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RESEARCH PAPER

Estimating Marginal Rates of Substitution in Two-Stage Processes With Undesirable Factors

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Abstract
Trade-offs between production factors such as marginal rates of substitution are significant aspects for decision makers and managers. Due to the complexity of processes and the presence of undesirable measures in many real-world applications, in this study, the relative efficiency and marginal rates of substitution are calculated in two-stage structures including weakly disposable undesirable intermediate measures. Actually, an approach based on the directional distance function is provided for this purpose. Therefore, the effect of the changes of a measure in other measures such as the effect of the changes of intermediate factors on the output of the first stage and second stage is measured by maintaining efficiency, and the rate of these changes is calculated. To elaborate in details, marginal rates of substitution in two-stage processes are dealt with using the proposed two-stage data envelopment analysis (DEA) approach while undesirable intermediate components are presented. A real data set is also used to clarify the proposed method herein.

Keywords: Data envelopment analysis, Efficiency, Marginal rates of substitution, Network structure, Undesirable factors.

Introduction

One of the most important techniques used by managers and decision makers to analyze the performance is data envelopment analysis (DEA). In this technique, which is based on linear programming, the relative efficiency of each system is assessed. Traditional DEA approaches, initially developed by Charnes et al. (1978) and extended by Banker et al. (1984), focus on the black box systems with the supposition of decreasing the inputs and increasing the outputs. Due to the presence of undesirable outputs in many real-world applications and complex structures of most of production systems, DEA has recently made a considerable contribution in analyzing undesirable factors and investigating network processes.

Seiford et al. (2002) provided an alternative method with desirable and undesirable factors, based on the BCC model of Banker et al. (1984). A decrease in the undesirable outputs and an increase in the desirable outputs were shown by them. The necessary conditions for estimating inputs and outputs in DEA were considered by Jahanshahloo et al. (2004) when undesirable factors were presented. They explained how to control the input/output level changes of particular decision-making units in order to maintain the DMU efficiency. Then, they resolved the problem of multi-objective linear programming with undesirable factors. Amirteimoori et al.

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(2006) developed a DEA model that could be utilized to treat the relative performance when decreasing undesirable outputs and increasing undesirable inputs. Kordrostami and Amirteimoori (2005) analyzed the relative efficiency of a set of dependent decision-making sub-units (DMSU) for devising a larger DMU in the presence of undesirable and desirable factors. In addition, undesirable inputs are increased and undesirable outputs are decreased to improve the performance of decision-making units. In the production process, there are also techniques that have the advantages of imposing weak disposability hypothesis on the functional form of the fundamental technology. Hailu and Veeman (2001) investigated undesirable outputs as inputs, expanded an approach with the weak disposability definition provided by Shephard (1974), and used a classical DEA model to evaluate the relative efficiency of DMUs in production technology. Then, Färe and Grosskopf (2003) asserted that considering the undesirable outputs as inputs is repugnant with physics rules and the standard principles of production processes, and used the weak disposability assumption of Shephard (1974). Färe and Grosskopf (2003) utilized a single abatement factor for all outputs. Then, Kuosmanen (2005) asserted that the right performance of the principle of weak disposability required the utilization of different factors for each firm. He mentioned a simple formulation of weak disposability that used non-uniform abatement factors, and maintained the linear structure of the model. Kuosmanen and Podinovski (2009) extended the issue of weak disposability under a relaxed supposition about convexity in DEA. Recently, many researchers studied undesirable factors with weak disposability assumption. Amirteimoori et al. (2017) proposed an alternative explanation of the weak disposability of outputs. Then, in the presence of undesirable outputs, a principle foundation has been presented to make a new production technology. Mehdiloozad and Podinovski (2018) suggested the supposition of weak input disposability. They indicated that Shephard technology was not convex, and so it introduced bias in the assessment of congestion. In addition, they supplied more axiomatic handling, and got a set of production technologies all of which indicated weak input disposability.

In addition, network DEA models enable decision makers to assess the relative efficiency of their internal processes in addition to the generic performance evaluation. Kao (2009) proposed a relational network DEA model to assess the system efficiency and the process efficiencies. Cook et al. (2010) provided linear models to assess the performance of multi-stage processes under the constant returns to scale assumption. They showed the overall efficiency as an additive weighted average of the components efficiencies. Liang et al. (2008) presented models utilizing game theory concepts. They handled intermediate measure connecting the two stages. Both the centralized and non-cooperative models allocated the same outcomes as applying the standard DEA model of the two stages distinctly. Then, the efficiency decomposition was found to be unique. Lozano et al. (2013) provided an approach for airports performance evaluation by the introduction of undesirable outputs into the network DEA. In addition, they proposed directional distance method to network DEA problems, in which the procedures could produce not only desirable ultimate outputs, but also undesirable outputs. Maghbouli et al. (2014) studied the performance of two-stage processes in the presence of undesirable intermediate factors with the weak disposability. Furthermore, the aspects, including cooperative and non-cooperative game theories (leader-follower), were independently investigated. Then, the relative performance of the units was evaluated. Liu et al. (2015) provided DEA models in a two-stage process with undesirable intermediate outputs. Especially, they used the free-disposal principles to build the production possibility sets. They used an envelopment framework like most of the available theoretical studies for DEA with undesirable factors. Wu et al. (2015) proposed an additive DEA method to assess the overall efficiency of a two-stage network structure, and also, more efficiency decomposition of the unique system. The results of the mentioned model could assist decision makers find the shortcoming of the proposed two-stage system, so that more appropriate proposals were made to improve the system performance. Then, this approach was implemented in the industrial production of the 30 provinces in China. Khalili-Damghani et al. (2015) proposed a customized network DEA model to assess the efficiency of electric power production and distribution processes. Electric power production and distribution network comply with producing and distributing the electric power, respectively. In the production case, plants consume fuels such as oil and gas to produce the electricity. In the distribution case, area electricity corporations transmit and distribute the electricity.

On the other hand, calculating marginal rates are also significant, because it provides information to the manufacturer or consumer that allows them to make alternatives or trade-offs at the input or output levels, while the efficiency is maintained. Moreover, by calculating marginal rates, suggestions can be made for better performance and achievement of expected conditions, so that the unit can economically function well. Rosen et al. (1998) proposed a comprehensive framework for analyzing directional derivatives in particular and marginal rates in general on DEA frontiers. A significant characterization of these derivatives at given points can be offered at the intervals they can take; equivalently, these intervals meet the derivatives right and left at these points. They utilized two methods for calculation. The first method was the dual equivalent computation of maximum and minimum multiplier ratios, and the second one was a modified simplex tableau method. Asmild et al. (2006) extended methods for assessing larger (non-marginal) trade-offs among variables in DEA. The methods were capable of investigating both group and non-group changes. Eventually, the methods for assessing both basic trade-offs were extended in order to evaluate the effect of one or more indexes of the change in one or more of the other indexes. Khoshandam et al. (2015) proposed a DEA-based method to compute the group marginal rates of substitution of DMUs treated as black boxes when undesirable outputs are presented. In addition, the computation of the directional marginal rates of a group of variables to another group has been shown. It is clear that because of the presence of two-stage processes in many environments, trade-off analysis of them is a significant topic.

As far as we know, analyzing marginal rates of substitution a measure with another measure has not been investigated in DEA two-stage systems with undesirable materials. Thus, in this paper, an approach is proposed to assess the efficiency scores of two-stage systems in the presence of undesirable intermediate measures. Then, Asmild et al.'s approach (Asmild et al., 2006) is extended to address the marginal rates of substitutions and calculate the marginal rate of substitutions in two-stage systems with undesirable measures. To illustrate in more details, the changes of throughputs against the increase or decrease of other throughputs in the specified amounts that are defined by decision makers or analysts are assessed.

At this stage, the following four schemes are calculated: In scheme 1, the marginal rates of the substitution of desirable output with desirable input from the first stage of the two-stage process are calculated. In scheme 2, the marginal rates of the substitution of desirable output with desirable input from the second stage of the two-stage process are computed. In scheme 3, the marginal rates of the substitution of desirable output with undesirable intermediate measure from the second stage of the two-stage process are found. In scheme 4, the marginal rates of the substitution of desirable output with undesirable intermediate measure from the first stage of the two-stage process are obtained. The above four schemes are calculated by maintaining efficiency decision-making units. Then, the proposed approach is provided.

The remainder of this paper is organized as follows. In Section 2, a review of the concepts and models is provided. Then Section 3 gives in an approach to measure the relative efficiency of two-stage processes with undesirable factors. A DEA-based approach is described to calculate the substitution marginal rates in two-stage processes in Section 4. Extension to multi-index for calculating marginal rates of substitution is given in Section 5. An illustrative application is presented in Section 6. Finally, Section 7 presents the conclusion of the paper.

Preliminaries

In this section, the existing DEA-based methods to address the weak disposability undesirable factors and the marginal rates of substitution are described briefly.

Undesirable Outputs

We consider n DMUs, $DMU_k : k=1,\dots,n$, with the vectors of inputs $\mathbf{x}_k=(x_{1k},\dots,x_{mk})\geq 0$, undesirable outputs $\mathbf{w}_k=(w_{1k},\dots,w_{jk})\geq 0$, and desirable outputs $\mathbf{v}_k=(v_{1k},\dots,v_{sk})\geq 0$. Production technology is shown by $p(x)=\{(\mathbf{v},\mathbf{w}) \mid \mathbf{x} \text{ can produce } (\mathbf{v},\mathbf{w})\}$.

Definition 1. Desirable and undesirable outputs $(\mathbf{v},\mathbf{w})$ are weakly disposable if and only if $(\mathbf{v},\mathbf{w})\in p(x)$ for all $0\leq\psi\leq 1$ imply $(\psi\mathbf{v},\psi\mathbf{w})\in p(x)$ (See Shephard, 1974).

Färe and Grosskopf (2003) proposed the following technology with weak disposability:

$$T_{FG}=\{(\mathbf{x},\mathbf{v},\mathbf{w}) \mid \mathbf{x}\geq\sum_{k=1}^{n}\lambda_k\mathbf{x}_k, \mathbf{v}\leq\psi\sum_{k=1}^{n}\lambda_k\mathbf{v}_k, \mathbf{w}=\psi\sum_{k=1}^{n}\lambda_k\mathbf{w}_k$$
$$\sum_{k=1}^{n}\lambda_k=1, \quad \lambda_k\geq 0 \quad, k=1,\dots,n, \quad 0\leq\psi\leq 1\} \tag{1}$$

ψ indicates the contraction factor in technology (1). This factor contracts good and bad outputs, simultaneously. The technology proposed by Kuosmanen (2005) is as follows:

$$T_{K}=\{(\mathbf{x},\mathbf{v},\mathbf{w}) \mid \mathbf{x}\geq\sum_{k=1}^{n}\lambda_k\mathbf{x}_k, \mathbf{v}\leq\sum_{k=1}^{n}\lambda_k\psi_k\mathbf{v}_k, \mathbf{w}=\sum_{k=1}^{n}\lambda_k\psi_k\mathbf{w}_k$$
$$\sum_{k=1}^{n}\lambda_k=1, \quad \lambda_k\geq 0 \quad, k=1,\dots,n, \quad 0\leq\psi_k\leq 1\} \tag{2}$$

Obviously, technology (2) is nonlinear, though it can be linearized by changing variables $\lambda_k=z_k+\mu_k, z_k=\lambda_k\psi_k$.

$$T_{K}=\{(\mathbf{x},\mathbf{v},\mathbf{w}) \mid \mathbf{x}\geq\sum_{k=1}^{n}(z_k+\mu_k)\mathbf{x}_k, \mathbf{v}\leq\sum_{k=1}^{n}z_k\mathbf{v}_k, \mathbf{w}=\sum_{k=1}^{n}z_k\mathbf{w}_k$$
$$\sum_{k=1}^{n}(z_k+\mu_k)=1, \quad z_k\geq 0 \quad, \mu_k\geq 0 \quad, k=1,\dots,n \quad \} \tag{3}$$

Undesirable Inputs

In some production processes, inputs increase to improve the performance. The previous research (e.g., Mehdiloozad & Podinovski, 2018) has investigated undesirable inputs. Actually, Mehdiloozad and Podinovski (2018) considered weakly disposable undesirable inputs. Briefly, the approach is explained herein.

For $DMU_k : k=1,\dots,n$, the vectors of desirable inputs, undesirable inputs, and desirable outputs are denoted by $\mathbf{x}_k=(x_{1k},\dots,x_{mk})\geq 0$, $\mathbf{x}'_k=(x'_{1k},\dots,x'_{jk})\geq 0$ and $\mathbf{y}_k=(y_{1k},\dots,y_{sk})\geq 0$, respectively. Production technology is shown by $p(\mathbf{y})=\{(\mathbf{x},\mathbf{x}') \mid (\mathbf{x},\mathbf{x}') \text{ can produce}(\mathbf{y})\}$.

Definition 2. Desirable and undesirable inputs $(\mathbf{x},\mathbf{x}')$ are weakly disposable if and only if $(\mathbf{x},\mathbf{x}') \in p(\mathbf{y})$ for all $\delta \geq 1$ implies $(\mathbf{x},\delta\mathbf{x}') \in p(\mathbf{y})$.

Mehdiloozad and Podinovski (2018) proposed the following technology with weak disposability assumption:

$$T_M = \{(\mathbf{x},\mathbf{x}',\mathbf{y}) \mid \mathbf{x} \geq \sum_{k=1}^{n} z_k \mathbf{x}_k, \sum_{k=1}^{n} z_k \delta_k \mathbf{x}'_k = \mathbf{x}', \quad 0 \leq \mathbf{y} \leq \sum_{k=1}^{n} z_k \mathbf{y}_k$$
$$\sum_{k=1}^{n} z_k = 1, \quad z_k \geq 0 \quad, k = 1,\dots,n, \quad \delta_k \geq 1\} \tag{4}$$

δ_k shows a gain factor, and is used for undesirable inputs. Obviously, technology (4) is nonlinear. However, it can be linearized by changing the variable $z_k\delta_k = z_k + \mu_k$. Therefore, we have:

$$T_M = \{(\mathbf{x},\mathbf{x}',\mathbf{y}) \mid \mathbf{x} \geq \sum_{k=1}^{n} z_k \mathbf{x}_k, \sum_{k=1}^{n} (z_k + \mu_k)\mathbf{x}'_k = \mathbf{x}', \quad 0 \leq \mathbf{y} \leq \sum_{k=1}^{n} z_k y_k$$
$$\sum_{k=1}^{n} z_k = 1, \quad z_k \geq 0 \quad, k = 1,\dots,n, \quad \mu_k \geq 0\} \tag{5}$$

Marginal Rates of Substitution

In production processes, changing an index affects one or more other indexes. Therefore, calculating the marginal rates of substitution is a crucial aspect in the production processes. In this subsection, trade-offs between input and output factors of DMUs are dealt with considering n units, $DMU_k : k = 1,\dots,n$, including vectors inputs $\mathbf{x}_k = (x_{1k},\dots,x_{mk}) \geq 0$ and outputs $\mathbf{y}_k = (y_{1k},\dots,y_{sk}) \geq 0$. The marginal rates of substitution are estimated using an index vector $\mathbf{f}_k = (-x_k, y_k)^t$.

Definition 3. Suppose that the point $\mathbf{f}_o = (-\mathbf{x}_o, \mathbf{y}_o)^t$ is efficient in the production process. The marginal rate of substitution of k^{th} index to t^{th} index at f_o is calculated as follows:

$$MR^{+}_{kt}(\mathbf{f}_o) = \left(\frac{\partial f_{ko}}{\partial f_{to}}\right)_{f_o^{+}} \quad , \quad MR^{-}_{kt}(\mathbf{f}_o) = \left(\frac{\partial f_{ko}}{\partial f_{to}}\right)_{f_o^{-}} \tag{6}$$

In fact, the left and right derivations in the definition 3 are at the point on the graph, which are the same as left and right marginal rates of substitutions. Asmild et al. (2006) used the following four-step method to calculate the marginal rates of substitution:

1. Consider the small incremental amount h for t^{th} index.

2. Acquire f^{*}_{ko} by solving the following linear programming:

$$\max f^{*}_{ko}$$
$$\text{s.t.} \quad (f_{1o},\dots,f_{to},\dots,f^{*}_{ko},\dots,f_{(m+s)o}) \tag{7}$$

3. Compute the marginal rate of substitution from the right as follows:

$$MR^{+}_{kt}(\mathbf{f}_o) = \frac{f^{*}_{ko} - f_{ko}}{h} \tag{8}$$

4. Similarly, repeating steps 2 and 3 for h=-h, the marginal rate of substitution from the left is estimated as follows:

$$MR^{-}_{kt}(\mathbf{f}_o) = \frac{f^{*}_{ko} - f_{ko}}{h} \quad (9)$$

Model (7) is computed for the k^{th} member of index, when the t^{th} member of the index is shifted by *h*, and the maximization objective function f^{*}_{ko} is on the efficient frontier.

Efficiency Analysis in the Two-Stage Process

Assume that there are k DMU_k with external inputs $\mathbf{x}_k = x_{ik}, i=1,2,...,m$ and external desirable outputs $\mathbf{v}_k = v_{lk}, l=1,2,...,L$, from the first stage, and *P* intermediate measures $\mathbf{w}_k = w_{pk} = z_{pk}, p=1,2,...,P$. This intermediate measure plays a mutual role that it is deemed as an undesirable output for the first stage and as an undesirable input for the second stage. External desirable inputs $\mathbf{f}_k = f_{tk}, t=1,2,...,T$ and external desirable outputs $\mathbf{y}_k = y_{rk}, r=1,2,...,s$ are presented in stage 2, too. The structure under evaluation is illustrated in Figure 1.

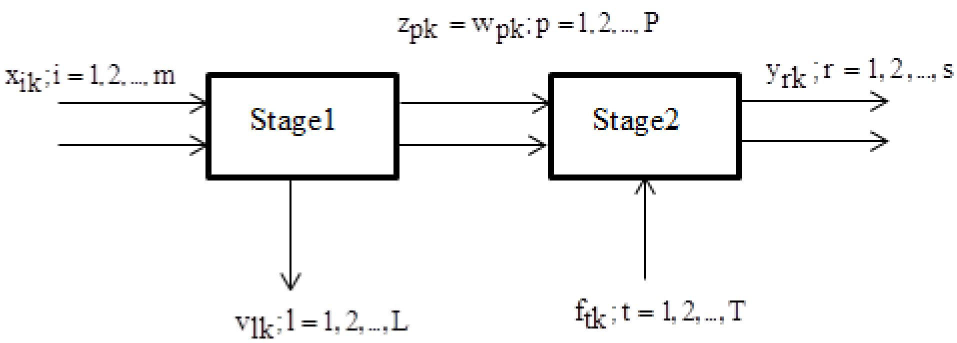

Figure 1. Two-Stage Process of DMU_k With Undesirable Outputs

Considering the above assumptions and Kuosmanen's approach (2005), the production technologies of stage 1 (T_1) and stage 2 (T_2) are defined as follows:

$$\begin{aligned} T_1 = \{(\mathbf{x},\mathbf{w},\mathbf{v}) \mid & \sum_{k=1}^{K} (\rho^k + \mu^k) x_i^k \le x_n && , i = 1,...,m \\ & \sum_{k=1}^{K} \rho^k w_p^k = w_p && , p = 1,...,P \\ & \sum_{k=1}^{K} \rho^k v_l^k \ge v_l && , l = 1,...,L \\ & \sum_{k=1}^{K} (\rho^k + \mu^k) = 1 \\ & \rho^k \ge 0 \;,\; \mu^k \ge 0 && , k = 1,...,K\} \end{aligned} \quad (10)$$

The above linear technology is based on unknown variables of ρ and μ. To illustrate in more details, it is under the variable returns to scale (VRS) assumption and the weak disposability of undesirable outputs. The purpose is to measure the efficiency of DMU_0 based on the abatement potential in undesirable outputs. This is obtained as the optimal value of the following model:

$$
\begin{aligned}
&\text{Min } \upsilon \\
&\text{s.t.} \\
&\sum_{k=1}^{K} (\rho^k + \mu^k) x_i^k \le x_i^o && , i = 1,\dots,m \\
&\sum_{k=1}^{K} \rho^k w_p^k = \upsilon w_p^o && , p = 1,\dots,P \\
&\sum_{k=1}^{K} \rho^k v_l^k \ge v_l^o && , l = 1,\dots,L \\
&\sum_{k=1}^{K} (\rho^k + \mu^k) = 1 \\
&\rho^k \ge 0 \;,\; \mu^k \ge 0 && , k = 1,\dots,K
\end{aligned}
\tag{11}
$$

Model (11) is a linear programming problem and is always feasible and bounded. Model (11) can be written as model (12):

$$
\begin{aligned}
&\text{Max } \omega \\
&\text{s.t.} \\
&\sum_{k=1}^{K} (\rho^k + \mu^k) x_i^k \le x_i^o && , i = 1,\dots,m \\
&\sum_{k=1}^{K} \rho^k w_p^k = (1-\omega) w_p^o && , p = 1,\dots,P \\
&\sum_{k=1}^{K} \rho^k v_l^k \ge v_l^o && , l = 1,\dots,L \\
&\sum_{k=1}^{K} (\rho^k + \mu^k) = 1 \\
&\rho^k \ge 0 \;,\; \mu^k \ge 0 \;, 0 \le \omega < 1 && , k = 1,\dots,K
\end{aligned}
\tag{12}
$$

In the directional distance function context, if $(d_x, d_w, d_v) = (0, -w^o, 0)$, model (11) can be written as model (12) (see Toloo et al., 2018). As model (12) admits, ω^* can play an optimal value and it can get only nonnegative values that means $\omega^* \ge 0$.

Definition 4. If $\omega^* = 0$, DMU_0 is efficient. If $0 < \omega^* < 1$, DMU_0 is inefficient (see Chambers et al., 1984). In other words, the efficiency can be defined as $e^1 = 1 - \omega^*$.

The production technology of the second stage is illustrated as follows:

$$
\begin{aligned}
T_2 = \{(\mathbf{f}, \mathbf{w}, \mathbf{y}) \mid &\sum_{k=1}^{K} \lambda^k f_t^k \le f_t && , t = 1,\dots,T \\
&\sum_{k=1}^{K} (\lambda^k + z^k) w_p^k = w_p && , p = 1,\dots,P \\
&\sum_{k=1}^{K} \lambda^k y_r^k \ge y_r && , r = 1,\dots,S \\
&\sum_{k=1}^{K} \lambda^k = 1 \\
&\lambda^k \ge 0 \;,\; z^k \ge 0 && , k = 1,\dots,K\}
\end{aligned}
\tag{13}
$$

It is clear that technology (13) is linear. It is also under the VRS technology and weak disposability of undesirable outputs. λ^k and z^k, $k = 1,\dots,K$ are, furthermore, unknown weights. Therefore, the efficiency of stage 2 can be calculated as follows:

$$\begin{aligned}
&\text{Max} \quad \theta \\
&\text{s.t.} \\
&\sum_{k=1}^{K} \lambda^k f_t^k \le (1-\theta) f_t^o \qquad ,t=1,\dots,T \\
&\sum_{k=1}^{K} (\lambda^k + z^k) w_p^k = (1+\theta) w_p^o \qquad ,p=1,\dots,P \\
&\sum_{k=1}^{K} \lambda^k y_r^k \ge (1+\theta) y_r^o \qquad ,r=1,\dots,S \\
&\sum_{k=1}^{K} \lambda^k = 1 \\
&\lambda^k \ge 0 \ , \quad z^k \ge 0, \quad 0 \le \theta < 1 \qquad ,\ k=1,\dots,K
\end{aligned} \tag{14}$$

Model (14) is a directional distance function model. So, the efficiency score can be obtained as $e^2 = 1-\theta^*$. Based on Figure 1 and aforementioned concepts, the following approach is proposed to assess the overall efficiency of two-stage systems:

$$\begin{aligned}
&\max \quad \frac{1}{2}(\theta+\omega) \\
&\text{s.t.} \\
&\sum_{k=1}^{K} (\rho^k + \mu^k) x_i^k \le x_i^o \qquad ,i=1,\dots,m \qquad (15.1) \\
&\sum_{k=1}^{K} \lambda^k y_r^k \ge (1+\theta) y_r^o \qquad ,r=1,\dots,S \qquad (15.2) \\
&\sum_{k=1}^{K} \rho^k v_l^k \ge v_l^o \qquad ,l=1,\dots,L \qquad (15.3) \\
&\sum_{k=1}^{K} \rho^k w_p^k = (1-\omega) w_p^o \qquad ,p=1,\dots,P \qquad (15.4) \\
&\sum_{k=1}^{K} (\lambda^k + z^k) w_p^k = (1+\theta) w_p^o \qquad ,p=1,\dots,P \qquad (15.5) \\
&\sum_{k=1}^{K} \lambda^k f_t^k \le (1-\theta) f_t^o \qquad ,t=1,\dots,T \qquad (15.6) \\
&\sum_{k=1}^{K} \lambda^k = 1 \qquad (15.7) \\
&\sum_{k=1}^{K} (\rho^k + \mu^k) = 1 \qquad (15.8) \\
&(\rho^k + \mu^k) \ge 0 \qquad (15.9) \\
&\rho^k \ge 0, \mu^k \ge 0, \lambda^k \ge 0, \quad z^k \ge 0 \quad ,k=1,\dots,K \\
&0 \le \theta < 1 \qquad ,0 \le \omega < 1
\end{aligned} \tag{15}$$

The linear programming problem (15) is always feasible. Consider an arbitrary solution for model (15) as follows:

$$\theta = \omega = 0$$

$$\lambda^k = \rho^k = 0 \quad ,\forall k \quad k \ne 0$$

$$\lambda^o = \rho^o = 1$$

$$z^k = \mu^k = 0 \qquad ,\forall k$$

It is obvious that the arbitrary solution is a feasible solution for model (15). In model (15), undesirable outputs are minimized in the first stage and undesirable inputs are maximized in second stage by implementing the direction vectors $(d_x, d_w, d_v) = (0, -w^o, 0)$ and

$(d_f, d_w, d_y) = (-f^o, w^o, y^o)$. The objective function of this model is defined as $\max \frac{1}{2}(\theta + \omega)$, which maximizes both abatement and gain of the undesirable intermediate factor. To illustrate, this factor plays the undesirable output role as the constraint (4) is satisfied. On the other hand, it plays the role of undesirable input for the second stage as the constraint (5) shows. By computing model (15), the overall efficiency and stage efficiencies are obtained. If $e^{o*} = 0$ (i.e. $\theta^{o*} = \omega^{o*} = 0$) in model (15), DMU_0 is efficient in each stage and in general. Besides, it is inefficient provided that at least one of θ^{o*} and ω^{o*} is not equal to zero. Therefore, the efficiency value can be estimated as $e^* = 1 - \frac{1}{2}(\theta^* + \omega^*)$.

Calculating Marginal Rates of Substitution in the Two-Stage Process

In order to estimate the impact of changes of a throughput in other throughputs in two-stage processes, in this section, marginal rates of substitution are calculated in four cases. Selected schemes can be made in any way to investigate the effect of a change from certain indicators (input or output) to other indicators. Therefore, calculating marginal rates of substitution with different schemes is important because the information that it provides to manufacturers or consumers allows them to make alternatives in the inputs or outputs while maintaining the efficiency. For instance, it is important to increase the desirable output in the production process. Thus, we increase or decrease the other factors to see their effect on the desirable output of the first or second stage. Similarly, other schemes can be considered to decrease the desirable input. Therefore, in order to perform better and achieve the expected conditions, suggestions are given to the two-stage system under evaluation so that the unit can show the performance that is economically viable for it. In fact, the changes made are adopted in a way that is favorable to the decision maker. The manager changes a set of indicators in any way she/he desires and measures the impact of these changes on another set of indicators.

The marginal rate of substitution in the two-stage structure under consideration is calculated based on linear programming (15).

- **Scheme 1:** In this scheme, we calculate the marginal rates of substitution desirable output *d* to desirable input *b* from the first stage of the two-stage process maintaining efficiency. Suppose the sub-units $(\mathbf{x}^o, \mathbf{v}^o, \mathbf{w}^o)$ in stage 1 and $(\mathbf{w}^o, \mathbf{f}^o, \mathbf{y}^o)$ stage 2 are on the frontier. It is defined by:

$$MR^{\pm}_{db}(\mathbf{x}^o, \mathbf{v}^o, \mathbf{w}^o) = \left[\frac{\partial v^o_d}{\partial x^o_b} \right]$$

We use the following four steps to calculate the marginal rates of substitution.

1. Considering a small incremental amount *h* for b^{th} index.
2. Acquiring v^{o*}_d by solving linear programming model (16).

$$
\begin{aligned}
\max \quad & v_d^{o*} \\
\text{s.t.} \quad & \\
& \sum_{k=1}^{K} (\rho^k + \mu^k) x_i^k \le x_i^o && , i = 1,\dots,m, \quad i \ne b \\
& \sum_{k=1}^{K} (\rho^k + \mu^k) x_b^k \le x_b^o + h && \\
& \sum_{k=1}^{K} \lambda^k y_r^k \ge y_r^o && , r = 1,\dots,S \\
& \sum_{k=1}^{K} \rho^k v_l^k \ge v_l^o && , l = 1,\dots,L, \quad l \ne d \\
& \sum_{k=1}^{K} \rho^k v_d^k \ge v_d^{o*} && \\
& \sum_{k=1}^{K} \rho^k w_p^k = w_p^o && , p = 1,\dots,P \\
& \sum_{k=1}^{K} (\lambda^k + z^k) w_p^k = w_p^o && , p = 1,\dots,P \\
& \sum_{k=1}^{K} \lambda^k f_t^k \le f_t^o && , t = 1,\dots,T \\
& \rho^k \ge 0, \mu^k \ge 0, \lambda^k \ge 0, z^k \ge 0, \; k = 1,\dots,K
\end{aligned}
\tag{16}
$$

Constraints (15.7), (15.8), and (15.9) of model (15) are added to the rest of constraints of model (16).

3. Computing the marginal rate of substitution from the right as follows:

$$MR_{db}^{+}(\mathbf{x}^o, \mathbf{v}^o, \mathbf{w}^o) = \frac{v_d^{o*} - v_d^o}{h} \tag{17}$$

4. Steps 2 and 3 are repeated for *h*=-*h*, e.g., the marginal rate of substitution is calculated from the left as follows:

$$MR_{db}^{-}(\mathbf{x}^o, \mathbf{v}^o, \mathbf{w}^o) = \frac{v_d^{o*} - v_d^o}{h} \tag{18}$$

- **Scheme 2:** In the second scheme, we calculate the marginal rates of substitution of desirable output *d* to desirable input *b* from the second stage of the two-stage process maintaining efficiency. Assume the sub-units $(\mathbf{x}^o, \mathbf{v}^o, \mathbf{w}^o)$ in stage 1 and $(\mathbf{w}^o, \mathbf{f}^o, \mathbf{y}^o)$ stage 2 are on the frontier. It is defined by:

$$MR_{db}^{\pm}(\mathbf{w}^o, \mathbf{f}^o, \mathbf{y}^o) = \left[\frac{\partial y_d^o}{\partial f_b^o} \right] \tag{19}$$

We use the four steps to calculate marginal rates in the similar way of the scheme 1.

- **Scheme 3**: In the third case, we calculate the marginal rates of substitution desirable output *d* to undesirable intermediate measure *b* from the second stage of the two-stage process maintaining efficiency. Assume the sub-units $(\mathbf{x}^o, \mathbf{v}^o, \mathbf{w}^o)$ in stage 1 and $(\mathbf{w}^o, \mathbf{f}^o, \mathbf{y}^o)$ stage 2 are on the frontier. The marginal rates of substitution are identified by:

$$MR_{db}^{\pm}(\mathbf{w}^o, \mathbf{f}^o, \mathbf{y}^o) = \left[\frac{\partial y_d^o}{\partial w_b^o} \right] \tag{20}$$

The four steps similar to scheme 1 are used to calculate the marginal rates of substitution.

- **Scheme 4:** In the fourth case, we calculate the marginal rates of substitution desirable output *d* to undesirable intermediate measure *b* from the first stage of the two-stage process maintaining efficiency. Assume the sub-units $(\mathbf{x}^o, \mathbf{v}^o, \mathbf{w}^o)$ in stage 1 and $(\mathbf{w}^o, \mathbf{f}^o, \mathbf{y}^o)$ stage 2 are on the frontier. The marginal rates of substitution are identified by:

$$MR^{\pm}_{db}(\mathbf{x}^o,\mathbf{v}^o,\mathbf{w}^o)=\left[\frac{\partial v^o_d}{\partial w^o_b}\right] \tag{21}$$

Analogous to scheme 1, the four steps are used to calculate the marginal rates of substitution.

In this section, we analyzed the marginal rates of substitution in four cases. Nevertheless, other cases can be considered and estimated, similarly. In addition, the h value is considered too small and arbitrary according to the data in the production process. Actually, the change of a factor in the size of h should be considered whose effect is on the same hyperplane (efficient facet). In the next section, we address the marginal rates of the substitutions of one set of variables with another set.

Extension to Multi-Index for Calculating Marginal Rates of Substitution

In many case studies, it is useful to assess the effect of changes in a set of variables on the other set. Assume $M=\{a_1,a_2,...,a_q\}$ and $N=\{u_1,u_2,...,u_f\}$; we developed the above method to calculate the marginal rate of the substitution of index in M with index in N. The four steps are the same as before, but the problem is multi-objective linear programming (MOLP). At this stage, scheme 3 (as an instance of the four schemes) is developed as follows:

1. Select the small increment h.
2. Obtain $y^{o*}_d : d\in M$ by solving the problem (22) and consider $\max\{y^{o*}_d : d\in M\}$ by maintaining the efficiency:

$$\begin{aligned}
\max\quad & \{y^{o*}_d : d\in M\} \\
\text{s.t.} \quad & \\
& \sum_{k=1}^{K}(\rho^k+\mu^k)x^k_i \le x^o_i && ,i=1,...,m \\
& \sum_{k=1}^{K}\lambda^k y^k_r \ge y^o_r && ,r=1,...,S,\quad r\notin M \\
& \sum_{k=1}^{K}\lambda^k y^k_d \ge y^{o*}_d && ,d\in M \\
& \sum_{k=1}^{K}\rho^k v^k_l \ge v^o_l && ,l=1,...,L \\
& \sum_{k=1}^{K}\rho^k w^k_p = w^o_p && ,p=1,...,P \\
& \sum_{k=1}^{K}(\lambda^k+z^k)w^k_p = w^o_p && ,p=1,...,P,\quad p\notin N \\
& \sum_{k=1}^{K}(\lambda^k+z^k)w^k_b = w^o_b+h && ,b\in N \\
& \sum_{k=1}^{K}\lambda^k f^k_t \le f^o_t && ,t=1,...,T, \\
& \rho^k\ge 0,\mu^k\ge 0,\lambda^k\ge 0,z^k\ge 0,\ k=1,...,K
\end{aligned} \tag{22}$$

Constraints (15.7), (15.8), and (15.9) of model (15) are incorporated to the rest of the constraints of model (22).

If $\varphi=\min\left\{y^{o*}_d : d\in M\right\}$, we have $\varphi\le y^{o*}_d : \forall d\in M$. By introducing the variable φ, the MOLP is transformed by the following single objective linear programming:

$$\begin{aligned} &\max \quad \varphi \\ &\text{s.t.} \quad \varphi \le y_d^{o*} : \forall d \in M \end{aligned} \tag{23}$$

The rest of the constraints remain in the model (22).

Now, Steps 3 and 4 are like the Scheme 3.

The marginal rate of substitution in schemes 1, 2, 3, and 4 is considered by the example in the next section.

The Illustrative Application

To emphasize the capabilities of the proposed method, the two-stage process is based on a set of actual data for industrial production in the 30 provincial levels of mainland China. This data set is available in Wu et al. (2015).

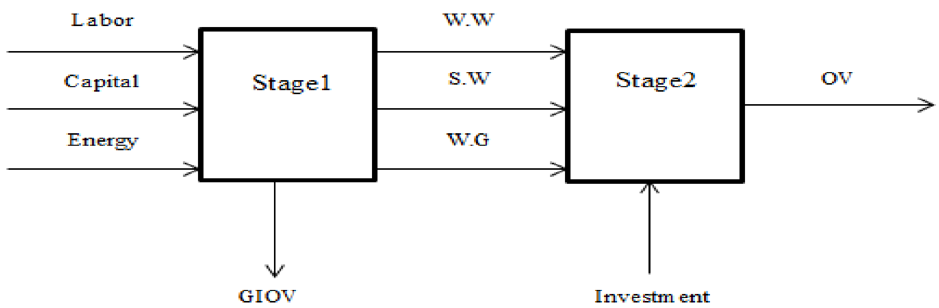

Figure 2. Two-Stage Industrial Production Processes for DMU_k

Actually, the introduced approach is used in order to assess the performance and calculate the marginal rates of substitution in this empirical application. Figure 2 shows the structure under assessment. It indicates that the first stage includes three desirable inputs (labor, capital, and energy) and four outputs (gross industrial output value (GIOV), wastewater (W.W), solid waste (S.W), and waste gas (W.G)), among which GIOV is the external output from stage 1. Moreover, W.W, S.W, and W.G are undesirable outputs in stage 1, and are the undesirable inputs of stage 2. In other words, they are undesirable intermediate measure products. Moreover, in stage 2, the investment in the external input is to treat industrial wastewater, solid waste, and waste gas. Therefore, in the second stage, the output includes desirable output (OV) (the output value of products made from wastewater, solid waste, and waste gas). The performance measures are described in Table 1.

Table 1. Variables and Units

Subsystem	**Factor**	**Variables**	**Units**
First	Desirable input	Labor	10000 persons
		Energy	10000 tons of coal
		Capital	100 million RMB Yuan
	Desirable output	Gross industrial output value	100 million RMB Yuan
	Undesirable intermediate measure	Waste water discharge	10000 tons
		Solid waste generated	10000 tons
		Waste gas emission	Billion standard cu.m
Second	Desirable input	Investment	100 million RMB Yuan
	Desirable output	Output value of products made from W.W, S.W, and W.G	100 million RMB Yuan

First, the overall and stages efficiencies are measured by the proposed approach that is model (15), model (12), and model (14). The overall efficiency measures are indicated in Table 2.

Based on the overall efficiency scores in Table 2, 10 efficient DMUs have been introduced generally. Actually, units 1, 3, 4, 11, 14, 20, 21, 24, 25, and 29 are efficient in the production process (i.e. stages 1 and 2). Furthermore, 19 units in the first stage and 10 units in the second stage have been identified as efficient. The overall and stage-based efficiencies of industrial production of 30 provincial level regions are also depicted in Figure 3.

In the next stage, we calculate the marginal rates of substitution for overall efficient regions by considering four cases stated in Section 4.

Table 2. Industrial Efficiencies for 30 Districts Across Provinces

DMU_K	Region	e^1	e^2	e^*
1	Beijing	1.0000	1.0000	1.0000
2	Tianjin	1.0000	0.3982	0.6991
3	Hebei	1.0000	1.0000	1.0000
4	Shanxi	1.0000	1.0000	1.0000
5	Inner Mongolia	1.0000	0.9562	0.9781
6	Liaoning	1.0000	0.1825	0.5913
7	Jilin	1.0000	0.4336	07168
8	Heilongjiang	0.4691	0.5110	0.4900
9	Shanghai	1.0000	0.3375	0.6688
10	Jiangsu	1.0000	0.7194	0.8597
11	Zhejiang	1.0000	1.0000	1.0000
12	Anhui	0.5331	0.6978	0.6155
13	Fujian	0.9769	0.2236	0.6062
14	Jiangxi	1.0000	1.0000	1.0000
15	Shandong	1.0000	0.4701	0.7350
16	Henan	0.8091	0.4050	0.6377
17	Hubei	0.6131	0.2432	0.4281
18	Hunan	0.7745	0.4301	0.7151
19	Guangdong	1.0000	0.2200	0.6100
20	Guangxi	1.0000	1.0000	1.0000
21	Hainan	1.0000	1.0000	1.0000
22	Chongqing	0.3573	0.3202	0.3387
23	Sichuan	0.5453	0.4375	0.4914
24	Guizhou	1.0000	1.0000	1.0000
25	Yunnan	1.0000	1.0000	1.0000
26	Shaanxi	0.3873	0.0973	0.2423
27	Gansu	0.5173	0.3890	0.4532
28	Qinghai	1.0000	1.0000	0.7372
29	Ningxia	1.0000	1.0000	1.0000
30	Xinjiang	0.2605	0.3814	0.3210

Results from scheme 1 are summarized in Table 3. The right marginal rates of substitution ($h = 10$) of ten units are zero, which are obtained from Eq. (17). Actually, the desirable outputs of these DMUs in stage 1 remain unchanged. Also, based on the left marginal rate of substitution ($h = -10$), all units remain infeasible except for one unit which has positive marginal rates. Outcomes were estimated using Eq. (18).

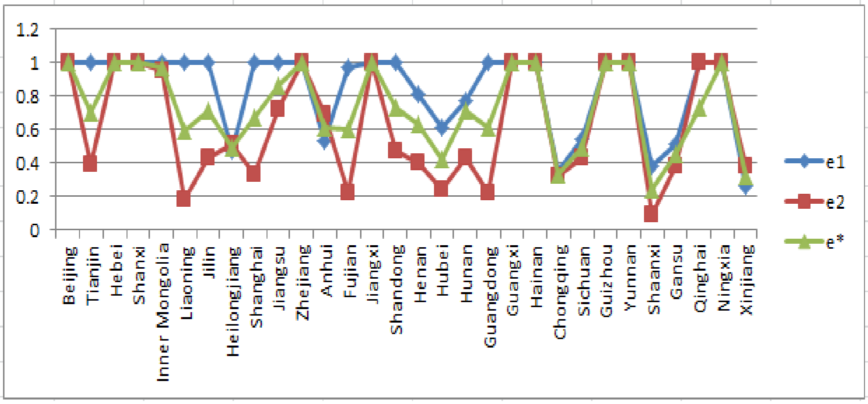

Figure 3. Efficiency Scores of Stages 1, 2, and the overall efficiency

In Table 4, the results from scheme 2 are provided briefly. The value $h = \pm 0.1$ is considered and the results for 10 efficient $DMUs$ are shown in this table. Right marginal rates of substitution of all units are zero. Indeed, desirable outputs of $DMUs$ in stage 2 remain unchanged with maintaining the efficiency. In addition, based on the left margin rate of substitution, all units remain infeasible except for two units, that they have positive marginal rates. These findings have been achieved by solving Eq. (19).

The findings of scheme 3 are briefly stated in Table 5. By considering $h = \pm 10$, they are shown in Table 5 for 10 efficient DMUs. The right marginal rates of substitution of the five units are positive and they are infeasible for another five units through changing the first undesirable intermediate measure. Furthermore, for the variation of the second undesirable intermediate factor, the right marginal rates of substitution of the three units are negative and they are infeasible for other units. In the direction of changing the third undesirable intermediate measure, furthermore, the right marginal rates of substitution of two units are positive, they are negative for three units, and the infeasibility is observed for the rest. In changing the two undesirable intermediate measures, wastewater and solid waste, the left marginal rates of substitution of the three units are positive, negative in one unit, and infeasible for the other six units. The left marginal rates of substitution of the four units are negative and they found infeasible for others when the variation of the third undesirable intermediate material is considered. These results were found using Eq. (20).

Table 3. Results of Marginal Rates of Substitution for Scheme 1

	No. DMU_k $MR_1^+ > 0$,$(MR_1^+ = 0)$ $MR_1^+ < 0$	No. DMU_k $MR_2^+ > 0$,$(MR_2^+ = 0)$ $MR_2^+ < 0$	No. DMU_k $MR_3^+ > 0$,$(MR_3^+ = 0)$ $MR_3^+ < 0$	No. Infeasibility In MR_1^+ ,(MR_2^+) ,MR_3^+
$h = 10$	0,(10),0	0,(10),0	0,(10),0	0,(0),0
	No. DMU_k $MR_1^- > 0$,$(MR_1^- = 0)$ $MR_1^- < 0$	No. DMU_k $MR_2^- > 0$,$(MR_2^- = 0)$ $MR_2^- < 0$	No. DMU_k $MR_3^- > 0$,$(MR_3^- = 0)$ $MR_3^- < 0$	No. Infeasibility In MR_1^- ,(MR_2^-) ,MR_3^-
$h = -10$	1,(0),0	1,(0),0	1,(0),0	9,(9),9

Results from scheme 4 (by regarding $h = \pm 10$ for 10 efficient $DMUs$) are summarized in Table 6. To explain, by using Eq. (21), the following outcomes are derived:

Table 4. Results of Marginal Rates of Substitution for Scheme 2

	No. DMU_k $MR_1^+ > 0, (MR_1^+ = 0), MR_1^+ < 0$	No. Infeasibility In MR_1^+
$h = 0.1$	0,(10),0	0
	No. DMU_k $MR_1^- > 0, (MR_1^- = 0), MR_1^- < 0$	No. Infeasibility In MR_1^-
$h = -0.1$	2,(0),0	8

When the first undesirable intermediate factor changes, the right marginal rates of substitution of six units are obtained zero, while they are infeasible for the other four units. Moreover, the right marginal rates of substitution of all ten units are infeasible in consideration of the alteration of the second undesirable intermediate factor.

Table 5. Results of Marginal Rates of Substitution for Scheme 3

	No. DMU_k $MR_1^+ > 0$,$(MR_1^+ = 0)$ $MR_1^+ < 0$	No. DMU_k $MR_2^+ > 0$,$(MR_2^+ = 0)$ $MR_2^+ < 0$	No. DMU_k $MR_3^+ > 0$,$(MR_3^+ = 0)$ $MR_3^+ < 0$	No. Infeasibility In MR_1^+ ,(MR_2^+) ,MR_3^+
$h = 10$	5,(0),0	0,(0),3	2,(0),3	5,(7),5
	No. DMU_k $MR_1^- > 0$,$(MR_1^- = 0)$ $MR_1^- < 0$	No. DMU_k $MR_2^- > 0$,$(MR_2^- = 0)$ $MR_2^- < 0$	No. DMU_k $MR_3^- > 0$,$(MR_3^- = 0)$ $MR_3^- < 0$	No. Infeasibility In MR_1^- ,(MR_2^-) ,MR_3^-
$h = -10$	3,(0),1	3,(0),1	0,(0),4	6,(6),6

Furthermore, the right marginal rates of substitution of two units are zero through the change of waste gas emission. Moreover, eight units of them are infeasible. By considering the changes of wastewater and solid waste, the left marginal rates of substitution of nine units are infeasible, while it is zero for one unit. The left marginal rates of substitution of the four units are zero, and they are infeasible for the other six units by assuming the variation of the third undesirable intermediate measure.

Table 6. Results of Marginal Rates of Substitution for Scheme 4

	No. DMU_k $MR_1^+ > 0$,$(MR_1^+ = 0)$ $MR_1^+ < 0$	No. DMU_k $MR_2^+ > 0$,$(MR_2^+ = 0)$ $MR_2^+ < 0$	No. DMU_k $MR_3^+ > 0$,$(MR_3^+ = 0)$ $MR_3^+ < 0$	No. Infeasibility In MR_1^+ ,(MR_2^+) ,MR_3^+
$h = 10$	0,(6),0	0,(0),0	0,(2),0	4,(10),8
	No. DMU_k $MR_1^- > 0$,$(MR_1^- = 0)$ $MR_1^- < 0$	No. DMU_k $MR_2^- > 0$,$(MR_2^- = 0)$ $MR_2^- < 0$	No. DMU_k $MR_3^- > 0$,$(MR_3^- = 0)$ $MR_3^- < 0$	No. Infeasibility In MR_1^- ,(MR_2^-) ,MR_3^-
$h = -10$	0,(1),0	0,(1),0	0,(4),0	9,(9),6

In the production process, increasing/decreasing a throughput always does not lead to an increase/decrease in another throughput. It may remain unchanged or infeasible.

Conclusions

In this paper, a two-stage DEA approach in the presence of weakly disposable undesirable materials has been provided to assess the performance of the overall system and processes. The technique was founded upon the directional distance function. In addition, some algorithms have been presented to calculate the marginal rates of the substitution of variables in two-stage structures. Actually, the effect of the changes of a throughput in other throughputs such as the effect of changes of intermediate measures on the output of the first stage and second stage is measured by maintaining efficiency. These changes in economics and production management provide beneficial information that help with making a better decision. Moreover, the approach was applied to data from industrial production in the 30 provinces of mainland China. Findings show that the overall efficiency and stage efficiencies of two-stage DEA network can be estimated using the rational computational approach proposed herein. Furthermore, the results obtained from calculating the marginal rates of substitution for cases under investigation will be beneficial for better decision making and planning.

For future research, the marginal rates of substitution in multi-stage structures (series and parallel) can be studied. In addition, calculating the non-marginal rates of substitution in multi-stage processes would be crucial for studying.

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