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A new approach based on state conversion to stability analysis and control design of switched nonlinear cascade systems

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ABSTRACT

In this paper, the problems of control and stabilization of switched nonlinear cascade systems is investigated. The so called simultaneous domination limitation (SDL) is introduced in previous works to assure the existence of a common quadratic Lyapunov function (CQLF) for switched nonlinear cascade systems. According to this idea, if all subsystems of a switched system satisfy the SDL, a CQLF can be constructed by employing the back-stepping approach. The major shortcoming of the SDL is that this limitation cannot be satisfied for complicated switched nonlinear systems. Therefore, a CQLF cannot be constructed by employing the back-stepping approach. Moreover, if SDL is satisfied, only stabilization problem can be solved. In this paper, a new approach based on state conversion is introduced to solve the stabilization and control problems of switched nonlinear cascade systems without any limitation. Several simulation and experimental studies are provided to show the effectiveness of the proposed approach.

1. Introduction

A system whose behavior depends on finite or infinite connections between discrete and continuance dynamical systems is called a hybrid dynamical system. The gear shift control, computer control systems, room temperature control by thermostat, social networks, multi-agent systems, vehicular networks, traffic control, etc. are examples of hybrid dynamical systems [1-8]. Switched systems are an important class of hybrid dynamical systems consisting of a set of subsystems and a rule specifying the switching action between them [9-11].

In recent decades, a large amount of research works is carried out on stability analysis and control design of switched systems. These researches can be categorized from several aspects of view: stabilization analysis [12-14], control and tracking problems [15-16], optimal performance control [17-18], stability analysis under time delay [19-20], switched stochastic systems [22], etc. Two general approaches are employed for stability analysis of switched systems. 1) Common Lyapunov function (CQLF) for arbitrary switching [5,6,23] and 2) multiple Lyapunov function (MLF) for constrained switching [16,24].

A switched system under arbitrary switching is globally asymptotically stable (GAS) if all subsystems share a CQLF [9, 13, 27]. A switched system under constrained switching is GAS if all subsystems are GAS and the switching actions are slow such that the switching periods are larger than the dwell time [9, 25].

The stability analysis and control design of switched nonlinear systems includes more complexity relative to the switched linear systems. Therefore, fewer results are provided about switched nonlinear systems in the literature. Most of the proposed researches are carried out on stabilization, control, time delay, optimal performance and other analyses on switched linear systems [8, 28-30].

The so called simultaneous domination limitation (SDL) at first was presented in [31] to stability analysis of switched nonlinear systems in cascade form. According to this limitation, if all subsystems of a switched cascade nonlinear system are simultaneously dominatable, a CQLF can be constructed by employing the back-stepping method [13, 16, 32].

In back-stepping control method, the virtual control law associated with the k-th step stabilizes the all previous k states [33]. For a switched cascade system, the virtual control law of the k-th step should stabilize the previous k states for all subsystems [13, 31]. A switched nonlinear cascade system is called simultaneously dominatable if at each step of back-stepping control design there exists a virtual control law that makes all

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subsystems stable [13, 31, 34]. If the structure of a nonlinear cascade system be complicated, finding a virtual control law satisfying SDL is very difficult or even impossible. Although, a great deal of researchers have used the SDL idea in previous researches, but they did not present any procedure to find virtual control laws satisfying the SDL [13, 16, 31, 32, 35, 36]. On the other hand, in the case of satisfying SDL, only the stabilization problem can be solved and all approaches based on SDL cannot solve the tracking problem [13, 16, 31, 32, 34, 35]. Whereas the SDL idea is broadly employed to solve different problems arisen of switched nonlinear cascade systems, but its fundamental restrictions for complicated switched systems have not been discussed.

Since sustaining SDL in complicated switched nonlinear cascade systems is very difficult or impossible, in this study, a new method is presented to eliminate the limitations of this approach. Afterwards, the new method is generalized to the tracking problem. The most contributions of this paper is presenting a new approach to solve the stability analysis and tracking control of complicated switched nonlinear cascade systems without the limitations of SDL-based approaches. At first, the limitations of SDL idea are discussed by presented some examples. Afterwards, the new idea is introduced and is generalized to both stabilization and tracking problems.

The rest of this paper is structured as follows. In section 2, the general form of switched nonlinear cascade systems is presented and the SDL idea is introduced. Afterwards, the limitation of this idea are discussed by presenting an example. In section 3, a new method based on state conversion is introduced to eliminate the shortcomings of SDL idea and solve both stabilization and tracking control problems. In section 4, several numerical and experimental studies are provided to show the effectiveness of the proposed approach. Finally, this paper is concluded in section 5.

2. Preliminaries and problem description

Consider the following switched nonlinear system

$$\dot{\mathbf{x}} = \mathbf{g}_{\sigma(t)}(\mathbf{x}) + \mathbf{h}^T_{\sigma(t)}(\mathbf{x})\mathbf{u}\ , \quad \sigma(t) \in \{1,2,...,\mathrm{m}\} \tag{1}$$

where $\mathbf{g}_{\sigma(t)}(\mathbf{x})$ and $\mathbf{h}_{\sigma(t)}(\mathbf{x})$ are switching functions, $\mathbf{x}$ is the state vector of system, $\mathbf{u}$ is the vector of inputs, $\sigma(t)$ is the switching signal and m is the number of subsystems. The smooth scalar positive definite function $V(\mathbf{x})$, is called a CQLF for (1) if for all $\sigma(t)$, the following inequality is satisfied [9]

$$\frac{\partial V}{\partial \mathbf{x}}\left(\mathbf{g}_{\sigma(t)}(\mathbf{x}) + \mathbf{h}^T_{\sigma(t)}(\mathbf{x})\mathbf{u}\right) < 0 \tag{2}$$

Theorem 1. If there exists a CQLF satisfying (2), the system (1) is GAS under arbitrary switching [9,13].

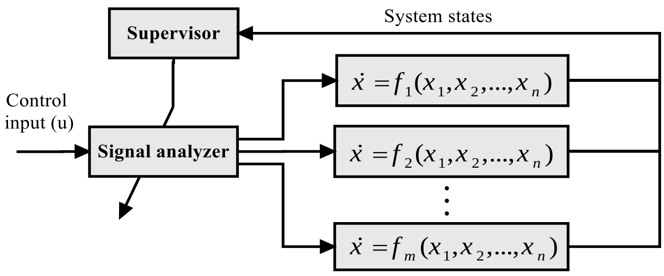

Figure 1. General structure of a switching system.

Fig. 1 depicts the general structure of a switching system with m subsystems. At each instant, the supervisor block based on output of system generates the switching signal $\sigma(t)$. According to this signal, the signal analyzer activates the related subsystem.

Consider the following general form of switched nonlinear cascade systems [31]

$$\begin{cases} \dot{x}_1 = x_2 + \mu_{1,\sigma(t)}(x_1) \\ \quad\vdots \\ \dot{x}_i = x_{i+1} + \mu_{i,\sigma(t)}(\mathbf{x}_i) \qquad ; \quad y = x_1 \\ \quad\vdots \\ \dot{x}_n = g_{\sigma(t)}(\mathbf{x}_n) + \mathbf{h}^T_{\sigma(t)}(\mathbf{x}_n)\mathbf{u} \end{cases} \tag{3}$$

where $x_i, i = 1,2,...,n$ and y are states and output of system, respectively. $\mu_{i,\sigma(t)}, g_{\sigma(t)}(\mathbf{x})$ and $\mathbf{h}_{\sigma(t)}(.) = \left[h_{1,\sigma(t)}(.), h_{2,\sigma(t)}(.), ..., h_{r,\sigma(t)}(.)\right]$ are smooth switching functions, $\mathbf{u} = [u_1, u_2, ..., u_r]$ is the vector of r control inputs $u_1, u_2, ..., u_r$ and $\mathbf{x}_i = [x_1, x_2, ..., x_i]$. It is assumed that the origin is an equilibrium point of (3) therefore, $\mu_{i,\sigma(t)}(\mathbf{0}) = 0, i = 1,2,...,n-1$ and $g_{\sigma(t)}(\mathbf{0}) = 0$. Moreover, we assume that $h_{i,\sigma(t)}(\mathbf{0}) \neq \mathbf{0}, i = 1,...,r$. For simplicity, the switching signal $\sigma(t)$ is shown by σ in the rest of this paper.

To show the major drawback of SDL idea, the back-stepping approach is considered for the switched nonlinear cascade system (3). At the first step, consider the first equation of (3)

$$\dot{x}_1 = x_2 + \mu_{1,\sigma}(x_1) \tag{4}$$

At this step, the CQLF and its time derivative is in the following form

$$V_1 = \frac{1}{2}x_1^2, \quad \dot{V}_1 = x_1\left[x_2 + \mu_{1,\sigma}(x_1)\right] \tag{5}$$

As discussed in [13,16,31], if there exists a virtual control law $x_2 = \alpha_1$ satisfying $\dot{V}_1 < 0$ for all $\sigma \in \{1,...,\mathrm{m}\}$, all of the first-order subsystems (4) are simultaneously dominatable. According to SDL idea, at the i-th step of back-stepping method it is essential to find a virtual control law assuring the simultaneous domination limitation.

This back-stepping approach which is based on SDL idea contains two major drawbacks. 1- For complicated switched nonlinear systems, finding a proper virtual control law making all subsystems simultaneously dominatable is hard or even impossible. 2- If there exists a virtual control law assuring SDL, this approach only can stabilize the switched nonlinear cascade systems and cannot be employed to solve the tracking problem [13,31,34].

For more explanation, two different switched nonlinear cascade systems are presented as follow. For the first example, consider the following switched system with three subsystems.

$$\begin{cases} \dot{x}_1 = x_2 + \mu_\sigma(x_1) \\ \dot{x}_2 = u \end{cases} ; \qquad \sigma \in \{1,2,3\} \tag{6}$$

where $\mu_1 = -3x_1^5, \mu_2 = x_1^5, \mu_3 = 6x_1^5$. These subsystems have very similar structural nonlinearities. Therefore by choosing the virtual control law $x_2 = \alpha_1 = -7x_1^5$ all subsystems are simultaneously dominatable.

$$x_2 = \alpha_1 = -7x_1^5 \Rightarrow \begin{cases} \sigma = 1: \ \dot{V}_1 = x_1\left(x_2 - 3x_1^5\right) = -11x_1^6 < 0 \\ \sigma = 2: \ \dot{V}_1 = x_1\left(x_2 + x_1^5\right) = -6x_1^6 < 0 \\ \sigma = 3: \ \dot{V}_1 = x_1\left(x_2 + 6x_1^5\right) = -x_1^6 < 0 \end{cases} \tag{7}$$

Therefore, all subsystems are simultaneously dominatable and the virtual control law α_1 stabilizes x_1 for all subsystems. But as mentioned previously, this approach only solves the stabilization problem and cannot be applied on tracking problem.
For the second system, consider the following switched cascade system with complicated nonlinearities.

$$\begin{cases} \dot{x}_1 = x_2 + \mu_\sigma(x_1) \\ \dot{x}_2 = u \end{cases}; \qquad \sigma \in \{1,2,3\} \tag{8}$$

where

$\mu_1 = 3x_1(x_1 - 1), \mu_2 = -x_1^3 \sin(x_1 - 0.6), \mu_3 = -x_1 - 2\sin(x_1).$

Since finding an explicit virtual control law satisfying following inequalities is impossible, SDL cannot be achieved for switched nonlinear system (8).

$$x_2 = \alpha_1 \Rightarrow \begin{cases} \sigma = 1: \ \dot{V}_1 = x_1\left(\alpha_1 + 3x_1^2 - 3x_1\right) \overset{?}{<} 0 \\ \sigma = 2: \ \dot{V}_1 = x_1\left(\alpha_1 - x_1^3 \sin(x_1 - 0.6)\right) \overset{?}{<} 0 \\ \sigma = 3: \ \dot{V}_1 = x_1\left(\alpha_1 - x_1 - 2\sin(x_1)\right) \overset{?}{<} 0 \end{cases} \tag{9}$$

The above examples show that the SDL can be satisfied only for switched nonlinear systems with very simple nonlinearities. Moreover, by this method only the stabilization problem may be solved. Since SDL cannot be achieved for switched nonlinear systems with complicated nonlinearities, it is not possible to tracking control or even stabilize these systems by back-stepping approach.

Motivated by the mentioned drawbacks, in the next section a new method based on state conversion is presented which eliminate these shortcomings.

3. State conversion approach

In this section, a new method based on state conversion and back-stepping method is introduced to construct a CQLF for switched nonlinear cascade system (3). Then it will be shown that the proposed system under arbitrary switching is GAS without limitation.

Consider the following state conversion.

$$\begin{cases} p_1 = x_1 \\ p_2 = \dot{x}_1 = x_2 + \mu_{1,\sigma} \\ p_3 = \ddot{x}_1 = x_3 + \mu_{2,\sigma} + \left(x_2 + \mu_{1,\sigma}\right)\dfrac{\partial \mu_{1,\sigma}}{\partial x_1} \\ \vdots \\ p_n = x_1^{(n-1)} = \bar{F}_\sigma(\mathbf{x}) \end{cases} \tag{10}$$

where $\bar{F}_\sigma(\mathbf{x})$ is a complicated switched function. System (3) in the new coordinate $[p_1, p_2, ..., p_n]$ is represented in the following form

$$\begin{cases} \dot{p}_1 = p_2 \\ \dot{p}_2 = p_3 \\ \vdots \\ \dot{p}_n = \bar{g}_\sigma(\mathbf{x}_n) + \mathbf{h}_\sigma^T(\mathbf{x}_n)\mathbf{u} \end{cases}; \quad y = p_1 \tag{11}$$

which $\bar{g}_\sigma(\mathbf{x}_n)$ is a complicated switched nonlinear function of $\mathbf{x}_n$, $\mu_{i,\sigma}(\mathbf{x}_i)$ and $g_\sigma(\mathbf{x})$. The following subsections present the stabilization and tracking problems based on the new state conversion.

3.1. Stabilization of switched nonlinear cascade systems based on the state conversion

In this section, by following the back-stepping control design [37], the stabilization problem of switched nonlinear cascade systems is investigated.

Step 1. For switched nonlinear cascade system (11) define that $z_1 = p_1$. Therefore, we will have $\dot{z}_1 = p_2$. At the first step, the CQLF V_1 and its time derivative is in the following form

$$V_1 = \frac{1}{2}z_1^2 \Rightarrow \dot{V}_1 = z_1 p_2 \tag{12}$$

By choosing the first virtual control law $\alpha_1 = p_2 = -c_1 z_1$ we will have $\dot{V}_1 = -c_1 z_1^2$. Where c_1 is a positive constant.

Step 2. By defining $z_2 = p_2 - \alpha_1$, the first two equations of (11) can be written in the following form

$$\begin{cases} \dot{z}_1 = z_2 + \alpha_1 \\ \dot{z}_2 = p_3 - \dfrac{\partial \alpha_1}{\partial p_1} p_2 \end{cases} \tag{13}$$

For the second step, the CQLF V_2 and its time derivative along (13) are in the following form

$$\begin{aligned} &V_2 = V_1 + \frac{1}{2}z_2^2 = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 \Rightarrow \\ &\dot{V}_2 = z_1(z_2 + \alpha_1) + z_2\left[p_3 - \frac{\partial \alpha_1}{\partial p_1} p_2\right] \end{aligned} \tag{14}$$

The following second intermediate control law is defined.

$$p_3 = \alpha_2 = -c_2 z_2 + \frac{\partial \alpha_1}{\partial p_1} p_2 - z_1 \tag{15}$$

where c_2 is a positive value. Therefore, we have $\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2$.

Step r $(r = 3, ..., n-1)$. At this step, by defining $z_r = p_r - \alpha_{r-1}$, the first r equations of (11) are represented in terms of new variables $z_1, z_2, ..., z_r$ as shown in the following form.

$$\begin{cases} \dot{z}_1 = z_2 + \alpha_1 \\ \dot{z}_2 = z_3 + \alpha_2 - \dfrac{\partial \alpha_1}{\partial p_1} p_2 \\ \vdots \\ \dot{z}_r = p_{r+1} - \displaystyle\sum_{i=1}^{r-1} \frac{\partial \alpha_{r-1}}{\partial p_i} p_{i+1} \end{cases} \tag{16}$$

For this step, the CQLF and its time derivative are in the following form

$$\begin{aligned} &V_r = \frac{1}{2}\sum_{i=1}^{r} z_i^2 \Rightarrow \\ &\dot{V}_r = -\sum_{i=1}^{r-1} c_i z_i^2 + z_r\left[z_{r-1} + p_{r+1} - \sum_{i=1}^{r-1} \frac{\partial \alpha_{r-1}}{\partial p_i} p_{i+1}\right] \end{aligned} \tag{17}$$

For this step we consider that

$$p_{r+1} = \alpha_r = -c_r z_r - z_{r-1} + \sum_{i=1}^{r-1} \frac{\partial \alpha_{r-1}}{\partial p_i} p_{i+1} \tag{18}$$

where c_r is a positive value. Therefore, $\dot{V}_r = -\sum_{i=1}^{r} c_i z_i^2$.

Step n. At the final step, by defining that $z_n = p_n - \alpha_{n-1}$ the system (11) is represented in the following form

$$\begin{cases} \dot{z}_1 = z_2 + \alpha_1 \\ \vdots \\ \dot{z}_r = p_{r+1} - \sum_{i=1}^{r-1} \frac{\partial \alpha_{r-1}}{\partial p_i} p_{i+1} \\ \vdots \\ \dot{z}_n = \bar{g}_\sigma(\mathbf{x}_n) + \mathbf{h}_\sigma^T(\mathbf{x}_n)\mathbf{u} - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial p_i} p_{i+1} \end{cases} \tag{19}$$

The CQLF for switched system (19) and its time derivative will be in the following form

$$V = \frac{1}{2}\sum_{i=1}^{n} z_i^2 \Rightarrow$$
$$\dot{V}_n = -\sum_{i=1}^{n-1} c_i z_i^2 + z_n \left[z_{n-1} + \bar{g}_\sigma(\mathbf{x}_n) + \mathbf{h}_\sigma^T(\mathbf{x}_n)\mathbf{u} - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial p_i} p_{i+1} \right] \tag{20}$$

Defining the following control law

$$\mathbf{u} = -\frac{\mathbf{h}_\sigma^T(\mathbf{x})}{\mathbf{h}_\sigma(\mathbf{x})\mathbf{h}_\sigma^T(\mathbf{x})} \left[c_n z_n + \bar{g}_\sigma(\mathbf{x}) - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial p_i} p_{i+1} + z_{n-1} \right] \tag{21}$$

yields $\dot{V} = -\sum_{i=1}^{n} c_i z_i^2$ where c_n is a positive value.

Theorem 1. Under the control input (21) the switched nonlinear cascade system (3) is GAS under arbitrary switching.

Proof. Time derivative of the CQLF (20) along (19) and applying (21) will result to $\dot{V} = -\sum_{i=1}^{n} c_i z_i^2$ which is a negative definite function. Therefore, all subsystems of (19) are GAS. On the other hand, V is a CQLF. So that, the stability of switched nonlinear cascade system (19) in switching instants is assured. To prove the asymptotic stability in x-domain, consider that since z_1 ($z_1 = p_1$) is asymptotically stable, from the first equation of (10) we conclude that x_1 is also asymptotically stable $\left(\lim_{t\to\infty} x_1(t) = 0\right)$. On the other hand, since $\mu_{1,\sigma(t)}(0) = 0$ from the first equation of (3) $\left(\dot{x}_1 = x_2 + \mu_{1,\sigma(t)}(x_1)\right)$ we conclude that x_2 is asymptotically stable. By following this logic for $i = 2,...,n$, it is inferred that under the control input (21) the switched nonlinear cascade system (3) is GAS under arbitrary switching without the limitations of the SDL-based approaches [13,16,31,32,34,35].

3.2. Tracking problem of switched nonlinear cascade systems based on the state conversion

In this subsection, the tracking problem is studied for switched system (3). It is assumed that the output signal x_1 tracks the desired smooth signal x_d.

Step 1. Define that $z_1 = p_1 - x_d$. The first equation of (11) is represented as $\dot{z}_1 = p_2 - \dot{x}_d$. The CQLF and the virtual control law are in the following form for this step.

$$V_1 = \frac{1}{2} z_1^2 \Rightarrow \dot{V}_1 = z_1(p_2 - \dot{x}_d) \Rightarrow p_2 = \alpha_1 = -c_1 z_1 + \dot{x}_d \tag{22}$$

Step r $(r = 2,...,n-1)$. By considering $z_r = p_r - \alpha_{r-1}$, which α_{r-1} is the virtual control law of previous step. The first r equations of (11) in terms of z_i can be written as follows.

$$\begin{cases} \dot{z}_1 = z_2 + \alpha_1 - \dot{x}_d \\ \dot{z}_2 = z_3 + \alpha_2 - \frac{\partial \alpha_1}{\partial p_1} p_2 - \ddot{x}_d \\ \vdots \\ \dot{z}_r = p_{r+1} - \sum_{i=1}^{r-1} \frac{\partial \alpha_{r-1}}{\partial p_i} p_{i+1} - x_d^{(r)} \end{cases} \tag{23}$$

For this step, the CQLF and its time derivative along (23) are in the following form

$$V_r = \frac{1}{2}\sum_{i=1}^{r} z_i^2 \Rightarrow$$
$$\dot{V}_r = -\sum_{i=1}^{r-1} c_i z_i^2 + z_r \left[z_{r-1} + p_{r+1} - \sum_{i=1}^{r-1} \frac{\partial \alpha_{r-1}}{\partial p_i} p_{i+1} - x_d^{(r)} \right] \tag{24}$$

The virtual control law of this step is in the following form

$$p_{r+1} = \alpha_r = -c_r z_r - z_{r-1} + \sum_{i=1}^{r-1} \frac{\partial \alpha_{r-1}}{\partial p_i} p_{i+1} + x_d^{(r)} \tag{25}$$

Therefore, we will have $\dot{V}_r = -\sum_{i=1}^{r} c_i z_i^2$.

Step n. By defining $z_n = p_n - \alpha_{n-1}$, the switched system (11) is converted to

$$\begin{cases} \dot{z}_1 = z_2 + \alpha_1 - \dot{x}_d \\ \vdots \\ \dot{z}_r = z_{r+1} + \alpha_r - \sum_{i=1}^{r-1} \frac{\partial \alpha_{r-1}}{\partial p_i} p_{i+1} - x_d^{(r)} \\ \vdots \\ \dot{z}_n = \bar{g}_\sigma(\mathbf{x}) + \mathbf{h}_\sigma^T(\mathbf{x})\mathbf{u} - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial p_i} p_{i+1} - x_d^{(n)} \end{cases} \tag{26}$$

Finally, the CQLF and its time derivative along (26) will be in the following form

$$V = \frac{1}{2}\sum_{i=1}^{n} z_i^2 \Rightarrow$$
$$\dot{V}_n = -\sum_{i=1}^{n-1} c_i z_i^2 + z_n \left[z_{n-1} + \bar{g}_\sigma(\mathbf{x}_n) + \mathbf{h}_\sigma^T(\mathbf{x}_n)\mathbf{u} - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial p_i} p_{i+1} - x_d^{(n)} \right] \tag{27}$$

Choosing the following control law

$$\mathbf{u} = -\frac{\mathbf{h}_\sigma^T(\mathbf{x})}{\mathbf{h}_\sigma(\mathbf{x})\mathbf{h}_\sigma^T(\mathbf{x})} \left[c_n z_n + \bar{g}_\sigma(\mathbf{x}) - \sum_{i=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial p_i} p_{i+1} + z_{n-1} - x_d^{(n)} \right] \tag{28}$$

yields $\dot{V} = -\sum_{i=1}^{n} c_i z_i^2$. The results are presented in the next theorem.

Theorem 2: The output of switched nonlinear cascade system (3) tracks the desired signal x_d asymptotically under the control input (28) and arbitrary switching.

Proof. Replacing (28) in (27) yields $\dot{V} = -\sum_{i=1}^{n} c_i z_i^2$. Therefore, the individual tracking error of each subsystem of (3) converges to zero asymptotically. On the other hand, since V is a CQLF, the convergence of tracking error in switching instants is assured.

4. Verification studies

In this section, numerical and experimental studies are provided to verify the effectiveness of the proposed approaches presented in theorems 1 and 2. The numerical and experimental studies investigate the results of theorems 1 and 2, respectively.

4.1. Simulation study (stabilization problem)

For simulation study, the following switched nonlinear cascade system with three subsystems is considered

$$\begin{cases} \dot{x}_1 = x_2 + \mu_{1,\sigma}(x_1) \\ \dot{x}_2 = x_3 + \mu_{2,\sigma}(\mathbf{x}_2) \qquad ; \quad \sigma \in \{1,2,3\} \\ \dot{x}_3 = g_{\sigma(t)}(\mathbf{x}) + h_\sigma(\mathbf{x})u \end{cases} \tag{29}$$

where

$$\mu_{1,1} = x_1, \mu_{1,2} = x_1^3, \mu_{1,3} = x_1^2, \mu_{2,1} = \sin(x_2), \mu_{2,2} = x_1x_2, \mu_{2,3} = x_1x_2^2$$
$$g_1 = x_1^2, g_2 = x_1x_3, g_3 = x_3^2 + x_2, h_1 = 2 + x_1^2, h_2 = 5, h_3 = 7 + x_2^4 \tag{30}$$

It is obvious that the subsystems of (29) are not simultaneously dominatable. Therefore, the idea of SDL cannot be employed to control design and stability analysis of this system. According to (21), the control input is as follows

$$\mathbf{u} = -\frac{1}{\mathbf{h}_\sigma(\mathbf{x})}\left[c_3 z_3 + \bar{g}_\sigma(\mathbf{x}) - \sum_{i=1}^{2} \frac{\partial \alpha_2}{\partial p_i} p_{i+1} + z_2 \right] \tag{31}$$

where

$$z_3 = p_3 - \alpha_2, \quad \alpha_2 = -c_2 z_2 + \frac{\partial \alpha_1}{\partial p_1} p_2 - z_1, \quad z_2 = p_2 - \alpha_1,$$
$$\alpha_1 = -c_1 z_1, \; z_1 = p_1, \quad c_1, c_2, c_3 > 0$$
$$\bar{g}_\sigma(\mathbf{x}) = g_\sigma(\mathbf{x}) + \left(x_3 + \mu_{2,\sigma}\right)\left(\frac{\partial \mu_{1,\sigma}}{\partial x_{1,\sigma}} + \frac{\partial \mu_{2,\sigma}}{\partial x_{2,\sigma}}\right) +$$
$$+\left(x_2 + \mu_{1,\sigma}\right)\left(\frac{\partial \mu_{2,\sigma}}{\partial x_1} + \left(\frac{\partial \mu_{1,\sigma}}{\partial x_1}\right)^2\right) + \left(x_2 + \mu_{1,\sigma}\right)^2$$

A high frequency signal is applied to system (29) according to Fig. 2. The behavior of states x_1, x_2 and x_3 are depicted in Fig. 3. As this figure indicates, all states are GAS under the control input (31) and arbitrary switching. Fig. 4 shows the control input of switched nonlinear system (29).

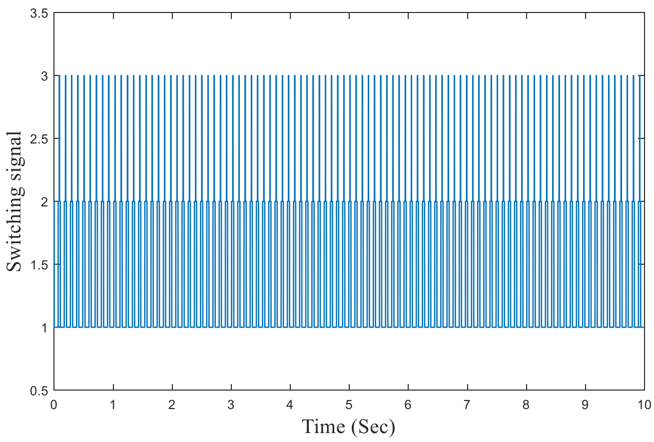

Figure 2. Switching signal: stabilization problem.

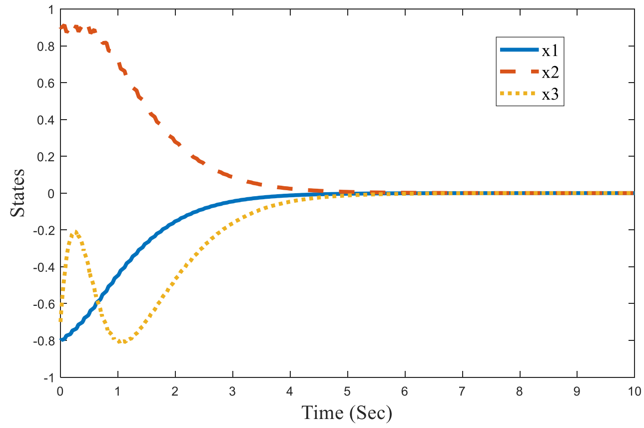

Figure 3. State responses: stabilization problem.

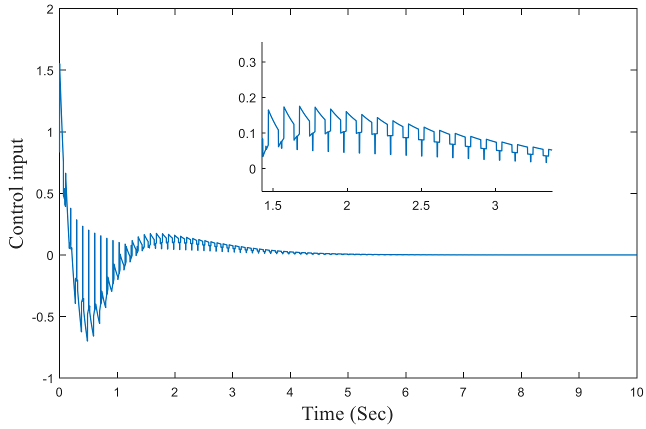

Figure 4. Control input: stabilization problem.

4.2. Experimental study (tracking problem)

To evaluate practically the results of theorem 2, an experimental setup of shape memory alloy actuators is designed as shown in Fig. 5. This setup consists of a rotational disk and two shape memory alloy wires connecting differentially. A $5K\Omega$ rotational potentiometer is employed in the center of the disk to measure the angular displacement. At the end of both wires, two load cells are used to measure each SMA wire forces. Each load cell contains a $0.9K\Omega$ Weston bridge resistance in unloading situation. Two identical indicator clocks are employed to show the rotational displacement of the disk. Fig. 6 depicts a schematic structure of this setup. As shown in this figure, an electrical diode is used in path of each wire rectifying the electric current.

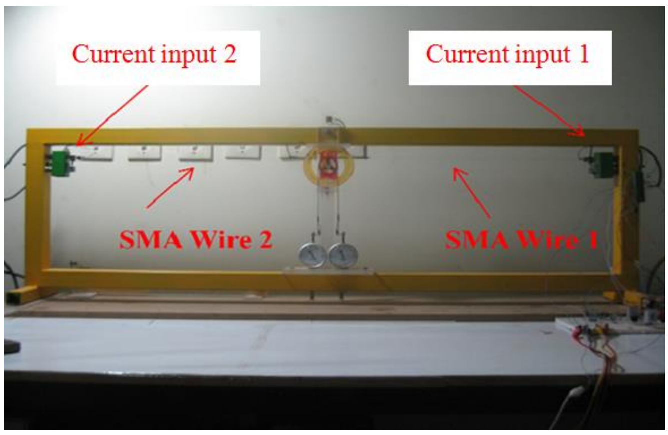

Figure 5. Experimental setup of differential shape memory alloy wires.

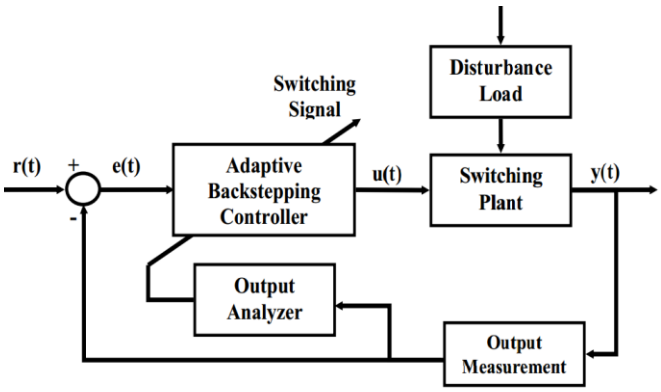

Figure 6. The closed-loop system of SMA mechanism.

The STM-32 microcontroller used in this setup belongs to Cortex M3 family which generates the appropriate commands to control the angular position of mechanism. The maximum processing clock-rate and the flash memory of this microcontroller are 53 MHz and 256 Kbytes, respectively.

The equation of motion of the rotating disk is as follows

$$\ddot{\theta} = h_k u + g_k \dot{\theta} + d_k \,, \quad k \in \{1,2,3\} \tag{32}$$

where $\theta, \dot{\theta}$ and $\ddot{\theta}$ are angular position, velocity and acceleration of the disk, $u = i^2$, i is the current of SMA wire, k is the number of active subsystem and d_k is the disturbance signal. For switched nonlinear system (32), the switching functions are in the following form

$$\begin{aligned} h_1 &= \frac{Ar\alpha}{I}\frac{\Omega\xi_{T,1}+\theta_t}{1-\Omega\xi_{\sigma,1}},\ g_1 = -\left(\frac{Ar^2}{Il_0}\frac{D}{1-\Omega\xi_{\sigma,1}} + \frac{kr^2}{I}\right), \\ d_1 &= -\frac{Ar}{I}\frac{\Omega\xi_{T,1}+\theta_t}{1-\Omega\xi_{\sigma,1}}\beta(T-T_\infty) \end{aligned} \tag{33}$$

$$\begin{aligned} h_2 &= \frac{Ar\alpha}{I}\frac{\Omega\xi_{T,2}+\theta_t}{1-\Omega\xi_{\sigma,2}},\ g_2 = -\left(\frac{Ar^2}{Il_0}\frac{D}{1-\Omega\xi_{\sigma,2}} + \frac{kr^2}{I}\right), \\ d_2 &= -\frac{Ar}{I}\frac{\Omega\xi_{T,2}+\theta_t}{1-\Omega\xi_{\sigma,2}}\beta(T-T_\infty) \end{aligned} \tag{34}$$

$$\begin{aligned} h_3 &= \frac{Ar\alpha}{I}\theta_t,\ g_3 = -\left(\frac{Ar^2}{Il_0}D + \frac{kr^2}{I}\right), \\ d_3 &= -\frac{Ar}{I}\theta_t\beta(T-T_\infty) \end{aligned} \tag{35}$$

More details of the above parameters can be found in [38]. The characteristics of SMA wires used in this paper, are same as [38]. In [38] a robust control is presented to control the antagonistic pair of SMA actuators. As a new approach, we present a new switched robust adaptive back-stepping control method to solve the tracking problem of the switched nonlinear system (32).

By defining that $x_1 = \theta, x_2 = \dot{\theta}$ and $x_3 = \ddot{\theta}$, and the new state variables $z_1 = x_1 - x_d$, $z_2 = x_2 - \alpha_1$ and $z_3 = x_3 - \alpha_2$, the state space form of the governing equation (32) is presented as

$$\begin{cases} \dot{z}_1 = z_2 + \alpha_1 - \dot{x}_d \\ \dot{z}_2 = z_3 + \alpha_2 - \dfrac{\partial \alpha_1}{\partial x_1}x_2 - \dfrac{\partial \alpha_1}{\partial t} \\ \dot{z}_3 = h_k u + g_k x_2 + d_k - \dfrac{\partial \alpha_2}{\partial x_1}x_2 - \dfrac{\partial \alpha_2}{\partial x_2}x_3 - \dfrac{\partial \alpha_2}{\partial t} \end{cases} \tag{36}$$

where $\alpha_1 = -c_1 z_1 + \dot{x}_d$ and $\alpha_2 = -z_1 - c_2 z_2 + \dfrac{\partial \alpha_1}{\partial z_1}x_2 + \dfrac{\partial \alpha_1}{\partial t}$. Since the disturbance signals d_k is a function of natural and geometrical characteristics of wires, they are bounded. Therefore, we have $|d_k| \prec D_k$ where D_k are the upper bounds of disturbance signals. The time variation of switched functions h_k and g_k are very slowly in each subsystems. Therefore, they can be considered as adaptive parameters. By employing the adaptive back-stepping control design presented in the previous works of authors [13,33] the CQLF associate with the switched nonlinear system (32) will be constructed as follows

$$V = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2}z_3^2 + \frac{1}{2\gamma_1}\tilde{h}^2 + \frac{1}{2\gamma_2}\tilde{g}^2 \tag{37}$$

where $\tilde{h} = h_k - \hat{h}$ and $\tilde{g} = g_k - \hat{g}$. $\hat{h}$ and $\hat{g}$ are the estimations of h and g, respectively. Moreover, γ_1 and γ_2 are positive values. Taking time derivative of (37) along the switched system (36) and performing some algebraic manipulations lead to the following control and adaptation rules.

$$u = \begin{cases} -\dfrac{1}{\hat{h}_k}\begin{bmatrix} \hat{g}_k x_2 + sign(z_3)D_k - \dfrac{\partial \alpha_2}{\partial x_1}x_2 \\ -\dfrac{\partial \alpha_2}{\partial x_2}x_3 - \dfrac{\partial \alpha_2}{\partial t} + z_2 - c_3 z_3 \end{bmatrix}, & \hat{h}_k \neq 0 \\ 0, & \hat{h}_k = 0 \end{cases} \tag{38}$$

$$\dot{\hat{h}}_k = \gamma_1 z_3 u, \qquad \dot{\hat{g}}_k = \gamma_2 z_3 x_2 \tag{39}$$

To study the tracking problem, the desired signal $x_d = 20\cos\left(\frac{\pi}{8}t\right) + 20\sin\left(\frac{\pi}{8}t + \frac{\pi}{4}\right)$ is considered. Fig. 7 shows the tracking problem for switched nonlinear system(32). According to this figure, the system output tracks the desired signal asymptotically. Figs. 8 and 9 show the switching signal and control input of the proposed switched nonlinear system.

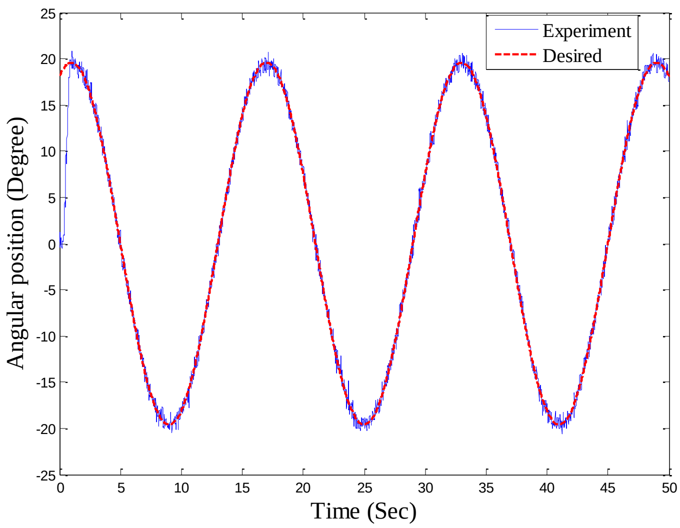

Figure 7. Tracking problem for SMA mechanism.

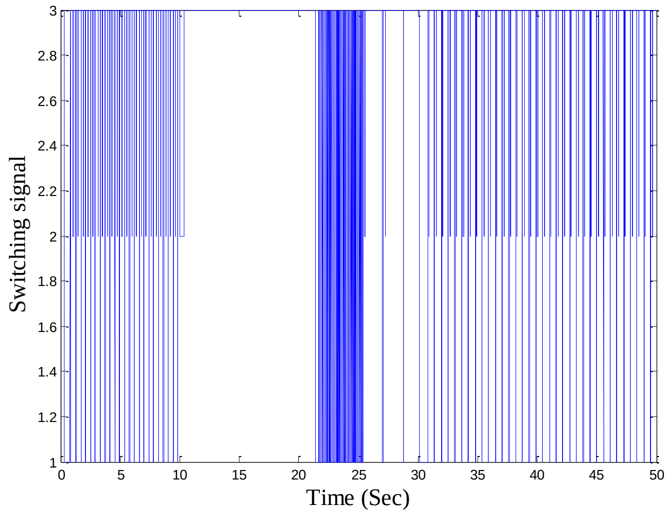

Figure 8. Switching signal: tracking problem.

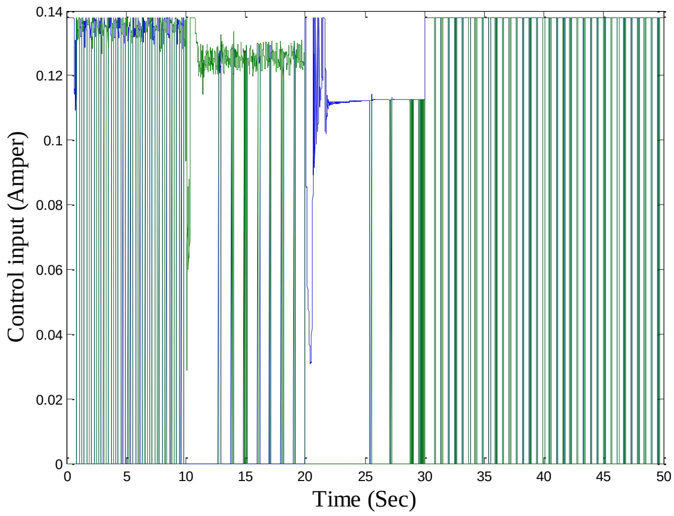

Figure 9. Control input: tracking problem.

5. Conclusion

In this paper, a highly usage approach based on simultaneous domination limitation to construct a CQLF for switched nonlinear cascade systems was reviewed. By presenting an example, it was shown that the simultaneous domination limitation cannot be satisfied in switched nonlinear cascade systems with complicated structure. To overcome this restriction, a new backstepping approach based on state conversion was introduced. It was shown that this new approach solves both problems of stabilization and tracking of switched nonlinear cascade systems without any limitation. Numerical and experimental studies were provided to show the effectiveness of the proposed method. The results confirm the effectiveness of the proposed approach compared with the previous method.

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