \end{vmatrix} > 0 \quad (18)$$

The optimal parameter values are obtained via (19) – (22).

$$\alpha_1 = \frac{\begin{vmatrix} -k_1 & k_{12} & k_{1P} & k_{1L} \\ -k_2 & 2k_{22} & k_{2P} & k_{2L} \\ -k_P & k_{2P} & 2k_{PP} & k_{PL} \\ -k_L & k_{2L} & k_{PL} & 2k_{LL} \end{vmatrix}}{|M|} \tag{19}$$

$$\alpha_2 = \frac{\begin{vmatrix} 2k_{11} & -k_1 & k_{1P} & k_{1L} \\ k_{12} & -k_2 & k_{2P} & k_{2L} \\ k_{1P} & -k_P & 2k_{PP} & k_{PL} \\ k_{1L} & -k_L & k_{PL} & 2k_{LL} \end{vmatrix}}{|M|} \tag{20}$$

$$\alpha_P = \frac{\begin{vmatrix} 2k_{11} & k_{12} & -k_1 & k_{1L} \\ k_{12} & 2k_{22} & -k_2 & k_{2L} \\ k_{1P} & k_{2P} & -k_P & k_{PL} \\ k_{1L} & k_{2L} & -k_L & 2k_{LL} \end{vmatrix}}{|M|} \tag{21}$$

$$\alpha_L = \frac{\begin{vmatrix} 2k_{11} & k_{12} & k_{1P} & -k_1 \\ k_{12} & 2k_{22} & k_{2P} & -k_2 \\ k_{1P} & k_{2P} & 2k_{PP} & -k_P \\ k_{1L} & k_{2L} & k_{PL} & -k_L \end{vmatrix}}{|M|} \tag{22}$$

Spatial dynamic illustration

A typical illustration of a simulated spatial and dynamic devlopment of a CCF beech forest with adaptive management control is given in Figures 1., 2. and 3.. The simulation is based on the model developed in Lohmander (2018b). The figures show the positions and sizes of the trees in a forest area of one hectare. The situation in year 35 is found in Figure 1.. Then, the trees grow until year 69 (Figure 2.). Some of the largest trees in Figure 1. are harvested before we come to the situation in Figure 2.. The trees are affected by local competition and the largest trees grow better than the smaller trees. Between year 69 and year 70, timber prices are very high. As a result, most of the large trees in year

69 (Figure 2.) have been harvested before we reach year 70 (Figure 3.). In the simulation model, new, young and small trees are dynamically generated from seeds in random locations.

When the parameters of the harvest control function were optimized, Lohmander (2018b) defined the numerical model with 10 hectares of forest, in the form of 10 independent one-hectare forests. The illustrations in this chapter show the development of one of these one-hectare forests.

It was possible to change the selected number, 10, of hectares in the model, to any other number. Numerical tests showed that the optimized values of the harvest control function parameters usually converge to stable solutions also with rather low number of hectares. Already with 5 hectares, the optimized control function parameter values were almost identical to the optimized control function parameter values obtained with 10 hectares. The excecution time is, however, almost proportional to the number of hectares and every research project has some time limit. For these reasons, it was considered satisfactory to use 10 hectares in the optimizations. Of course, since the speed of computers increases over time, and since future researchers may be interested in even higher parameter precision, future applications of the model may be based on even larger forests.

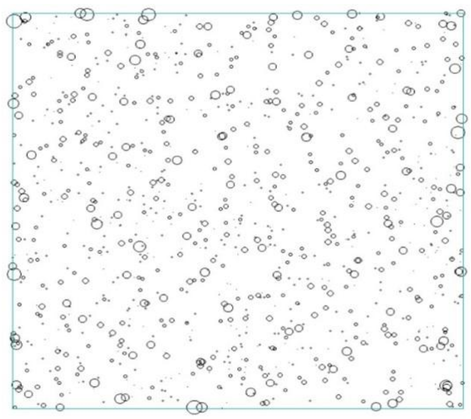

Figure 1. Spatial map of simulated trees in a CCF forest in year 35. The map represents one hectare where the four sides of the square are 100 meters each. The circles represent trees and the diameters of the circles are proportional to the diameters of the trees.

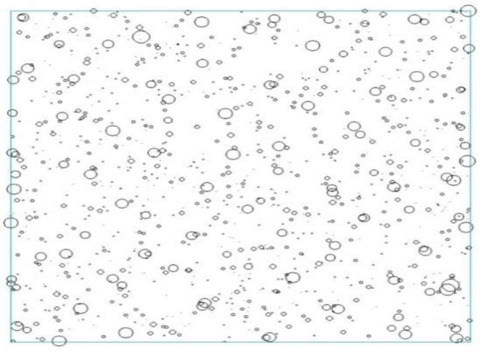

Figure 2. Spatial map of simulated trees in a CCF forest in year 69.

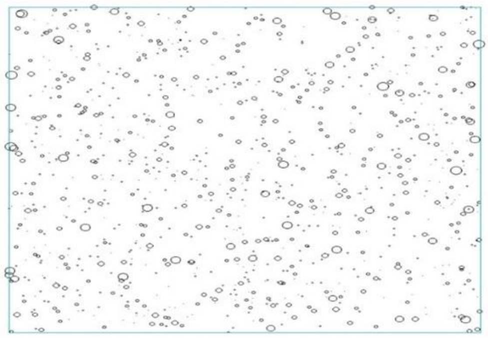

Figure 3. Spatial map of simulated trees in a CCF forest in year 70.

A detailed numerical case

Continuous forest management has earlier usually been optimized at the stand level via deterministic and stochastic dynamic programming and optimal control theory. Lohmander (1986), (1987), (1988) and (1990) and Lohmander and Mohammadi (2008) are such examples. In Lohmander (1992b) and (1992c), optimal control functions are derived via quasi gradient methods in combination with stochastic simulation. In these studies, it has been found that the optimal harvest level is an increasing function of the price and stock levels and that the expected present value is an increasing function of the risk in the price process, as long as we have adaptive harvest decisions and stationary price processes. Now, we may use these general principles as background when we optimize forestry decisions at a more detailed level. Concrete transformations of the earlier findings from stand level optimizations to preliminary principles in forestry with tree level decisions, H1 – H3, will now be suggested. Fundamental production economics principles give the following hypotheses:

H1: In case the market price is equal to the expected price and in case the tree has no competition, then there is a unique limit diameter α_m such that $\alpha_m > 0 \quad m \in \{1,2\}$.

H2: It is economically rational to harvest more during periods with higher prices and less during periods with lower prices. This means that it is optimal to harvest a tree at a smaller diameter than α_m in case the price is very high and to harvest the tree at a larger diameter if the price is very low. Hence: $\alpha_P < 0$.

H3: Trees may have strong or weak competition with neighboring trees. (Strong competition corresponds to a high stock level and weak competition corresponds to a low stock level.) If the competition is weak in a local area, the trees grow well and it is profitable to let them develop to larger diameters than if the competition is strong. If the competition is strong in a local area, it is economically rational to harvest the trees at a smaller diameter. This way, the remaining trees will grow better than under strong competition. Hence: $\alpha_L < 0$.

Now, a particular case with empirical background presented in Lohmander et al. (2017) will be analyzed. The forest and the adaptively controlled stochastic simulation model contain trees of only one species, Norway spruce, in Sweden. However, different trees have different timber qualities. The timber quality of a particular tree, $Q(t,i)$, is high $Q(t,i)=1$ or low $Q(t,i)=0$. Trees that have higher quality also have higher prices per cubic meter. The notation is adjusted to fit the application. The parameters are defined in Table 1.

Table 1. Adaptive control function parameters in the particular numerical case.

Parameter name:	Corresponding parameter in the general model:	Constant in the limit diameter function	Derivative of limit diameter with respect to:	These transformations simplify the analysis:
dlim_0	α_1	Yes		dlim_0 = 0.1x
dlim_c	α_L	No	Local competition	dlim_c = 0.001z
dlim_q		No	Timber quality of the particular tree	
dlim_p	α_P	No	Timber price – expected timber price	dlim_p = 0.001y

In order to make this section easier to follow, the optimization of the parameter dlim_q is excluded. An even more detailed pre-analysis of the problem gave an optimal value of dlim_q. This parameter is included in the finally documented adaptive control function.

The approximating function is $\Phi(.)$. Using the transformations described in Table 1. and treating dlim_q as an already known parameter, we have the approximating function $\Phi = \Phi(x, y, z)$. This function is specified to have this functional form:

$$\Phi = k_x x + k_{xx} x^2 + k_{xy} xy + k_{yyy} y^3 + k_z z + k_{zz} z^2 + k_{xz} xz \quad (23)$$

Multiple regression analysis was used to test the function and to estimate the parameter values. To create the data set, 448 alternative parameter combinations with stochastic full system simulations were used. The approximating function fits the data well. The R2 value exceeds 99.2%. All of the p-values of the estimated parameters of the approximating function obtained values below the 5% significance limit. The highest of these values was 0,037697081. Five of the seven p-values were far below 0,001.

Table 2. Regression statistics

Multiple-R	**0,996060268**
R2	0,992136058
Adjusted R2	0,989761492
Standard error of estimate	14,5934915
Number of observations	448

Table 3. ANOVA

	fg	KvS	MKv	F	p-value for F
Regression	7	11849170,53	1692738,647	7948,24949	0
Residual	441	93919,76743	212,9699942		
Total	448	11943090,3			

Table 4. Parameter estimations

Parameter	Coefficient	Standard error	t-ratio	p-value
Constant	0			
x	92,23657812	1,637862818	56,31520363	1,6304E-203
xx	-15,12419474	0,379080306	-39,89707323	1,9827E-148
xy	-3,971394886	0,168170542	-23,61528261	2,69321E-80
yyy	0,162419153	0,012052529	13,4759396	6,60883E-35
z	-10,87340641	2,854717482	-3,80892557	0,000159361
zz	-1,247730986	0,598599346	-2,084417556	0,037697081
xz	1,272859441	0,561028884	2,268794845	0,023763276

After the regression analysis, we make this approximating function:

$$\Phi = 92.237x - 15.124x^2 - 3.9714xy + 0.16242y^3 - 10.873z - 1.2477z^2 + 1.2729xz \quad (24)$$

In the rest of the analysis, we consider this approximation to be a correct representation of the expected present value per hectare, π. Hence,

$$\pi = 92.237x - 15.124x^2 - 3.9714xy + 0.16242y^3 - 10.873z - 1.2477z^2 + 1.2729xz \quad (25)$$

The first order optimum conditions are:

$$\begin{cases} \dfrac{d\pi}{dx} = 92.237 - 30.248x - 3.9714y + 1.2729z = 0 \\ \dfrac{d\pi}{dy} = -3.9714x + 0.48726y^2 = 0 \\ \dfrac{d\pi}{dz} = -10.873 + 1.2729x - 2.4954z = 0 \end{cases} \quad (26)$$

Note that one of the equations is not linear. Hence, we can not apply the linear equation solution via Cramers rule which was described in the earlier section. We will start with some variable eliminations.

$$\left(\frac{d\pi}{dy} = -3.9714x + 0.48726y^2 = 0 \right) \Rightarrow \left(x = 0.12269y^2 \right) \quad (27)$$

$$\left(\frac{d\pi}{dz} = -10.873 + 1.2729x - 2.4954z = 0 \right) \Rightarrow \left(z = -4.3572 + 0.51010x \right) \quad (28)$$

$$\begin{pmatrix} x = 0.12269y^2 \\ z = -4.3572 + 0.51010x \end{pmatrix} \Rightarrow \left(z = -4.3572 + 0.062584y^2 \right) \quad (29)$$

$$\left(\begin{array}{l} \frac{d\pi}{dx} = 92.237 - 30.248x - 3.9714y + 1.2729z = 0 \\ x = 0.12269y^2 \\ z = -4.3572 + 0.062584y^2 \end{array} \right) \Rightarrow \quad (30)$$

$$\left(\begin{array}{l} 92.237 - 30.248\left(0.12269y^2\right) - 3.9714y \\ +1.2729\left(-4.3572 + 0.062584y^2\right) = 0 \end{array} \right)$$

Now, we have a quadratic equation:

$$-3.6315y^2 - 3.9714y + 86.691 = 0 \quad (31)$$

We follow the standard procedure to solve it.

$$y^2 + 1.0936y - 23.872 = 0 \quad (32)$$

$$y = -\frac{1.0936}{2} \pm \sqrt{\left(\frac{1.0936}{2}\right)^2 + 23.872} \quad (33)$$

$$y = -0.5468 \pm 4.9164 \quad (34)$$

So, two different values of y meet the first order optimum conditions:

$$\left(y_1, y_2\right) = (4.3639, -5.4632) \quad (35)$$

Most likely, one of these solutions represents a maximum and one a minimum. We want to investigate the second order maximum conditions. We have a three-variable quadratic form.

$$\begin{vmatrix} \pi_{xx} & \pi_{xy} & \pi_{xz} \\ \pi_{yx} & \pi_{yy} & \pi_{yz} \\ \pi_{zx} & \pi_{zy} & \pi_{zz} \end{vmatrix} = \begin{vmatrix} -30.248 & -3.9714 & 1.2729 \\ -3.9714 & 0.97452y & 0 \\ 1.2729 & 0 & -2.4954 \end{vmatrix} \quad (36)$$

Note that we find y in one place in the quadratic form. The second order maximum conditions of the three principal minors are:

$$|\pi_{xx}| < 0$$

$$\begin{vmatrix} \pi_{xx} & \pi_{xy} \\ \pi_{yx} & \pi_{yy} \end{vmatrix} > 0 \qquad (37)$$

$$\begin{vmatrix} \pi_{xx} & \pi_{xy} & \pi_{xz} \\ \pi_{yx} & \pi_{yy} & \pi_{yz} \\ \pi_{zx} & \pi_{zy} & \pi_{zz} \end{vmatrix} < 0$$

$$|\pi_{xx}| = |-30.248| = -30.248 < 0 \qquad (38)$$

Observation a:

The first maximum condition is always met.

$$\begin{vmatrix} \pi_{xx} & \pi_{xy} \\ \pi_{yx} & \pi_{yy} \end{vmatrix} = \begin{vmatrix} -30.248 & -3.9714 \\ -3.9714 & 0.97452y \end{vmatrix} > 0 \qquad (39)$$

$$\begin{vmatrix} -30.248 & -3.9714 \\ -3.9714 & 0.97452y \end{vmatrix} = -30.248(0.97452y) - (-3.9714)^2 > 0 \qquad (40)$$

$$-29.477 - 15.772y > 0 \qquad (41)$$

$$y < -0.53506 \qquad (42)$$

Observation b:

The second maximum condition is met if $y < -0.53506$.

$$\begin{vmatrix} \pi_{xx} & \pi_{xy} & \pi_{xz} \\ \pi_{yx} & \pi_{yy} & \pi_{yz} \\ \pi_{zx} & \pi_{zy} & \pi_{zz} \end{vmatrix} = \begin{vmatrix} -30.248 & -3.9714 & 1.2729 \\ -3.9714 & 0.97452y & 0 \\ 1.2729 & 0 & -2.4954 \end{vmatrix} < 0 \qquad (43)$$

$$\begin{vmatrix} \pi_{xx} & \pi_{xy} & \pi_{xz} \\ \pi_{yx} & \pi_{yy} & \pi_{yz} \\ \pi_{zx} & \pi_{zy} & \pi_{zz} \end{vmatrix} = 39.357 + 71.979y < 0 \qquad (44)$$

$$y < -0.54678 \qquad (45)$$

Observation c:

The third maximum condition is met if y<-0.54678 . So, if we want to make sure that we have a maximum, then:

$$\left(\begin{vmatrix} \pi_{xx} & \pi_{xy} \\ \pi_{yx} & \pi_{yy} \end{vmatrix} > 0 \right) \Rightarrow (y < -0.53506) \qquad (46)$$

$$\left(\begin{vmatrix} \pi_{xx} & \pi_{xy} & \pi_{xz} \\ \pi_{yx} & \pi_{yy} & \pi_{yz} \\ \pi_{zx} & \pi_{zy} & \pi_{zz} \end{vmatrix} < 0 \right) \Rightarrow \left(y < -0.54678 \right) \qquad (47)$$

Since the first order optimum conditions are met by $\left(y_1, y_2\right) = (4.3639, -5.4632)$, we now know that $y = -5.4632$ gives a unique maximum. We may mark the maximizing values with stars and also calculate the maximizing values of x and z . $y^* = -5.4632$, $x^* = 0.12269\left(y^*\right)^2 \approx 3.6619$,

$z^* = -4.3572 + 0.062584\left(y^*\right)^2 \approx -2.4893$. This point gives the globally unique maximum:

$$(x^*, y^*, z^*) = (3.6619, -5.4632, -2.4893) \qquad (48)$$

The optimal values of (x^*, y^*, z^*) calculated with more decimals are: x^* = 3.661887475, y^* = -5.463160161, z^* = -2.489293673. If we use these values, we may calculate the expected present value (the unit is kSEK/hectare).

$$\pi^* = \pi(x^*, y^*, z^*) = 195{,}6554283 \qquad (49)$$

Now, it is time to calculate the optimal parameters of the the limit diameter function:

$$\text{dlim\_0} \approx \frac{x^*}{10} \approx 0.366 \qquad (50)$$

$$\text{dlim\_p} \approx \frac{y^*}{1000} \approx -0.00546 \qquad (51)$$

$$\text{dlim\_c} \approx \frac{z^*}{1000} \approx -0.00249 \qquad (52)$$

The optimized, numerically specified version of the limit diameter function is:

$$D_i(t) = 0.366 - 0.00546\left(P(S(i),t) - E\left(P(S(i),t)\right)\right) \qquad (53)$$
$$-.00249\,L(t,i) + 0.0600\,Q(t,i)$$

$D_i(t)$ is given in the unit meters, $P(S(i),t)$ has the unit SEK and $Q(t,i) \in \{0,1\}$, 0 for the second highest and 1 for the highest timber quality, respectively. The estimation and the precise definition of the expression for $L(t,i)$ may be studied in Lohmander et al. (2017). (The details of the particular function $L(t,i)$ have no general interest and they would complicate this text too much to be included here.).

The approximation of the objective function is illustrated in Figures 4., 5. and 6.. It may be noted that the maximum value is approximately 195.7 kSEK/ha in all of the figures. Furthermore, the calculated values of dlim_0, dlim_p and dlim_c seem to be correct and the expected present value function looks strictly concave in the three graphs.

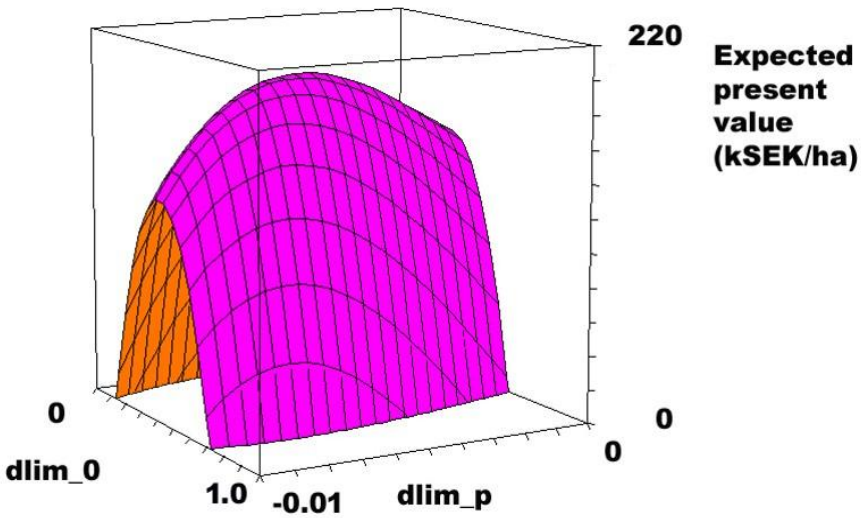

Figure 4. The approximated objective function, the expected present value, as a function of two of the adaptive control function parameters, dlim_0 and dlim_p. The other parameters in the adaptive control function have been given the optimal values. It may be noted that in case the price variations are not used to determine the optimal harvest decisions (dlim_p = 0), then the objective function decreases from approximately 195.7 kSEK/ to 150.6 kSEK/ha. The reduction is 23%. Obviously, it is very important to consider prices when harvest decisions are taken.

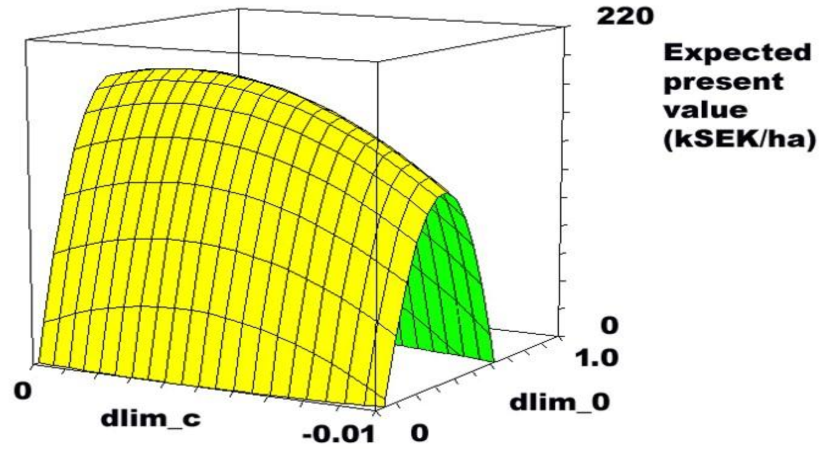

Figure 5. The approximated objective function, the expected present value, as a function of two of the adaptive control function parameters, dlim_0 and dlim_c. The other parameters in the adaptive control function have been given the optimal values. In case local competition is not used to determine the optimal harvest decisions (dlim_c = 0), then the objective function decreases from approximately 195.7 kSEK/ to 188.1 kSEK/ha. The reduction is 3.9 %.

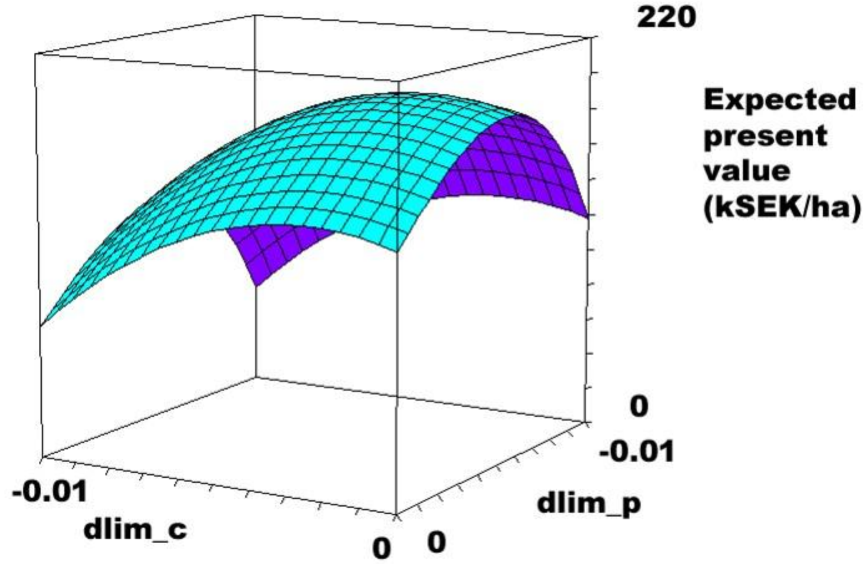

Figure 6. The approximated objective function, the expected present value, as a function of two of the adaptive control function parameters, dlim_c and dlim_p. The other parameters in the adaptive control function have been given the optimal values. In case prices and local competition are not used to determine the optimal harvest decisions (dlim_p = 0 and dlim_c = 0), then the objective function decreases from approximately 195.7 kSEK/ to 140.6 kSEK/ha. The reduction is 28.2 %.

Results

We have considered forests with large numbers of trees of different sizes and species. Prices of different species are stochastic. The total expected present value has been maximized. The problem is solved using an adaptive control function. The parameters of the control function are optimized via the first order optimum conditions of a multivariate polynomial approximation of the objective function. Three general results are:

R1. A tree should be harvested at a smaller diameter if the local competition from other trees is high than if the local competition is low.

R2. A tree should be harvested at a larger diameter if the timber quality is high and not low.

R3. A tree should be harvested at a smaller diameter if the market net price for wood is high and not low.

General observations:

O1. The definitions and results derived from the general function model are relevant as long as the introduced functional forms are relevant. The general model may serve as a tool to be used in general, theoretical analysis and optimization of forest management. In case alternative functional forms can be motivated in particular cases, the analysis should be adjusted in the same way.

O2. The conclusions from the numerical model are dependent on a particular growth function, particular price process and cost function assumptions, timber quality premium figures and many other detailed model assumptions relevant to natural regeneration. With other assumptions, the numerical results will in general be different. However, the general tendencies reported here, R1 – R3, are expected to be valid.

Conclusions

It is possible to find optimal solutions even if the management problems have large numbers of integer variables, nonlinearities and stochastic processes. The introduced and tested methods are quite general and can be applied to many other kinds of problems in other sectors. The present approach makes it possible to determine optimal adaptive control rules and to estimate the economic values of mixed forests with trees in many size classes and of many species. With traditional forest management planning methods, the market price variations, locally relevant competition information, multi-species management options and variations in timber quality are not considered in the optimal way. It is important to make market adapted harvest decisions. If the stochastic price variations are not considered when the harvest decisions are taken, the expected present value is reduced by 23%. As a result, the economic values of optimally managed forests are underestimated via traditional calculation methods.

Discussion

In the introduction, the central question of this study was presented: “What is the best way to sequentially update the information and adaptively determine the management decisions?” Now, we have seen that it is possible to obtain a significant improvement of the expected present value, if the timber market prices and tree sizes are sequentially monitored and harvest decisions are adaptively optimized via the derived optimal control function. The optimal harvest decisions are functions of the observed market prices and the tree sizes. It is quite clear that the earlier studies by Hessenmöller et al. (2018) and by Schütz (2006) suggested forest management decisions that were insensitive to market price changes. In the example presented in this study, the expected present value is reduced by 23% if the stochastic price variations are not considered when the harvest decisions are taken. Obviously, the earlier methods underestimated the economic results and gave less rational management guidelines.

In order to widen the perspective on the presented topic, optimal continuous cover forestry, we should consider model assumptions and limitations. First, as always, we have to be aware that every model is just a model. No model exists that covers every detail of real problems. Mostly, in applied operations research, the choice has been to represent real world problems in ways that suit some standard optimization methods, or, in some way that makes it possible to use simulation. In the first case, with standard optimization methods, it has sometimes been possible to derive solutions that can be proved to be optimal, perhaps even globally optimal. However, the transformations of a real problem to some format needed by some optimization method may have made it questionable if the derived solution really is the optimal solution to the original real problem.

In the second case, when the simulation approach is used, the problem structure and many details of the original problem may be considered and the dynamic model behavior may resemble the real world very well. However, nothing has really been optimized; just simulated. In the present study, a methodology has been suggested and tested that makes it possible to really maintain most details of the structure of the real problem and still obtain an optimized solution. This may be considered a general improvement obtained via this approach.

Some of the model properties may, of course, be questioned such as the use of expected present value as the objective function. Expected value analysis is relevant under the assumption of risk neutrality. In applications, other attitudes towards risk may prevail. However, with perfectly diversified portfolios, risk neutrality can be considered relevant, even if the investors are not risk neutral. Present value maximization can be shown to be optimal if the capital market is perfect, in the sense that it is possible to borrow and save arbitrarily selected amounts of money with an exogenously determined rate of interest. Clearly, this assumption is seldom fully met in real applications. Nevertheless, present value maximization is standard in almost all economic production theory.

There are many assumptions concerning functional forms and parameter values in the forest growth equations, timber price and harvest cost equations and stochastic market price equations that may be discussed and questioned. The best available empirical and statistical methods have been used to derive these equations. Unfortunately, it is impossible to present and discuss all of these many details within the framework of one article.

Two suggestions for future research are the following:

- The method should be used in combination with new empirical forest and market data from different regions. It is possible to derive optimal forest management policies to be used for alternative combinations of tree species in different parts of the world.
- Another ambition of this paper has been to highlight the methodological approach that can also be useful in other kinds of applied problems, and not only in forestry. Hopefully, the reader will consider new applications of the general methodology in the near future.

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