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Risk Management in Oil Market: A Comparison between Multivariate GARCH Models and Copula-based Models

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Abstract

High price volatility and risk are the main features of commodity markets. One way to reduce this risk is to apply the hedging policy in future contracts. In this regard, in this paper, we will calculate the optimal hedging ratios for OPEC oil. In this study, besides the multivariate GARCH models, for the first time, we use conditional copula models for modeling the dependence structure between OPEC oil and WTI future contract with different maturities and estimating hedging ratios for OPEC oil by using WTI future contracts. The results of this study show that the dependence structure between OPEC oil and WTI future contract in three maturities is asymmetric. In addition, results indicate that during the studied period (2003-2017), Copula-based models have more efficient in applying the hedging policy than multivariate GARCH models. The average optimal hedge ratio increases with a rise in the maturity of contracts. On the other hand, the highest performance of hedging strategies was achieved by using WTI *futures contracts with six months' maturity.*

Keywords: Asymmetric Dependence, Optimal Hedge Ratio, Copula-based Models, OPEC Oil.

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1. Introduction

Commodity markets such as the global oil market always endure price volatility. Over the years price volatility has become one of the most important and challenging issues in these markets. However, price

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volatility in the global oil market, due to its strategic role in the world economy, has detrimental effects on the economies of both oil exporters and oil importers as well as the economies of oil and petrochemical related businesses like refineries. A solution to overcome these effects is to implement hedging policy by derivatives (such as future contract).

Earlier studies performed by Edrington (1979) and Myers and Thompson (1989) provided estimates of optimal static hedge ratio with Ordinary Least Squares method (OLS). After the introduction of GARCH models, many researchers have used the models such as multivariate GARCH models to estimate optimal dynamic hedge ratios. For instance, Haigh and Holt (2002) using the BEKK-GARCH model developed by Engle and Kroner (1995), estimated the optimal dynamic hedge ratio for West Texas Intermediate (WTI) crude oil, heating oil, and gasoline. Jalali-Naini and Kazemi-Manesh (2006) using the BEKK-GARCH model and spot return and future return of WTI in one to four maturities, estimated the optimal dynamic hedge ratios for WTI crude oil. Khodadadian (2010) using the BEKK-GARCH model, estimated the optimal dynamic hedge ratios for Iranian oil and crude WTI crude oil future contract. In addition, Chang et al. (2011) compared the effectiveness of the models such as DCC-GARCH, CCC-GARCH, and BEKK-GARCH in the WTI crude oil hedging strategy.

All aforementioned dynamic models which are based on the multivariate GARCH models, assume that the joint distribution between spot and future returns is a normal or t-Student distribution. This assumption means that the dependence structure between spot and future returns is a symmetric linear structure and is solely indicated by the correlation coefficient. This assumption has been challenged by some researchers like Ang and Chen (2002) and Patton (2006). They showed that the dependence structure between return on assets is asymmetric, which means the degree of dependence of markets during the time when there is good news is different from the time when there is bad news (critical situation). Since the efficiencies of hedging policies are closely related to appropriate modeling of the dependence structure, an imprecise modeling of the dependence structure may cause the implementation of inefficient hedging policies.

It has been proposed that this limiting assumption can be left aside using Copula functions. Therefore, in order to overcome the limitations of the other methods Copula functions have been used. Copula functions are flexible in the modeling of dependence structure, so enable us to investigate probable dependence structures between return on assets, like asymmetric tail dependence. Then, by considering this true structure, one can calculate hedge ratios. After the introduction of conditional copula by Patton (2006), some researchers employed these functions to estimate optimal dynamic hedge ratio. Hsu et al. (2008), using three different conditional copula models such as normal copula, Gumbel copula, and Clayton copula, estimated optimal dynamic hedge ratios for S&P 500 index, the FTSE 100 index, and the Swiss franc. They also made a comparison in the performance of hedging strategy between multivariate GARCH models and copula based models. Their results showed that copula based models give better performance in hedging strategy. Chang (2012) investigated the dependence structure between WTI spot and future returns using Copula functions, and then calculated optimal dynamic hedge ratios. It was concluded that the dependence structure between WTI spot and future returns is asymmetric. Such an asymmetric dependence structure influences hedging strategy. Therefore, a model in which this asymmetry is considered shows better performance in hedging strategy. Ghorbel and Trabelsi (2012), using different Copula functions, calculated the optimal dynamic hedge ratios for WTI crude oil, heating oil, and propane. Their results indicated that in comparison with traditional models, Copula functions gives better performance in hedging strategy. Doifli and Ghorbel (2015) calculated the optimal dynamic hedge ratios for WTI crude oil and heating oil, and concluded that Joe copula leads to the best Hedging Strategy. Sukcharoen and Leatham (2017) estimated the optimal dynamic hedge ratios for WTI and heating oil and concluded that the vine copula based model improves the efficiency of hedging strategy.

However, heretofore, there have been no research on the investigating the dependence structure between OPEC's oil price and futures contract prices of other base oils using different Copula functions. Moreover, there have been no research on the calculation of optimal dynamic hedge ratios of OPEC's oil using Copula functions yet. Therefore, in this article, we will try to find answers to the following questions: Whether the dependence structure between OPEC's oil and WTI crude oil future contract is symmetric or not. Does using Copula functions improve the hedging strategy of OPEC's oil in comparison with multivariate GARCH models? In other words, whether leaving aside the assumption that the joint distribution of asset returns is a normal or t-Student distribution improves the hedging strategy? In order to answer these questions well, we will calculate the optimal dynamic hedge ratios of OPEC’s oil by both multivariate GARCH models and different Copula functions. Afterwards, based on these calculations, we will compare the two models in the performance of hedging strategy. The paper is organized as follows: Section 2 discusses the theoretical framework of hedging; methodological frameworks are discussed in Section 3; Section 4 discusses the data analysis; Section 5 discusses estimating of models parameters; Section 6 discusses calculation of hedge ratios and finally Section 7 concludes.

2. Theoretical Framework

One of the most popular topics within risk management field is determining optimal hedging strategy. Without uncertainty spot and future rate of return, maybe equal but by having uncertainty, it is appeared that there is uncertainty gap between them. The minimum variance of portfolio return strategy is one of hedging strategies that has been most commonly used in the financial literature. Hsu et al. (2008), Lee (2009), Lai et al. (2009), and Chang (2012) in their researches, using the minimum variance of portfolio return strategy and copula based models, calculated hedge ratios. Similarly, in the current study we will use the minimum variance of portfolio return strategy. According to this strategy, we will determine the optimal hedge ratio (i.e. the value of a futures contract that should be sold in the futures market in order to reduce the variation of every dollar of spot asset). When an oil producer hedges and sells futures contracts in the futures market, then its portfolio return which consists of spot crude oil and short position in oil futures contract can be written as:

$$r_{H,t} = r_{s,t} - \delta_t r_{f,t} \rightarrow S_t = \frac{r_{st} - r_{ut}}{r_f} \quad (1)$$

Where $r_{H,t}$ is the return of hedged portfolio in the period from t-1 to t, $r_{s,t}$ is spot market return in the period from t-1to t, $r_{f,t}$ is futures market return in the period from t-1 to t, and δ_t represents hedge ratio. The optimal hedge ratios occur when conditional variance of hedged portfolio is minimized.

By differentiating Eq. (1) and setting the derivative equal to zero, the optimal hedge ratios can be calculated as:

$$\delta_t^* \left| \Omega_{t-1} = \frac{\operatorname{cov}\left(r_{s,t}, r_{f,t} \left| \Omega_{t-1}\right.\right)}{\operatorname{var}\left(r_{f,t} \left| \Omega_{t-1}\right.\right)} \right. \quad (2)$$

Due to the Eq. N(1), hedge ratio follows indirectly the rate of return of future contract.

3. Methodological Framework

3.1 Multivariate GARCH Models

In multivariate GARCH model, Vector autoregressive (VAR) model, is used to modeling conditional mean of returns (Vector error correction model (VECM) is used when time series are cointegrated). The three multivariate GARCH models BEKK, DCC and DCC are used to estimate returns volatilities. If we represent the vector of the return of oil spot ($r_{s,t}$) and the return of crude oil futures contract ($r_{f,t}$) in the period t by $r_t = \begin{bmatrix} r_{s,t} \\ r_{f,t} \end{bmatrix}$, then

$$r_t = \mu_t + \varepsilon_t \quad (3)$$

where μ_t is the return conditional mean vector, and ε_t is the disturbance terms vector. Considering that the VAR model is used here, μ_t may be written as

$$\mu_t = A_0 + \sum_{i=1}^{p} A_i r_{t-i} \tag{4}$$

where A_0 is the vector of constants and $A_i = \begin{bmatrix} A_{is} & A_{if} \end{bmatrix}$ is the matrix of coefficients for the lag i, which A_{is} and A_{if} is coefficients of $r_{s,t-i}$ and $r_{f,t-i}$, respectively. In the case of using the VECM model, μ_t may be written as:

$$\mu_t = A_0 + B_0 (ECT_{t-1}) + \sum_{i=1}^{p} A_i r_{t-i} \tag{5}$$

in which the error correction term is added. In order to model the disturbance terms, the following equation may be used:

$$\varepsilon_t = H_t^{1/2} z_t \tag{6}$$

where z_t has the following features: $E(z_t) = 0$ and $\mathrm{var}(z_t) = I_n$. Different methods used to the model H_t, namely CCC, BEKK, and DCC. The so-called BEKK model, introduced by Engle and Kroner in 1995, has been used in numerous researches. In the BEKK model, the conditional covariance matrix is given by the following equation:

$$H_t = C'C + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + B'H_{t-1}B \tag{7}$$

In the CCC model, the conditional covariance matrix can be expressed as follows:

$$H_t = D_t R D_t \tag{8}$$

where $D_t = diag(h_{11,t}^{1/2}, ..., h_{nn,t}^{1/2})$ denotes a diagonal matrix of conditional standard deviations of the returns and correlation matrix is assumed to be constant over time $\left(R_t = R\right)$.The limitation of this model is that the correlation coefficients are assumed to be constant. To avoid this limitation, Engle and Sheppard (2001) introduced the DCC model. The conditional covariance matrix of DCC model can be expressed as follows:

$$H_t = D_t R_t D_t \tag{9}$$

and the conditional correlation matrix is time varying and is modeled as follows: $R_t = Q_t^{*-1} Q_t Q_t^{*-1}$ and $Q_t = (1-\alpha-\beta)\overline{Q} + \alpha\varepsilon_{t-1}\varepsilon_{t-1}^{'} + \beta Q_{t-1}$ (10)

where Q_t^* is a diagonal matrix consisting of the square roots of the diagonal elements of the matrix Q_t .

3.2 Copula Based Models

In the modeling literature, Sklar introduced the notion of Copula in 1959. Sklar's theorem essentially states that any n-dimensional joint distribution function decomposes into its n-marginal distributions and a copula function. The copula function provides a description of the dependence between all n variables.

Kendall's tau and tail dependence are dependence indicators in Copula literature and have been extensively used in experimental researches. Following, we will discuss about these two dependency indicators.

Kendall's tau is defined as the difference between the probabilities of concordance (the variables moves in same direction) and discordance (the variables moves in opposite direction) between the two variables. Kendall's tau is given by:

$$\tau = \Pr\left[\left(x_i - x_j\right)\left(y_i - y_j\right) \succ 0\right] - \Pr\left[\left(x_i - x_j\right)\left(y_i - y_j\right) \prec 0\right] \quad (11)$$

If the Kendall's tau is positive (negative), the probability of the movement of the variables in same direction is greater (lower) than the probability of the movement of the variables in opposite direction.

The concept of tail dependence embedded within the copula theory is understood as the dependence between the variables at extreme values. Upper tail dependence and lower tail dependence are given by:

$$\lambda_u \equiv \lim_{u\to 1^-} P\left[r_{f,t} \succ F_f^{-1}\left(u \middle| \Omega_{t-1}\right) \middle| r_{s,t} \succ F_s^{-1}\left(u \middle| \Omega_{t-1}\right)\right] \quad (12)$$

$$\lambda_l \equiv \lim_{u\to 0^+} P\left[r_{f,t} \prec F_f^{-1}\left(u \middle| \Omega_{t-1}\right) \middle| r_{s,t} \prec F_s^{-1}\left(u \middle| \Omega_{t-1}\right)\right]$$

when λ_u is positive and close to 1, it indicates that, under the condition that both returns have extreme positive values, the degree of co-movement in the same direction is large. Analogously, when λ_l is positive and close to 1, it implies that the probability of an extreme negative futures return conditional on the extreme negative spot return is large.

Until 2006, the copula theory has been defined only for unconditional distributions. However, in 2006, Patton in his important publication developed the copula theory to cover conditional distributions. He defined the relationship between conditional copula function and conditional distribution function as follows:

$$F(r_{s,t}, r_{f,t} | \Omega_{t-1}) = C(u_{s,t}, u_{f,t} | \Omega_{t-1}) \tag{13}$$

where $r_{s,t}$ is spot market return, $r_{f,t}$ is futures market return, F is joint distribution function, Ω_{t-1} is all information available up to time t-1, C is the copula function, $u_{s,t}$ is the value of distribution function of spot return, and $u_{f,t}$ is the value of distribution function of future return. From Eq. (14), one can obtain the relationship between joint density, Copula density, and marginal densities as follows:

$$f\left(r_{s,t},\ r_{f,t} | \Omega_{t-1}\right) = \left\{f_s\left(r_{s,t} | \Omega_{t-1}\right) \times f_f\left(r_{f,t} | \Omega_{t-1}\right)\right\} \times c\left(u_{s,t},\ u_{f,t} | \Omega_{t-1}\right) \tag{14}$$

where f and c represent the Probability density function and Copula density, respectively.

In order to estimate the parameters of Copula-based models, we will use the inference function for margins (IFM) approach introduced by Patton. IFM is basically a two-stage approach**.** In the first stage, the parameters of marginal density functions are estimated using univariate GARCH models. In the second stage, based on the estimates performed in the first stage, the parameters of Copula density function are estimated.

In modeling literature, Copula functions are divided into two main classes, namely elliptical copula functions and Archimedean copula function.

3.2.1 Elliptical Copula

The normal Copula is the most commonly used elliptical Copula. The normal Copula is based on the multivariate normal distribution function and the general formula for its bivariate density is:

$$c_t^N(u_{s,t},u_{f,t};\rho_t)=\phi_\rho\left(\phi^{-1}(u_{s,t}),\phi^{-1}(u_{f,t})\right)$$
$$=\frac{1}{\sqrt{1-\rho_t^2}}\exp\left(-\frac{\rho_t^2\left\{(\phi^{-1}(u_{s,t}))^2+(\phi^{-1}(u_{f,t}))^2\right\}-2\rho_t\phi^{-1}(u_{s,t})\phi^{-1}(u_{f,t})}{2(1-\rho_t^2)}\right)$$

(15)

where ϕ_ρ is the normal joint probability density function with a Pearson's correlation coefficient of ρ and Φ^{-1} is inverse univariate standard normal distribution function. Similar to Patton (2006), the dynamics of correlation coefficient over time is given by the following equation:

$$\rho_t = H\left(b_0^n + b_1^n\rho_{t-1} + b_2^n.\frac{1}{10}\sum_{j=1}^{10}\phi^{-1}(u_{s,t-j}).\phi^{-1}(u_{f,t-j})\right) \quad (16)$$

where $H(z)=(1-e^{-z})/(1+e^{-z})$.

In order to consider any persistence in correlation coefficient, this specification includes the prior period correlation coefficient. Moreover, in order to consider any variation in correlation process, the average value of multiplication of $\phi^{-1}(u_{s,t-j})$ and $\phi^{-1}(u_{f,t-j})$ in last ten periods has been used. It should be noted that elliptical copula assumes a linear relationship between two variables. Also, it cannot model an asymmetric dependency. In order to overcome these limitations, Archimedean copulas are used.

3.2.2 Archimedean Copula

Since Archimedean copulas make it possible to model non-linear and asymmetric dependencies, they have been extensively used in experimental researches. Archimedean copulas are defined as follows: Let $\varphi:[0,1]\rightarrow[0,\infty]$ be a continuous, strictly decreasing, convex function with $\varphi(0)=\infty$ and $\varphi(1)=0$, then the function:

$$C\left(u_1,...,u_d\right)=\psi^{-1}\left(\psi(u_1)+\psi(u_2)+...+\psi(u_d)\right) \quad (17)$$

is called Archimedean copula (for 0≤u, v≤1). The function φ is called generator function. In order to compare different dependency structures, in this study the three most common Archimedean copulas including Clayton copula, Gumbel copula, and symmetrized Joe-Clayton Copula have been used. Gumbel copula allows to investigate the upper tail dependence, while preassumes the lower tail dependence as 0. The Gumbel copula density function is given by:

$$c^G(u_{s,t},u_{f,t};\alpha_t^G)=\frac{C^G(u_{s,t},u_{f,t};\alpha_t^G)(\ln u_{s,t}\ \ln u_{f,t})^{\alpha_t^G-1}\left[(-\ln u_{s,t})^{\alpha_t^G}+(-\ln u_{f,t})^{\alpha_t^G}\right]^{\frac{1}{\alpha_t^G}}+\alpha_t^G-1}{u_{s,t}u_{f,t}\left[(-\ln u_{s,t})^{\alpha_t^G}+(-\ln u_{f,t})^{\alpha_t^G}\right]^{2-(1/\alpha_t^G)}} \tag{18}$$

where

$$C^G(u_{s,t},u_{f,t};\alpha_t^G)=\exp\left(-\left[(-\ln u_{s,t})^{\alpha_t^G}+(-\ln u_{f,t})^{\alpha_t^G}\right]^{\frac{1}{\alpha_t^G}}\right) \tag{19}$$

The relationship between α_t^G and τ_t^G is given by $\alpha_t^G=\left(1-\tau_t^G\right)^{-1}$ and the dynamic process of Kendall's tau is specified by:

$$\tau_t^G=\Gamma(b_0^g+b_1^g\tau_{t-1}^g+b_2^g.\frac{1}{10}\sum_{j=1}^{10}\left|u_{s,t-j}-u_{f,t-j}\right|) \tag{20}$$

where $\Gamma\left(z\right)=\frac{1}{\left(1+e^{-z}\right)}$.

In contrast with Gumbel copula, Clayton copula allows to investigate the lower tail dependence, while presumes the right tail dependence as 0.The Clayton copula density function is given by:

$$c^C(u_{s,t},u_{f,t};\alpha_t^C)=\frac{(1+\alpha_t^C)(u_{s,t}^{-\alpha_t^C}+u_{f,t}^{-\alpha_t^C}-1)^{-2-(1/\alpha_t^C)}}{(u_{s,t}u_{f,t})^{\alpha_t^C+1}} \tag{21}$$

Where $\alpha_t^C > 0$ and the relationship between α_t^C and τ_t^C is given by $\alpha_t^C = \frac{2\tau_t^C}{\left(1-\tau_t^C\right)}$. Also, the dynamic process of Kendall's tau is specified by:

$$\tau^C = \Gamma(b_0^c + b_1^c \tau_{t-1}^c + b_2^c . \frac{1}{10}\sum_{j=1}^{10} \left| u_{s,t-j} - u_{f,t-j} \right|) \tag{22}$$

Symmetrized Joe-Clayton copula allows us to investigate the probable asymmetry in the upper and lower tail dependence. In contrast with Clayton and Gumbel copulas, Symmetrized Joe-Clayton copula is a two parameter copula, and specified by:

$$C_{SJC}(u,v \left| \tau^U, \tau^L \right.) = 0.5 \cdot \left(C_{JC}(u,v \left| \tau^U, \tau^L \right.) + C_{JC}(1-u, 1-v \left| \tau^U, \tau^L \right.) + u + v - 1 \right) \tag{23}$$

where:

$$C_{JC}\left(u,v \middle| \tau^U, \tau^L\right) = 1 - \left(1 - \left\{ \left[1-(1-u)^\kappa\right]^{-\gamma} + \left[1-(1-v)^\kappa\right]^{-\gamma} - 1 \right\}^{-1/\gamma} \right)^{1/\kappa} \tag{24}$$

$$\kappa = 1 \big/ \log_2(2-\tau^U) \quad \gamma = -1 \big/ \log_2(\tau^L) \quad \tau^U, \tau^L \in (0,1)$$

In Symmetrized, Joe-Clayton copula τ^U and τ^L are model parameters and indicate upper and lower tail dependence. The dynamics of these parameters are modeled as:

$$\tau_t^U = \Lambda\left(\omega_U + \beta_U \tau_{t-1}^U + \alpha_U . \frac{1}{10}\sum_{J=1}^{10} \left| u_{s,t-j} - u_{f,t-j} \right| \right)$$

$$\tau_t^L = \Lambda\left(\omega_L + \beta_L \tau_{t-1}^L + \alpha_L . \frac{1}{10}\sum_{J=1}^{10} \left| u_{s,t-j} - u_{f,t-j} \right| \right) \tag{25}$$

where, in order to keep both τ^U, τ^L constant in the range (0,1), we used $\Lambda(x) = (1+e^{-x})^{-1}$ transformation.

4. Data Analysis

In this research, we use weekly data on OPEC's oil price and the price of WTI crude oil future contracts in NYMEX with maturities of one, three, six, and nine months. These data belong to a period of time from beginning of January 2003 to the end of August 2017, and are available in Www.opec.org, Www.eia.gov, and www.quandl.com. By taking logarithms of these data and then taking the first differences of these logarithms, one can calculate returns of OPEC's oil and WTI crude oil future contract. Table 1 summarizes the statistical characteristics of these rates.

Table 1: Descriptive Statistics

Return	mean	Std deviation	max	min	kurtosis	skewness	Jarque-Bera statistic	P-value of Jarque-Bera
OPEC	0.0627	4.49	19.24	-25.28	6.08	-0.6	350	0.00
Wti 1	0.0483	5.06	24.12	-31.32	7.41	-0.58	664	0.00
Wti 3	0.058	4.5	21.13	-24.33	5.57	-0.5	243	0.00
Wti 6	0.075	4.06	18.59	-21.04	5.31	-0.51	204	0.00
Wti 9	0.084	3.78	16.59	-19.30	5.37	-0.54	217	0.00

In order to generate the models properly, it is required to perform stationary test, cointegration test, structural breaks test, and determine optimal lag length in multivariate GARCH and copula based GARCH models. This will be discussed following.

According to the results obtained from ADF unit root test given in Table 2, all price levels are nonstationary but their differences are stationary. Johansen cointegration test was used to evaluate the cointegration between OPEC's oil price and the prices of future contracts with different maturities. The results are presented in Table 2 and imply that prices are not cointgrated.

Table 2: ADF and Johansen Cointegration Test

		Price of OPEC	Price of wti1	Price of wti3	Price of wti6	Price of wti9
ADF	**t statistic**	**-1.7537**	**-1.9699**	**-1.9427**	**-1.9591**	**-1.9572**
	P value	**0.4**	**0.3**	**0.31**	**0.3**	**0.3**
Result		Non stationary	Non stationary	Non stationary	Non stationary	Non stationary

cointegration test	Statistic	3.248	2.92	2.8	2.92
	P value	0.071	0.083	0.094	0.0875
Result in 5% significant level		no cointegration	no cointegration	no cointegration	no cointegration

In order to evaluate the structural breaks in level of oil prices, Bai-Perron test was used. The results obtained from this test imply four breaks in each series of OPEC and wti future contract price level in different maturities. Due to occurring structural breaks, Zivot-Andrews unit root test was used to evaluate price stationary. The results (assuming the structural break in intercept, structural break in time trend, and structural break in both) imply that even in the presence of structural breaks, crude oil price time series are nonstationary. Moreover, due to occurring structural breaks, the Gregory Hansen cointegration test was used to evaluate the cointegration between OPEC's oil price and the prices of future contracts in different maturities. According to the results obtained from this test, taking into account the structural breaks, OPEC's oil price and the price of WTI crude oil future contract with a maturity of one month are cointegrated, but with maturities of three, six, and nine months, they are not cointegrated. [1] Considering this fact, in estimating multivariate GARCH models, we will employ VECM method to model OPEC's oil returns and returns of WTI future contract with a maturity of one month.

The next stage is to determine the optimal number of lags in VAR model for different maturities. The SBC and AIC criteria were used to determine the number of lags. Number of lag selected by each criterion is reported in table 3.

Table 3: Optimal Lags of VAR Model

	OPEC and wti1 return	OPEC and wti3 return	OPEC and wti6 return	OPEC and wti9 return
Optimal lags based on SBC	2	1	1	1
Optimal lags based on AIC	8	8	1	1

1. Due to space constraints, the results of Bai-Perron, Zivot-Andrews, and Gregory Hansen tests have not been presented by detail.

Considering that in this study, in order to calculate the optimal hedge ratios, we focus on the estimation of the covariance matrix coefficients, in order to prevent the estimation of many parameters, in models including the future contract by one and three month maturity, we determine the number of lags of mean model based on the SBC criterion. But in the in models including the future contract by six and nine month maturity, both criteria indicate that one lag is optimal, so we will use the one lags in modeling mean of these models.

The first step in the modeling copula based models is to model the returns by ARMA approach. The initial guess of the number of optimal lags for modeling the conditional mean of returns is done by taking into account the ACF and partial ACF between the components of time series. Then, using the SBC information criterion, and also considering the point that the number of lags should be determined such that neither ACFs nor partial ACFs have significant values, the ultimate model is chosen. According to the structures of ACFs and partial ACFs, models given in Table 4 were guessed for each time-series.

Table 4: Guessed Models for Conditional Mean

Guessed model	Model 1	Model 2	Model 3	P-value of residuals Ljung box statistic
OPEC	$r_t = a_0 + a_1 r_{t-1} + \varepsilon_t + b_3 \varepsilon_{t-3}$ SBC=5.86679	$r_t = a_0 + a_1 r_{t-1} + a_3 r_{t-3} + \varepsilon_t$ SBC=5.86477	$r_t = a_0 + a_1 r_{t-1} + \varepsilon_t$ SBC=5.8630	Q(5)=0.58 Q(15)=0.61
Wti1	$r_t = a_0 + a_7 r_{t-7} + \varepsilon_t + b_8 \varepsilon_{t-8}$ SBC=6.07522	$r_t = a_0 + a_7 r_{t-7} + a_8 r_{t-8} + \varepsilon_t$ SBC=6.0785	$r_t = a_0 + \varepsilon_t + b_7 \varepsilon_{t-7} + b_8 \varepsilon_{t-8}$ SBC=6.07543	Q(5)=0.77 Q(15)=0.88
Wti3	$r_t = a_0 + \varepsilon_t + b_8 \varepsilon_{t-8}$ SBC=5.8514	$r_t = a_0 + a_8 r_{t-8} + \varepsilon_t$ SBC=5.8530	-	Q(5)=0.82 Q(15)=0.82
Wti6	$r_t = a_0 + \varepsilon_t + b_8 \varepsilon_{t-8}$ SBC=5.6532	$r_t = a_0 + a_8 r_{t-8} + \varepsilon_t$ SBC=5.6542	-	Q(5)=0.79 Q(15)=0.76
Wti9	$r_t = a_0 + \varepsilon_t + b_8 \varepsilon_{t-8}$ SBC=5.51153	$r_t = a_0 + a_8 r_{t-8} + \varepsilon_t$ SBC=5.5123	-	Q(5)=0.73 Q(15)=0.88

We used Ljang-box Q statistics to perform diagnostic tests for conditional mean. P-values of these statistics implied that the correlation between residuals is insignificant in all lags. In addition, the results obtained from Lagrange multiplier test implies the presence of conditional heteroscedasticity in all time-series. Therefore, in order to model the conditional variances of aforementioned models, the following GJR-GARCH(1,1) model will be used:

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \lambda d_{t-1}\varepsilon_{t-1}^2 + \beta\ h_{t-1} \tag{26}$$
$$\varepsilon_t = h_t^{1/2} z_t$$

where h_t is the conditional variance of return and z_t is standardized residual. In order to estimate the different parameters of selected models, conditional mean and variance models will be estimated simultaneously. Finally remember that, the Jarque-Bera test statics, and skewness values of returns in table 1 show that there is a considerable difference between the distribution of returns and normal distribution. This is why t-student distribution will be used to estimate the aforementioned models.

5. Estimating Models Parameters

In this section, we will estimate crude oil optimal hedge ratio using the aforementioned models. For the save of space, we have not included the detailed results obtained from CCC, DCC, and BEKK specifications of multivariate GARCH models. Note that since eigenvalues of parameter vectors in all models estimated by different specifications are less than unity, they are stability models.

After estimating the coefficients of multivariate GARCH models, we may want to estimate the coefficients of Copula-based models. The first step in estimating the coefficients of Copula-based models is to estimate ARMA-GJR-GARCH for each time-series. The results are given in Table 5. As can be seen, in the model estimated for OPEC's oil, all parameters of the conditional variance are significant. In the models estimated for WTI crude oil future contracts with different maturities, the GARCH parameters are significant, which implies the heteroscedasticity in the residual of conditional mean model.

Table 5: ARMA-GJR-GARCH (1, 1) Result

	OPEC	Wti1	Wti3	Wti6	Wti9

		OPEC	Wti1	Wti3	Wti6	Wti9
Mean	a_0	0.128 (0.902)	0.146 (0.89)	0.15 (1)	0.19 (1.41)	0.21^{***} (1.71)
	a_1	0.094^{**} (2.49)	-	-	-	-
	a_7	-	-0.026 (-0.72)	-	-	-
	a_8	-	-	0.097^{*} (2.76)	-	-
	b_8	-	0.107^{*} (2.97)	-	0.085^{**} (2.38)	0.072^{**} (2.04)
Variance	α_0	0.256^{***} (1.74)	0.94^{**} (2.34)	0.88^{**} (2.39)	$0. 44^{***}$ (1.82)	$0. 33^{***}$ (1.77)
	α_1	0.0547^{***} (1.64)	0.0078 (0.28)	0.012 (0.39)	0.03 (1.02)	0.05 (1.58)
	λ	0.085^{**} (2.23)	0.11^{*} (2.94)	0.096^{**} (2.53)	0.058^{***} (1.8)	0.038 (1.16)
	β	0.89^{*} (36)	0.88^{*} (26)	0.88^{*} (24)	0.9^{*} (29)	0.9^{*} (30)
P-value of squared residual Ljung box		Q(5)=0.75 Q(15)=0.51	Q(5)=0.58 Q(15)=0.66	Q(5)=0.46 Q(15)=0.63	Q(5)=0.74 Q(15)=0.96	Q(5)=0.68 Q(15)=0.95
Dof		8.25	8.8	10.02	8.02	7.04

Note: value in Parentheses are t statistic and ***, **and* indicate significance at 10%, 5% and 1% levels.

Moreover, the parameter λ which represents the asymmetry effect of positive and negative shocks on conditional variance is significant in all models except in the model estimated for future contract with a maturity of nine month. This means that compared with positive shocks, negative shocks (bad news) have a greater influence on conditional variance. Diagnostic tests for conditional variance performed using Ljung-Box Q statistic, indicates that the correlation between the squared residuals in all lags in ARMA-GARCH model is insignificant. The P-value of Ljung-Box Q statistic for lags 5 and 15 are given in the table 5. Moreover, after estimating the model, we used Lagrange Multiplier test to investigate the remaining GARCH effects in the model. The results showed that the F statics of this test was insignificant.

After estimating the univariate GARCH models for each time-series, we estimate the coefficients of copulas using residuals of these models. These estimations are performed using maximum likelihood estimation (MLE) and MATLAB software.

Table 6: Copula Model Result

		OPEC and wti1	**OPEC and wti3**	**OPEC and wti6**	**OPEC and wti9**
Normal copula	b_0	-2.18*	10.32*	13.4*	13.08*
		(-2.75)	(4.66)	(22.23)	(25.21)
	b_1	5.679*	-8.04*	-11.36*	-11.06*
		(6.57)	(-3.25)	(-21.9)	(-23.39)
	b_2	-0.109***	-0.274	-0.097	-0.11
		(-1.77)	(-1.24)	(-1.03)	(-1.27)
Clayton copula	b_0	2.396**	4.563*	1.768**	1.579**
		(2.22)	(8.17)	(2.23)	(2.43)
	b_1	-0.279	-0.889*	0.325	0.385
		(-0.48)	(-6.51)	(1.15)	(1.55)
	b_2	-7.16***	-14.99*	-5.76***	-5.33**
		(-1.86)	(-3.21)	(-1.88)	(-2.03)
Gumbel copula	b_0	4.1*	6.98*	7.1*	6.77*
		(3.77)	(5.23)	(9.39)	(8.43)
	b_1	-0.434	-0.85*	-0.99*	-0.94*
		(-1.23)	(-3.63)	(-16.09)	(-12.19)
	b_2	-9.77**	-21.02*	-14.85*	-15*
		(-2.03)	(-2.75)	(-3.55)	(-2.63)
Symmetrized Joe Clayton copula	ω_U	2.8*	0.03	1.84**	-0.14**
		(2.75)	(0.18)	(2.09)	(3.26)
	β_U	-1.81	1.5*	-0.58	2.08*
		(-1.27)	(14)	(-0.41)	(6.7)
	α_U	-4.57*	-1.01*	-3.08	-2.92*
		(-3.22)	(-9.26)	(-0.66)	(-4.7)
	ω_L	1.57*	1.97*	5.91***	2.74*
		(8.7)	(25.7)	(1.77)	(5.4)
	β_L	-0.2	0.02*	-5.84	-1.8*
		(-0.17)	(3.55)	(-1.43)	(-15.3)
	α_L	-4.04*	-7.86*	0.16	1.07*
		(-4.43)	(-12)	(0.06)	(-5.4)

Note: value in Parentheses are z statistic and ***, **and* indicate significance at 10%, 5% and 1% levels.

Table 6 represents the results obtained from the estimation of Copula functions. According to these results, in normal Copula, the coefficient b_1 is significant, which indicates the persistence of the influence of a given shock on dynamic correlation process.
However, the coefficient b_2 which presents the variation in dynamic correlation process is only significant in the model containing WTI crude oil future contract with one month maturity.

The comparison between the results of estimating Gumbel and Clayton copulas show that in the Kendall’s tau dynamic process of Clayton copula, the coefficient b_1 is only significant for the model including OPEC's oil return and WTI crude oil future contract with one-month maturity. However, in Gumble copula, this coefficient is significant in all models except in the model including OPEC's oil returns and WTI crude oil future contract with one-month maturity. The coefficient b_2 which presents the variation in the Kendall’s tau dynamic process, is significant in all models estimated using the both copulas, and its value is always negative as expected.

In addition, the results obtained from estimations performed by symmetrized Joe-Clayton Copula show that the coefficient β_L , which indicates the persistence of influence of a given shock on lower tail dependence dynamics, is significant in the models estimated for WTI future contracts with three, and nine months maturities. Similarly, the coefficient β_U , which indicates the persistence of influence of a given shock on upper tail dependence dynamics, is significant in the models estimated for WTI future contracts with three, and nine months maturities. Moreover, the parameter α_L which indicates the variation in lower tail dependence dynamics is significant for models containing WTI future contracts with one, three, and nine months maturity. Similarly, the parameter α_U which indicates the variation in upper tail dependence dynamics is significant for models containing WTI future contracts with one, three, and nine months maturity.

Table 7 represents the mean values of Kendall’s tau for Gumbel and Clayton copulas. As it can be seen, for all maturities, the mean values of Kendall’s tau are higher for Gumbel Copula. This means that the difference between the probability of the movement of the variables in same direction and the probability of the movement of the variables in opposite direction for Gumbel Copula is greater than that for Clayton Copula.

Table 7: Average of Kendall Tau

	copula	Average of Kendall Tau
Wti1 and OPEC	Gumble	0.66
	Clayton	0.57
Wti3 and OPEC	Gumble	0.69
	Clayton	0.61
Wti6 and OPEC	Gumble	0.7
	Clayton	0.63
Wti9 and OPEC	Gumble	0.63
	Clayton	0.62

In addition, the comparison between lower and upper tail dependence for symmetrized Joe-Clayton Copula is given in Table 8.

Table 8: Average of Upper and Lower Tail Dependence

OPEC and wti1	τ^U	0.72
	τ^L	0.72
OPEC and wti3	τ^U	0.73
	τ^L	0.76
OPEC and wti6	τ^U	0.74
	τ^L	0.78
OPEC and wti9	τ^U	0.75
	τ^L	0.77

As can be seen, in model containing WTI future contract with one-month maturity, there is no difference between the degree of dependence in lower and upper tails. However, for models containing other maturities of WTI future contracts, the degree of dependence in lower tail is greater than in upper tail. This means that the probability of the movement of OPEC's oil price and WTI futures contracts with maturities of three, six, and nine months in same direction during the time when there is bad news (critical situation) is higher than such an probability during the time when there is good news. Therefore, symmetric dependence between lower and upper tails occurs only for model containing futures contract with one-month maturity. However, the tail dependence is asymmetric between OPEC and WTI futures contract with three, six, and nine months maturities.

6. Calculation of Hedge Ratios

As it can be seen in Eq.(2), for calculating optimal hedge ratios, the conditional covariances between the returns of OPEC's oil and futures contracts are required. These values will be given as a part of model output for multivariate GARCH models and normal copula. However, in Gumbel copula, Clayton copula, and symmetrized Joe-Clayton Copula, the conditional covariance is not included in model output. Therefore, similar to the studies performed by Hsu et al. (2008) and Power et al. (2013), the conditional covariance in Copula-based model is calculated using the following relationship:

$$\operatorname{cov}(r_{f,t}, r_{s,t}) = \operatorname{cov}(\varepsilon_{f,t}, \varepsilon_{s,t}) = h_{s,t} h_{f,t} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} z_{s,t} z_{f,t} f(z_{s,t}, z_{f,t} \,|\, \Omega_{t-1}) drdw \tag{27}$$

Where $z_{s,t}$ and $z_{f,t}$ are standardized residuals of univariate GARCH model and $f(z_{s,t}, z_{f,t} \,|\, \Omega_{t-1})$ can be calculated using Eq. (15). The expression in the integral in the above relation denotes the covariance between standardized variables $z_{s,t}$ and $z_{f,t}$. We know that the covariance between standardized variables is equal to the correlation coefficient between the main variables (not standardized). Thus, the expression inside the integral represents the correlation coefficient between the variables $\varepsilon_{f,t}, \varepsilon_{s,t}$ and by multiplying it in the conditional deviations of each of these variables, the covariance between them are obtained.

Afterwards, using the covariances calculated using Eq.(28), the optimal hedge ratios of Copula-based models will be calculated.

Table 9 presents the average of hedge ratios and the efficiency of hedging strategies in different models.

In order to evaluate the efficiency of hedging strategies in different models, we will use Edrington hedge efficiency index. This index is calculate using the following relationship:

$$HE = \frac{\text{var}_{unhedged} - \text{var}_{hedged}}{\text{var}_{unhedged}} \tag{28}$$

where $\text{var}_{\text{unhedged}}$ and $\text{var}_{\text{hedged}}$ are the variances of the return of OPEC's oil and return of hedged portfolio, respectively. The model with maximum decrease in variance (higher amount of index) gives better performance in hedging strategy.

Table 9: Hedge Efficiency of Different Models

		Average of hedge ratio	Average of hedged portfolio variance	Hedge efficiency
OPEC and wti1	CCC model	0.7779	4.68	0.756
	DCC model	0.7648	5.09	0.7253
	BEKK model	0.7701	5.01	0.7348
	Normal copula	0.7882	5.509	0.7599
	Gumble copula	0.799	4.849	0.7793
	Clayton copula	0.7493	6.894	0.6891
	Symmetrized Joe Clayton copula	0.8149	4.2002	0.81
OPEC and wti3	CCC model	0.8556	4.062	0.79
	DCC model	0.8417	4.204	0.7678
	BEKK model	0.8513	3.8939	0.7825
	Normal copula	0.8811	4.3117	0.8039
	Gumble copula	0.8847	4.074	0.8103
	Clayton copula	0.8277	6.2285	0.7096
	Symmetrized Joe Clayton copula	0.8943	3.692	0.8279
OPEC	CCC model	0.9465	3.9529	0.7944

		Average of hedge ratio	Average of hedged portfolio variance	Hedge efficiency
and wti6	DCC model	0.9392	3.8274	0.7869
	BEKK model	0.9444	3.731	0.7898
	Normal copula	0.9741	3.8434	0.8223
	Gumble copula	0.9747	3.7572	0.8232
	Clayton copula	0.9138	5.855	0.7237
	Symmetrized Joe Clayton copula	0.9901	3.208	0.8492
OPEC and wti9	CCC model	1.013	4.183	0.7822
	DCC model	1.007	4.057	0.777
	BEKK model	1.011	4.002	0.7782
	Normal copula	1.041	3.9425	0.8178
	Gumble copula	1.03	4.087	0.8068
	Clayton copula	0.975	5.925	0.7186
	Symmetrized Joe Clayton copula	1.053	3.435	0.8364

The results obtained from estimating hedging ratios, show that all Copula-based models (except Clayton copula) give better performance in hedging strategy when compared to multivariate GARCH models for all maturities. Therefore, using Copula-based models instead of multivariate GARCH models improves the performance of hedging strategy. For all maturities, Clayton copula gives the worst performance in hedging strategy. In addition, according to the results, symmetrized Joe-Clayton Copula gives the highest performance in hedging strategy for all maturities. This is because symmetrized Joe-Clayton Copula is the only Copula which assumes asymmetric dependence between OPEC's oil return and the return of WTI futures contracts with different maturities.

Moreover, comparison between the mean hedge ratios estimated using symmetrized Joe-Clayton Copula leads to the conclusion that the optimal dynamic hedge ratio increases as maturity length increases. Considering that in calculating the optimal hedge ratios, the conditional variance of the performance of futures contracts is incorporated in the denominator, the increasing dynamic hedge ratio with maturity length can be related to Samuelson hypothesis of maturity effect. Based on this hypothesis, as the maturity length of a future contract increases, the fluctuation of the contract decreases. This leads to an increase in hedge ratio.

Comparison between the performances of optimal models for each maturity shows that the highest hedging performance is achieved by using futures contract with six months maturity. This means that if it is possible to hedge without any constraint for selection of maturity length, then the best choice is using WTI future contract with six-month maturity and modeling dependence structure by symmetrized Joe-Clayton Copula.

7. Conclusions and Suggestions

In this research, in order to investigate the dependence structure between OPEC's oil and WTI future contract and also to estimate the optimal dynamic hedge ratios for OPEC's oil during a period of time from beginning of January 2014 to the end of August 2017, conventional GARCH-based models and newly developed Copula-based models were used. The results shows that dependence structure between OPEC's oil and WTI futures contracts with maturities of three, six, and nine months is asymmetric. It has been shown that Copula-based models give better performance in hedging strategy when compared to multivariate GARCH models. This means that using Copula models and leaving aside the assumption that the joint distribution of asset returns is a normal or t Student distribution improves the hedging strategy. It can also be shown that symmetrized Joe-Clayton Copula gives the highest performance in hedging strategy for all maturities. Moreover, the highest performance of hedging strategies achieved by using WTI futures contract with six months maturity. Therefore, in order to decrease the fluctuation of oil revenues, it is preferred to perform hedge policy by futures contract with six months maturity.

In addition, the results showed that the optimal hedge ratio increases with maturity length of futures contract, which is in accordance with Samuelson hypothesis of maturity effect.

Therefore, in order to take appropriate hedging strategies for future contracts of OPEC's oil, it is recommended to use Copula-based models instead of multivariate GARCH models. Moreover, in order to achieve the highest performance in hedging strategy, and reduce the fluctuation of oil revenues, the maturity length of the contract should be taken into account as an important parameter. The results are useful for decision makers such as OPEC members and the major of energy and council.

References

Ang, A., & Chen, J. (2002). Asymmetric Correlations of Equity Portfolios. *Journal of Financial Economics, 63*, 443-494.

Ansari Ardali, Z., & Mousavi, M., & Kurdbacheh, H. (2017). Estimating Optimal Hedge Ratio and Comparing Their Effectiveness in the Natural Gas Market. *Quarterly Journal of Energy Economics, 13*(53), 35-60.

Chang, K. (2012). The Time-varying and Asymmetric Dependence between Crude Oil Spot and Futures Markets: Evidence from the Mixture Copula-based ARJI–GARCH Model. *Economic Modelling, 29*, 2298-2309.

Chang, C., McAleer, M., & Tansuchat, R. (2011). Crude Oil Hedging Strategies Using Dynamic Multivariate GARCH. *Energy Economics, 33*(5), 912-923.

Doifli, I., & Ghorbel, A. (2015). Optimal Hedging Strategy with Futures Oil Markets via FI-EGARCH Copula Model. *American Journal of Finance and Accounting, 4*(1), 1-28.

Ghorbel, A., & Trabelsi, A. (2012). Optimal Dynamic Hedging Strategy with Futures Oil Markets via FIEGARCH-EVT Copula Models. *International Journal of Managerial and Financial Accounting, 4*, 1-28.

Haigh, M. S., & Holt, M. (2002). Crack Spread Hedging: Accounting for Time-varying Spillovers in the Energy Futures Markets. *Journal of Applied Econometrics, 17*, 269-289.

Hsu, C. C., Tseng, C. P., & Wang, Y. H. (2008). Dynamic Hedging with Futures: A Copula Based GARCH Model. *Journal of Futures Markets, 28*(11), 1095-1116.

Khodadadian, B. (2010). *Hedging of Oil Price Risk Using Future Contracts: Case Study of Iran* (Unpublished Master's Thesis). University of Tarbiat Modarres, Iran.

Kotkatvuori-Örnberg, J. (2016). Dynamic Conditional Copula Correlation and Optimal Hedge Ratios with Currency Futures. *International Review of Financial Analysis, 47*(C), 60-69.

Lai, Y., Chen, W. S., & Gerlach, R. (2009). Optimal Dynamic Hedging via Copula-Threshold GARCH Models. *Mathematics and Computers in Simulation, 79*, 2609-2624.

Lai, Y. S. (2018). Dynamic Hedging with Futures: A Copula-based GARCH Model with High-frequency Data. *Review of Derivatives Research, 21*, 307-329.

Lee, H. T. (2009). A Copula-based Regime Switching GARCH for Optimal Futures Hedging. *Journal of Future Market, 29*, 946-972.

Liu, P., Vedenov, D., & Power, G. J. (2017). Is Hedging the Crack Spread No Longer All it is Cracked up to be? *Energy Economics, 63*, 31-40.

Mehrara, M., & Naebi, F. (2017). Assessing the Effectiveness of Fixed and Dynamic Hedge Ratio for Gold Coin Future Contracts of Iranian Commodity Exchangers. *Economic Research, 52*(3), 711-734.

Nasserian, M., & Esmaeeli, A. (2014). *Hedging of Iran's Crude Oil Price Risk by Futures Contracts in the NYMEX Using the mv-GARCH Method* (Unpublished Master's Thesis). Islamic Azad University, Central Tehran Branch, Iran.

Nikoloulopoulos, A. K., Joe, H., & Li, H. (2012). Vine Copulas with Asymmetric Tail Dependence and Applications to Financial Return Data. *Computational Statistics & Data Analysis, 56*(11), 3659-3673.

Ornberg, J. (2016). Dynamic Conditional Copula Correlation and Optimal Hedge Ratios with Currency Futures. *International Review of Financial Analysis, 47*, 60-69.

Patton, A. J. (2006). Modeling Asymmetric Exchange Rate Dependence. *International Economic Review*, *47,* 527-556.

Sukcharoen, K., & Leatham, D. J. (2017). Hedging Downside Risk of Oil Refineries: A Vine Copula Approach. *Energy Economics*, *66,* 493-507.