JCAMECH

Vol. 51, No. 2, December 2020, pp 472-481
DOI: 10.22059/jcamech.2020.313559.570

Pull-in behaviors of micro-beam made of bidirectional functionally grade materials based on modified couple stress theory

Esmail Zarezadeh [a], and Ali Shirpay [b]*

[a] *Department of Electrical Engineering, Amirkabir University of Technology, Tehran, Iran*
[b] *Department of physics, center of basic science khatam ol-anbia (PBU) university, Tehran, Iran*

ARTICLE INFO

Article history:
Received: 12 October 2020
Accepted: 21 November 2020

Keywords:
pull-in
couple stress
Bi directional functionally graded materials
size effect
Galerkin

ABSTRACT

In this paper, pull-in behavior of cantilever micro/nano beams made of bi directional functionally graded materials (BDFGM) with small scale effects under electrostatic force is investigated. Couple stress theory is employed to study the influence of small-scale on pull-in behavior. The material properties except Poisson's ratio obey the arbitrary function in the thickness and length direction. The approximate analytical solutions for the pull-in voltage and pull-in displacement of the microbeams are derived using the Galerkin method. Comparison between the results of present work with other article for pull-in behavior of a microbeams made of isotropic material reveals the accuracy of this study. Numerical results explored the effects of material length scale parameter, inhomogeneity constant, gap distance and dimensionless thickness.

1. Introduction

Recently, micro and nanotechnology has found a special place in various sciences such as medical and engineering [1]. So the attention of scientists has been attracted to this science. In this regard, mechanical engineers have done a lot of research to decipher the ambiguities of nanotechnology. In nano and micro scale the material properties is dependent on size [2-6]. Some of these theories are nonlocal elasticity theory [7-10], strain gradient theory [11], couple stress theory [12-14] and surface stress theory. Couple stress theory is one of the most important non-classical theories in the field of micro/nano-mechanics. The couple stress theory was introduced by Toupin [12], Mindlin and Tiersten [13] and Koiter [14]. Modified couple stress theory has been introduced by Yang et al. [15]. In this theory two higher order material length scale parameters are introduced in addition to the two Lame constants. One of the good aspects of this theory is that the four additional parameters in the micropolar theory and five additional parameters in the strain gradient theory were reduced to two additional parameters. This property has attracted some researchers in recent years to derive formulations of mechanical analyzing for micro-beams and micro-plates and investigate their mechanical behavior based on this theory. The formulations and mechanical behavior of homogeneous linear micro-beams [16-18] homogenous nonlinear micro-beams [19, 20], functionally graded (FG) linear micro-beams [21], functionally graded nonlinear micro-beams [22], linear micro-plates [23, 24], nonlinear micro-plates [25, 26] and composite laminated beams [27] have been presented in the framework of the modified couple stress theory.

Functionally graded material (FGM) is one of latest concept in the composite material design field. The material properties of functionally graded material continuously vary from point to another point. In other words, material properties are functions of location. The use of FG materials reduces the weight and increases the strength of structures. A number of papers considering various aspects of FGM have been published in recent years [28-53]. It should be noted that most of the above-mentioned analyses are related to FGMs with material properties varying in one direction only. However, there are practical occasions which require tailored grading of properties in two or even three directions. As reported by Steinberg [54], the fuselage of an aerospace craft undergoes an extremely high temperature field with excessive temperature gradient on the surface and through the thickness, when the plane sustains flight at a speed of Mach 8 and at an altitude of 29 km. In this circumstance, the conventional unidirectional FGMs may not be so appropriate to resist multi-directional severe variations of temperature. Therefore, it is of great significance to develop novel FGMs with properties varying in two or three directions (2D or 3D FGMs) to withstand a more general temperature field.

Consider a beam-type actuator contains two conductive electrodes which one is fixed and the other one is movable. A voltage difference is applying between these two electrodes causes the movable electrode to deflect towards the fixed electrode

* Corresponding author. E-mail: zare40087@gmail.com

(ground electrode), because of the electrostatic forces. A voltage which is causes the electrode becomes unstable and pulls-in onto the ground electrode [55] is known as pull-in voltage. Currently, Yang et al. [56] investigated the pull-in behavior of beams using Eringen's nonlocal elasticity theory and the small-scale effect on the pull-in voltage was captured. Rahaeifard et al. [21] studied the deflection and static pull-in behavior of micro-cantilevers by utilizing the modified couple stress theory and numerical results were presented. Approximation solution for pull-in voltage of clamped-clamped and clamped-free microbeams via Rayleigh–Ritz method by Kong [16]. In recent years, a lot of research on pull-in phenomena [55, 56] and micro/nano-structures [2, 3, 34, 57-63] had been done.

This paper presents couple stress theory for solids predicting the small-scale effect on the pull-in behavior of electrostatically actuated bi directional functionally graded microbeams.

2. Analysis

As one could see in the schematic view of the system in Figure 1, the system under study is a microbeam with a rectangle cross-section with *L* length, *b* width, and *h* thickness. In this figure, the distance between the electrode and the microbeam is *d*. *E* is Young's modulus. The system's coordinate system is *xyz*, and it is assumed that the electric force is applied to the microbeam in the *z*-direction. *w* (*x*) shows the transverse displacement of the middle axis of the beam.

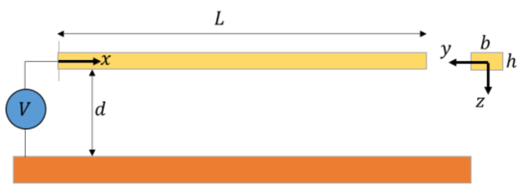

Fig. 1 Schematic of an electrostatic actuated FGM micro beam

The differential equation will be extracted taking into account the assumptions governing the Euler-Bernoulli beam. In order to model the beam, the effects of gravity and shear deformation have been ignored. Also, the cross-sectional area along the beam is constant, and in this research, the axial displacement and traction of the middle plate in modeling will be considered.

According to modified couple stress theory, the strain energy for the linear elastic material resulting from the displacement field at volume *v* is both dependent on the strain tensor and dependent on the second derivative of the displacement, which is presented below.

$$\delta U = \int_V \left(\sigma_{ij}\delta\varepsilon_{ij} + m_{ij}\delta\chi_{ij}\right)dv \tag{1}$$

In the above relation, the Cauchy stress tensor "σ", the $\boldsymbol{\varepsilon}$ strain tensor, the couple-stress deviation component "m", and the symmetric curvature tensor "χ" (the symmetrical part of the rotational gradients) are defined as follows.

$$\sigma_{ij} = \lambda\ \varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} \tag{2}$$

$$m_{ij} = 2\mu l^2\chi_{ij} \tag{3}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{4}$$

$$\chi_{ij} = \frac{1}{2}\left(\theta_{i,j} + \theta_{j,i}\right) \tag{5}$$

In the above relations, λ is the first constant of Lameh, μ is shear modulus, δ_{ij} is Kronecker delta. l is the length scale parameter and property of a material that is small compared to the dimensions of the object, and its effect becomes important when the dimensions of the object are on a micro-nano scale. The value of the length scale parameter is obtained using laboratory methods. u and θ are the vectors of displacement and rotation, and the relationship between them is as follows.

$$\theta_i = \frac{1}{2}u_{j,i}\varsigma_{ijk}e_k \tag{6}$$

According to Euler bernouli beam theory, the displacement of the beam in three directions *x*, *y* and z can be shown as follows.

$$\begin{cases} u_1 = u_0 - z\dfrac{dw}{dx} \\ u_2 = 0 \\ u_3 = w(x) \end{cases} \tag{7}$$

That u_1, u_2, *and* u_3 are the components of the displacement vector along the *x*, *y*, and *z* axes, respectively. According to the modified couple stress theory for Euler–Bernoulli beams, the components of θ is as follows:

$$\theta_x = 0,\ \theta_y = \frac{dw}{dx},\ \theta_z = 0 \tag{8}$$

By applying Equation (8) in Equation (5), χ is obtained as follows.

$$\chi = \frac{1}{2}\begin{bmatrix} 0 & -\dfrac{d^2w}{dx^2} & 0 \\ -\dfrac{d^2w}{dx^2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{9}$$

And by placing the relation (9) in relation (3) for *m*, the following relation is obtained.

$$m = \mu l^2\begin{bmatrix} 0 & -\dfrac{d^2w}{dx^2} & 0 \\ -\dfrac{d^2w}{dx^2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{10}$$

The strain component can be obtained from the following equation:

$$\varepsilon_{xx}=\frac{du_1}{dx}=\frac{du_0}{dx}-z\frac{d^2w}{dx^2}$$
$$\varepsilon_{yy}=\varepsilon_{zz}=\varepsilon_{xz}=\varepsilon_{yz}=\varepsilon_{xy}=0 \tag{11}$$

In this study, the arbitrary functional model in both *x* and *z* directions is considered for Young's modulus and coefficient of thermal expansion.

$$E(x,z)=f_1(x)g_1(z) \tag{12}$$

$$\alpha(x,z)=f_2(x)g_2(z) \tag{13}$$

Due to the above relationships, the non-zero stress components are calculated as follows.

$$\sigma_{xx}=\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}(\varepsilon_{xx}-\alpha T) \tag{14}$$

In relation (14), *E*, *α* and *ν* are the Young's modulus, the coefficient of thermal expansion, and the Poisson's coefficient, respectively. *T* is temperature changes. For slender beams with a large length to height ratio, the Poisson effect is small and can be ignored. By placing $\nu=0$, the equation (14) is simplified as follows.

$$\sigma_{xx}=f_1(x)g_1(z)\left[\frac{du_0}{dx}-z\frac{d^2w}{dx^2}-f_2(x)g_2(z)T\right] \tag{15}$$

Using the principle of account of changes and the principle of Hamilton, the equation and boundary conditions are obtained using $\delta U-\delta V=0$ relation in the longitudinal and transverse directions; where *U* and *V*, represent the potential energy and the work of the non-conservative forces respectively to be determined. By placing the above equations in Equation (1), the strain energy of the beam is obtained as follows.

$$\delta U=\int_V\left(\sigma_{xx}\delta\varepsilon_{xx}+m_{xy}\delta\chi_{xy}+m_{yx}\delta\chi_{yx}\right)dv=\int_V\left(\sigma_{xx}\delta\varepsilon_{xx}+2m_{xy}\delta\chi_{xy}\right)dv$$

$$=\int_0^L\left(N_{xx}\frac{d\delta u_0}{dx}-\left(M_{xx}-Y_{xy}\right)\frac{d^2\delta w}{dx^2}\right)dx \tag{16}$$

In the above relation, the resulting stresses are defined as follows:

$$N_{xx}=\int_A\sigma_{xx}dA \tag{17}$$

$$M_{xx}=\int_A z\sigma_{xx}dA \tag{18}$$

$$Y_{xy}=\int_A m_{xy}dA \tag{19}$$

Due to the fact that the system of this problem is under two electrostatic forces and temperature changes, the work of the external force is defined in two parts:

$$\delta V=\delta V_T+\delta V_{ele} \tag{20}$$

The Thermal field work is defined as follows:

$$\delta V_T=\int_0^L\left(N^T\frac{dw}{dx}\frac{d\delta w}{dx}\right)dx \tag{21}$$

which N^T is defined as follows:

$$N^T=\int_A E\alpha TdA=\int_A f_1f_2g_1g_2TdA=f_1f_2T\int_A g_1g_2dA=f_1f_2I_3T \tag{22}$$

where

$$I_3=\int_A g_1g_2dA \tag{23}$$

As a result, heat-induced work can be described as follows:

$$\delta V_T=\int_0^L f_1f_2I_3T\frac{dw}{dx}\frac{d\delta w}{dx}dx \tag{24}$$

The work caused by electrostatic force is as follows:

$$\delta V_{ele}=\int_0^L F_{ele}\delta wdx \tag{25}$$

The electrostatic force is as follows.

$$F_{ele}=\frac{\varepsilon bV^2}{2(d-w)} \tag{265}$$

In the above relation, ε is Vacuum permittivity, b is the beam width, V is the applied voltage and *d* is the distance to the potential level of zero. Using the minimum potential relationship, and the above relations we have the following:

$$\int_0^L\left(N_{xx}\frac{d\delta u_0}{dx}-\left(M_{xx}-Y_{xy}\right)\frac{d^2\delta w}{dx^2}\right)dx-\int_0^L f_1f_2I_3T\frac{dw}{dx}\frac{d\delta w}{dx}dx$$
$$-\int_0^L\frac{\varepsilon bV^2}{2(d-w)}\delta wdx=0 \tag{27}$$

Using part by parts integral, the equilibrium equation and boundary conditions governing the problem is obtained.

Equilibrium equation:

$$\frac{dN_{xx}}{dx}=0$$
$$\frac{d^2}{dx^2}\left(M_{xx}+Y_{xy}\right)+\frac{d}{dx}\left(f_1 f_2 I_3 T\frac{dw}{dx}\right)-\frac{\varepsilon bV^2}{2\left(d-w\right)}=0 \tag{28}$$

Boundary conditions for two clamped beams:

$$w(0)=w(L)=0$$
$$\left.\frac{dw}{dx}\right|_{x=0,L}=0 \tag{29}$$

The resultant stress can be derived as follow:

$$Y_{xy}=\int_A m_{xy}dA=\int_A -\mu l^2\frac{d^2w}{dx^2}dA=-\int_A\frac{E}{2\left(1+\nu\right)}l^2\frac{d^2w}{dx^2}dA=$$

$$=-\int_A\frac{f_1g_1}{2\left(1+\nu\right)}l^2\frac{d^2w}{dx^2}dA=-\frac{f_1}{2\left(1+\nu\right)}I_0l^2\frac{d^2w}{dx^2} \tag{30}$$

$$M_{xx}=\int_A z\sigma_{xx}dA=-f_1I_2\frac{d^2w}{dx^2} \tag{31}$$

$$N_{xx}=\int_A \sigma_{xx}dA=f_1I_0\frac{du_0}{dx}-f_1f_2I_3T \tag{32}$$

where

$$\begin{Bmatrix}I_0\\ I_2\\ I_3\end{Bmatrix}=\int_A\begin{Bmatrix}g_1\\ z^2g_1\\ g_1g_2\end{Bmatrix}dA \tag{33}$$

Now, using the equilibrium equation and the resultant stress the Navier equation of problem can be obtained:

$$\left(I_2+I_0s\right)\left[\frac{d^2f_1}{dx^2}\frac{d^2w}{dx^2}+2\frac{df_1}{dx}I_2\frac{d^3w}{dx^3}+f_1I_2\frac{d^4w}{dx^4}\right]+$$

$$+\left(\frac{df_1}{dx}f_2+f_1\frac{df_2}{dx}\right)I_3T\frac{dw}{dx}+f_1f_2I_3T\frac{d^2w}{dx^2}-\frac{\varepsilon bV^2}{2(d-w)^2}=0 \tag{34}$$

For convenience, the following nondimensionalizations are used

$$\bar{w}=\frac{w}{d},\ \bar{x}=\frac{x}{L},\ \bar{T}=\frac{I_3L^2T}{I_2},\ \bar{I}_0=\frac{I_0L^2}{I_2},\ \bar{l}=\frac{l}{L}$$
$$\bar{V}^2=\frac{\varepsilon bL^4V^2}{2I_0d^3} \tag{35}$$

According to the dimensionless parameters, the Navier equation on the problem becomes dimensionless as follows:

$$\left(1+\bar{I}_0 s\bar{l}^2\right)\left[\frac{d^2 f_1}{d\bar{x}^2}\frac{d^2\bar{w}}{d\bar{x}^2}+2\frac{df_1}{d\bar{x}}\frac{d^3\bar{w}}{d\bar{x}^3}+f_1\frac{d^4\bar{w}}{d\bar{x}^4}\right]+$$

$$+\left(\frac{df_1}{d\bar{x}}f_2+f_1\frac{df_2}{d\bar{x}}\right)\bar{T}\frac{d\bar{w}}{d\bar{x}}+f_1 f_2\bar{T}\frac{d^2\bar{w}}{d\bar{x}^2}-\frac{\bar{V}^2}{(1-\bar{w})^2}=0 \tag{36}$$

To obtain an approximate response, Equation (36) is solved using the Galerkin method. To solve this equation, a uniform shape mode is used. The static response is assumed to be as follows:

$$\bar{w}=\sum_{i=1}^{k} a_i\varphi_i\left(\bar{x}\right) \tag{37}$$

where a_i is the constant coefficients and $\varphi_i(x)$ is the symmetrical shape mode of the clamped beam. k is also the number of shape mode required to solve the equation. In order to make the algebraic equation of a_i appear simpler and more compound, first the denominator of the sentence related to the electrostatic force $(1-w(x))^2$ is multiplied by the sides of the equation. Then, by placing the relation 37 and multiplying φ_i (x) as a weight function and integrating it in the range x = 0-1, the couple algebraic equations are obtained as follows:

$$eq=\left(1-\sum_{i=1}^{k}a_i\varphi_i\right)^2\left[\left(1+\bar{I}_0 s\bar{l}^2\right)\left[\frac{d^2 f_1}{d\bar{x}^2}\frac{d^2}{d\bar{x}^2}+2\frac{df_1}{d\bar{x}}\frac{d^3}{d\bar{x}^3}+f_1\frac{d^4}{d\bar{x}^4}\right]\sum_{i=1}^{k}a_i\varphi_i+\right.$$
$$\left.+\left(\frac{df_1}{d\bar{x}}f_2+f_1\frac{df_2}{d\bar{x}}\right)\bar{T}\frac{d}{d\bar{x}}\sum_{i=1}^{k}a_i\varphi_i+f_1 f_2\bar{T}\frac{d^2}{d\bar{x}^2}\sum_{i=1}^{k}a_i\varphi_i\right]-\bar{V}^2 \tag{38}$$

Here's how to do it according to Galerkin method:

$$\int_0^1 \varphi_i\left(\bar{x}\right).eq d\bar{x}=0 \tag{39}$$

In this way, the spatial coordinate is removed and the differential equation becomes an algebraic equation:

$$\int_0^1\left\{\varphi_i\left(1-\sum_{i=1}^{k}a_i\varphi_i\right)^2\left[\left(1+\bar{I}_0 s\bar{l}^2\right)\left[\frac{d^2 f_1}{d\bar{x}^2}\frac{d^2}{d\bar{x}^2}+2\frac{df_1}{d\bar{x}}\frac{d^3}{d\bar{x}^3}+f_1\frac{d^4}{d\bar{x}^4}\right]\sum_{i=1}^{k}a_i\varphi_i+\right.\right.$$
$$\left.\left.+\left(\frac{df_1}{d\bar{x}}f_2+f_1\frac{df_2}{d\bar{x}}\right)\bar{T}\frac{d}{d\bar{x}}\sum_{i=1}^{k}a_i\varphi_i+f_1 f_2\bar{T}\frac{d^2}{d\bar{x}^2}\sum_{i=1}^{k}a_i\varphi_i\right]-\bar{V}^2\right\}d\bar{x}=0 \tag{40}$$

The a_i coefficient is obtained by numerical solution of this system of equations. In this way, static deformation can be calculated for different a_i as a result of different DC voltages.

3. Results

In this section, the numerical results from the previous section are examined. First, the validity of the results is checked. Past articles will be used for this purpose. In the following, various parameters such as heterogeneity coefficients, temperature field effect, and size effects will be studied. To verify the accuracy of the results, the present paper is compared with Stuberg et al. [64]for homogeneous microbeam with different lengths in Figure 2. In the article by Stoeberg et al., the properties of materials are considered as $E=169Gpa$ ، $\nu=0.3$ ، $b=50\mu m$ ، $l=0$ ، $d=1\mu m$. It could be observed that the answer to the present project is very accurate.

The E elastic modulus and α coefficient of thermal expansion are considered as the arbitrary function that changes in both length and thickness according to the following. The Poisson ratio is also assumed to be constant.

$$E\left(x,z\right)=f_1(x)g_1(z)$$

$$\alpha\left(x,z\right)=f_2(x)g_2(z)$$

In direction of the thickness of the beam, the modulus of elasticity and the coefficient of thermal expansion are written according to the rule of mixtures:

$$g_1\left(z\right)=E_cV_c+E_mV_m$$

$$g_2\left(z\right)=\alpha_cV_c+\alpha_mV_m$$

The m and c indexs are the metal and ceramic, respectively. Volume fraction of metal and ceramics is presented as follows:

$$V_c(z)=\left(\frac{2z+h}{2h}\right)^n$$

$$V_m(z)+V_c(z)=1$$

Also, the properties of the microbeam in terms of the length of the material are assumed based on the exponential function:

$$f_1(x)=e^{\frac{n_1}{L}x}$$

$f_2(x) = e^{\frac{n_2}{L}x}$

As a result, the modulus of elasticity and coefficient of thermal expansion in terms of thickness are obtained as follows:

$$g_1(z) = \left[E_c \left(\frac{2z+h}{2h} \right)^{n_3} + E_m \left(1 - \left(\frac{2z+h}{2h} \right)^{n_3} \right) \right]$$

$$g_2(z) = \left[\alpha_c \left(\frac{2z+h}{2h} \right)^{n_4} + \alpha_m \left(1 - \left(\frac{2z+h}{2h} \right)^{n_4} \right) \right]$$

In order to examine the problem parameters in all the following results, the metal and ceramic parts of FG material are considered aluminum and zirconia, respectively. The properties of these materials are presented in Table 1 [65].

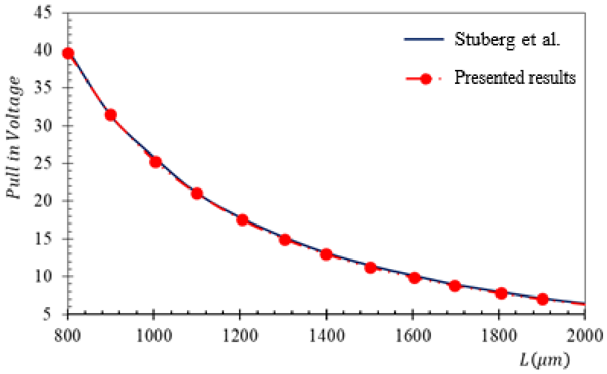

Figure 2. Comparison of Presented results obtained for pull-in voltage with those presented in the literatures ($E = 169Gpa$ ، $v = 0.3$ ، $b = 50\mu m$ ، $l = 0$ ، $d = 1\mu m$ و $h = 14.4\mu m$)

Table 1. Material properties of microbeam

Material	Properties	
	$E(Gpa)$	υ
Aluminum(AL)	70	0.3
Zirconia(ZrO_2)	200	0.3

Figure 3 and 4 show the dimentionless modulus of elasticity along the length and thickness of thebeam for different values of n_1 and n_3. If n_1 is considered zero, the beam is full ceramic and if n_1 is infinite, it is full metal. The ceramic and metal alterations is linear for $n_1 = 1$.

Figure 3 shows the distribution of the dimensionless modulus of elasticity based on the z/h in $x = 0$, which shows the distribution of the modulus of elasticity along the thickness of the microbeam. It is observed that with the increase of the gradient index of the (n_1) material, the modulus of elasticity decreases, which is due to the fact that with the increase of the n_1 metal phase, the modulus of elasticity of the metal is less than that of the ceramic in this sample. Figure 4 also shows the distribution of dimensionless elastic modulus based on the x/L in $z = -h/2$, which shows the distribution of modulus of elasticity along the length of the beam. It is observed that with increasing β the modulus of elasticity increases exponentially. For β = 0, the modulus of elasticity distribution is constant. The effect of the temperature field on the pull-in instability voltage is shown in Figure 5. As the temperature rises, the deflection of the beam increases at a constant voltage. In addition, with increasing temperature, the beam at low voltage becomes unstable and causes a short circuit in the electrical circuit. By lowering the temperature and cooling, the pull-in instability voltage can be delayed and it can be controlled.

Figure 6 examines the effect of the length scale parameter on pull-in instability voltage. As can be seen in this figure, the deflection of microbeam decreases with increasing length scale parameter. This means that as the size of the microbeam smaller, the structure hardens and shows more rigid behavior than the balk material, and the experimental results of previous research confirm this. At the same time, as the length scale parameter increases, the voltage of the pull-in instability increases. If the length scale parameter equal to zero is the classic response, which reports lower the pull-in voltage instability.

To investigate the effect of beam thickness, the frequency ratio parameter (V_r) is defined as follows:

$$V_r = \frac{V_{Couple\ Stress}}{V_{Classic}}$$

Figure 7 shows the effect of beam thickness on V_r. This figure shows that in small sizes, the two classical theory and the couple stress theory have significant differences and the effects of size cannot be ignored. But in large sizes, the magnitude V_r tends to 1, indicating that two theories give the same answer, and that the theory of the couple stress can be omitted. Couple stress theory also predicts the stiffness compared to classical theory.

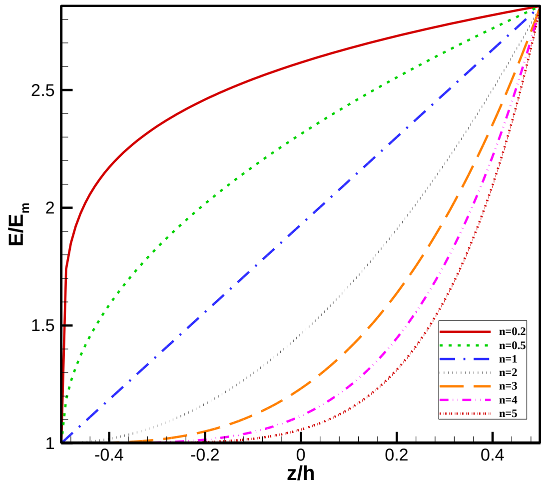

Figure 3. Distribution of module of elasticity versus *z/h* at *x*=0

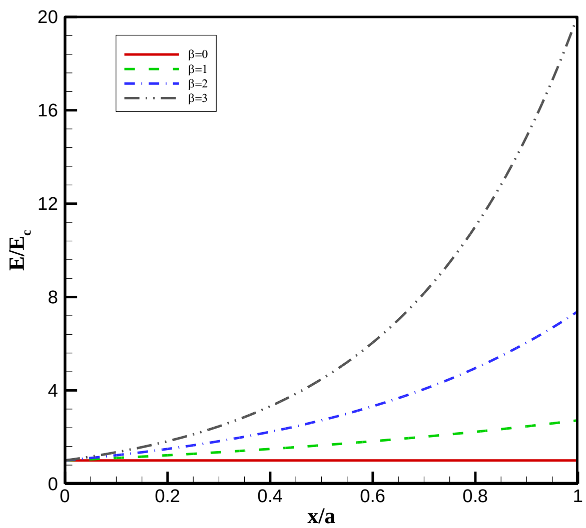

Figure 4. Distribution of module of elasticity versus *x/L* at *z*=-*h/2*

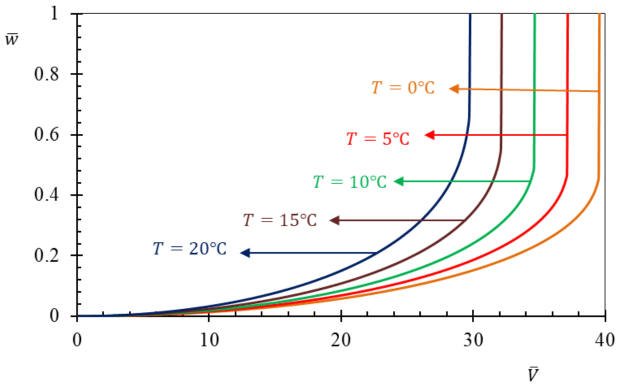

Figure 5. Efect of temperature on pull-in instability voltage ($E_c = 70Gpa$ ، $E_m = 70Gpa$ ، $\alpha_m = 11\times10^{-6}\,1/^{\circ}C$ $\alpha_c = 23.1\times10^{-6}\,1/^{\circ}C$ ، $\nu = 0.3$ ، $b = 1\mu m$ ، $L = 100\mu m$ ، $\bar{l} = 0.01$ ، $d = 1\mu m$ ، $h = 1\mu m$ ، $n_1 = 2$ ، $n_2 = 2$ ، $n_3 = 2$ ، $n_4 = 2$)

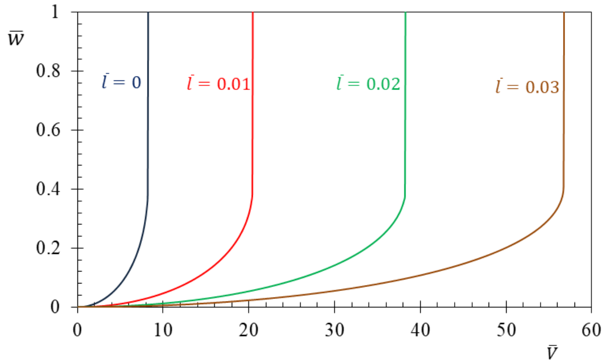

Figure 6. Efect of size parameter on pull-in instability voltage ($E_c = 70Gpa$ ، $E_m = 70Gpa$ ، $\alpha_m = 11\times10^{-6}\,1/^{\circ}C$ $\alpha_c = 23.1\times10^{-6}\,1/^{\circ}C$ ، $\nu = 0.3$ ، $b = 1\mu m$ ، $L = 100\mu m$ ، $T = 2^{\circ}C$ ، $d = 1\mu m$ و $h = 1\mu m$ ، $n_1 = 0.25$ ، $n_2 = 0.25$ ، $n_3 = 0$ ، $n_4 = 0$)

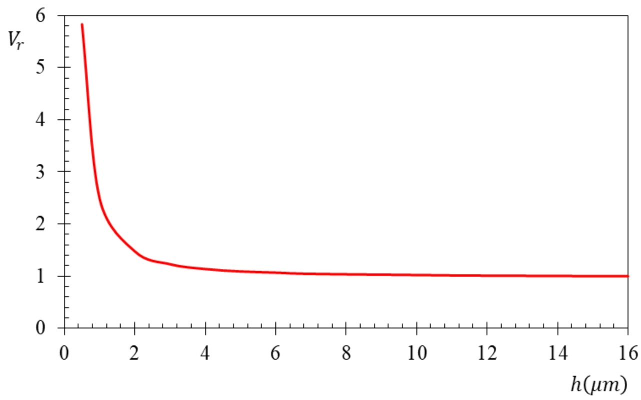

Figure 7. Efect of thicknesson pull-in instability voltage ($E_c = 70Gpa$ ، $E_m = 70Gpa$ ، $\alpha_m = 11\times10^{-6}\,1/^{\circ}C$ $\alpha_c = 23.1\times10^{-6}\,1/^{\circ}C$ ، $\nu = 0.3$ ، $b = 1\mu m$ ، $L = 100\mu m$ ، $T = 2^{\circ}C$ ، $\bar{l} = 0.01$ ، $d = 1\mu m$ ، $n_1 = 0.25$ ، $n_2 = 0.25$ ، $n_3 = 0$ ، $n_4 = 0$)

The effect of the n_1 parameter on the pull-in instability voltage is shown in Figure 8. The results of this figure show that by increasing n_1 the pull-in instability voltage is delayed, which is due to the fact that with increasing n_1 the modulus of elasticity increases and naturally the polarity instability voltage increases. In addition, at a constant voltage, with increasing n_1, the deflection of the beam decreases. The effect of the n_2 parameter on the polarity instability voltage is shown in Figure 9. The results of this figure show that with increasing n_2 the polarity instability voltage occurs earlier, which is due to the fact that with increasing n_2 the coefficient of thermal expansion increases and naturally the polarity voltage decreases. By increasing n_2, the thermal stresses help the pullen, which causes the polarity of the voltage to become more stable. Meanwhile, at a constant voltage, with increasing n_2, the rise of the beam increases. Also, the effect of n_2 is less than n_1 on the polarity voltage. The effect of the n_3 parameter on the polarity instability voltage with the dimension is shown in figure 10. The results of this figure show that with increasing n_3 the polarity instability voltage is delayed, which is due to the fact that with increasing n_3 the modulus of elasticity increases and naturally the polarity instability voltage increases. Meanwhile, at a constant voltage, with increasing n_3, the rise of the beam decreases.

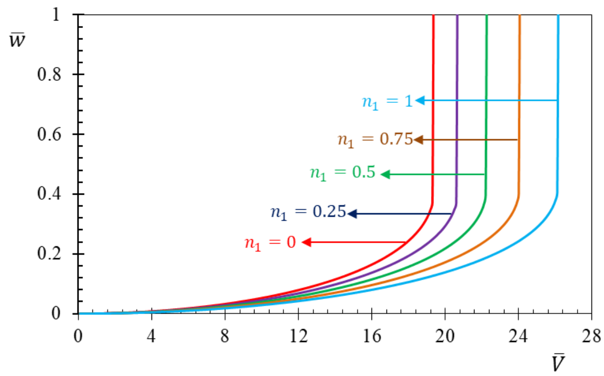

Figure 8. Effect of n_1 pull-in instability voltage ($E_c = 70Gpa$ ، $E_m = 70Gpa$ ، $\alpha_m = 11\times10^{-6}\,1/^{\circ}C$ $\alpha_c = 23.1\times10^{-6}\,1/^{\circ}C$ ، $\nu = 0.3$ ، $b = 1\mu m$ ، $L = 100\mu m$ ، $h = 1\mu m$ ، $T = 0^{\circ}C$ ، $\bar{l} = 0.01$ ، $d = 1\mu m$ ، $n_2 = 0$ ، $n_3 = 0$ ، $n_4 = 0$)

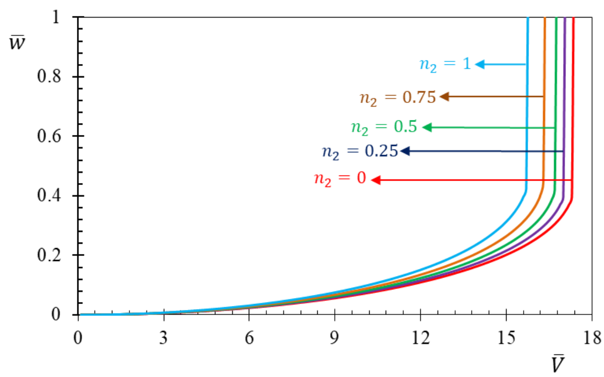

Figure 9. Effect of n_2 pull-in instability voltage ($E_c = 70Gpa$ ، $E_m = 70Gpa$ ، $\alpha_m = 11\times10^{-6}\,1/^{\circ}C$ $\alpha_c = 23.1\times10^{-6}\,1/^{\circ}C$ ، $\nu = 0.3$ ، $b = 1\mu m$ ، $L = 100\mu m$ ، $h = 1\mu m$ ، $T = 20^{\circ}C$ ، $\bar{l} = 0.01$ ، $d = 1\mu m$ ، $n_1 = 0$ ، $n_3 = 0$ ، $n_4 = 0$)

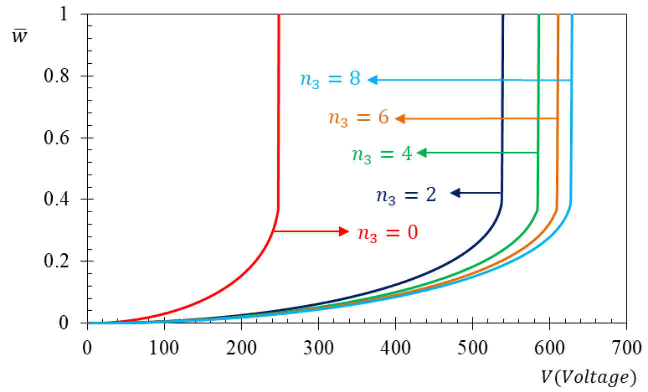

Figure 10. Efect of n_3 pull-in instability voltage ($E_c = 70Gpa$ ، $E_m = 70Gpa$ ، $\alpha_m = 11\times10^{-6} 1/{}^{\circ}C$ $\alpha_c = 23.1\times10^{-6} 1/{}^{\circ}C$ ، $\nu = 0.3$ ، $b = 1\mu m$ ، $L = 100\mu m$ ، $h = 1\mu m$ ، $T = 5^{\circ}C$ ، $\bar{l} = 0.01$ ، $d = 1\mu m$ ، $n_1 = 0$ ، $n_2 = 0$ ، $n_4 = 0$)

4. Conclusions

The present paper has discussed the applicability of a non-classical continuum theories by couple stress theory to obtain the size dependent on pull-in behavior of electrostatically actuated cantilever micro/nano beams made of bi directional functionally graded materials. Comparison of the results obtained from couple stress theory with those of obtained from other paper solution proved the efficiency and accuracy of this method. Finally, some numerical results are presented to compare the results of the proposed model with those predicted by Classical theory. It is also shown that small scale effects significantly contribute to the pull-in behavior of electrostatically actuated cantilever micro/nano beams and cannot be neglected. Further, pull-in voltage decreases with the increase in the size scale parameter value.

References

[1] A. Hadi, A. Rastgoo, A. Bolhassani, N. Haghighipour, Effects of stretching on molecular transfer from cell membrane by forming pores, *Soft Materials*, Vol. 17, No. 4, pp. 391-399, 2019.

[2] M. Hosseini, H. H. Gorgani, M. Shishesaz, A. Hadi, Size-dependent stress analysis of single-wall carbon nanotube based on strain gradient theory, *International Journal of Applied Mechanics*, Vol. 9, No. 06, pp. 1750087, 2017.

[3] M. M. Adeli, A. Hadi, M. Hosseini, H. H. Gorgani, Torsional vibration of nano-cone based on nonlocal strain gradient elasticity theory, *The European Physical Journal Plus*, Vol. 132, No. 9, pp. 393, 2017.

[4] M. Hosseini, A. Hadi, A. Malekshahi, M. Shishesaz, A review of size-dependent elasticity for nanostructures, *Journal of Computational Applied Mechanics*, Vol. 49, No. 1, pp. 197-211, 2018.

[5] A. Soleimani, K. Dastani, A. Hadi, M. H. Naei, Effect of out-of-plane defects on the postbuckling behavior of graphene sheets based on nonlocal elasticity theory, *Steel and Composite Structures*, Vol. 30, No. 6, pp. 517-534, 2019.

[6] A. Hadi, A. Rastgoo, N. Haghighipour, A. Bolhassani, Numerical modelling of a spheroid living cell membrane under hydrostatic pressure, *Journal of Statistical Mechanics: Theory and Experiment*, Vol. 2018, No. 8, pp. 083501, 2018.

[7] A. C. Eringen, 2002, *Nonlocal continuum field theories*, Springer Science & Business Media,

[8] A. C. Eringen, Nonlocal polar elastic continua, *International Journal of Engineering Science*, Vol. 10, No. 1, pp. 1-16, 1972/01/01/, 1972.

[9] A. C. Eringen, On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves, *Journal of applied physics*, Vol. 54, No. 9, pp. 4703-4710, 1983.

[10] A. C. Eringen, Theory of micromorphic materials with memory, *International Journal of Engineering Science*, Vol. 10, No. 7, pp. 623-641, 1972/07/01/, 1972.

[11] D. C. C. Lam, F. Yang, A. C. M. Chong, J. Wang, P. Tong, Experiments and theory in strain gradient elasticity, *Journal of the Mechanics and Physics of Solids*, Vol. 51, No. 8, pp. 1477-1508, 8//, 2003.

[12] R. A. Toupin, Elastic materials with couple-stresses, *Archive for Rational Mechanics and Analysis*, Vol. 11, No. 1, pp. 385-414, January 01, 1962.

[13] R. D. Mindlin, H. F. Tiersten, Effects of couple-stresses in linear elasticity, *Archive for Rational Mechanics and Analysis*, Vol. 11, No. 1, pp. 415-448, January 01, 1962.

[14] W. Koiter, Couple stresses in the theory of elasticity, I & II, *Proceedings of the Koninklijke Nederlandse Akademie van Wetenschappen*, Vol. 67, pp. 17-44, 1964.

[15] F. Yang, A. C. M. Chong, D. C. C. Lam, P. Tong, Couple stress based strain gradient theory for elasticity, *International Journal of Solids and Structures*, Vol. 39, No. 10, pp. 2731-2743, 2002/05/01/, 2002.

[16] S. Kong, Size effect on pull-in behavior of electrostatically actuated microbeams based on a modified couple stress theory, *Applied Mathematical Modelling*, Vol. 37, No. 12, pp. 7481-7488, 2013/07/01/, 2013.

[17] H. M. Ma, X. L. Gao, J. N. Reddy, A microstructure-dependent Timoshenko beam model based on a modified couple stress theory, *Journal of the Mechanics and Physics of Solids*, Vol. 56, No. 12, pp. 3379-3391, 2008/12/01/, 2008.

[18] S. Park, X. Gao, Bernoulli–Euler beam model based on a modified couple stress theory, *Journal of Micromechanics and Microengineering*, Vol. 16, No. 11, pp. 2355, 2006.

[19] W. Xia, L. Wang, L. Yin, Nonlinear non-classical microscale beams: Static bending, postbuckling and free vibration, *International Journal of Engineering Science*, Vol. 48, No. 12, pp. 2044-2053, 2010/12/01/, 2010.

[20] M. Asghari, M. H. Kahrobaiyan, M. T. Ahmadian, A nonlinear Timoshenko beam formulation based on the modified couple stress theory, *International Journal of Engineering Science*, Vol. 48, No. 12, pp. 1749-1761, 2010/12/01/, 2010.

[21] M. Rahaeifard, M. H. Kahrobaiyan, M. Asghari, M. T. Ahmadian, Static pull-in analysis of microcantilevers based on the modified couple stress theory, *Sensors and Actuators A: Physical*, Vol. 171, No. 2, pp. 370-374, 2011/11/01/, 2011.

[22] L.-L. Ke, Y.-S. Wang, J. Yang, S. Kitipornchai, Nonlinear free vibration of size-dependent functionally graded microbeams, *International Journal of Engineering Science*, Vol. 50, No. 1, pp. 256-267, 2012.

[23] E. Jomehzadeh, H. R. Noori, A. R. Saidi, The size-dependent vibration analysis of micro-plates based on a modified couple stress theory, *Physica E: Low-dimensional Systems and Nanostructures*, Vol. 43, No. 4, pp. 877-883, 2011/02/01/, 2011.

[24] L. Yin, Q. Qian, L. Wang, W. Xia, Vibration analysis of microscale plates based on modified couple stress theory, *Acta Mechanica Solida Sinica*, Vol. 23, No. 5, pp. 386-393, 2010.

[25] M. Asghari, Geometrically nonlinear micro-plate formulation based on the modified couple stress theory,

International Journal of Engineering Science, Vol. 51, No. Supplement C, pp. 292-309, 2012/02/01/, 2012.

[26] M. Farajpour, A. Shahidi, A. Hadi, A. Farajpour, Influence of initial edge displacement on the nonlinear vibration, electrical and magnetic instabilities of magneto-electro-elastic nanofilms, *Mechanics of Advanced Materials and Structures,* Vol. 26, No. 17, pp. 1469-1481, 2019.

[27] M. Mohammad-Abadi, A. R. Daneshmehr, Modified couple stress theory applied to dynamic analysis of composite laminated beams by considering different beam theories, *International Journal of Engineering Science,* Vol. 87, No. Supplement C, pp. 83-102, 2015/02/01/, 2015.

[28] M. Z. Nejad, A. Hadi, A. Rastgoo, Buckling analysis of arbitrary two-directional functionally graded Euler–Bernoulli nano-beams based on nonlocal elasticity theory, *International Journal of Engineering Science,* Vol. 103, pp. 1-10, 2016.

[29] M. Z. Nejad, A. Hadi, Non-local analysis of free vibration of bi-directional functionally graded Euler–Bernoulli nano-beams, *International Journal of Engineering Science,* Vol. 105, pp. 1-11, 2016/08/01/, 2016.

[30] M. Z. Nejad, A. Hadi, Eringen's non-local elasticity theory for bending analysis of bi-directional functionally graded Euler–Bernoulli nano-beams, *International Journal of Engineering Science,* Vol. 106, pp. 1-9, 2016/09/01/, 2016.

[31] A. Daneshmehr, A. Rajabpoor, A. Hadi, Size dependent free vibration analysis of nanoplates made of functionally graded materials based on nonlocal elasticity theory with high order theories, *International Journal of Engineering Science,* Vol. 95, pp. 23-35, 2015.

[32] A. Hadi, M. Z. Nejad, M. Hosseini, Vibrations of three-dimensionally graded nanobeams, *International Journal of Engineering Science,* Vol. 128, pp. 12-23, 2018.

[33] M. Z. Nejad, A. Hadi, A. Farajpour, Consistent couple-stress theory for free vibration analysis of Euler-Bernoulli nano-beams made of arbitrary bi-directional functionally graded materials, *Structural Engineering and Mechanics,* Vol. 63, No. 2, pp. 161-169, 2017.

[34] M. Hosseini, M. Shishesaz, K. N. Tahan, A. Hadi, Stress analysis of rotating nano-disks of variable thickness made of functionally graded materials, *International Journal of Engineering Science,* Vol. 109, pp. 29-53, 2016.

[35] Z. Mazarei, M. Z. Nejad, A. Hadi, Thermo-elasto-plastic analysis of thick-walled spherical pressure vessels made of functionally graded materials, *International Journal of Applied Mechanics,* Vol. 8, No. 04, pp. 1650054, 2016.

[36] A. Hadi, M. Z. Nejad, A. Rastgoo, M. Hosseini, Buckling analysis of FGM Euler-Bernoulli nano-beams with 3D-varying properties based on consistent couple-stress theory, *Steel and Composite Structures,* Vol. 26, No. 6, pp. 663-672, 2018.

[37] M. Gharibi, M. Zamani Nejad, A. Hadi, Elastic analysis of functionally graded rotating thick cylindrical pressure vessels with exponentially-varying properties using power series method of Frobenius, *Journal of Computational Applied Mechanics,* Vol. 48, No. 1, pp. 89-98, 2017.

[38] E. Zarezadeh, V. Hosseini, A. Hadi, Torsional vibration of functionally graded nano-rod under magnetic field supported by a generalized torsional foundation based on nonlocal elasticity theory, *Mechanics Based Design of Structures and Machines,* Vol. 48, No. 4, pp. 480-495, 2020.

[39] M. Shishesaz, M. Hosseini, K. N. Tahan, A. Hadi, Analysis of functionally graded nanodisks under thermoelastic loading based on the strain gradient theory, *Acta Mechanica,* Vol. 228, No. 12, pp. 4141-4168, 2017.

[40] M. Zamani Nejad, M. Jabbari, A. Hadi, A review of functionally graded thick cylindrical and conical shells, *Journal of Computational Applied Mechanics,* Vol. 48, No. 2, pp. 357-370, 2017.

[41] M. Zamani Nejad, A. Rastgoo, A. Hadi, Effect of exponentially-varying properties on displacements and stresses in pressurized functionally graded thick spherical shells with using iterative technique, *Journal of Solid Mechanics,* Vol. 6, No. 4, pp. 366-377, 2014.

[42] M. Hosseini, M. Shishesaz, A. Hadi, Thermoelastic analysis of rotating functionally graded micro/nanodisks of variable thickness, *Thin-Walled Structures,* Vol. 134, pp. 508-523, 2019.

[43] M. Z. Nejad, A. Hadi, A. Omidvari, A. Rastgoo, Bending analysis of bi-directional functionally graded Euler-Bernoulli nano-beams using integral form of Eringen's non-local elasticity theory, *Structural Engineering and Mechanics,* Vol. 67, No. 4, pp. 417-425, 2018.

[44] M. Z. Nejad, N. Alamzadeh, A. Hadi, Thermoelastoplastic analysis of FGM rotating thick cylindrical pressure vessels in linear elastic-fully plastic condition, *Composites Part B: Engineering,* Vol. 154, pp. 410-422, 2018.

[45] M. Mohammadi, M. Hosseini, M. Shishesaz, A. Hadi, A. Rastgoo, Primary and secondary resonance analysis of porous functionally graded nanobeam resting on a nonlinear foundation subjected to mechanical and electrical loads, *European Journal of Mechanics-A/Solids,* Vol. 77, pp. 103793, 2019.

[46] A. Hadi, A. Rastgoo, A. Daneshmehr, F. Ehsani, Stress and strain analysis of functionally graded rectangular plate with exponentially varying properties, *Indian Journal of Materials Science,* Vol. 2013, 2013.

[47] A. Barati, A. Hadi, M. Z. Nejad, R. Noroozi, On vibration of bi-directional functionally graded nanobeams under magnetic field, *Mechanics Based Design of Structures and Machines,* pp. 1-18, 2020.

[48] R. Noroozi, A. Barati, A. Kazemi, S. Norouzi, A. Hadi, Torsional vibration analysis of bi-directional FG nano-cone with arbitrary cross-section based on nonlocal strain gradient elasticity, *Advances in nano research,* Vol. 8, No. 1, pp. 13-24, 2020.

[49] A. Barati, M. M. Adeli, A. Hadi, Static torsion of bi-directional functionally graded microtube based on the couple stress theory under magnetic field, *International Journal of Applied Mechanics,* Vol. 12, No. 02, pp. 2050021, 2020.

[50] M. Najafzadeh, M. M. Adeli, E. Zarezadeh, A. Hadi, Torsional vibration of the porous nanotube with an arbitrary cross-section based on couple stress theory under magnetic field, *Mechanics Based Design of Structures and Machines*, pp. 1-15, 2020.

[51] K. Dehshahri, M. Z. Nejad, S. Ziaee, A. Niknejad, A. Hadi, Free vibrations analysis of arbitrary three-dimensionally FGM nanoplates, *Advances in nano research,* Vol. 8, No. 2, pp. 115-134, 2020.

[52] M. M. Khoram, M. Hosseini, A. Hadi, M. Shishehsaz, Bending Analysis of Bi-Directional FGM Timoshenko Nano-Beam subjected to mechanical and magnetic forces and resting on Winkler-Pasternak foundation, *International Journal of Applied Mechanics*, 2020.

[53] A. Hadi, A. Rastgoo, N. Haghighipour, A. Bolhassani, F. Asgari, S. Soleymani, Enhanced gene delivery in tumor cells using chemical carriers and mechanical loadings, *Plos one,* Vol. 13, No. 12, pp. e0209199, 2018.

[54] M. A. Steinberg, Materials for aerospace, *Scientific American,* Vol. 255, No. 4, pp. 66-73, 1986.

[55] Y. T. Beni, M. R. Abadyan, A. Noghrehabadi, Investigation of Size Effect on the Pull-in Instability of Beam-type NEMS under van der Waals Attraction, *Procedia Engineering,* Vol. 10, No. Supplement C, pp. 1718-1723, 2011/01/01/, 2011.

[56] J. Yang, X. Jia, S. Kitipornchai, Pull-in instability of nano-switches using nonlocal elasticity theory, *Journal of Physics D: Applied Physics,* Vol. 41, No. 3, pp. 035103, 2008.

[57] F. Ebrahimi, M. R. Barati, A. Dabbagh, A nonlocal strain gradient theory for wave propagation analysis in temperature-dependent inhomogeneous nanoplates, *International Journal of Engineering Science,* Vol. 107, pp. 169-182, 10//, 2016.

[58] Ç. Demir, Ö. Civalek, On the analysis of microbeams, *International Journal of Engineering Science,* Vol. 121, pp. 14-33, 12//, 2017.

[59] M. Shishesaz, M. Hosseini, K. Naderan Tahan, A. Hadi, Analysis of functionally graded nanodisks under thermoelastic loading based on the strain gradient theory, *Acta Mechanica*, August 01, 2017.

[60] A. Koochi, A. S. Kazemi, A. Noghrehabadi, A. Yekrangi, M. Abadyan, New approach to model the buckling and stable length of multi walled carbon nanotube probes near graphite sheets, *Materials & Design,* Vol. 32, No. 5, pp. 2949-2955, 5//, 2011.

[61] F. Ebrahimi, M. R. Barati, A nonlocal higher-order refined magneto-electro-viscoelastic beam model for dynamic analysis of smart nanostructures, *International Journal of Engineering Science,* Vol. 107, pp. 183-196, 10//, 2016.

[62] B. Akgöz, Ö. Civalek, Thermo-mechanical buckling behavior of functionally graded microbeams embedded in elastic medium, *International Journal of Engineering Science,* Vol. 85, pp. 90-104, 12//, 2014.

[63] H. Mohammad-Sedighi, A. Koochi, M. Keivani, M. Abadyan, Microstructure-dependent dynamic behavior of torsional nano-varactor, *Measurement,* Vol. 111, pp. 114-121, 12//, 2017.

[64] P. M. Osterberg, S. D. Senturia, M-TEST: a test chip for MEMS material property measurement using electrostatically actuated test structures, *Journal of Microelectromechanical systems,* Vol. 6, No. 2, pp. 107-118, 1997.

[65] S. Hosseini-Hashemi, M. Fadaee, S. R. Atashipour, A new exact analytical approach for free vibration of Reissner–Mindlin functionally graded rectangular plates, *International Journal of Mechanical Sciences,* Vol. 53, No. 1, pp. 11-22, 2011.