**Journal of Algorithms and Computation**

journal homepage: http://jac.ut.ac.ir

# $k$-Total difference cordial graphs

R. Ponraj*[1], S.Yesu Doss Philip†[2] and R. Kala‡[3]

[1]Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627412, Tamilnadu, India.
[2]Research Scholar,Department of Mathematics, Manonmaniam sundarnar university, Abishekapatti, Tirunelveli-627 012, Tamilnadu, India.
[3]Department of Mathematics Manonmaniam sundarnar university, Abishekapatti, Tirunelveli-627 012, Tamilnadu, India.

## ABSTRACT

Let $G$ be a graph. Let $f : V(G) \to \{0, 1, 2, \ldots, k-1\}$ be a map where $k \in \mathbb{N}$ and $k > 1$. For each edge $uv$, assign the label $|f(u) - f(v)|$. $f$ is called a $k$-total difference cordial labeling of $G$ if $|t_{df}(i) - t_{df}(j)| \leq 1$, $i, j \in \{0, 1, 2, \ldots, k-1\}$ where $t_{df}(x)$ denotes the total number of vertices and the edges labeled with $x$.A graph with admits a $k$-total difference cordial labeling is called a $k$-total difference cordial graphs. We investigate $k$-total difference cordial labeling of some graphs and study the 3-total difference cordial labeling behaviour of star,bistar,complete bipartiate graph,comb,wheel,helm,armed crown etc.

*Keyword:* Star, Bistar, Complete bipartiate,Comb, Wheel, Helm, Armed Crown

## ARTICLE INFO

*Article history:*
Received 14, May 2018
Received in revised form 3, April 2019
Accepted 8 May 2019
Available online 01, June 2019

AMS subject Classification: 05C78.

*Corresponding author: R. Ponraj Email: ponrajmaths@gmail.com
†jesuphilip09@gmail.com
‡karthipyi91@yahoo.co.in

# 1 Introduction

[1] introduced notion of cordial labeling of graphs. The cocept of $k$-difference cordial graph was introduced in [4] . Recently Ponraj etal [5] has been introduced the concept of $k$-total prime cordial graph . Motivated by this, we introduce k-total difference cordial labeling of graphs. Also we prove that every graph is a subgraph of a connected $k$-total differnce cordial graphs and investigate 3-total prime cordial labeling of sevarel graphs like path,star, bistar, complete bipartite graph etc .

# 2 $k$-Total difference cordial labeling

**Definition 2.1**Let $G$ be a graph. Let $f : V(G) \to \{0, 1, 2, \ldots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge $uv$, assign the label $|f(u) - f(v)|$. $f$ is called $k$-total difference cordial labeling of $G$ if $|t_{df}(i) - t_{df}(j)| \leq 1$, $i, j \in \{0, 1, 2, \ldots, k-1\}$ where $t_{df}(x)$ denotes the total number of vertices and the edges labelled with $x$.A graph with a k-total difference cordial labeling is called k-total difference cordial graph.
**Remark.** 2- total difference cordial graph is 2-total product cordial graph.

# 3 Preliminaries

**Definition 3.1** The corona of $G_1$ with $G_2$,$G_1 \odot G_2$ is the graph obtained by taking one copy of $G_2$ and $p_1$ copies of $G_2$ and joining the $i^{th}$ vertex of $G_1$ with an edge to every vertex in the $i^{th}$ copy of $G_2$.
**Definition 3.2** Armed crown $AC_n$ is the graph obtained from the cycle $C_n : u_1u_2 \ldots u_nu_1$ with $V(AC_n) = V(C_n) \cup \{v_i, w_i : 1 \leq i \leq n\}$ and $E(AC_n) = E(C_n) \cup \{u_iv_i, v_iw_i : 1 \leq i \leq n\}$.
**Definition 3.3**$C_n(m)$ denotes the one point union of $m$ copies of cycle $C_n$.
**Definition 3.4**An edge $x = uv$ of $G$ is said to be subdivided if it is replaced by the edges $uw$ and $wv$ where $w$ is a vertex not in $V(G)$.If every edge of $G$ is subdivided,the resulting graph is called the subdivision graph $S(G)$.

# 4 Main Results

**Theorem 4.1.**Let $G$ be a $(p, q)$graph.Then $G$ is a subgraph of a connected $k$-total different cordial graph.

*Proof.* Consider the graph $K_p$.Let $u_1, u_2, \ldots, u_n$ be the vertices of $K_p$.Let $m = p + \binom{p}{2}$ and Take $r = \begin{cases} \frac{m}{2}, & \text{if } m \text{ is even} \\ \frac{m-1}{2}, & \text{if } m \text{ is odd} \end{cases}$ . Consider $k-1$ copies of the star $K_{1,r}$.Let $K^i_{1,r}$ be the $i^{th}$ copy of the star $K_{1,r}$ and $V(K^i_{1,r})=\{u^i, v^i_j : 1 \leq j \leq r\}$,$E(K^i_{1,r})=\{u^iv^i_j : 1 \leq i \leq r\}$.The super graph $G^*$ is obtained from $K_p$ by identify $u_i$ with $u^i, 1 \leq i \leq k-1$ . We now assign the label to the vertices of $G^*$ as given below.Assign the label 0 to $u_1, u_2 \ldots, u_n$.Next assign the label $i$ to the vetices $v^i_1, v^i_2, \ldots, v^i_r$,$1 \leq i \leq k-1$.Clearly $t_{df}(0) = t_{df}(1) =$

.....$t_{df}(k-1) = m$ or $t_{df}(0) = m$,$t_{df}(1) = t_{df}(2) = .....t_{df}(k-1) = m-1$ according as $m$ is even or odd. □

**Theorem 4.2.**If $n \equiv 0(\mod k)$ then the star $K_{1,n}$ is $k$-total difference cordial.

*Proof.* Let $V(K_{1,n})$=$\{u, v_i : 1 \leq i \leq n\}$ and $E(K_1, n)$=$\{uv_i : 1 \leq i \leq n\}$. Let $n = kt$,$t \in N$ Assign the label 0 to the central vertex $u$ .We now move to the pendent vertices.Assign the label 0 to the first $t$ pendent vertices $v_1, v_2, \ldots, v_t$. Now assign the label 1 to the next $t$ pendent vertices $v_{t+1}, v_{t+2}, \ldots, v_{2t}$. Next assign the label to the pendent $v_{2t+1}, v_{2t+2}, \ldots, v_{3t}$.We now assign the label 2 to the next $t$ pendent vertices and so on.In this process,the vertices $v_{(k-1)t+1} \ldots v_{(k-1)t+t}$ receive the label $k-1$.Clearly $t_{df}(0) = t+1$,$t_{df}(1) = t_{df}(2) = t_{df}(3) = \ldots = t_{df}(k-1) = t$. □

**Theorem 4.3.** The path $P_n$ is 3-total difference cordial iff $n \neq 2$

*Proof.* Let $P_n$ be the path $u_1, u_2 \ldots, u_n$.
**Case 1.** $n \in \{1, 3, 4, 5, 6, 7, 8\}$.
3-total difference cordial labeling is given in table 1

| n | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ | $u_6$ | $u_7$ | $u_8$ |
|---|---|---|---|---|---|---|---|---|
| 0 | | | | | | | | |
| 1 | 0 | 2 | | | | | | |
| 1 | 0 | 2 | 0 | | | | | |
| 0 | 2 | 2 | 1 | 1 | | | | |
| 0 | 2 | 2 | 1 | 1 | 0 | | | |
| 0 | 2 | 2 | 1 | 1 | 0 | 2 | | |
| 0 | 2 | 2 | 2 | 2 | 1 | 0 | 1 | |

Table 1:

**Case 2.** $n = 2$.
Suppose $f$ is a 3-total difference cordial labeling of $P_2$.Then $t_{df}(0) = t_{df}(1) = t_{df}(2) = 1$.To get the label 2,2 must be the vertex label.Without loss of generality $f(u_1) = 2$.
**Subcase 1.** $f(u_2) = 0$.
Here,$t_{df}(1) = 0$,a contradiction.
**Subcase 2.** $f(u_2) = 1$.
In this case $t_{df}(0) = 0$ a contradiction.
**Subcase 3.** $f(u_2) = 2$.
Here,$t_{df}(1) = 0$,a contradiction.
**Case 3.** $n = 3t, t > 2$
**Subcase 1.** $n = 3t$,$t$ is odd.
Assign the label 1 to the vertices $u_1, u_2, \ldots, u_t$ and 2 to the vertices $u_2$,$u_4$,... ,$u_{t-1}$.
Next assign the label 1 to the vertices $u_{t+1}, u_{t+2}, \ldots, u_{\frac{3t+1}{2}}$.
Next assign the label 2 to the vertices $u_{\frac{3t+1}{2}}, u_{\frac{3t+3}{2}}, \ldots, u_n$.Clearly $t_{df}(0) = 2t-1$,$t_{df}(1) = 2t$,$t_{df}(2) = 2t$.

**Subcase 2.** $n = 3t$,$t$ is even. Assign the label 1 to the vertices $u_1, u_3, \ldots, u_{t-1}$,$u_{t+1}$ and 2 to the vertices $u_2, u_4, \ldots, u_t$. Next assign the label1to the vertices $u_{t+2}, u_{t+3}, \ldots, u_{\frac{3t+2}{2}}$. We now assign the label 2 to the next consequent vertices $u_{\frac{3t}{2}}, u_{\frac{3t+1}{2}}, \ldots, u_{3t-2}$.
Finally assign the 0 to the vertices $u_{3t+1}$ and $u_{3t}$. Clearly $t_{df}(0) = 2t-1$,$t_{df}(1) = 2t$,$t_{df}(2) = 2t$.
**Case 4.** $n = 3t + 1, t > 2$.
**Subcase 1.** $t$ is odd.
As in subcase 1 of case 3 as the label to the vertices $u_1, u_2, \ldots, u_{n-1}$.
Finally assign the label 0 to the vertex $u_n$.Clearly $t_{df}(0) = 2t$,$t_{df}(1) = 2t$,$t_{df}(2) = 2t + 1$.
**Subcase 2.** $t$ is even.
Let $f$ be the 3-difference cordial labels of subcase 2 of case 3. Define $g(u_{i+1}) = f(u_i)$,$1 \leq i \leq n$ and $g(u_1) = 0$.Clearly $t_{df}(0) = 2t$,$t_{df}(1) = 2t + 1$,$t_{df}(2) = 2t$.
**Case 5.** $n = 3t + 2.t > 2$
**Subcase 1.** $t$ is odd.
Let $f$ be the 3-difference cordial labels of subcase 2 of case 4 Define $g(u_{i+1}) = f(u_i)$,$1 \leq i \leq n$ and $g(u_1) = 1$.Clearly $t_{df}(0) = 2t + 1$,$t_{df}(1) = 2t + 1$,$t_{df}(2) = 2t$.
**Subcase 2.** $t$ is even.
As in subcase 2 case 4 assign the label to the vertices $u_1, u_2, \ldots, u_{n-1}$. Finally assign the label 2 to the last vetex $u_n$. Clearly $t_{df}(0) = 2t + 1$,$t_{df}(1) = 2t + 1$,$t_{df}(2) = 2$. □

**Theorem 4.4.**The bistar $B_{n,n}$ is 3-total different cordial iff $n \equiv 1, 2(\mod 3)$.

*Proof.* Let $V(B_{n,n}) = \{u, v, u_i, v_i : 1 \leq i \leq n\}$ and $E(B_{n,n})$=$\{uu_i, vv_i, uv : 1 \leq i \leq n\}$. Note that $B_{n,n}$ has $2n + 2$ vertices and $2n + 1$ edges.
**Case 1.** $n \equiv (1 \mod 3)$.

Let $n = 3t + 1$. Assign the label 0 to the central vertices $u$ and $v$. We now move to the pendent vertices $u_i$. Assign the label 1 to the vertices $u_1, u_2, \ldots, u_{2t}, u_{2t+1}$ and 0 to the vertices $u_{2t+2}, u_{2t+3}, \ldots, u_n$. Now we consider the vertices $v_1, v_2, \ldots, v_n$. Assign the label 2 to the vertices $v_1, v_2, \ldots, v_{2t+1}$ and 0 to the vertices $v_{2t+2}, v_{2t+3}, \ldots, v_n$.
**Case 2.** $n \equiv 2(\mod 3)$.
Let $n = 3t + 2$,$t \in N$.As in case 1 assign the label to the vertices $u$, $v$, $u_i$,$v_i$ $(1 \leq i \leq n-1)$.Finally assign the label 1 and 2 respectively to the vertices $u_n$ and $v_n$.The table given below establish that this vertex labeling pattern is a 3 total difference cordial labeling.

| Values of n | $t_{df}(0)$ | $t_{df}(1)$ | $t_{df}(2)$ |
|---|---|---|---|
| $3t + 1$ | $4t + 3$ | $4t + 2$ | $4t + 2$ |
| $3t + 2$ | $4t + 3$ | $4t + 4$ | $4t + 4$ |

Table 2:

**Case 3.** $n \equiv 0( \pmod 3)$.
Let $n = 3t$, $t \in \mathbb{N}$.Suppose $f$ is a 3- total difference cordial labeling.This implies $t_{df}(0) = t_{df}(1) = t_{df}(2) = 3t + 1$.

**Subcase 1.** $f(u) = f(v) = 0$. Clearly to get the edge label 0,the pendent vertices should be received the label 0.Since the edge $uv$ receive the label 0,We have 3 receive the label 0.That is the edge $uv$ together with the vertices $u$ and $v$.We need remain $3t-2$,0 labels.For the odd values of $t$,$3t-2$ is odd.So we can not label $\frac{3t-2}{2}$ vertices by 0.For the even values of $t$,assume $\frac{3t-2}{2}$ vertices of $u_1, u_2, \ldots, u_n$ is labelled by 0.In this case $t_{df}(2) = \frac{3t-2}{2} \leq 3t+1$ a contradiction.
**Subcase 2.** $f(u) = 0, f(v) = 1$. To get the edge label 2,2 should be label of the vertices $u_i$.Therefore the sum label 2 of the vertices and corresponding edge label is $3t+1$,a contradiction is odd.
**Subcase 3.** $f(u) = 0, f(v) = 2$ To get the edge label 1,0 and 1 are labels of adjacent vertices (or) 2 and 1 are the labels of adjacent vertices.Therefore the sum of label 1 of the vertices $u_i$ and corresponding edge label is $3t+1$ or the sum of label 1 of $u_i$ with corresponding edge label and label 1 of vertices $v_i$ with corresponding edge label is $3t+1$, a contradiction. □

**Theorem 4.5** The complete bipartite graph $K_{2,n}$ is 3-total difference cordial.

*Proof.* Let $V_1 = \{u, v\}$ and $V_2 = \{u_1, u_2 \ldots u_n\}$ where $(V_1, V_2)$ is the bipartition of $K_{2,n}$.We now give the vertex labeling.Assign the label 1 and 2 respectively to the vertices $u$ and $v$ of $V_1$.Next assign the label 0 to all the vertices $u_1, u_2, \ldots, u_n$ of $V_2$.It is easy to verify that,$t_{df}(1) = t_{df}(2) = n+1$ and $t_{df}(0) = n$ □

**Theorem 4.6** All combs are 3-total difference cordial.

*Proof.* Let $P_n \odot K_1$ be the comb with $P_n = u_1u_2\ldots u_n$ and $V(P_n \odot K_1) = V(P_n) \cup \{v_i : 1 \leq i \leq u\}$ and $E(P_n \odot K_1) = E(P_n) \cup \{u_iv_i : 1 \leq i \leq u\}$. clearly $|V(P_n \odot K_1)| + |E(P_n \odot K_1)| = 4n-1$
**Case 1.** $n \equiv 0(mod 3)$.
Let $n = 3t$,$t \in N$.Assign the label 2 to the all the path vertices $u_1, u_2, \ldots, u_n$.We now move to the pendent vertices.Assign the label 2 to the pendent vertices $v_1, v_2, \ldots, u_t$.Next assign the label 1 to the remaining pendent vertices $v_{t+1}, v_{t+2} \ldots v_{3t}$.
**Case 2.** $n \equiv 2(mod 3)$.
Let $n = 3t+2$.In this case assign the label 2 to the all the path vertices and to the pendent vertices $v_1, v_2, \ldots, v_{t+1}$. Next assign the label 1 to the remaining pendent vertices.The table given below shows that this labeling $f$ is a 3-total difference cordial labels.

| Values of n | $t_{df}(0)$ | $t_{df}(1)$ | $t_{df}(2)$ |
|---|---|---|---|
| $3t$ | $4t-1$ | $4t$ | $4t$ |
| $3t+2$ | $4t+2$ | $4t+2$ | $4t+3$ |

Table 3:

□

**Theorem 4.7** All Wheels are 3-total difference cordial.

*Proof.* Let $W_n = C_n + K_1$ where $C_n$ is the cycle $u_1u_2 \dots u_nu_1$ and $V(K_1) = \{u\}$.Assign the label 1 to the central vertex $u$ and assign the label 2 to the all the rim vertices $u_i(1 \leq i \leq n)$.Clearly $t_{df}(1) = n+1$ and $t_{df}(0) = t_{df}(2) = n$.Hence $W_n$ is 3-total difference cordial. □

**Theorem 4.8** Helms $H_n$ is 3-total difference cordial.

*Proof.* Helm $H_n$ is obtained from the wheel $W_n = C_n+K_1$ where $C_n$ is the cycle $u_1u_2 \dots u_n$ and $V(K_1) = \{u\}$ by attaching pendent edges to the rim vertices.Let $v_1, v_2, \dots, v_n$ be the pendent vertices adjacent to $u_1, u_2, \dots, u_n$ respectively.Assign label to the vertices $u$ and $u_i$ as in theorem 4.7.
**Case 1.** $n \equiv 0 \pmod 3$.
Let $n = 3t$,$t \in N$.Assign the label 0 to the vertices $u_1, u_4, \dots, u_{3t-2}$ and 1 to the vertices $u_2, u_5, \dots, u_{3t-1}$ and 2 to the vertices $u_3, u_6, \dots, u_{3t}$
**Case 2.** $n \equiv 1 \pmod 3$.
Let $n = 3t + 1$,$t \in N$.Assign the label to the vertices $u_1, u_2, \dots, u_{3t}$ as in case (1).Next assign the label 0 to the vertex $u_{3t+1}$.
**Case 3.** $n \equiv 2 \pmod 3$.
Let $n = 3t + 2$,$t \in N$.In this case,assign the label to the vertices $u_1, u_2, \dots, u_{3t}$,$u_{3t+1}$ as in case 2.Finally assign the label 1 to the vertex $u_{3t+2}$.
The table given below shows that this labeling $f$ is a 3-total difference cordial labelling of $H_n$.

| Values of n | $t_{df}(0)$ | $t_{df}(1)$ | $t_{df}(2)$ |
|---|---|---|---|
| $3t$ | $4t$ | $4t + 1$ | $4t$ |
| $3t + 1$ | $4t + 1$ | $4t + 1$ | $4t$ |
| $3t + 2$ | $4t + 2$ | $4t + 1$ | $4t + 1$ |

Table 4:

□

**Theorem 4.9**$AC_n$ is 3-Total difference cordial for all $n \geq 3$

*Proof.* Clearly $AC_n$ has 3 vertices and $3n$ edges.Assign the label 2 to the all the cycle vertices $u_1u_2 \dots u_n$.Neext assign the label 2 to the all the vertices with degree 2.That is assign the label 2 to the vertices $v_1, v_2, \dots, v_n$.Finally assign the label 1 to all the pendent vertices $w_1w_2 \dots w_n$.It is easy to verify that $t_{df}(0) = t_{df}(1) = t_{df}(2) = 2n$. □

Any star is $S(K_{1,n})$-total difference cordial.
Let $V(S(K_{1,n})) = \{u, u_i, v_i : 1 \leq i \leq n\}$ and
$E(S(K_{1,n}))$=$\{uu_i, u_iv_i : 1 \leq i \leq n\}$.
**Case 1.** $n = 3t$.
Assign the label $o$ to $u$,Next assign 0 the vertices $u_1, u_2, \dots, u_{2t}$ and 2 to $u_{2t+1}, u_{2t+2}, \dots, u_{3t}$.
Now consider the pendent vertices $v_1, v_2, \dots, v_m$.
Assign the label 2 to the vertices $v_1, v_2, \dots, v_m$.Finally assign the label 1 to the every

pendent vetices $v_{t+1}, v_{t+2}, \ldots, v_{2t}, \ldots, v_{3t}$.Clearly $t_{df}(0) = 4t+1$,$t_{df}(1) = 4t$,$t_{df}(2) = 4t$.

**Case 2.** $m = 3t + 1$.
Assign the label to the vertices $u, u_i, v_i$ $1 \le i \le 3t$ as in case 1.Finally assign the label 2 and 1 respectively to the vertices $u_n$ and $v_n$. Clearly $t_{df}(0) = 4t+1$,$t_{df}(1) = 4t+2$,$t_{df}(2) = 4t+2$.
**Case 3.** $m = 3t + 2$.
As in case 1 assign the label to the vertices $u, u_i, v_i$ $1 \le i \le 3t$.Finally assign the label 1,2 and 0 respectively to the vertices $u_{3t+1}$,$u_{3t+2}$ and $v_{3t+1}$ and $v_{3t+2}$. Clearly $t_{df}(0) = 4t+3$,$t_{df}(1) = 4t+3$,$t_{df}(2) = 4t+3$.
**Theorem 4.10** $C_4(m)$ is 3-total difference cordial for all even values of $m$.

*Proof.* Let $C_4 : u_1^i u_2^i u_3^i u_4^i u_1^i$ be the $i^t h$ copy of the cycle in $C_4^m$ and $u = u_1^1 = u_1^2 = \ldots = u_1^m$.Assign the label 0 to the central vertex $u$.Next assign the label 2 to the vertices $u_2^i$,$u_3^i$,$u_4^i$, $1 \le i \le \frac{m-2}{2}$.We now assign the label 1 to the vertices $u_2^i$,$u_3^i$,$u_4^i$, $\frac{m}{2} \le i \le m-2$.Finally assign the label $0, 1, 1, 0, 2$ and 2 respectively to the vertex $u_2^{m-1}$,$u_3^{m-1}$,$u_4^{m-1}$,$u_2^m$,$u_3^m$ and $u_4^t$.The tabel is establish that this labelling $f$ is a 3-total difference cordial labelling.

| Values of t | $t_{df}(0)$ | $t_{df}(1)$ | $t_{df}(2)$ |
|---|---|---|---|
| 6r | 14r+1 | 14r | 14r |
| 6r+2 | 14r+5 | 14r+5 | 14r+5 |
| 6r+4 | 14r+9 | 14r+10 | 14r+9 |

Table 5:

□

**Theorem 4.11**The subdivision of bistar $B_{n,n}$,$S(B_{n,n})$ is 3-total different cordial for all $n$.

*Proof.* Let $V(S(B_{n,n})) = \{u, w, v, u_i, v_i, x_i, y_i : (1 \le i \le n)\}$ and
$E(S(B_{n,n}))$ =$\{uu_i, u_ix_i, uw, wv, vv_i, v_iy_i : 1 \le i \le n\}$.

**Case 1.** $u = 3t$.
Assign the label 0 to the vertices $u$ and $v$. We now assign the label 0 to the vertices $u_1, u_2 \ldots u_{2t}$ and $v_1, v_2, \ldots, v_{2t}$. Now assign the label 2 to the vertices $u_{2t+1}, u_{2t+2}, \ldots, u_{3t}, u_{2t+1}, u_{2t+2}, \ldots, u_{3t}$.Assign the label 2 to the vertices $x_1, x_2, \ldots, x_t$ and $y_2, y_3, \ldots, y_t$.Next assign the label 1 to $y_1, y_{t+1}, \ldots, y_{2t}, \ldots, y_{3t}$.
**Case 2.** $n = 3t + 1$,$t \in N$.
As in case 1 assign the label to the vertices $u$, $v$,$w$,$u_i$,$v_i$,$x_i$,$y_i$ $(1 \le i \le n-1)$.Finally assign the label 2,2,0 and 1 respectively to the vertices $u_n, x_n, v_n$, and $y_n$.
**Case 3.** $n = 3t + 2$,$t \in N$.
Assign the label to the vertices $u$, $v$,$w$,$u_i$,$v_i$,$x_i$,$y_i$ $(1 \le i \le n-1)$ as in case 2 .Finally assign the label 2,0,1 and 0 respectively to the vertices $x_n, u_n, v_n$, and $y_n$.
The table given below establish that this vertex labeling pattern is a 3 total difference cordial labeling.

| Values of t | $t_{df}(0)$ | $t_{df}(1)$ | $t_{df}(2)$ |
|---|---|---|---|
| 3t | 8t+2 | 8t+2 | 8t+1 |
| 3t+1 | 8t+4 | 8t+4 | 8t+5 |
| 3t+2 | 8t+7 | 8t+7 | 8t+7 |

Table 6:

□

**Theorem 4.12** $P_n \odot 2K_1$ is 3-total difference cordial for all $n$

*Proof.* .
Let $P_n$ be the path $u_1, u_2, \ldots, u_n$.Let $v_i$,$w_i$ be the pendent vertices adjacent to $u_i$ $(1 \leq i \leq n)$.We divide the proof into two cases.

**Case 1.** $n$ is even..
Assign the label 0 to all the path vertices $u_1$,$u_2$, ..., $u_n$.Next we consider the pendent vertices.Assign the label 1 to the vertices $v_1$,$v_2$, ..., $v_{\frac{n}{2}}$, $w_1$,$w_2$, ..., $w_{\frac{n}{2}}$ and 2 to the vertices $v_{\frac{n}{2}+1}$,$v_{\frac{n}{2}+2}$, ..., $v_n$,$w_{\frac{n}{2}+1}$,$w_{\frac{n}{2}+2}$, ..., $w_n$.
**Case 2.** $n$ is odd.
Assign the label to the vertices $u_i$,$v_i$,$w_i$ $(1 \leq i \leq n-1)$ as in case 1.Finally assign the label 0,1 and 2 respectively to the vertices $u_n$,$v_n$,$w_n$.
Since $t_{fd}(0) = 2n - 1$,$t_{fd}(1) = t_{fd}(2) = 2n$,this labeling pattern is a 3-total difference cordial labeling.

□

# References


[1] Cahit, I., Cordial graphs:A weaker version of graceful and harmonious graphs, *Ars Combinatoria*, **23**(1987), 201-207.

[2] Gallian, J.A., A Dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, **19** (2017) #Ds6.

[3] Harary, F., Graph theory, *Addision wesley*, New Delhi (1969).

[4] Ponraj, R., Adaickalam M.Maria, and Kala R., $k$-difference cordial labeling of graphs, *International J.Math.combin*, **2**(2016), 121-131.

[5] Ponraj, R., Maruthamani J., and Kala, R., $k$-total prime cordial labeling of graphs, *Journal of Algorithms and Combutation*, **50**(2018), 143-149.