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A non-monetary valuation system for open-pit mine design

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ABSTRACT

In open-pit mining, different designs are created, such as optimal ultimate pit limit and production planning. In order to determine the ultimate pit limit, two approaches are generally used based on geological and economic block models. In this paper, according to the long-term trend of metals price and mining costs, some suggestions were made to design the ultimate pit limit using the geological block model. In addition, a grade-based objective function was presented for determining the ultimate pit limit. Then, in order to solve the problem, a heuristic algorithm was developed to simultaneously determine the ultimate pit limit and the sequence of block mining. For a 2D geological block model, the final pit was generated using the proposed algorithm. Furthermore, to validate the generated pit limit, the results of a 3D geological block model were compared with those of the Lerchs-Grossman algorithm. The comparison showed that the two pits corresponded to each other with an accuracy value of 97.7 percent.

Keywords : *Open-pit design, Ultimate pit, Non-monetary value, Optimization, Heuristic algorithm*

1. Introduction

Open-pit mines are the world's largest mineral producers. There are different open-pit mine designs, whose most important tasks are the ultimate pit design, and long-, medium-, and short-term production planning. The ultimate pit outline specifies the areas in which ore bodies could be mined economically. Some parameters are directly related to the final pit size, including the mine lifetime and the tonnage of minable waste and ore blocks. The cost, revenue, and the total cash flow of mining operations throughout the mine lifetime are estimated based on these parameters.

After the mineral exploration stage, a block model is made for the mineral deposit. The grades of blocks are calculated using different estimation methods, such as inverse distance, inverse squared distance, or Kriging, among others. After creating the grade (geological) block model, an economic block model is prepared based on the assumed economic parameters.

Two general approaches, i.e., geological and economic block models, are used for ultimate open-pit design and production planning. In most cases, the optimum ultimate pit is designed with maximizing the overall profit of the final pit outline. Several algorithms have been developed for this purpose, including floating cone [1], floating cone II [2], Korobov [3], 2D dynamic programming of Roman [4], Lerchs-Grossman [5] (LG), and network flow [6, 7]. These algorithms use the economic block model based on monetary block valuation. Among these methods, the LG algorithm, which is the most widely used method, determines the optimum pit outline mathematically based on graph theory. Then, production scheduling is planned to extract the blocks within the ultimate pit using a method such as parametric analysis [5].

In the design process, it is assumed that the economic parameters are constant throughout the mine life. Therefore, the ultimate pit remains optimum only if the economic conditions stay unchanged. If economic conditions change in the future, the optimal pit will no longer be valid.

Some methods use the geological block model as a non-monetary valuation for open-pit design. These methods include parameterization [8] and Wang & Sevim's [9, 10, 11] algorithms developed for simultaneous determination of ultimate pit and production planning. Similarly, Gershon [12] presented an algorithm only for production planning. One of the most important reasons for developing such methods is the instability of economic parameters over time.

Totally, there are two major categories of open-pit design algorithms based on their block model inputs. The first one includes monetary-based algorithms such as LG, Floating Cone, Korobov, network flow, and Parametric Analysis. The second category of algorithms uses the geological block model to determine the ultimate pit limit and production scheduling. These algorithms are grade-based methods and include Parameterization, Wang & Sevim's, and Gershon's algorithms. In fact, these algorithms use blocks grades as inputs.

In open-pit design, economic parameters and block grades are two sources of uncertainty. In this regard, many research studies have been conducted on the price, cost, and grade uncertainty in open-pit mining [13-25]. Some techniques, such as the real option [26-29], have been used to consider the economic uncertainty. These grade-based algorithms were developed to reduce the effect of economic uncertainty on open-pit mine design. The main advantage of grade-based methods, which use blocks grades as their non-monetary values, is reducing the main uncertainty resources into one. However, they do not take into account the operating costs of blocks in their valuations. The reason is that the value of waste blocks is defined as zero in this approach, but these blocks actually possess negative values. Thus, this valuation perspective cannot provide the net value of blocks; subsequently, the ultimate pit limit cannot be determined by it. Accordingly, this non-monetary technique should be improved to be more practical in open-pit mining. Since this grade-based valuation technique eliminates the economic parameters, the complex problem of including price, cost, and grade uncertainties all at once will be changed to a grade uncertainty problem.

In this paper, first, the long-term pattern of changes in metal price

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and mining costs were investigated, and then, an equivalent non-monetary value and a time-independent system were presented for blocks valuation. Afterward, a heuristic algorithm was developed which generates the ultimate pit limit and the sequence of block mining, simultaneously.

2. Long-term pattern of price and cost variations

Based on the data collected by Dehghan & Ataiepour [30] and Dehghani et al. [31], the pattern of changes in metal price and mining costs is similar for long-term open-pit and underground mining for some mineral deposits, such as copper deposits. Furthermore, according to data provided by Wellmer et al. [32], this similar pattern can be confirmed in gold and silver mines as well. The graph of these changes is shown in Figs 1 and 2 for open-pit and underground copper mines, respectively. Also, for each of these charts, the best fit lines have been plotted. The dramatic slope of these lines represents the high uncertainty and the upward trend of these economic parameters over time.

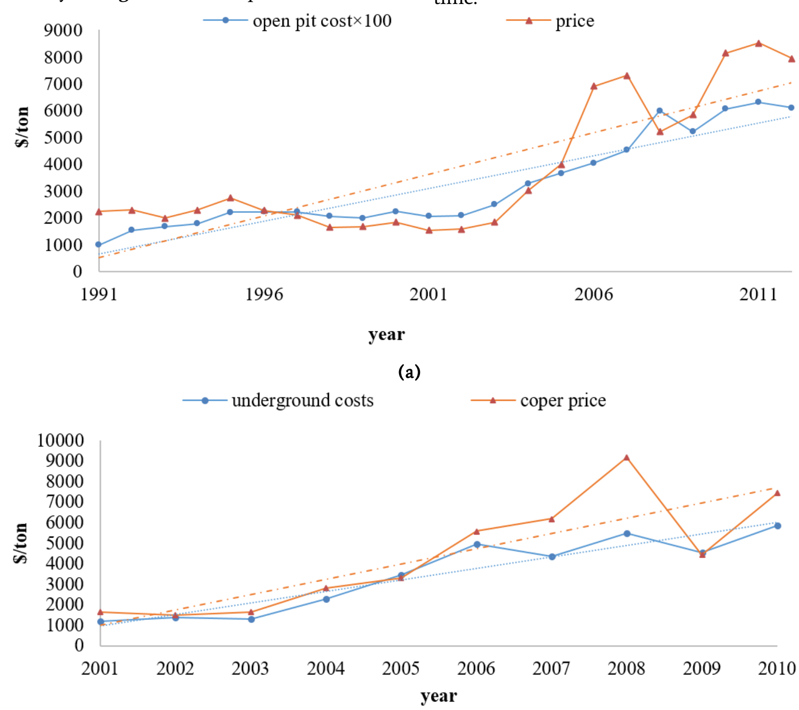

Fig. 1. Long-term variations of copper price and mining costs in (a) open-pit mines (modified from [31]) (b) underground mines (modified from [30]).

Generally, the mathematical relationship between metal price and mining costs can be expressed as equation (1).

$$C_{ton}(t) = P_{ton}(t) \times \alpha(t) \quad (1)$$

Where *t* is time, $C_{ton}(t)$ is the total cost of mining and processing the ore per ton at time t, $P_{ton}(t)$ is the price of metal per ton at time t, and $\alpha(t)$ is the conversion factor of cost and price at time t.

Figs 2 and 3, respectively show the best fit lines for the cost-to-price ratio for copper open-pit mines from 1991 to 2012 and the cost-to-price ratio for copper underground mines from 2001 to 2010. As seen, the slope of the average line is almost zero. This horizontal slope, unlike long-term slopes of price and cost variations, reflects the lower uncertainty of the cost-to-price ratio in long-term periods. Therefore, according to the similar pattern of price and cost variations, it can be assumed that the conversion coefficient of price and cost ($\alpha(t)$) for long-term goals is independent of time and is constant (α). According to this model, long-term price and cost variations can be expressed as equation (2).

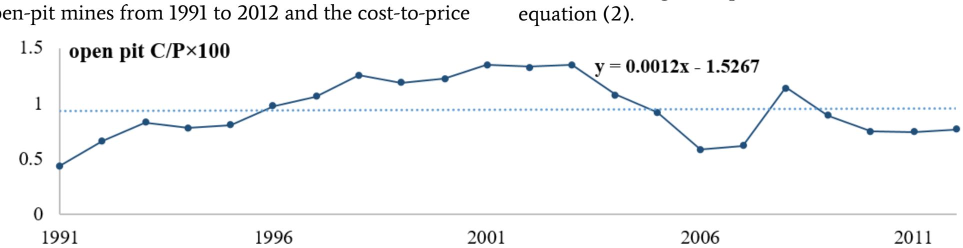

Fig. 2. Long-term variation of mining cost-to-price ratio in copper open-pit mines.

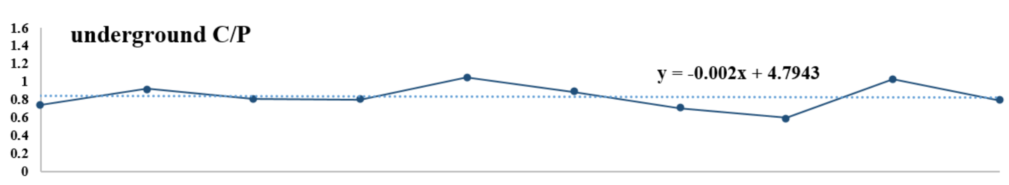

Fig. 3. Long-term variation of mining cost-to-price ratio in copper underground mines.

$$C(t) = P(t) \times \alpha \quad (2)$$

Equation (2) provides a long-term perspective on the variation of operating cost and metal price. The operating cost and metal price cannot be predicted in the long-term, but coefficient α can be used to forecast the cost-to-price ratio for long-term purposes.

3. Non-monetary gross values of blocks

As stated above, two types of monetary and non-monetary valuation are used to define the values of ore blocks. In the first approach, the monetary gross values of the ore blocks are calculated using equation (3) to create an economic block model. Then, by finding the mining and processing costs from the gross values, the monetary net values of the ore blocks are determined.

$$V_b = g_b \times P_{ton} \times T_b \times \underbrace{R_{min} \times R_{pro}}_{R_{total}} \quad (3)$$

where V_b is the gross value of a block in currency unit (CU), g_b is the ore block grade in percent, P_{ton} is the metal price in CU per ton, T_b is the tonnage of the block, R_{min}, R_{pro}, and R_{total} are respectively the mining operations, processing and total recovery in percent. If the contained metal (CM) of each block is used as a non-monetary value, the gross value of ore blocks is calculated from equation (4). In this equation, the recovery rate of mining and mineral processing operations is considered ideally at the best status, i.e., 100%, because the aim is performing a long-term analysis and the total recovery may be improved in the future.

$$V_b \equiv g_b \times T_b \quad (4)$$

If it is assumed that the densities of all blocks are equal, especially in low-grade ore bodies, the grades of ore blocks can be used as the equivalent concept of ore blocks CM by eliminating the weights of the blocks (T_b). Therefore, in equation (5), the grades of ore blocks are expressed as the equivalent concept of their gross values.

$$V_b \equiv g_b \% \quad (5)$$

The most important advantage of the definition of ore blocks' value is the elimination of uncertainty from economic parameters (price and costs). Therefore, the only remaining source of uncertainty risk is the grade of blocks.

On the other hand, by increasing the precision of grade estimations through simultaneous exploration along with the mining operations throughout the mine lifetime, the accuracy of estimation of blocks grade may improve as well.

So far, in the grade-based algorithms of ultimate pit design, like Wang & Sevim's or Gershon's algorithms, there has been no idea of defining and considering the cost of mining the blocks. In these methods, waste and ore-bearing blocks with values below the cutoff-grade are assigned a zero value. But in fact, these blocks impose mining costs on the whole mining operations and have to be considered in the non-monetary concept.

4. Non-monetary mining costs

Mining operation costs are paid from the revenue of raw-ore or metal. Thus, some parts of the CM can be used to equalize the costs. In this way, a part of the CM of blocks should be determined from the total CM to pay the mining costs. Accordingly, equation (6) can be used to determine the equivalent metal tonnage of mining and processing costs for each block. In this regard, the costs are assumed to be independent of block grades.

$$\underbrace{\left(C_{ton}^{min} + C_{ton}^{pro}\right)}_{C_{ton}^{total}} \times T_b = g_{cost} \times P_{ton} \times T_b \quad (6)$$

where, C_{ton}^{min} and C_{ton}^{pro} are mining and processing costs per ton in CU, respectively; C_{ton}^{total} is the total costs of mining and processing per ton in CU and g_{cost} is equivalent grade of the total cost in percent. By rearranging the equation (6) and assuming the equality of blocks weights, the equivalent concept of the total costs for each block is obtained through equation (7).

$$g_{cost} = C_{ton}^{total} / P_{ton} \% \quad (7)$$

As stated before, the long-term patterns of cost and price variations can be assumed equal. Consequently, according to equation (2), the operating cost-to-price ratio can be assumed constant in long-term periods. Therefore, in the case of using the aforementioned equivalent definition for mining costs, it is possible to disregard uncertainty in equation (7) and to assume this ratio as deterministic. This equation is, in fact, the definition of the cutoff grade. Hence, in other words, the non-monetary definition and the equivalent concept of mining costs can be defined as the cutoff-grade. It should be mentioned that the used assumptions were to model the unpredictable changes in economic parameters in a simplified form.

5. Non-monetary net values of blocks

According to the non-monetary definition for gross values of ore blocks and mining costs of all blocks, the non-monetary net values (NNVs) of ore blocks can be calculated by equation (8).

$$V_{b,net}^{ore} \equiv g_b^{ore} - g_{cutoff} \% \quad (8)$$

where $V_{b,net}^{ore}$, g_b^{ore} and g_{cutoff} are the equivalent non-monetary net value of ore blocks, the grade of ore block, and the cutoff grade in percent. Also, in this system of value definition, the cost of waste or ore blocks under cutoff grades are calculated from equation (9).

$$V_b^{waste} \equiv -g_{cutoff} \% \quad (9)$$

where V_b^{waste} is the equivalent non-monetary mining cost for waste blocks.

6. Ultimate pit limit determination

The optimum ultimate pit is an outline of the deposit in a way that its blocks are economically mineable. Accordingly, engineers maximize the total profit of the ultimate pit. The objective function of ultimate pit determination is equation (10) subject to the constraints (11) and (12).

$$Max \sum_{(i,j,k)\in OB} V_{ijk} \times x_{ijk} \quad (10)$$

Subject to:

$$x_{ijk} \le x_{i'j'k'} \qquad \forall (i,j,k) \in OB,\ (i',j',k') \in UB_{ijk} \quad (11)$$

$$x_{ijk} = 0 \text{ or } 1 \quad (12)$$

where i is the block index in columns (easting), j is the block index in rows (northing), k is the block index in levels (elevation), V_{ijk} is the net value of ore blocks in the CU, x_{ijk} is a binary decision variable of presence or absence of block ijk within the optimum ultimate pit, OB is the set of coordinates of all ore body blocks, UB_{ijk} is the set of coordinates of upper blocks which must be removed to facilitate the access to the block ijk.

An extension of equation (10) can be expressed subject to constraints (11) and (12), as follows:

$$Max\left(\sum_{(i,j,k)\in Ore}\left(P \times g_{ijk} \times T_{ijk} - C \times T_{ijk}\right) \times x_{ijk} - \sum_{(i,j,k)\in Waste} C \times T_{ijk} \times x_{ijk}\right) \quad (13)$$

where P is the metal price per ton in the CU, g_{ijk} is the grade of block ijk in percent, T_{ijk} is the tonnage of block ijk, C is the mining cost of blocks per ton in CU, Ore is the set of all ore blocks coordinates and $Waste$ is the set of all waste blocks coordinates.

Using equation (7) and the equivalent non-monetary concept of mining costs, equation (13) can be rewritten as equation (14).

$$Max\left(\sum_{(i,j,k)\in Ore}\left(P \times g_{ijk} \times T_{ijk} - P \times g_{cutoff} \times T_{ijk}\right) \times x_{ijk} - \sum_{(i,j,k)\in Waste} P \times g_{cutoff} \times T_{ijk} \times x_{ijk}\right) \quad (14)$$

Assuming the equality for weights of all blocks especially in low-grade deposits and eliminating the common positive coefficients P×T from equation (14), equation (15) subject to constraints (11) and (12) is obtained as the grade-based objective function of for determining the ultimate pit limit.

$$Max\left(\sum_{(i,j,k)\in Ore}\left(g_{ijk}-g_{cutoff}\right)\times x_{ijk}-\sum_{(i,j,k)\in Waste} g_{cutoff}\times x_{ijk}\right) \tag{15}$$

This objective function can be solved with the use of the LG algorithm.

7. Heuristic algorithm

In order to solve the objective function (15) subject to constraints (11) and (12), the complex mathematical algorithms such as Branch-and-Bound or the LG algorithm can be used. In this section, using the Downward Cone concept in Gershon's algorithm [12], a heuristic algorithm was developed to generate the ultimate pit limit. Gershon's algorithm is a grade-based heuristic algorithm proposed to determine an extracting sequence for the blocks within the ultimate pit limit. Therefore, to use this algorithm, first, the optimum pit limit must be determined. Having no mathematical proof, heuristic algorithms seek an appropriate solution according to the logic of the problem. Due to the huge number of decision variables and constraints in ultimate pit optimization and production planning problems, these issues are defined in the NP-hard category.

In order to present the heuristic algorithm, two concepts of the biggest possible pit and positional weight are introduced first. In the proposed algorithm, the search is performed initially by removing the unnecessary blocks in the pit and includes the furthest and deepest ore blocks. This pit is called the biggest possible pit (BPP). Also, the positional weight (PW) of a block is the sum of the grades of ore blocks within the downward cone of a block without considering its grade. The meaning of a block PW is the metal amount that can be obtained from the underneath blocks after this block in the extraction path. Fig 4 shows the BPP and PW values of the block (1, 5).

k \ i →	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	0	0	0	0
2	0	0	10	20	10	15	0	0	0	0
3	0	0	0	10	20	0	0	10	0	0
4	0	0	0	25	5	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0

g_{ij} ←BPP $PW_{(1,5)}=20+10+15+10+20+25+5=105$

Fig. 4. BPP determination and PW calculation.

The flowchart of the proposed algorithm is illustrated in Fig 5. First, the BPP should be defined for the deposit. To define the BPP for each ore block, an upward cone must be determined, then all of these upward cones should be collected. Then, the PW of all blocks within the BPP must be calculated as explained above (Fig 4). The algorithm uses a block-to-block search approach to determine the extraction sequence of blocks and to calculate the cumulative value of the blocks to find the highest value. The number of search steps is equal to the number of blocks within the BPP. In each step, two sub-steps of determining the technically minable blocks and then selecting the suitable block for extraction should be executed. A practically mineable block is one whose upper blocks mining sequence is determined or it is located on the topographic surface. At first, among the BPP blocks, the practically extractable blocks are selected as candidates. The technical condition for block extraction is removing its overlying nine or five blocks in the 3D block models and three overlying blocks in the 2D block models. The candidate blocks that are practically extractable and must be compared to each other to choose the most suitable block is shown in Fig 6. Further details will be provided in the following paragraphs using a numerical example.

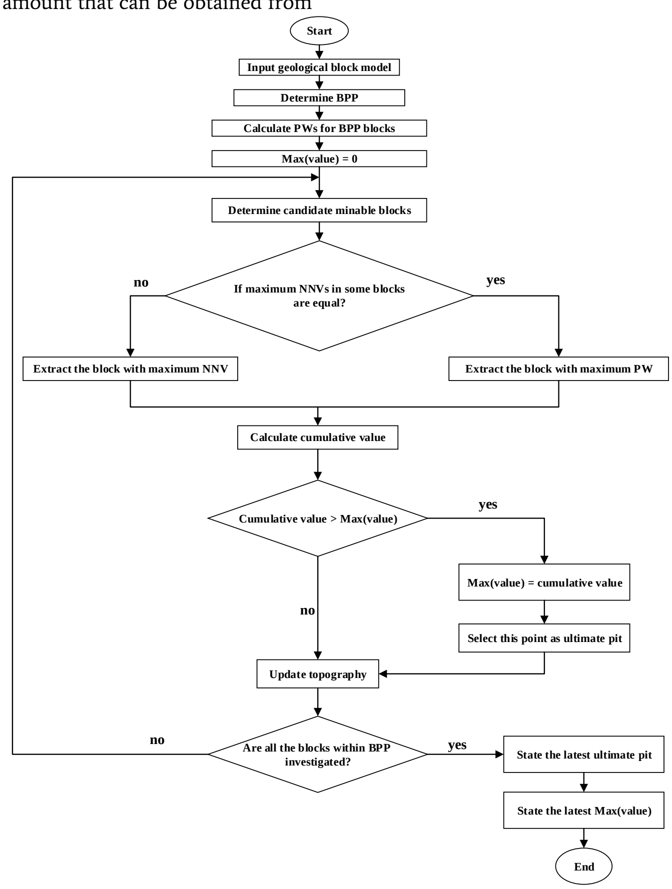

Fig. 5. Presented heuristic algorithm (PW: positional weight, NNV: non-monetary value of block).

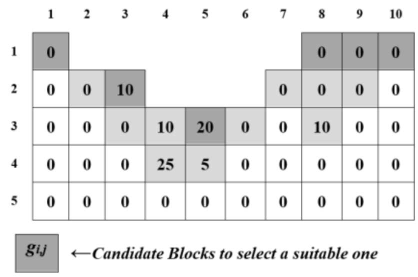

Fig. 6. Updated topography and technically candidate blocks in the 10th step.

The first parameter used to compare the candidate blocks in each step is the non-monetary net value (NNV). Among the candidate blocks, the block that has the highest non-monetary value (NNV) would be selected as the extracted block at this step. If the highest NNV of some candidate blocks is equal, their PWs should be compared. In this case, the block with the highest PW will be selected as the extracted block at this step. It should also be noted that at the beginning of the algorithm, the parameter of Max(value) must be determined with the temporary initial value of zero. After selecting the block in each step, the block value of this step should be added to the values of previous blocks. In each step, the cumulative value of the extracted blocks should be compared with the Max(value) value. If the cumulative value of this step is greater than Max(value), this cumulative value will be replaced with Max(value) and the blocks from the first to this step will be introduced as the temporary final pit. Otherwise, Max(value) and temporary values remain unchanged. In each step, the selected blocks must be removed from the block model and the topographic level should be updated for the next step. These steps will continue until the mining sequence of all BPP blocks is specified.

The algorithm was used for geological block modeling, as shown in Fig 7. In Fig 8, the non-monetary net value and the PW of the blocks are calculated with a cutoff grade of 3%.

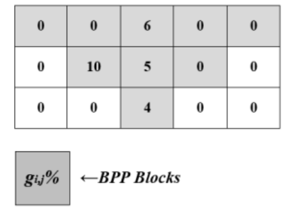

Fig. 7. Geological block model.

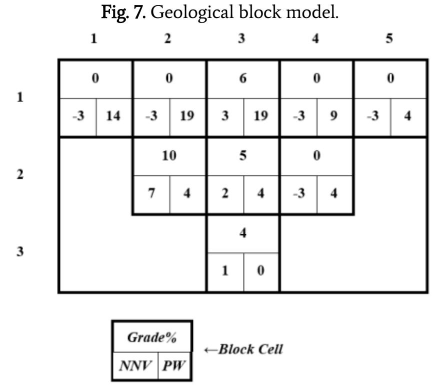

Fig. 8. Grades, NNVs and PWs of the geological block model shown in Fig 7.

Fig 9 shows the process of determining the most appropriate block to be extracted in each step, as the NNVs of the candidate blocks are compared and the block with the highest NNV is selected for extraction. For equal NNVs in steps 2, 3, and 5, the PW values of the blocks are compared. Finally, the block with the highest PW is selected.

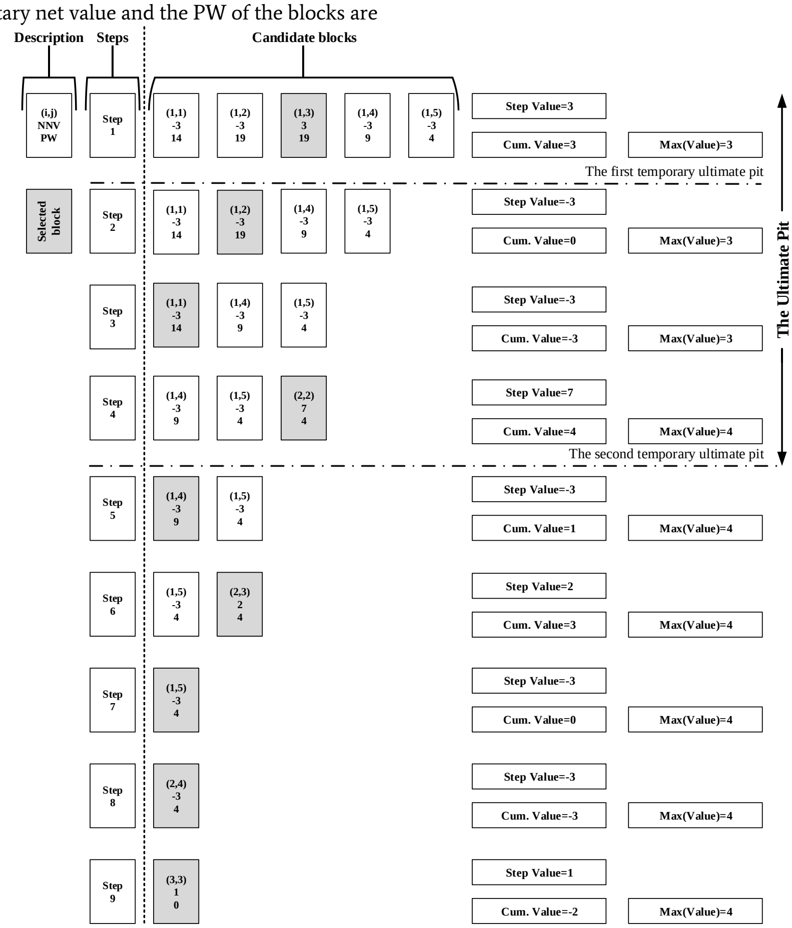

Fig. 9. Steps of the algorithm for sample block modeling.

3	2	1	5	7
0	4	6	8	0
0	0	9	0	0

←*UP Blocks*

Fig. 10. Ultimate pit and mining sequence for the geological block model shown in Fig 7.

Determining the ultimate pit and the extraction sequence of the blocks can be conducted simultaneously using this new algorithm. Based on the comparison of PW values for candidate blocks at each step, the extraction sequence will move towards the high-grade areas of the ore body. In fact, the algorithm attempts to extract more valuable blocks in a shorter period of time. Accordingly, the proposed sequence of this algorithm can maximize the net present value (NPV). In this algorithm, the point with the highest cumulative value is determined as the optimum final pit.

8. Numerical Two-Dimensional Example

In this section, the new algorithm is applied to the 2D geological block model shown in Fig 11 to generate the ultimate pit limit. To do this, after determining the BPP, the PWs and NNVs of blocks are determined. In this model, the cutoff grade is assumed at 0.1%. Fig 12 shows the grades, PWs, and NNVs of the BPP blocks.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0.5	0.25	0	0	0	0	0.5	0.5	0.75	2	0	0	0	0	0	0	0
3	0	0	0	1	1	1.5	2	1.25	1	0	0	0.5	0.25	2	2	1	1	0.5	0	0	0
4	0	0	0	1	2	2.5	3	2	1.5	1	1	0.5	1	3	1.5	2	0.25	0	0	0	0
5	0	0	0	0	2.25	1	2	3	3	1	1.5	1.5	1	2	1	0	0	0	0	0	0
6	0	0	0	0	0	1.25	2	1	1	0.25	0.2	0.2	0.25	0.5	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0.75	0.5	0.5	0.2	0.2	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0.2	0.5	0.2	0.15	0.15	0.2	0.2	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0.2	0	0	0	0	0	0	0	0

$g_{i,j}\%$ ←*BPP Blocks*

Fig. 11. The 2D geological block model.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	0 -0.1 | 5.45	0 -0.1 | 12.45	0 -0.1 | 19.65	0 -0.1 | 29	0 -0.1 | 37.05	0 -0.1 | 41.2	0 -0.1 | 43.6	0 -0.1 | 44.1	0 -0.1 | 42.6	0 -0.1 | 39.85	0 -0.1 | 39.35	0 -0.1 | 36.35	0 -0.1 | 36.35	0 -0.1 | 31.35	0 -0.1 | 27.6	0 -0.1 | 22.9	0 -0.1 | 17.5	0 -0.1 | 10.9	0 -0.1 | 6	0 -0.1 | 1.35	0 -0.1 | 0.4
2		0 -0.1 | 5.45	0 -0.1 | 12.45	0 -0.1 | 19.65	0.5 0.4 | 28.5	0.25 0.15 | 35.3	0 -0.1 | 37.25	0 -0.1 | 37.85	0 -0.1 | 33.85	0 -0.1 | 30.6	0.5 0.4 | 27.65	0.5 0.4 | 25.6	0.75 0.65 | 23.35	2 1.9 | 23.35	0 -0.1 | 21.4	0 -0.1 | 17	0 -0.1 | 10.9	0 -0.1 | 6	0 -0.1 | 1.35	0 -0.1 | 0.4	
3			0 -0.1 | 5.45	1 0.9 | 11.45	1 0.9 | 17.65	1.5 1.4 | 24	2 1.9 | 27.8	1.25 1.15 | 25.95	1 0.9 | 24.85	0 -0.1 | 22.35	0 -0.1 | 17.85	0.5 0.4 | 15.1	0.25 0.15 | 15.6	2 1.9 | 14.15	2 1.9 | 12.75	1 0.9 | 8.4	1 0.9 | 4.5	0.5 0.4 | 0.85	0 -0.1 | 0.4		
4				1 0.9 | 4.45	2 1.9 | 8.45	2.5 2.4 | 12.15	3 2.9 | 14.25	2 1.9 | 17.05	1.5 1.4 | 15.2	1 0.9 | 12.1	1 0.9 | 9.1	0.5 0.4 | 8.1	1 0.9 | 7.65	3 2.9 | 6.25	1.5 1.4 | 4.65	2 1.9 | 2.25	0.25 0.15 | 0.6	0 -0.1 | 0.4			
5					2.25 2.15 | 2.2	1 0.9 | 5.2	2 1.9 | 6.9	3 2.9 | 7	3 2.9 | 5.8	1 0.9 | 4.45	1.5 1.4 | 3.35	1.5 1.4 | 2.65	1 0.9 | 2.25	2 1.9 | 1.65	1 0.9 | 1.25	0 -0.1 | 0.6	0 -0.1 | 0.4				
6						1.25 1.15 | 0.95	2 1.9 | 1.95	1 0.9 | 2.65	1 0.9 | 2.25	0.25 0.15 | 2.3	0.2 0.1 | 1.8	0.2 0.1 | 1.3	0.25 0.15 | 0.9	0.5 0.4 | 0.75	0 -0.1 | 0.6	0 -0.1 | 0.4					
7							0.75 0.65 | 0.2	0.5 0.4 | 0.7	0.5 0.4 | 0.9	0.2 0.1 | 0.85	0.2 0.1 | 0.7	0 -0.1 | 0.7	0 -0.1 | 0.75	0 -0.1 | 0.6	0 -0.1 | 0.4						
8								0.2 0.1 | 0	0.5 0.4 | 0	0.2 0.1 | 0	0.15 0.05 | 0	0.15 0.05 | 0.2	0.2 0.1 | 0.2	0.2 0.1 | 0.2							
9													0.2 0.1 | 0								

Grade %	
NNV	PW

←*Block cell*

Fig. 12. Calculation of NNVs and PWs for the block model shown in Fig 11.

The suggested extraction sequence by the algorithm for the BPP blocks is shown in Fig 13.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	85	65	38	24	9	4	2	1	3	5	6	12	14	18	28	31	57	73	79	93	105
2		86	66	39	25	10	8	7	16	21	13	15	19	29	35	58	74	80	94	106	
3			87	67	40	26	11	17	22	32	43	20	30	36	59	75	81	95	107		
4				88	68	41	27	23	33	44	46	50	37	60	76	82	96	108			
5					89	69	42	34	45	47	51	52	61	77	83	101	109				
6						90	70	48	49	53	55	62	78	84	102	110					
7							91	71	54	56	63	98	97	103	111						
8								92	72	64	100	99	104	112							
9													113								

Fig. 13. Mining sequence for the block model shown in Fig 11.

The graph of cumulative values based on the mining sequence is shown in Fig 14, in which the maximum cumulative value is obtained at step 96. Therefore, the non-monetary value of the final pit limit generated by the algorithm is 61.1. This final pit is shown in Fig 15.

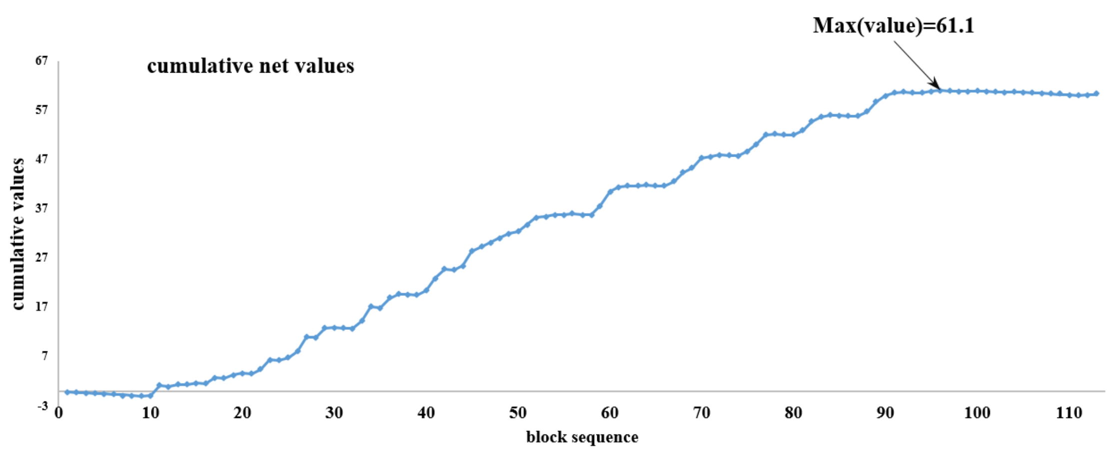

Fig. 14. Cumulative values of blocks and the Max(value).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	85	65	38	24	9	4	2	1	3	5	6	12	14	18	28	31	57	73	79	93	105
2		86	66	39	25	10	8	7	16	21	13	15	19	29	35	58	74	80	94	106	
3			87	67	40	26	11	17	22	32	43	20	30	36	59	75	81	95	107		
4				88	68	41	27	23	33	44	46	50	37	60	76	82	96	108			
5					89	69	42	34	45	47	51	52	61	77	83	101	109				
6						90	70	48	49	53	55	62	78	84	102	110					
7							91	71	54	56	63	98	97	103	111						
8								92	72	64	100	99	104	112							
9													113								

←BPP   Optimum pit by the Algorithm→

Fig. 15. The ultimate pit and mining sequence by the algorithm.

Also, in order to validate the new algorithm, the ultimate pit for the geological block model shown in Fig 16 was generated by the 2D dynamic programming algorithm [5]. The result shows that both methods produce the same ultimate pit with 96 blocks and a total value of 61.1.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1
2	0	-0.1	-0.1	-0.1	0.4	0.15	-0.1	-0.1	-0.1	-0.1	0.4	0.4	0.65	1.9	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1
3	0	0	-0.1	0.9	0.9	1.4	1.9	1.15	0.9	-0.1	-0.1	0.4	0.15	1.9	1.9	0.9	0.9	0.4	-0.1	0	0
4	0	0	0	0.9	1.9	2.4	2.9	1.9	1.4	0.9	0.9	0.4	0.9	2.9	1.4	1.9	0.15	-0.1	0	0	0
5	0	0	0	0	2.15	0.9	1.9	2.9	2.9	0.9	1.4	1.4	0.9	1.9	0.9	-0.1	-0.1	0	0	0	0
6	0	0	0	0	0	1.15	1.9	0.9	0.9	0.15	0.1	0.1	0.15	0.4	-0.1	-0.1	0	0	0	0	0
7	0	0	0	0	0	0	0.65	0.4	0.4	0.1	0.1	-0.1	-0.1	-0.1	-0.1	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0.1	0.4	0.1	0.05	0.05	0.1	0.1	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0.1	0	0	0	0	0	0	0	0

←BPP

Optimum pit by 2D Dynamic Programming Alg.→

Fig. 16. The optimum ultimate pit by the 2D Dynamic Programming algorithm.

9. Three-Dimensional Block Model

In this section, the developed valuation system and the presented algorithm were applied to a 3D grade block model of a massive sulfide copper ore body. The number of blocks toward the north and east was 60. Also, the number of blocks from the top of the blocky model to the floor was equal to 20. The size of blocks was 15 × 15 × 15 m with an average density of 2.8 ton/m^3, with a cutoff grade of 0.25 percent. This block model had 7340 ore blocks with grades higher than the cutoff grade. The number of BPP blocks was 9970. The optimum pit of the ore body with LG algorithm had 6586.6 non-monetary values. The presented heuristic algorithm was used to determine the ultimate pit and blocks sequences. The graph of cumulative values vs blocks sequence is shown in Fig 17. The run time of the algorithm for this block model in MATLAB was 40 seconds. The results showed that its ultimate pit had 9901 blocks with total a non-monetary value of 6436.1. The obtained pit is shown in Fig 17. The numbers within the plot are a summation of the blocks in the vertical direction. According to the results of LG and the presented algorithms, the accuracy of the new method is 97.7 percent in this case study.

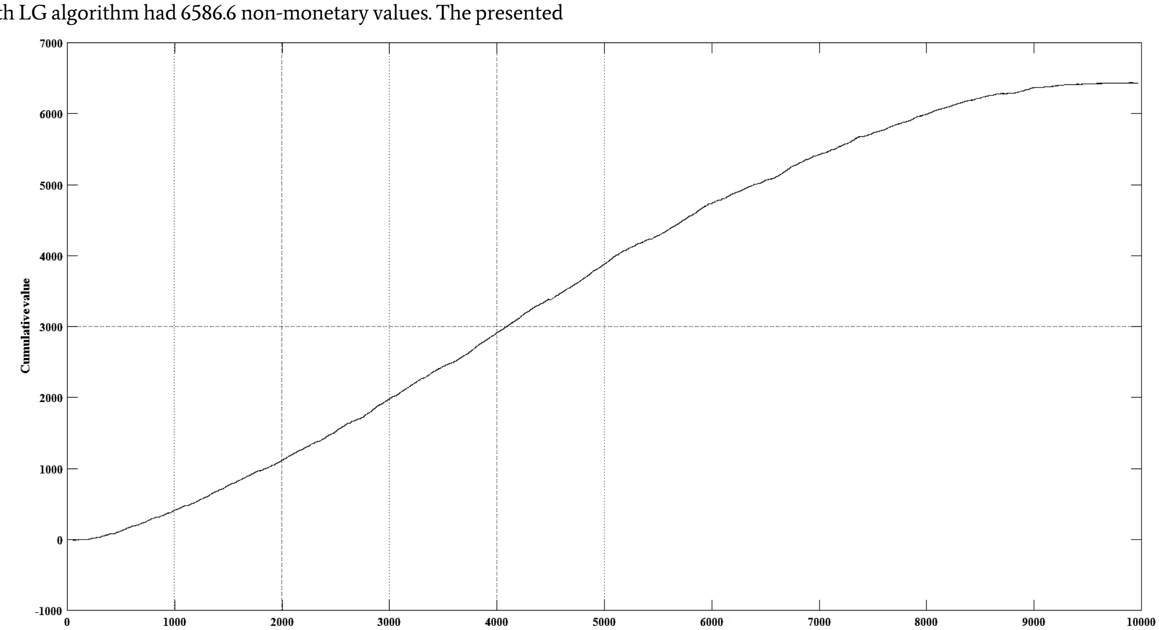

Fig. 17. The graph of results obtained from the presented algorithm for the 3D geological block model.

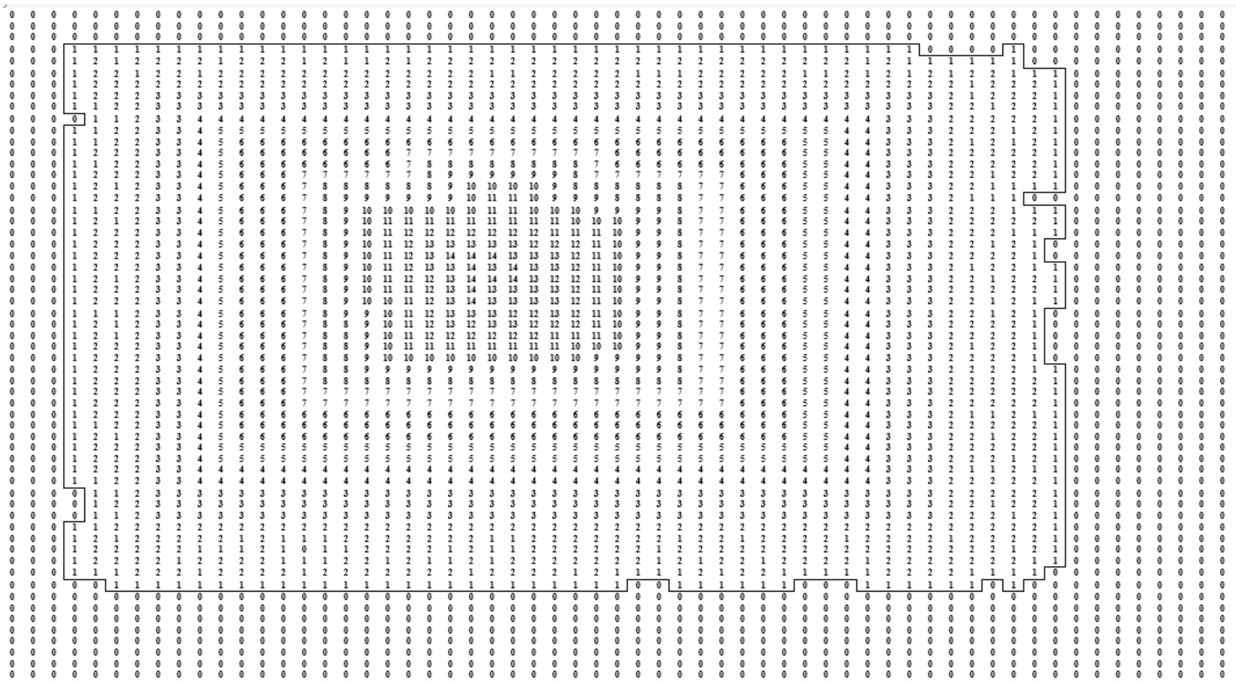

Fig. 18. The plot of the ultimate pit obtained from the algorithm for the 3D geological block model.

10. Discussion

It is a fact that the economic parameters (cost and metal price) are not stable over the lifetime of an open-pit mine. The economic parameters and the grades of blocks are two sources of uncertainty in open-pit design. Generally, taking both of these uncertainties into account is highly complicated in ultimate pit limit design and production planning. As said, according to equation (1), the operating cost-to-metal price ratio in each time t is $\alpha(t)$. Achieving a relation to predicting the behavior of this ratio is a key objective to consider the economic changes during the lifetime of a mine. Two average trends of descending and ascending along with the constant value over time are three possible statuses for the cost-to-price ratio over long-term periods. These trends are shown in Fig 19. Some of the traditional and simple assumptions in economical investigations in mining engineering suppose that the trend of mineral prices in ascending or the trend of mining costs is descending. Accordingly, these simple assumptions in unlimited long-term mine scheduling will mathematically support this notion that in mine designs, the prices will be so high and the mining costs will be near to zero. In other words, the metal price will have a huge effect on the mining project and the mining costs can be neglected. But in contrast, obviously, the present time is the long-term future of the last decades and nowadays the prices are not so huge and the costs are not near-zero. As mentioned earlier, the average cost-to-price ratio approximately remains unchanged over the long-term. This is evidence of the importance of simultaneous investigation of long-term variations of operating costs and mineral prices. The results of this paper support this idea that the long-term variations of operating costs and metal prices can be assumed the same. This means that $\alpha(t)$ has less amount of uncertainty than cost and price. As an important advantage, the presented non-monetary valuation system reduces the uncertainty sources into one. Accordingly, the only remained source of uncertainty is the grades of blocks. A comparison of uncertainties in monetary and non-monetary valuations is shown in Fig 20. The next most important advantage of the presented valuation system is that it can be used for ultimate pit determination. Therefore, the developed heuristic algorithm can determine the ultimate pit limit and the mining sequence of blocks, simultaneously. If the old grade-based valuation method is used for the presented algorithm, since the operating cost is not considered, the Max(value) parameter will increase following a quite straight line in the *cumulative value-to-block sequence* graph. Consequently, the algorithm is unable to find the optimum or near-optimum ultimate pit limit. The mentioned straight graph would not be able to determine an economic ultimate pit that has the optimum net value and it will determine the BPP as the ultimate pit. This problem raised from the old grade-based method shown in Fig 21. By adding the non-monetary cost of mining operations to the grade-based valuation method, the developed system can be used to determine the ultimate pit limit and block mining sequence.

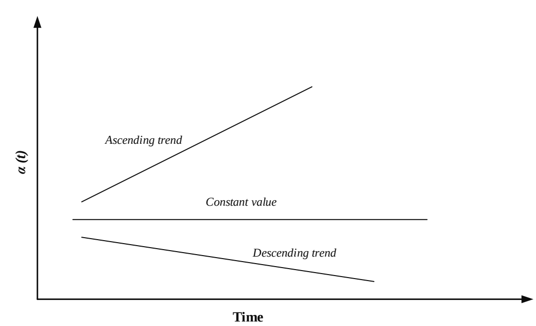

Fig. 19. The possible statuses for α (t) trending vs time.

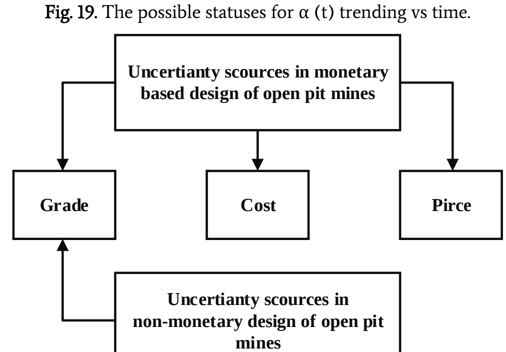

Fig. 20. A comparison of uncertainty sources in monetary and non-monetary valuation systems.

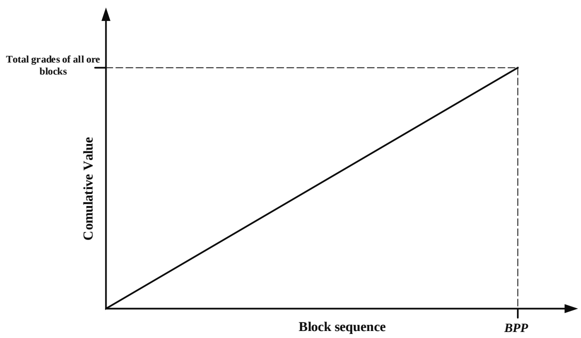

Fig. 21. The straight graph obtained from the algorithm based on the old grade-based valuation system.

The main goal of the ultimate pit is maximizing the net present value (NPV) not the undiscounted profit. The developed algorithm is steering the mining path toward the valuable blocks that have higher grades. The NPV of mining operations can be maximized through the obtained mining sequence indirectly from this algorithm. The developed method for open-pit mining design is independent of the discount rate. The presented algorithm can be applied to an economic block model for the simultaneous determination of the ultimate pit limit and block mining sequence. The steps of pit design through the developed non-monetary valuation system is shown in Fig 22.

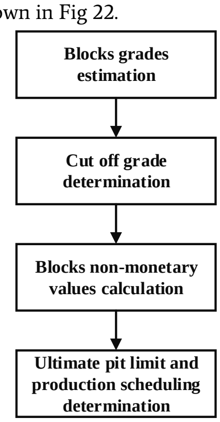

Fig. 22. The steps of open-pit design according to the non-monetary valuation system.

Totally, the developed non-monetary valuation system is based on the fact that the global economic environment changes as an integrated dynamic system. This means that the long-term variation of costs and prices are in a reciprocal relation together. Consequently, the obtained idea in this paper can be used for the long-term design of open-pit mines through the suggested block valuation system. It should be noted that the accuracy of this idea should be reviewed in mid-term and short-term designs like 6- or 12-month production planning.

11. Conclusions

The optimum open-pit outline is generally determined with the use of economic or geological block models. The ultimate pit, which is based on the economic block model, is valid until economic condition remains unchanged. The long-term variations of the metal price and mining costs involve severe uncertainty. In this paper, a non-monetary valuation system was developed to determine the optimum open-pit limit. For this aim, the long-term stability of the cost-to-price ratio was used to define the non-monetary valuation system. Consequently, the objective function of the ultimate pit limit determination was presented based on the blocks' non-monetary value. A new heuristic algorithm was developed to solve this objective function. This algorithm determines the ultimate pit limit and production planning simultaneously. The results obtained using this algorithm are consistent with those of the LG method. The new algorithm is more straightforward in understanding, calculating, and programming than the LG algorithm. Another advantage of the algorithm is providing a mining sequence so that the high-grade areas will be extracted more quickly. Since this algorithm applies a block-to-block search approach to the ore body, it is easy to incorporate variable slope angles.

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