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On the buckling analysis of functionally graded sandwich beams using a unified beam theory

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ABSTRACT

In this paper, a unified beam theory is developed and applied to study the buckling response of two types of functionally graded sandwich beams. In the first type (Type A), the sandwich beam has a hardcore whereas in the second type (Type B), the sandwich beam has a softcore. In both the type of beams, face sheets are made up of functionally graded material. The material properties of face sheets are varied through the thickness according to the power-law distribution. A unified beam theory developed in the present study uses polynomial and non-polynomial type shape functions in-terms of thickness coordinate to account for the effect of shear deformation. The present theory is built upon classical beam theory and shows a realistic variation of transverse shear stresses through the thickness of the beam. The governing equations are deduced based on the principle of virtual work. Analytical solutions for simply supported sandwich beams subjected to axial force are presented. The critical buckling load factors of two types of FG sandwich beams are investigated. The numerical results are obtained for various power law coefficients and face-core-face thickness ratios. The validity of the present theory is proved by comparing the present results with various available solutions in the literature.

1. Introduction

Laminated sandwich beams are often subjected to delamination and stress concentration problems due to the abrupt change in material properties at the layer interface. To overcome these problems of laminated sandwich beams, functionally graded (FG) sandwich beams can be the best suitable option. FG materials are formed by gradually varying the material properties with a specific gradient from one surface to another surface of the beam [1-10]. Beam type structural elements are often subjected to axial forces which cause buckling of these structures. Therefore, an accurate study of the buckling behaviour of FG sandwich beams is required to design it for the axial forces. Classical beam theories such as the Euler-Bernoulli beam theory (EBT) of Bernoulli [11] and the first-order beam theory (FBT) of Timoshenko [12] are used by many researchers in the past decades. However, these theories are not accurate enough for the analysis of thick beams due to neglecting the effect of transverse shear deformation. Also, the FBT shows the constant variation of shear strain across the thickness of the beam. These limitations of the classical theories force the researchers to develop more accurate theories for the analysis of thick beams. These theories are classified as higher order beam theories (HOBTs). The detailed and critical review of HOBTs can be found in Sayyad and Ghugal [13, 14]. Few review articles on FG materials and structures can also be found in the literature such as Jha et al. [15], Swaminathan et al. [16], and Sayyad and Ghugal [17]. A good number of research papers have been published in the literature on buckling analysis of single layer FG beams [18-29], however, the literature on the analysis of FG sandwich beams is limited.

Bhangale and Ganesan [30] studied thermal buckling and vibration behavior of the FG sandwich beam with constrained viscoelastic layer core using the finite element method. Zenkour et al. [31] investigated the bending response of FG viscoelastic sandwich beams using various refined beam theories. Vo et al. [32, 33] have developed a finite element model for the vibration and buckling analysis of FG sandwich beams based on third order polynomial type beam theory. Nguyen et al. [34, 35] have

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developed an inverse trigonometric beam theory for the vibration and buckling analysis of FG sandwich beams. Lanc et al. [36] have presented a buckling analysis of FG sandwich box beams using the Euler-Bernoulli beam theory and the Vlasov theory. Bennai et al. [37] developed a new hyperbolic beam theory considering the effects of shear and normal deformations for the free vibration and buckling analysis of FG sandwich beams under various boundary conditions. Tossapanon and Wattanasakulpong [38] have solved buckling and free vibration problems of FG sandwich beams using FBT based on the Chebyshev collocation method. Osofero et al. [39] presented an analytical solution for vibration and buckling of FG sandwich beams using various quasi-3D theories considering the effects of normal deformation. Kahya and Turan [40] have developed a finite element model for the vibration and buckling of FG sandwich beams based on FBT. Karamanli [41] studied the static behaviour of two-directional FG sandwich beams of different boundary conditions by using a third order polynomial type beam theory considering the effects of transverse shear and normal deformations based on the symmetric smoothed particle hydrodynamics method. Li et al. [42] developed a higher-order shear deformable mixed finite element model to determine displacements and stresses in FG sandwich beams. Sayyad and Ghugal [43] presented the nth order shear deformation theory for the analysis of composite laminates under cylindrical bending. Shinde et al. [44] applied the hyperbolic theory for the thermal analysis of isotropic plates. Sayyad and Avhad [45] presented a bending analysis of functionally graded sandwich beams using hyperbolic shear deformation beam theory.

Mohammadi et al. [46-57] and; Mohammadi and Abbas Rastgoo [58, 59] have presented bending, buckling and free vibration analysis of nanobeams and graphene sheets embedded in an elastic medium under mechanical and thermal environment using various nonlocal theories. A similar study is applied by Asemi et al. [60-63] and Farajpour et al. [64-71] for the piezoelectric nanostructures. One can also refer [72-77] for a similar approach.

Based on the aforementioned literature review, it is pointed out that the studies on buckling analysis of FG sandwich beams are limited in the literature. Therefore, in this article, we restrict our focus on the buckling analysis of sandwich beam with homogenous core and FG face sheets. Two types of homogenous cores (softcore and hardcore) are considered in the present study. Buckling analysis of sandwich beams is carried out using a unified beam theory which recovered various shear deformation beam models such as the third order beam theory of Reddy [78], the FBT of Timoshenko [12], the CBT of Bernoulli-Euler [11], etc. Few non-polynomial type shape functions are first time used in the present form of displacement field such as the trigonometric function of Levy [79], the hyperbolic function of Soldatos [80], and the exponential function of Karama et al. [81]. The governing equations are deduced based on the principle of virtual work. Buckling solutions are obtained by using Navier's technique for various power law coefficients, aspect ratios, and face-core-face thickness ratios.

2. Geometry and materials

Consider an FG sandwich beam as shown in Fig. 1. The beam has length L and rectangular cross-section $b \times h$; h_1, h_2, h_3, and h_4 are the thickness coordinates of each layer measured from the neutral axis. The width of the beam is considered as unity in the y-direction. The beam is subjected to axial forces. The four types of relations between face sheets to core thickness ratios are considered (1-0-1, 2-1-2, 1-1-1 and 1-2-1).

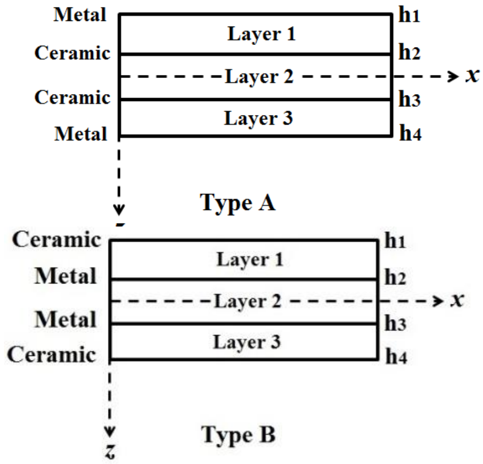
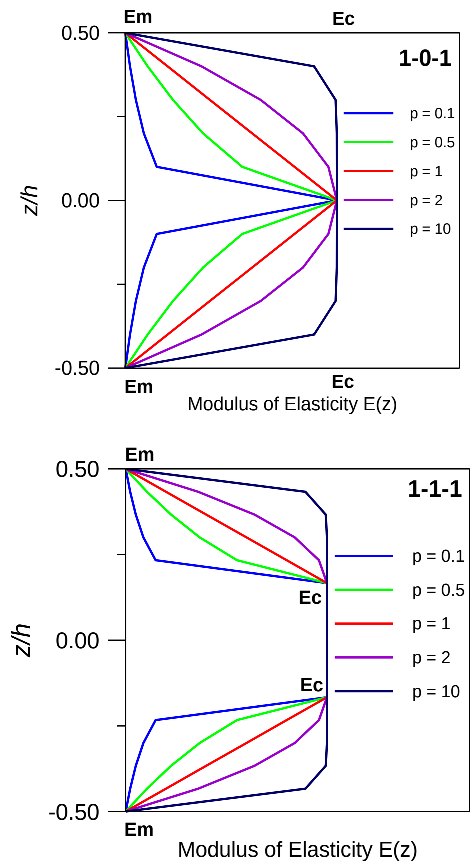


Figure 1. Material gradation of FG sandwich beams (Type A: Hardcore, Type B: Softcore)

Face sheets of the sandwich beams are made up of FG material in which elastic properties of the material are graded across the thickness of the beam. The simple rule of mixture i.e. the power-law is used for the gradation of material properties.

$$\begin{aligned} &\text{Type A(hardcore)}: \quad E(z)^{(n)} = E_m + (E_c - E_m)V^{(n)} \\ &\text{Type B(softcore)}: \quad E(z)^{(n)} = E_c + (E_m - E_c)V^{(n)} \end{aligned} \tag{1}$$

where $V^{(n)}$ represents the function of volume fraction for the n^{th} layer (n=1,2,3); E_m represents and Young's modulus of metal whereas E_c represents Young's modulus of ceramic.



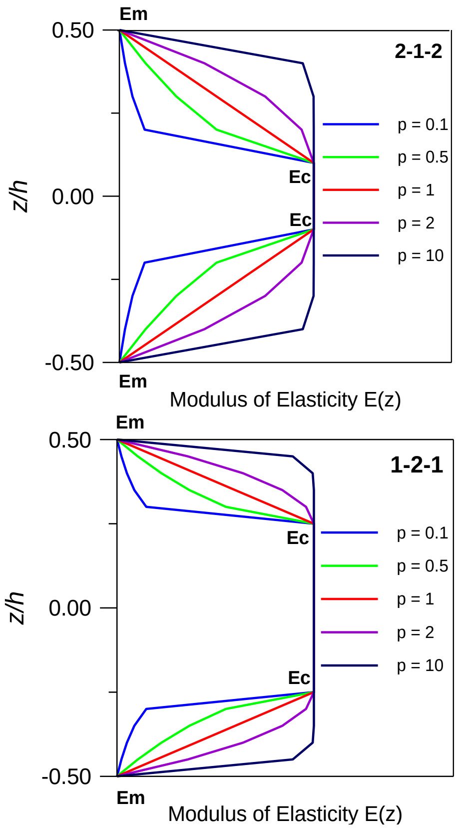


Figure 2. Through the thickness variation of Young's modulus of FG sandwich plate (Type A: Hardcore) for various skin-core-skin thickness ratios

Functions of volume fraction for FG face sheets and homogenous core are assumed as follows.

$$V^{(1)} = \left(\frac{z-h_1}{h_2-h_1}\right)^p \quad \text{for} \quad z \in [h_1, h_2]$$

$$V^{(2)} = 1 \quad \text{for} \quad z \in [h_2, h_3] \tag{2}$$

$$V^{(3)} = \left(\frac{z-h_4}{h_3-h_4}\right)^p \quad \text{for} \quad z \in [h_3, h_4]$$

where p is the power-law coefficient that indicates material variation profile through the thickness. Through-the-thickness variations of Young's modulus of FG sandwich beams made of ceramic hardcore and metallic softcore are shown in Figs. 2 and 3.

3. Development of theory

The kinematic formulation of the present unified beam theory is based on the following assumptions.

1) Axial displacement consists of extension, bending and shear components.
2) Axial displacement considered the effect of transverse shear deformation.
3) Transverse displacement consists of only bending component.
4) The effect of transverse normal strain is neglected.

The displacement field of the present unified beam theory is as follows

$$u(x,z) = u_0(x) - z\frac{\partial w_0}{\partial x} + R\left[\phi(x) + \frac{\partial w_0}{\partial x}\right] \tag{3}$$

$$w(x) = w_0(x)$$

where u_0 and w_0 are the axial and transverse displacements of the neutral axis in x and z-directions respectively; ϕ is the shear slope used to represent the effect of transverse shear deformation on the neutral axis of the beam. R represents transverse shear strain shape function in-terms of thickness coordinate (z) to satisfy traction free boundary conditions at the top and bottom surfaces of the beam. Different theories can be recovered by choosing their respective shape functions.

Parabolic beam theory (PBT): $R = z\left[1-(4/3)(\bar{z})^2\right], \quad \bar{z} = z/h$

Trigonometric beam theory (TBT): $R = (h/\pi)\sin(\pi\bar{z})$

Hyperbolic beam theory (HBT): $R = \left[z\cosh(1/2) - h\sinh(\bar{z})\right]$

Exponential beam theory (EBT): $R = z\exp\left[-2(\bar{z})^2\right]$

First-order beam theory (FBT): $R = z$

Classical beam theory (CBT): $R = 0$

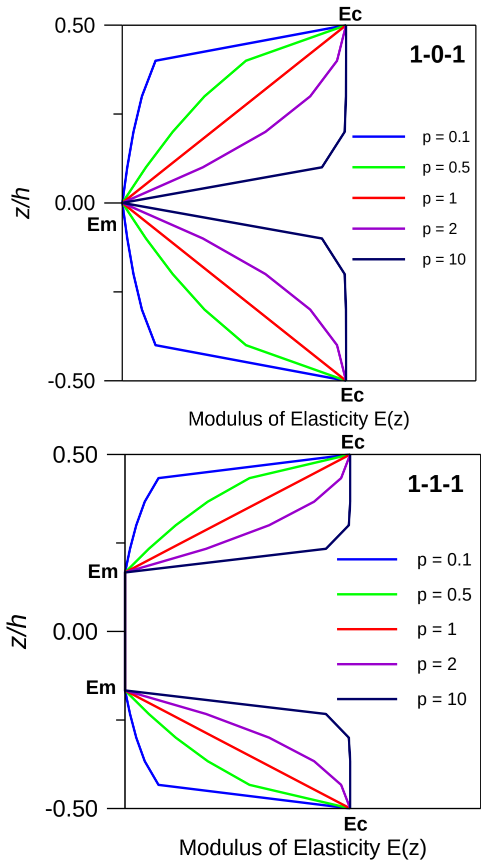
The strains associated with the present unified beam theory in Eq. (3) are obtained from the linear theory of elasticity.

$$\varepsilon_x = \varepsilon_x^0 + z\varepsilon_x^1 + R\left(\varepsilon_x^2 - \varepsilon_x^1\right), \quad \gamma_{xz} = \frac{dR}{dz}\gamma_{xz}^0 \tag{4}$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x}, \quad \varepsilon_x^1 = \frac{-\partial^2 w_0}{\partial x^2}, \quad \varepsilon_x^2 = \frac{\partial \phi}{\partial x}, \quad \gamma_{xz}^0 = \left(\phi + \frac{\partial w_0}{\partial x}\right) \tag{5}$$

The state of stress accounting for axial (σ_x) and transverse shear stresses (τ_{xz}) in the n^{th} layer of FG sandwich beams can be expressed as:



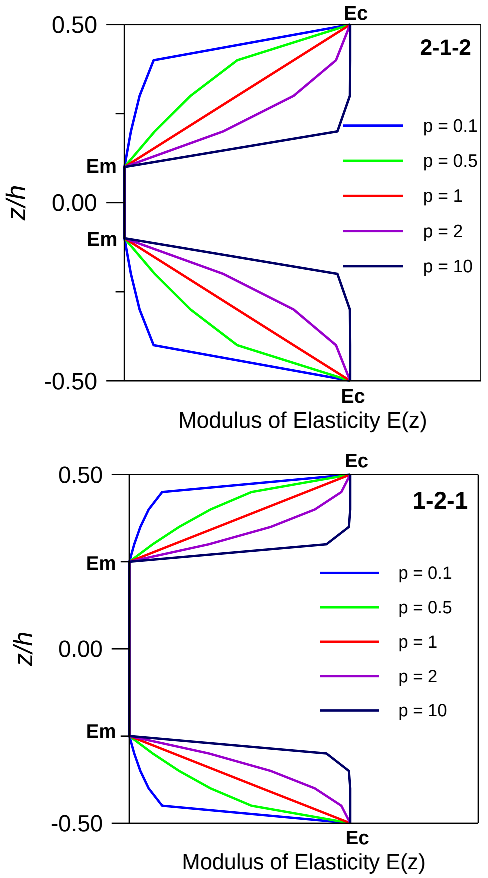


Figure 3. Through the thickness variation of Young's modulus of FG sandwich plate (Type B: Softcore) for various skin-core-skin thickness ratios

$$\sigma_x^n = E(z)^{(n)}\varepsilon_x^n \quad \text{and} \quad \tau_{xz}^k = \frac{E(z)^{(n)}}{2(1+\mu)}\gamma_{xz}^n \tag{6}$$

The axial force, shear force and moment resultants associated with the present theory are determined by integrating stress expressions over the thickness of the beam.

$$\begin{Bmatrix} N_x^c \\ M_x^c \\ M_x^s \end{Bmatrix} = \int_{-h/2}^{+h/2} \begin{Bmatrix} 1 \\ z \\ R \end{Bmatrix} \sigma_x \, dz, \quad Q_{xz}^s = \int_{-h/2}^{+h/2} \tau_{xz} \frac{dR}{dz} dz \tag{7}$$

The principle of virtual work stated in Eq. (8) is employed to derive the governing equations associate with the present unified beam theory.

$$\int_{-h/2}^{h/2}\int_0^L \left[\sigma_x^{(k)}\delta\varepsilon_x + \tau_{xz}^{(k)}\delta\gamma_{xz}\right]dx\,dz - \int_0^L q(x)\delta w\,dx + \int_0^L N_{xx}^0 \frac{\partial^2 \delta w}{\partial x^2}dx = 0 \tag{8}$$

where N_{xx}^0 is the axial force along the neutral axis and perpendicular to cross-section ($b\times h$). Integrating Eq. (8) by parts, collecting the coefficients of $\delta u_0, \delta w_0, \delta\phi$ and setting them equal to zero, one can obtain the following governing differential equations associated with the present unified beam theory.

$$\begin{aligned} &\delta u_0: \quad \frac{\partial N_x^c}{\partial x} = 0 \\ &\delta w_0: \quad \frac{\partial^2 M_x^c}{\partial x^2} - \frac{\partial^2 M_x^s}{\partial x^2} - \frac{\partial Q_{xz}^s}{\partial x} + q - N_{xx}^0 \frac{\partial^2 w_0}{\partial x^2} = 0 \\ &\delta\phi: \quad \frac{\partial M_x^s}{\partial x} - Q_{xz}^s = 0 \end{aligned} \tag{9}$$

The boundary conditions at x = 0, and x = L are of the form

Either $N_x^c = 0$ or $u_0 = 0$

Either $M_x^s - M_x^c = 0$ or $\partial w_0 / \partial x = 0$

Either $\left(\frac{\partial M_x^c}{\partial x} + \frac{\partial M_x^s}{\partial x}\right) = 0$ or $w_0 = 0$

Either $M_x^s = 0$ or $\phi = 0$

Using Eq. (7) and Eq. (9), the following governing equations in terms of unknown variables are derived.

$$\delta u_0: -A_{11}\frac{\partial^2 u_0}{\partial x^2} - \left(As_{11} - B_{11}\right)\frac{\partial^3 w_0}{\partial x^3} - As_{11}\frac{\partial^2 \phi}{\partial x^2} = 0 \tag{10}$$

$$\begin{aligned} &\delta w_0: \left(As_{11} - B_{11}\right)\frac{\partial^3 u_0}{\partial x^3} + \left(Ass_{11} - 2Bs_{11} + D_{11}\right)\frac{\partial^4 w_0}{\partial x^4} \\ &-Acc_{55}\frac{\partial^2 w_0}{\partial x^2} + \left(Ass_{11} - Bs_{11}\right)\frac{\partial^3 \phi}{\partial x^3} - Acc_{55}\frac{\partial \phi}{\partial x} \\ &= q(x,y) - N_{xx}^0 \frac{\partial^2 w_0}{\partial x^2} \end{aligned} \tag{11}$$

$$\begin{aligned} &\delta\phi: -As_{11}\frac{\partial^2 u_0}{\partial x^2} - \left(Ass_{11} - Bs_{11}\right)\frac{\partial^3 w_0}{\partial x^3} + Acc_{55}\frac{\partial w_0}{\partial x} \\ &-Ass_{11}\frac{\partial^2 \phi}{\partial x^2} + Acc_{55}\phi = 0 \end{aligned} \tag{12}$$

where various stiffness coefficients are defined as follows:

$$\begin{aligned} &\left\{A_{ij} \quad B_{ij} \quad D_{ij}\right\} = \int_{-h/2}^{+h/2} E(z)^{(n)}\left\{1 \quad z \quad z^2\right\}dz, \\ &\left\{As_{ij} \quad Bs_{ij}\right\} = \int_{-h/2}^{+h/2} E(z)^{(n)} R\left\{1 \quad z\right\}dz, \\ &\left\{Ass_{ij}\right\} = \int_{-h/2}^{+h/2} E(z)^{(n)} R^2 dz, \\ &\left\{Acc_{ij}\right\} = \int_{-h/2}^{+h/2} E(z)^{(n)}\left[\frac{dR}{dz}\right]^2 dz \end{aligned} \tag{13}$$

4. The Navier solution

Critical buckling load factors for the simply supported FG sandwich beams are obtained using the Navier's solution technique. The following are the boundary conditions for the simply supported ends:

$$w = M_x^c = M_x^s = \phi = 0 \quad \text{at } x = 0 \text{ and } x = L \tag{14}$$

FG sandwich beam is subjected to axial compressive force $\left(N_{xx}^0 = N_0\right)$ only (see Fig. 4); the transverse load is assumed as zero (q = 0).

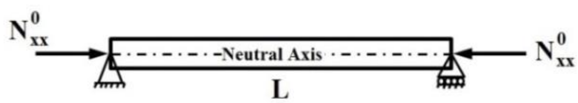


Figure 4. FG sandwich beams subjected to the axial compressive force

The governing equations and the boundary conditions of the present unified beam theory will be satisfied with the following form of unknowns ($\delta u_0, \delta w_0, \delta\phi$):

$$u_0(x) = \sum_{m=1,3,5}^{\infty} u_m \cos\alpha x$$
$$w_0(x) = \sum_{m=1,3,5}^{\infty} w_m \sin\alpha x \qquad (15)$$
$$\phi(x) = \sum_{m=1,3,5}^{\infty} \phi_m \cos\alpha x$$

where $\alpha = m\pi / a$ and m is the half-wave number along the x-direction. u_m, w_m and ϕ_m are the unknown coefficients to be determined. Substituting values of the unknown from Eq. (15) into the set of governing equations (10)-(12) yields the following equations from which one can obtained critical buckling load factors for the simply supported FG sandwich beam.

$$\left\{ \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} - N_0 \begin{bmatrix} 0 & 0 & 0 \\ 0 & N_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} \begin{Bmatrix} u_{mn} \\ w_{mn} \\ \phi_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (16)$$

where

$$K_{11} = A_{11}\alpha^2,$$
$$K_{12} = \left(As_{11} - B_{11}\right)\alpha^3,$$
$$K_{13} = As_{11}\alpha^2 + As_{66}\beta^2,$$
$$K_{22} = \left(Ass_{11} - 2Bs_{11} + D_{11}\right)\alpha^4 + Acc_{55}\alpha^2, \qquad (17)$$
$$K_{23} = \left(Ass_{11} - Bs_{11}\right)\alpha^3 + Acc_{55}\alpha,$$
$$K_{33} = \left(Ass_{11}\alpha^2 + Acc_{55}\right),$$
$$N_{33} = k\alpha^2$$

From a non-trivial solution of Eq. (16), one can obtain the critical buckling loads factors for the FG sandwich beam.

5. Numerical results and discussion

In this section, buckling analysis for four types of symmetric F G sandwich beams is presented using unified beam theory. Th e beam is taken to be made of alumina and aluminum with the following properties.

Ceramic (Alumina, Al2O3): E_c=380 GPa, μ=0.3

Metal (Aluminum, Al): E_m=70 GPa, μ=0.3

The four types of layer configurations (LC) are considered for the detailed numerical study (see Table 1).

Table 1. Thickness coordinates of four types of FG sandwich beams.

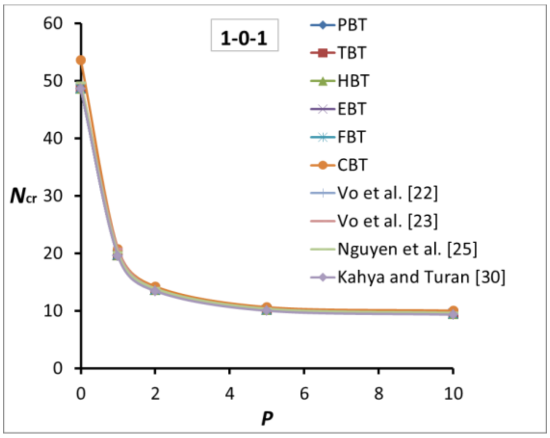
LC	Thickness coordinates
1-0-1	$h_1 = -h/2$, $h_2=0$, $h_3=0$ and $h_4=h/2$
2-1-2	$h_1 = -h/2$, $h_2=-h/10$, $h_3=h/10$ and $h_4=h/2$
1-1-1	$h_1 = -h/2$, $h_2=-h/6$, $h_3=h/6$ and $h_4=h/2$
1-2-1	$h_1 = -h/2$, $h_2=-h/4$, $h_3=h/4$ and $h_4=h/2$

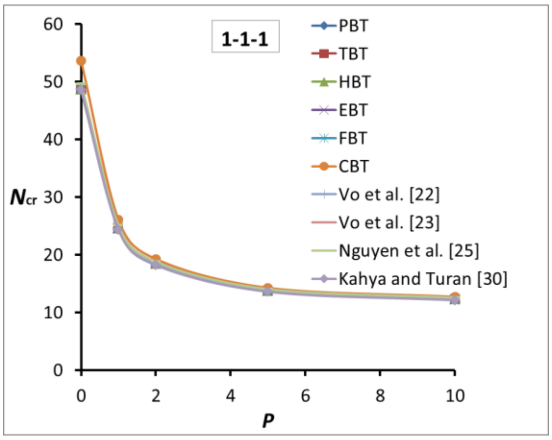
The following non-dimensional forms are used to present the critical buckling load factors.

$$N_{cr} = \frac{12 N_0 a^2}{E_m h^3} \qquad (18)$$

In this study, critical buckling load factors are obtained for FG sandwich beams subjected to axial compressive force as shown in Fig. 4. Four types of layer configurations (1-0-1, 2-1-2, 1-1-1 and 1-2-1) with a homogenous softcore and hardcore have been solved. Effects of power law coefficients (p=0,1,2,5,10), face-core-face ratios and aspect ratios (L/h=5,10,20,50,100) have been studied. Numerical results are presented in Tables 2 and 5. The present results are compared with previous solutions available in the literature such as Vo et al. [32, 33], Nguyen et al. [35] and Kahya and Turan [40]. Examination of Tables 2 and 3 reveals that non-dimensional critical buckling load factors obtained by using all the present models (PSDT, TSDT, HSDT, ESDT) are in excellent agreement with reference solutions, whereas FSDT and CPT overestimate the same for all layer configurations and power law coefficients. Also, it is observed that the non-dimensional critical buckling load factors are found to increase with an increase in the values of power law coefficients for homogenous softcore whereas it is found to decrease with an increase in the values of power law coefficients for homogenous hardcore. Similar relation can be observed between the thickness of the core and the non-dimensional critical buckling load factors. In the case of hardcore, an increase in the thickness of the core increases the values of critical buckling load factors, i.e. minimum for 1-0-1 and maximum for 1-2-1. Whereas, in the case of softcore, an increase in the thickness of core decreases the values of critical buckling load factors, i.e. (maximum for 1-0-1 and minimum for 1-2-1). Figs. 5 and 6 show the effect of power-law coefficients on the critical buckling load factors using the present models as well as reference solutions.

Tables 4 and 5 summarize the effect of L/h ratios on the critical buckling load factors. The numerical results are presented for L/h=5,10,20,50,100. Table 4 represent critical buckling load factors for Type B sandwich beams whereas Table 5 represent critical buckling load factors for Type A sandwich beams. The numerical results are presented for four types of symmetric FG sandwich beams. It is pointed out from these tables that the non-dimensional critical buckling load factor is minimum for thick beam and maximum for a thin beam.





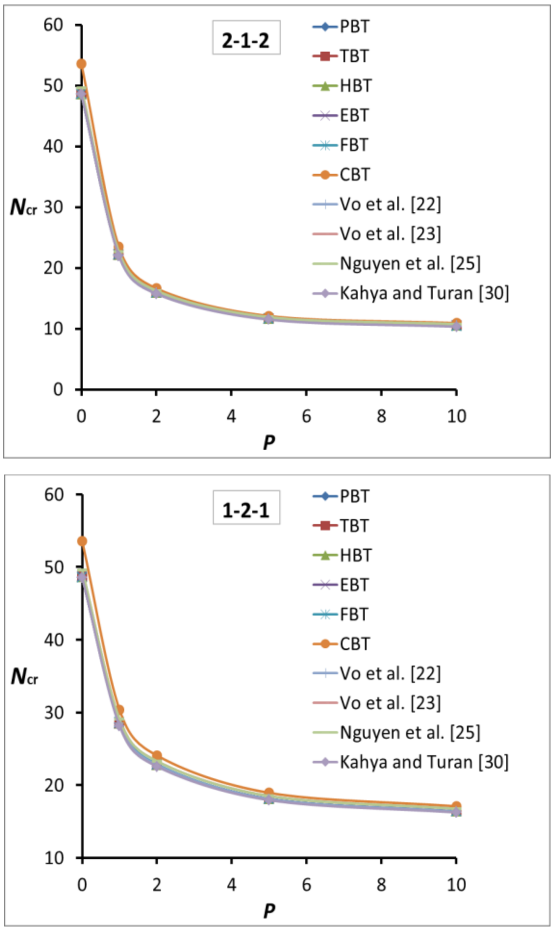
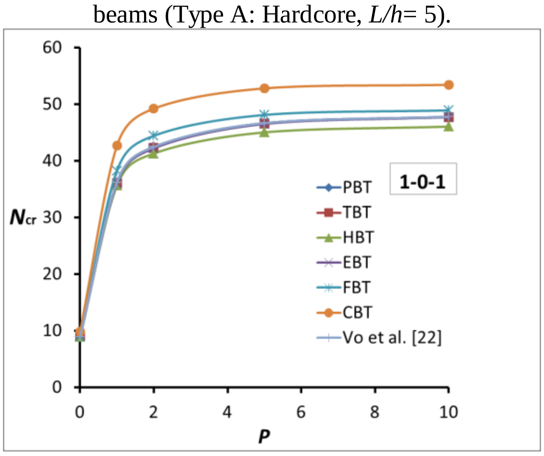


Figure 5. Effects of the power law coefficients on the non-dimensional critical buckling load factor (N_{cr}) of FG sandwich beams (Type A: Hardcore, L/h= 5).



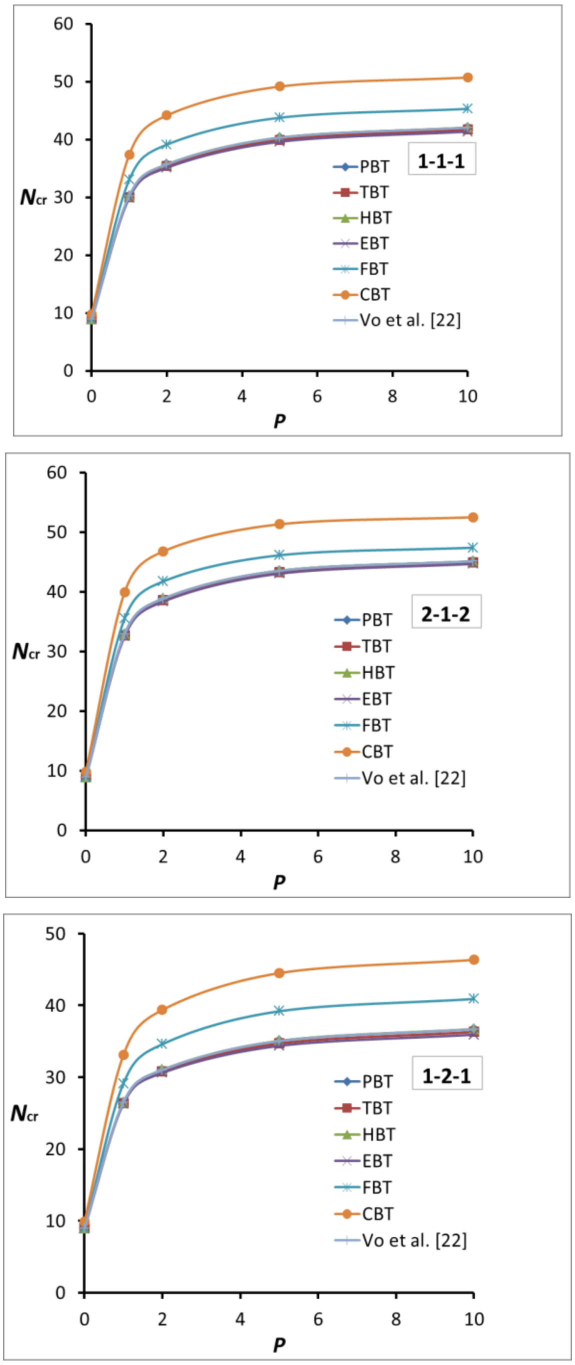


Figure 6. Effects of the power law coefficients on the non-dimensional critical buckling load factor (N_{cr}) of FG sandwich beams (Type B: Softcore, L/h= 5)

Table 2. Non-dimensional critical buckling load factor (N_{cr}) for a hardcore FG sandwich beam (Type A, L/h=5).

		Present Models						Reference Solution			
LC	p	PBT	TBT	HBT	EBT	FBT	CBT	Ref. [32]	Ref. [33]	Ref. [35]	Ref. [40]
1-0-1	0	48.595	48.603	48.628	48.683	48.590	53.577	48.595	49.590	49.597	48.590
	1	19.652	19.661	19.651	19.673	19.692	20.796	19.652	20.742	20.089	19.485
	2	13.580	13.590	13.579	13.601	13.564	14.240	13.580	13.883	13.885	13.436
	5	10.145	10.158	10.144	10.172	10.113	10.650	10.146	10.367	10.370	10.012
	10	9.4515	9.4654	9.4503	9.4810	9.4380	10.022	9.4515	9.6535	9.6573	9.3292
2-1-2	0	48.595	48.603	48.628	48.683	48.590	53.577	48.595	49.590	49.597	48.590
	1	22.210	22.218	22.210	22.228	22.266	23.506	22.210	22.706	22.706	22.017
	2	15.915	15.923	15.914	15.933	15.905	16.653	15.915	16.276	16.276	15.762
	5	11.667	11.677	11.666	11.688	11.624	12.117	11.667	11.930	11.932	11.517
	10	10.534	10.546	10.533	10.558	10.483	10.933	10.534	10.768	10.771	10.354
1-1-1	0	48.595	48.603	48.628	48.683	48.590	53.577	48.595	49.590	49.597	48.590
	1	24.559	24.564	24.559	24.571	24.649	26.057	24.559	25.107	25.106	24.326
	2	18.358	18.364	18.358	18.371	18.373	19.258	18.358	18.777	18.775	18.190
	5	13.721	13.728	13.720	13.736	13.693	14.263	13.721	14.035	14.035	13.583
	10	12.260	12.269	12.259	12.278	12.220	12.711	12.260	12.539	12.540	12.112
1-2-1	0	48.595	48.603	48.628	48.683	48.590	53.577	48.595	49.590	49.597	48.590
	1	28.444	28.442	28.444	28.445	28.616	30.357	28.444	29.075	29.072	28.142
	2	22.786	22.785	22.786	22.786	22.868	24.074	22.786	23.304	23.300	22.571
	5	18.091	18.092	18.091	18.095	18.113	18.943	18.091	18.509	18.505	17.941
	10	16.378	16.381	16.378	16.384	16.380	17.090	16.378	16.757	16.755	16.244

Table 3. Non-dimensional critical buckling load factor (N_{cr}) for a softcore FG sandwich beam (Type B, L/h=5).

		Present Models						Reference Solution
LC	p	PBT	TBT	HBT	EBT	FBT	CBT	Vo et al. [32]
1-0-1	0	8.9519	8.9533	8.9579	8.9681	8.9508	9.8696	8.9519
	1	36.210	36.091	35.624	35.985	38.252	42.650	36.210
	2	42.450	42.326	41.293	42.213	44.415	49.207	42.450
	5	46.650	46.574	45.022	46.509	48.105	52.797	46.650
	10	47.782	47.743	46.043	47.717	48.918	53.425	47.782
2-1-2	0	8.9519	8.9533	8.9579	8.9681	8.9508	9.8696	8.9519
	1	32.897	32.717	32.914	32.547	35.506	39.940	32.897
	2	38.858	38.615	38.881	38.375	41.757	46.794	38.858
	5	43.533	43.295	43.555	43.053	46.137	51.330	43.533
	10	45.114	44.909	45.132	44.701	47.403	52.514	45.114
1-1-1	0	8.9519	8.9533	8.9579	8.9681	8.9508	9.8696	8.9519
	1	30.245	30.064	30.262	29.900	33.063	37.389	30.244
	2	35.705	35.420	35.732	35.143	39.139	44.188	35.705
	5	40.323	39.980	40.354	39.631	43.790	49.184	40.323
	10	42.069	41.733	42.098	41.386	45.326	50.736	42.069
1-2-1	0	8.9519	8.9533	8.9579	8.9681	8.9508	9.8696	8.9519
	1	26.480	26.369	26.491	26.286	29.126	33.089	26.480
	2	31.015	30.793	31.036	30.603	34.604	39.372	31.015
	5	35.035	34.693	35.067	34.372	39.192	44.504	35.035
	10	36.687	36.302	36.722	35.930	40.903	46.356	36.687

Table 4. Non-dimensional critical buckling load factor (N_{cr}) for a softcore FG sandwich beam for different aspect ratios (Type B, p=2).

		Present Models					
LC	L/h	PBT	TBT	HBT	EBT	FBT	CBT
1-0-1	5	42.450	42.326	41.293	42.213	44.415	49.207
	10	47.321	47.282	46.938	47.246	47.915	49.207
	20	48.722	48.711	48.618	48.702	48.879	49.207
	50	49.124	49.126	49.112	49.123	49.146	49.207
	100	49.170	49.195	49.183	49.187	49.178	49.207
2-1-2	5	38.858	38.615	38.881	38.375	41.757	46.794
	10	44.518	44.437	44.525	44.356	45.424	46.794
	20	46.203	46.181	46.205	46.159	46.444	46.794
	50	46.700	46.695	46.699	46.692	46.739	46.794
	100	46.776	46.766	46.770	46.765	46.781	46.794
1-1-1	5	35.705	35.420	35.732	35.143	39.139	44.188
	10	41.707	41.608	41.716	41.510	42.808	44.188
	20	43.540	43.514	43.543	43.487	43.835	44.188
	50	44.084	44.079	44.084	44.074	44.127	44.188
	100	44.167	44.154	44.162	44.162	44.168	44.188
1-2-1	5	31.015	30.793	31.036	30.603	34.604	39.372
	10	36.884	36.804	36.891	36.734	38.061	39.372
	20	38.719	38.697	38.721	38.678	39.036	39.372
	50	39.266	39.262	39.267	39.259	39.316	39.372
	100	39.339	39.341	39.346	39.346	39.344	39.372

Table 5. Non-dimensional critical buckling load factor (N_{cr}) for a hardcore FG sandwich beam for different aspect ratios (Type A, p=2).

		Present Models					
LC	L/h	PBT	TBT	HBT	EBT	FBT	CBT
1-0-1	5	13.580	13.590	13.579	13.601	13.564	14.240
	10	14.069	14.072	14.069	14.075	14.065	14.240
	20	14.197	14.198	14.197	14.199	14.196	14.240
	50	14.234	14.231	14.234	14.228	14.233	14.240
	100	14.241	14.241	14.239	14.236	14.236	14.240
2-1-2	5	15.915	15.923	15.914	15.933	15.905	16.653
	10	16.462	16.464	16.462	16.467	16.459	16.653
	20	16.605	16.605	16.605	16.606	16.605	16.653
	50	16.645	16.643	16.645	16.645	16.643	16.649	16.653
	100	16.656	16.647	16.651	16.641	16.670	16.653
1-1-1	5	18.358	18.364	18.358	18.371	18.373	19.258
	10	19.025	19.026	19.025	19.029	19.029	19.258
	20	19.200	19.199	19.200	19.201	19.201	19.258
	50	19.246	19.249	19.249	19.245	19.251	19.258
	100	19.255	19.243	19.256	19.239	19.272	19.258
1-2-1	5	22.786	22.785	22.786	22.786	22.868	24.074
	10	23.739	23.739	23.739	23.739	23.761	24.074
	20	23.989	23.990	23.990	23.989	23.995	24.074
	50	24.057	24.058	24.061	24.054	24.065	24.074
	100	24.050	24.054	24.071	24.053	24.079	24.074

6. Conclusions

Analytical solutions for the buckling analysis of two types of FG sandwich beams are presented in this paper using a unified beam theory. Various higher order beam theories can be recovered using the proposed theory. The effects of power-law coefficient, span-to-depth ratio and face-core-face thickness ratios on the critical buckling load factors are discussed. Based on the comparison of numerical results with previously published results, it is concluded that all the models recovered from the present unified theory are accurate and efficient in predicting the buckling behaviors of the FG sandwich beams with both soft as well as hard cores.

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