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# Parametric study of nonlinear buckling capacity of short cylinders with Hemispherical heads under hydrostatic pressure

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ABSTRACT

This study investigates the buckling behavior of short cylindrical shells with hemispherical heads subjected to hydrostatic pressure. It is assumed that the length of the cylindrical part is smaller than or equal to its diameter while its material may be different from that of hemispherical heads. Finite element analysis was used to seek out the effect of geometric parameters such as thickness, length, and volume of the tank on the ultimate buckling load. Results indicate that the buckling load is directly proportional with the thickness and inversely proportional with the volume of the vessel. A close examination of the buckling modes reveals that under uniform hydrostatic pressure, the cylindrical part undergoes the most critical deformation compared with its hemispherical heads. This behavior was observed for the two loading cases of (a), a hydrostatic pressure applied to the whole structure and (b), the hydrostatic pressure was only applied to the cylindrical part of the vessel.

## 1. Introduction

Shell, plate and beam are important structures in engineering [1-13]. A shell is a type of structural element which is characterized by its geometry, being a three-dimensional solid whose thickness is very small when compared with other dimensions [9, 14-19]. Thin structural shells are used to reduce the overall weight and increase the buckling load to weight ratio. Shells are structures whose thicknesses are small compared with the other dimensions and unlike the plates, they have an initial curvature. As thin structures, shells have many applications from water and oil tanks, pipelines, silos, wind turbine towers, to nanotubes. For this reason, the application of these structures becomes very important. Based on this need, many researchers have studied shell like tanks, including cylindrical, spherical and conical tanks, using finite element analysis. Their analytical and numerical results have shown good agreements with the experimental data. Consequently, numerical studies such as finite element analysis have proved to be reliable methods for predicting the buckling behavior of such structures.

On this basis, Galletly and Machut [20] presented a code for predicting the failure of torispherical shells under internal pressure due to buckling. Błachut and Wang [21], studied the buckling behavior of mild steel barreled shells under external hydrostatic pressure. They obtained failure loads using BOSOR5 and ABQUES. Jasion and Magnucki [22] investigated elastic buckling of barreled shells of revolution with constant mass and volume under external pressure. They used the finite element method (FEM) to present the buckling loads for a family of shells. In another work [23], they calculated the elastic buckling load of clothoidal–spherical

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shells and the related buckling modes under external pressure using ANSYS finite element commercial software. Wang *et al.* [24] used finite element analysis to determine the buckling loads of thin-walled torispherical heads in a residential electric water heater tank and compared their results with the laboratory buckling loads. Portela and Godoy [25] presented an experimental – computationa lstrategy for evaluating the buckling loads of steel tanks with domed roofs subjected to wind pressure. Błachut [26] utilized a numerical method along with experimental results to determine the elastic and elastic-perfectly plastic buckling loads of knuckle troispherical shells subjected to external pressure. In another study, Błachut and Vu [27] used ABAQUS finite element commercial software to obtain the burst pressure of shallow spherical caps and troisphers under uniform static pressure. The effect of geometric and material properties on the elastic-plastic buckling loads of torispherical shells under internal pressure was investigated by Galletly and Radhamohan [28]. Evkin *et al*. [29] presented an asymptotic solution for buckling loads of spherical shells subjected to external pressure and compared their results with those of ANSYS commercial software. Athiannan and Palaninathan [30] experimentally studied the buckling behavior of cylindrical shells subjected to axial and transverse shear loads. Bennur *et al.* [31] presented a 2-D static stress analysis using ANSYS. Nonlinear buckling behavior of spherical shells was studied by Hutchinson and Thompson [32]. Magnucki and Jasion [33] proposed an analytical model for pre-buckling and elastic buckling of barreled shells with constant thickness, made of a homogeneous and isotropic material, subjected to uniform radial pressure. Błachut [34] performed a comparison between the experimental and numerical results for buckling of cylindrical steel shells under external load. Niezgodzinski and Swiniarski [35] used the finite element method to investigate the static and dynamic stability of thin-walled spherical shells under external pressure. A series of experimental tests on the buckling of torispherical shells under internal pressure were conducted by Adachi and Benicek [36]. Additional work on the buckling behavior of the shells, by other researchers, can be found in Refs. [37-50].

The literature survey performed by the authors indicates that the studies performed on the buckling behavior of cylindrical shells with hemispherical heads are very limited. This may be due to the complexity of the resulting governing equations in such structures which leads to a lack of an analytical solution. For this reason, many researchers have used the finite element method (available in many software programs) as an alternative method to seek out a solution. Following this idea, the present work investigates the buckling behavior of short cylindrical shells with two hemispherical heads, subjected to uniform hydrostatic pressure. It is worth to mention that in this analysis, the material properties of the cylindrical shell and its hemispherical heads are allowed to be different.

## 2. Geometry, materials, and method

The short tank of interest is made of a cylindrical shell with two hemispherical heads on both sides, as shown in Fig. 1. The radius, thickness, and length of the cylindrical part are r, h, and l, respectively. Both shells (hemispherical heads and cylindrical body) are made of homogeneous and isotropic materials. The material property of the cylindrical part is allowed to be different from that of the hemispherical heads. The length of the cylindrical shell is considered to be smaller than or equal to its diameter. In other words,

$$l \leq 2r \tag{1}$$

Additionally, using the dimensions shown in Fig. (1), one can write,

$$H = l + 2r \tag{2}$$

The minimum total potential energy principle is used to derive the equilibrium equation of the tank. The change in potential energy due to the hydrostatic load can be written as;

$$P_1 = p\Delta V \tag{3}$$

where $\Delta V$ is the change in volume of the tank (vessel) and $P$ is the hydrostatic pressure.

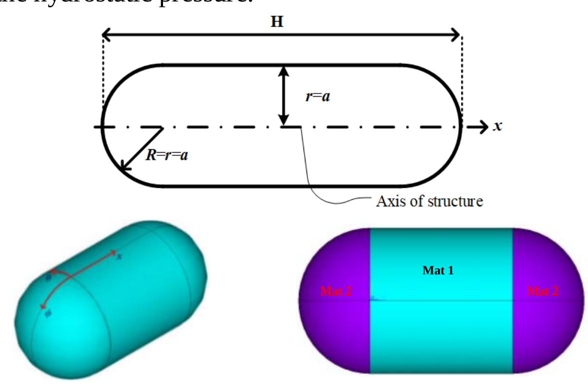



**Fig. 1**. Geometry and material of the tank.

Additionally, work down by the principle stresses on the strains can be written as;

$$\begin{aligned} &\text{Cylindrical shell; } P_{2C} = \iint \begin{bmatrix} N_1\varepsilon_1 \\ +N_2\varepsilon_2 \end{bmatrix} a\,dx\,d\theta \\ &\text{Spherical shell; } P_{2S} = \iint \begin{bmatrix} N_1\varepsilon_1 \\ +N_2\varepsilon_2 \end{bmatrix} R^2 \sin\varphi\, d\varphi\, d\theta \end{aligned} \tag{4}$$

where $\varepsilon_i$ and $N_i$ are the strains and membrane stresses, respectively. In addition, the strain energy due to displacements for cylindrical and spherical shells are as;

Cylindrical Shell part;

$$P_{3C} = \frac{Eh}{2\left(1-\upsilon^2\right)} \iint \begin{bmatrix} \varepsilon_1^2 + \varepsilon_2^2 + 2\upsilon\varepsilon_1\varepsilon_2 \\ +2\left(1-\upsilon\right)\varepsilon_{12}^2 \end{bmatrix} a\,dx\,d\theta + \frac{Eh^3}{24\left(1-\upsilon^2\right)} \iint \begin{bmatrix} \kappa_1^2 + \kappa_2^2 + 2\upsilon\kappa_1\kappa_2 \\ +2\left(1-\upsilon\right)\kappa_{12}^2 \end{bmatrix} a\,dx\,d\theta$$

Spherical Shell part;

$$P_{3S}=\frac{Eh}{2\left(1-\upsilon^2\right)}\iint\begin{bmatrix}\varepsilon_1^2+\varepsilon_2^2+2\upsilon\varepsilon_1\varepsilon_2\\+2\left(1-\upsilon\right)\varepsilon_{12}^2\end{bmatrix}R^2\sin\varphi\,d\varphi\,d\theta+\frac{Eh^3}{24\left(1-\upsilon^2\right)}\iint\begin{bmatrix}\kappa_1^2+\kappa_2^2+2\upsilon\kappa_1\kappa_2\\+2\left(1-\upsilon\right)\kappa_{12}^2\end{bmatrix}R^2\sin\varphi\,d\varphi\,d\theta \tag{5}$$

In these equations, $h$, $E$, $\upsilon$ and $\kappa_i$ are the shell thickness, elasticity modulus, and curvatures, respectively. The strains and curvatures in the cylindrical part are;

$$\begin{aligned}
\varepsilon_x&=\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\\
\varepsilon_\theta&=\frac{1}{a}\left(\frac{\partial v}{\partial\theta}-w\right)+\frac{1}{2a^2}\left(v+\frac{\partial w}{\partial\theta}\right)^2\\
\varepsilon_{x\theta}&=\frac{1}{2a}\left(a\frac{\partial v}{\partial x}+\frac{\partial u}{\partial\theta}+\frac{\partial w}{\partial x}\left(\frac{\partial w}{\partial\theta}+v\right)\right)\\
\kappa_x&=\frac{\partial^2 w}{\partial x^2}\\
\kappa_\theta&=\frac{1}{a^2}\left(\frac{\partial^2 w}{\partial\theta^2}+\frac{\partial v}{\partial\theta}\right)\\
\kappa_{x\theta}&=\frac{1}{2a}\frac{\partial v}{\partial x}+\frac{1}{a}\frac{\partial^2 w}{\partial x\partial\theta}+\frac{1}{4a}\left(\frac{\partial v}{\partial x}-\frac{1}{a}\frac{\partial u}{\partial\theta}\right)\\
&=\frac{1}{a}\frac{\partial^2 w}{\partial x\partial\theta}+\frac{3}{4a}\frac{\partial v}{\partial x}-\frac{1}{4a^2}\frac{\partial u}{\partial\theta}
\end{aligned}\tag{6}$$

while for the spherical shell we have;

$$\begin{aligned}
\varepsilon_\phi&=\frac{1}{R}\left[\frac{\partial u}{\partial\phi}-w+\frac{1}{2R}\left(u+\frac{\partial w}{\partial\phi}\right)^2\right]\\
\varepsilon_\theta&=\frac{1}{R}\begin{bmatrix}\dfrac{1}{\sin\phi}\dfrac{\partial v}{\partial\theta}+u\cot\phi-w\\+\dfrac{1}{2R}\left(v+\dfrac{1}{\sin\phi}\dfrac{\partial w}{\partial\theta}\right)^2\end{bmatrix}\\
\varepsilon_{\phi\theta}&=\frac{1}{2R}\begin{bmatrix}\dfrac{\partial v}{\partial\phi}+\dfrac{1}{\sin\phi}\dfrac{\partial u}{\partial\theta}-v\cot\phi\\+\dfrac{1}{2R}\left(u+\dfrac{\partial w}{\partial\phi}\right)\left(v+\dfrac{1}{\sin\phi}\dfrac{\partial w}{\partial\theta}\right)\end{bmatrix}\\
\kappa_{\phi\phi}&=\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial\phi^2}+\frac{\partial u}{\partial\phi}\right)\\
\kappa_{\theta\theta}&=\frac{1}{R^2}\left[\frac{1}{\sin^2\phi}\frac{\partial^2 w}{\partial\theta^2}+\frac{1}{\sin\phi}\frac{\partial v}{\partial\theta}+\left(\frac{\partial w}{\partial\theta}+u\right)\cot\phi\right]\\
\kappa_{\theta\phi}&=\frac{1}{2R^2}\begin{bmatrix}\dfrac{2}{\sin\phi}\dfrac{\partial^2 w}{\partial\theta\partial\phi}-\dfrac{2\cos\phi}{\sin^2\phi}\dfrac{\partial w}{\partial\theta}\\+\dfrac{\partial v}{\partial\phi}+\dfrac{1}{\sin\phi}\dfrac{\partial u}{\partial\theta}-v\cot\phi\end{bmatrix}
\end{aligned}\tag{7}$$

According to the principle of minimum total potential energy, at the onset of buckling, the first variation in the total potential energy must be zero. That is,

$$\delta\pi=\delta U+\delta V=\delta(P_{1C}+P_{1S}+P_{2C}+P_{2S}+P_{3C}+P_{3S})=0 \tag{8}$$

in which $\delta U$ and $\delta V$ represent the variations in strain energy and virtual potential energy of membrane stresses, respectively. Solving Eq. (8) leads to the equilibrium equations and the associated boundary conditions, results in very complex relations that are very difficult to solve. Consequently, a finite element scheme is implemented to determine the buckling loads, the results of which are presented in the next section.

## 3. Results and discussions

During modeling process, it is assumed that the tank is subjected to hydrostatic pressure. Accordingly, the buckling load is obtained for different thicknesses and tank volumes. The selected material properties for the heads and cylindrical wall are given in Table 1. SHELL281 which is suitable for analyzing thin to moderately-thick shell structures was used to mesh the model and investigate the shell's buckling behavior. This element has eight nodes, each of which has six degrees of freedom and is well-suited for linear, large rotation, and/or large strain nonlinear applications [51].

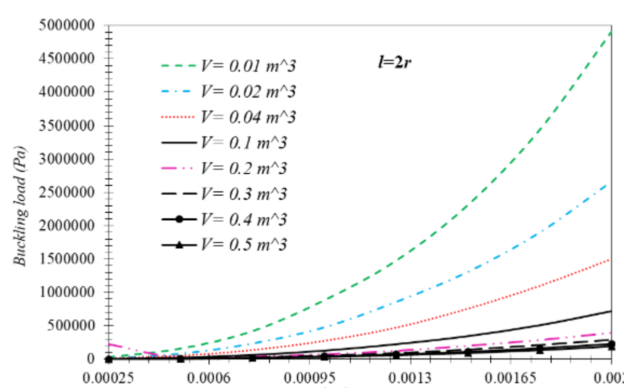
**Table 1.** Material properties of selected materials.

| Material | Young's modulus (GPa) | Poisson's ratio ($\upsilon$) |
|---|---|---|
| **Steel (cylindrical wall and/or hemispherical heads)** | 210 | 0.25 |
| **Aluminum (cylindrical wall and/or hemispherical heads)** | 71 | 0.33 |

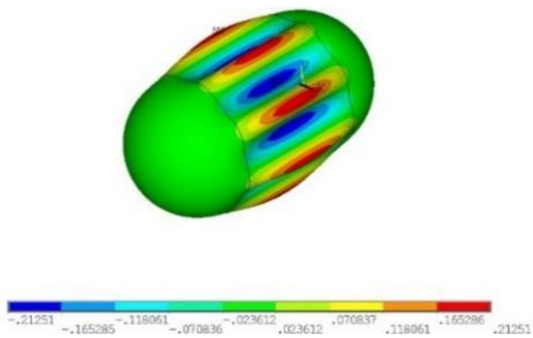
### *3.1. Length effect of the cylindrical part*

#### *3.1.1 l = 2r*

In the first model, it is assumed that the hemispherical shell is made of aluminum and steel cylindrical shell is made of steel. The buckling loads for different vessel volumes are shown in Fig. 2. As the thickness of the shell increases the buckling load increases, with any change in buckling mode. Also, the buckling load has an inverse relation with the volume of the tank.



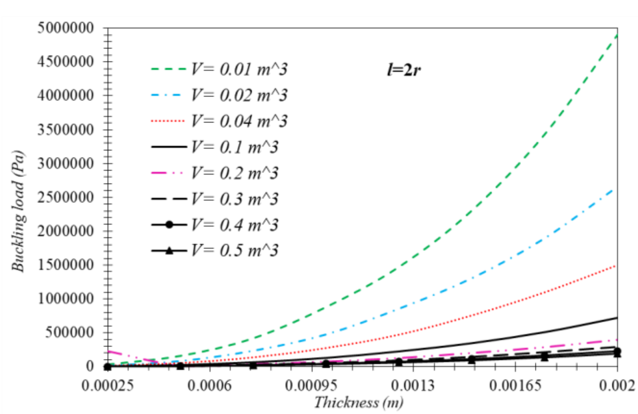

(a)



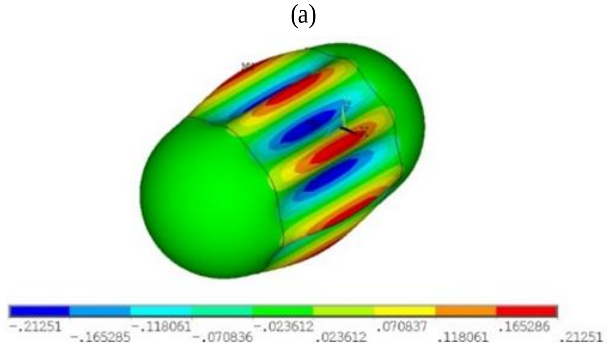

(b)

**Fig. 2.** The buckling load and buckling mode in a tank with *l*=2*r*. (a) Buckling load for different tank thicknesses and volumes and, (b) Buckling mode. The hemispherical shell is made of aluminum and the cylindrical part is made of steel.

In the second model, the whole structure was made of steel. The buckling load and buckling mode for various thicknesses and volumes are shown in Fig. 3. As in the first model, increasing the thickness and volume of the vessel, directly and inversely affect the buckling load, respectively. Comparing the buckling loads of the two models reveals that any change in the elastic modulus of the hemispherical heads does not have a substantial effect on the buckling behavior of the tank (vessel). Consequently, one may conclude that the buckling behavior of the tank mainly depends on the material of the cylindrical part rather than that of the hemispherical heads. This result has manifested itself in buckling mode which has mainly occurred in the cylindrical part.




(a)

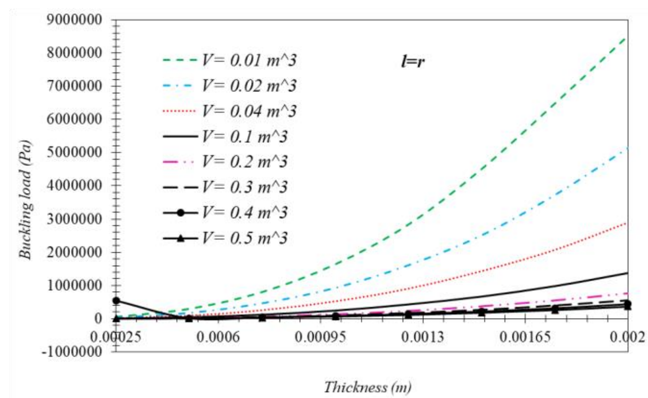



(b)

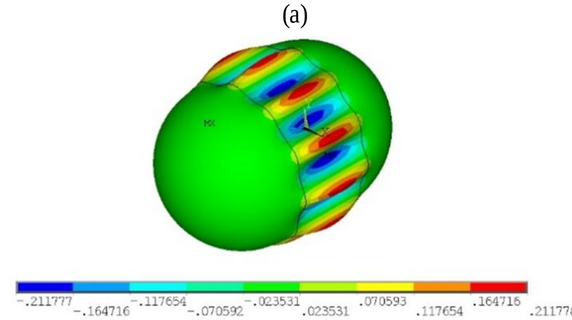
**Fig. 3.** Buckling load and buckling mode in a tank with l=2r, (a) buckling load for different tank thicknesses and volumes and, (b) buckling mode. The whole structure is made of steel.

### 3.1.2. *l* = *r*

The buckling load and buckling mode for a tank with an aluminum hemispherical head and steel cylindrical shell are shown in Fig. 4. Comparing Fig. 4 and Fig. 2, it can be observed that the reduction in length of the cylindrical part leads to an increase in buckling load. However, the change in length of the cylindrical part from *l* = 2*r* to *l* = *r* does not affect the buckling mode, when the same hydrostatic pressure is applied to the whole structure.




(a)




(b)

**Fig. 4.** The buckling load and buckling mode in a tank with l=r, (a) buckling load for different tank thicknesses and volumes and, (b) buckling mode. The hemispherical shell is made of aluminum and the cylindrical part is made of steel.

### *3.2. Comparison between buckling modes under different external pressure*

A comparison between the buckling modes of a tank with hydrostatic pressure applied to its whole structure, and the one with a similar load applied to only to its hemispherical load is shown in Fig. 5. As observed, for different thicknesses and volumes, there is only one buckling mode for the case in which the whole structure sustains a uniform hydrostatic pressure. However, for hemispherical heads being pressurized under a uniform load, different buckling modes occur.

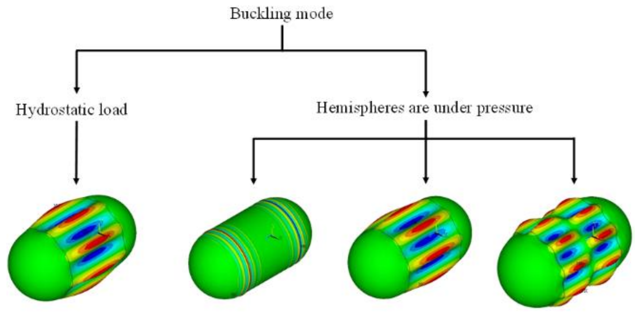



**Fig. 5.** A comparison between the buckling modes, based on different external loads.

## 4. Conclusions

Nonlinear buckling behavior of short cylindrical shell (tank) with two hemispherical heads was investigated. The finite element models were prepared and subjected to uniform hydrostatic pressures. The material of the cylindrical wall was allowed to be different from that of the hemispherical heads while its length was assumed to be smaller than or equal to its diameter. Results show that when the vessel is externally pressurized, the buckling load increases with an increase in the tank thickness. However, the adverse effect was observed as the tank volume increased. In addition, results show that the critical deformations occur in the cylindrical part (with only one buckling mode), provided the whole structure is subjected to uniform hydrostatic pressure. Moreover, if the hemispherical heads are the only pressurized components, depending on the magnitude of the applied load, different buckling modes occur in the cylindrical body of the tank. Such behavior means that in a short thin vessel, the critical portion of the whole structure appears to be the cylindrical wall (as opposed to its hemispherical heads). Also, based on a uniform pressure, the buckling mode remained intact for different vessel thicknesses, lengths, and volumes, provided the whole structure was hydrostatically pressurized. According to these results, to increase the overall buckling load capacity of a short vessel, it is recommended to use higher elastic moduli for the cylindrical shell, or even apply reinforcement ribs and/or struts to strengthen this portion of the structure.

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